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Bloch-Nordsieck summation and partonic distributions in impact parameter space

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Abstract

A model for the parton distributions of hadrons in impact parameter space has been constructed using soft gluon summation. This model incorporates the salient features of distributions obtained from the intrinsic transverse momentum behaviour of hadrons. Under the assumption that the intrinsic behaviour is dominated by soft gluon emission stimulated by the scattering process, the b-spectrum becomes softer and softer as the scattering energy increases. In minijet models for the inclusive cross-sections, this will counter the increase from $\sigma_{\rm jet}$.

The impact of parton scattering on the rise with energy of inclusive cross-sections was suggested by Cline and Halzen [1], after such rise was first observed in proton-proton collisions at ISR. Minijet [2–4] and eikonal minijet models [5–8] were subsequently developed to include an increasing number of partonic collisions in QCD resulting from the rapid rise in gluon densities. Recent measurements of photo- and hadro-production total cross-sections [9,10] in energy regions where QCD processes dominate, confirmed the trend and were confronted with theoretical predictions obtaining varying degrees of success [11]. Typically, the eikonal (unitary) models for total hadron-hadron cross-sections are written as

$$\sigma_{\rm total} = 2\int d^2\boldsymbol{b}[1 - e^{-n(b,s)/2}] \tag{1}$$

where the average number of collisions at impact parameter b is given by

$$n(b,s) = A(b)\sigma(s) \tag{2}$$

In mini-jet models, $\sigma(s) = \sigma_{\text{non-pert}}(s) + \sigma_{\rm jet}(s)$, with $\sigma_{\rm jet}$ to be calculated from perturbative QCD. A key ingredient of all models with a QCD component is the overlap function $A(b)$, which describes matter distribution in impact parameter space. In some models [5], it has been assumed to be the Fourier transform of the product of hadronic form factors of the colliding particles. In other models [12], a gaussian shape has been assumed, thus relating $A(b)$ to the intrinsic transverse momentum distribution of partons in the colliding hadrons. In either case, detailed information on $A(b)$ relies on parameters to be determined case by case. Moreover, while direct measurements of the EM form factors are available for nucleons and pseudoscalar mesons, experimental information regarding photons or other hadrons such as vector mesons is lacking. The same observation applies to

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the intrinsic-p_t interpretation for the spatial distribution of partons in vector mesons or photons. There is clearly need for a theoretical description of these distributions which would allow for a calculation of the mini-jet total cross-section (in the eikonal approximation) for photo-production and gamma-gamma collisions at future colliders.

The aim of this paper is to provide a model for $A(b)$ which could be applied to various cases of interest. We shall use Bloch-Nordsieck techniques to sum soft gluon transverse momentum distributions to all orders and compare our results with both the intrinsic transverse momentum approach as well as the form factor approach. In what follows, we shall first illustrate the Bloch-Nordsieck result and show that it gives a gaussian fall-off with an intrinsic transverse size consistent with Monte Carlo models [12]. We then calculate the relevant distributions and discuss their phenomenological application.

We propose that in hadron-hadron collisions, the b-distribution of partons in the colliding hadrons is the Fourier transform of the transverse momentum distribution resulting from soft gluon radiation emitted by quarks as the hadron breaks up because of the collision. This distribution is obtained by summing soft gluons to all orders, with a technique amply discussed in the literature [13,14]. The resulting expression is [15,16]

$$\mathcal{F}_{\rm BN}(K_\perp) = \frac{1}{2\pi}\int b db J_0(bK_\perp) e^{-h(b;M,\Lambda)} \tag{3}$$

with

$$h(b;M,\Lambda) = \frac{2c_F}{\pi}\int_0^M \frac{dk_\perp}{k_\perp}\alpha_s\left(\frac{k_\perp^2}{\Lambda^2}\right) \times \ln\frac{M+\sqrt{M^2-k_\perp^2}}{M-\sqrt{M^2-k_\perp^2}}[1-J_0(k_\perp b)] \tag{4}$$

The choice of the hadronic scale M, which accounts for the maximum energy allowed in a single ($k^2=0$) gluon emission, plays a crucial role, just as it did for the Drell-Yan process, where the above expression has been successfully [17–20] used to describe the transverse momentum distribution of the time-like virtual photon or W-boson. In these cases [17,19], the scale M was found to be energy dependent and to vary between $\sqrt{Q^2}/4$ and $\sqrt{Q^2}/2$. We shall return to this determination later.

The definition given in Eqs. (1), (2), requires for its consistency a normalized b-distribution, i.e.

$$\int d^2 b A(b) = 1. \tag{5}$$

Our proposed Bloch-Nordsieck expression for the overlap function $A(b)$, satisfying the above normalization, reads

$$A_{\rm BN} = \frac{e^{-h(b;M,\Lambda)}}{2\pi\int b db e^{-h(b;M,\Lambda)}} \tag{6}$$

An inspection of Eq. (4), immediately poses the problem of extending the known asymptotic freedom expression for α_s to the very small $k_\perp$ region. To avoid the small $k_\perp$ divergence in Eq. (4), it has been customary to introduce a lower cut-off in $k_\perp$ and freeze α_s at $k_\perp = 0$, i.e. to put

$$\alpha_s(k_\perp^2) = \frac{12\pi}{33-2N_f}\frac{1}{\ln[(k_\perp^2+a^2\Lambda^2)/\Lambda^2)]} \tag{7}$$

with $a=2$ in Ref. [20]. For applications where the scale M is large (e.g., W-transverse momentum distribution calculations) Eq. (4) is dominated by the (asymptotic) logarithmic behaviour and the small $k_\perp$-limit, albeit theoretically crucial, is not very relevant phenomenologically. However, this is not case in the present context, where we are dealing with soft gluon emission in low-p_t physics (responsible for large cross-sections). The typical scale of such peripheral interactions, is that of the hadronic masses, i.e. we expect $M \sim \mathcal{O}(1\text{–}2\ \text{GeV})$ and the small $k_\perp$ limit plays a basic role. This can be appreciated on a qualitative basis, by considering the limit $bM \ll 1$ of Eq. (4). In this region, we can approximate $1-J_0(kb) \approx b^2k^2/4$, to obtain

$$h(b;M,\Lambda) \approx b^2\, A \tag{8}$$

with

$$A = \frac{c_F}{4\pi}\int dk^2 \alpha_s\left(\frac{k^2}{\Lambda^2}\right)\ln\frac{4M^2}{k^2} \tag{9}$$

We obtain a function $h(b;M,\Lambda)$ with a gaussian fall-off as in models where $A(b)$ is the Fourier transform of an intrinsic transverse momentum distribution of partons, i.e. $\exp(-k_\perp^2/4A^2)$. Note that the relevance of an integral similar to the one in Eq. (9) has been recently discussed in connection to hadronic event shapes [21].

Our choice for the infrared behaviour of α_s for a quantitative description of the distribution in Eq. (4), does not follow Eq. (7), but is inspired by the Richardson potential for quarkonium bound states [22]. In a number of related applications [23,24], we have proposed to calculate the above integral using the following expression for α_s:

$$\alpha_s(k_\perp) = \frac{12\pi}{(33-2N_f)} \frac{p}{\ln[1+p(\frac{k_\perp}{\Lambda})^{2p}]} \tag{10}$$

which coincides with the usual one-loop expression for large (relative to Λ) values of $k_\perp$, while going to a singular limit for small $k_\perp$. For the special case $p = 1$ such an α_s coincides with one used in the Richardson potential [22], and which incorporates – in a compact expression – the high-momentum limit demanded by asymptotic freedom as well as linear quark confinement in the static limit. We have generalized Richardson's ansatz to values of $p \leq 1$. For $1/2 < p \leq 1$, this corresponds to a confining potential rising less than linearly with the interquark distance r. The range $p \neq 1$ has an important advantage, i.e., it allows the integration in Eq. (4) to converge for all values of $k_\perp$. A qualitative argument to justify the use of such (less singular) values for the parameter p follows.

Assume a confining potential (in momentum space) given by "one-loop gluon" exchange term

$$\tilde{V}(Q) = K(\frac{\alpha_s(Q^2)}{Q^2}) \tag{11}$$

where K is a constant calculable from the asymptotic form of α_s. Let us choose the simple form

$$\alpha_s(Q^2) = \frac{1E}{b\ \ln[1+(Q^2/\Lambda^2)^p]} \tag{12}$$

so that $\tilde{V}(Q)$ for small Q goes as

$$\tilde{V}(Q) \to Q^{-2(1+p)} \tag{13}$$

Eq. (13) implies, for the potential, in coordinate space

$$V(r) = \int \frac{d^3Q}{(2\pi)^3} e^{i\mathbf{Q}\cdot\mathbf{r}} \tilde{V}(Q) \tag{14}$$

as

$$V(r) \to (1/r)^3 \cdot r^{(2+2p)} \sim C\ r^{(2p-1)} \tag{15}$$

for large r (C is another constant). A simple check is that for p equal to zero, the usual Coulomb potential is regained. Notice that for a potential rising with r, one needs $p > 1/2$. A determination of p can be made following an argument due to Polyakov [25]. We minimize the potential, consistent with the constraint that the potential energy (or mass, M) increases as $J^{(1/2)}$ for large J. (A linear Regge trajectory means that the angular momentum $J \sim M^2$). The "confinement" energy plus the rotational energy (all for large r and large J) reads

$$V(r,J) \sim (\frac{J(J+1)}{r^2}) + Cr^{(2p-1)} \tag{16}$$

Since the first term goes down with r and the second grows with r (for confinement, $p > (1/2)$ is necessary), a minimum is guaranteed (for a fixed J). So we minimize the above potential, and find the position $r = r_0(J)$ where the minimum is, as a function of J,

$$r_0(J) \sim J^{2/(2p+1)}, \text{ for large } J \tag{17}$$

Inserting this value of r_0, we find the minimum effective potential (or rest mass) as a function of J

$$U_{\min}(J, r_0) \sim J^{(\frac{4p-2}{2p+1})} \tag{18}$$

A mass spectrum with linearly rising Regge trajectories, for which $M^2 \sim J$, implies the right side to grow like $J^{(1/2)}$, which gives

$$p = (5/6) \tag{19}$$

For the motivations given in [23] and repeated above, the value $p = 5/6$ was chosen in previous calculations of the transverse momentum distribution of Drell-Yan pairs [23,19]. Compared to Eq. (7), we consider our present choice (i.e., Eq. (10)) physically more transparent as it leads to a direct quantitative estimate of soft-gluon generated intrinsic transverse momentum of partons in hadrons. With our ansatz, the intrinsic transverse momentum becomes a calculable quantity rather than being an assumed one (see related discussion in Nakamura et al. [23]). It can be obtained from the decomposition

$$h(b; M, \Lambda) = h_{\text{intrinsic}} + \Delta(b; M, \Lambda) \tag{20}$$

with

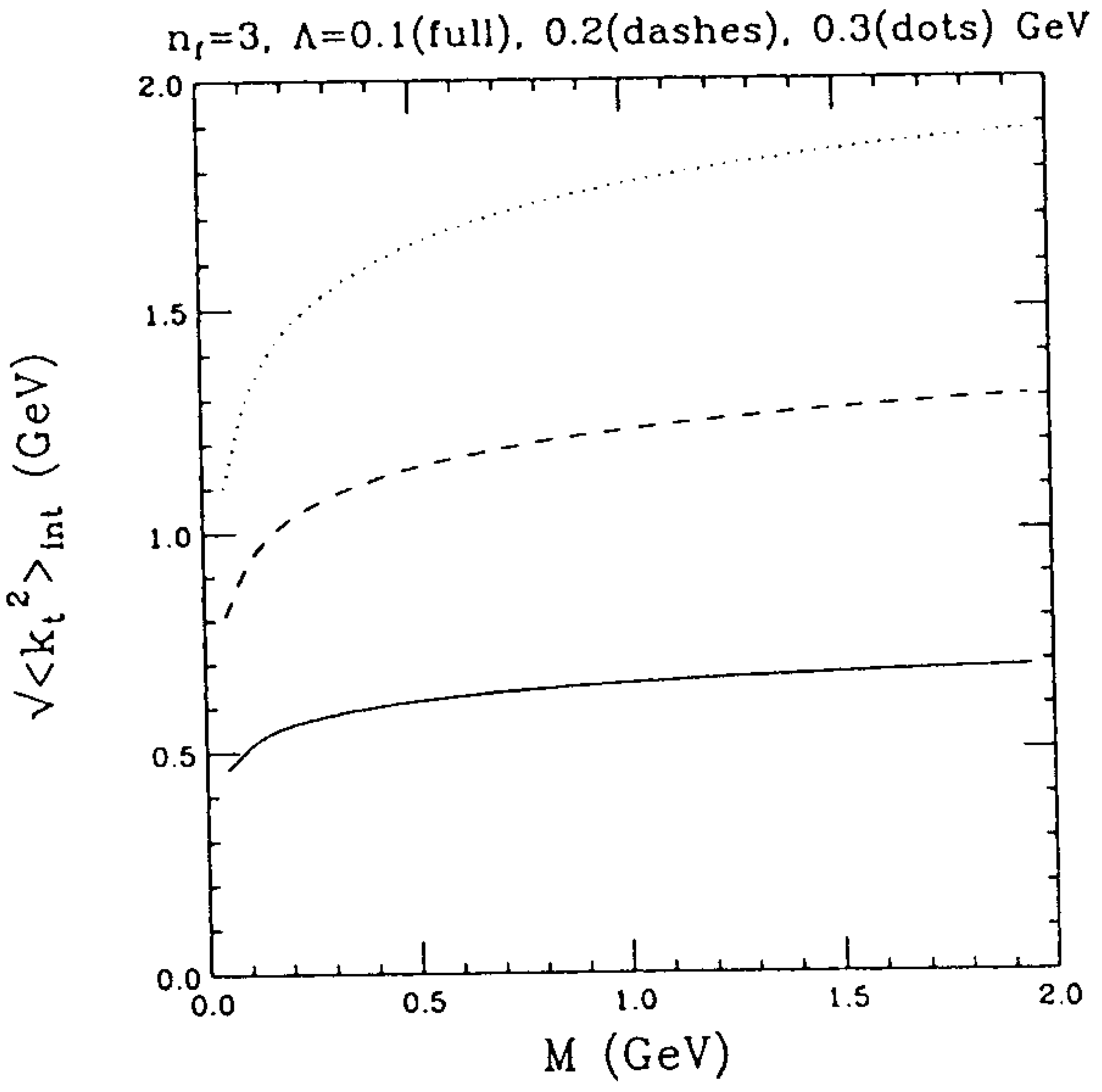

Fig. 1. Intrinsic transverse momentum of partons in a hadron for different hadron scales M, in units of Λ, for $p = 5/6$.

$$h_{\text{intrinsic}} = \frac{32}{33 - 2N_f} \int_0^{\Lambda} \frac{dk}{k} \frac{p}{\ln(1 + p(\frac{k^2}{\Lambda^2})^p)} \times \ln \frac{M + \sqrt{M^2 - k^2}}{M - \sqrt{M^2 - k^2}} (1 - J_0(bk)) \tag{21}$$

where the contribution to the integral from the region $k_\perp \le \Lambda$ determines the form and type of the "intrinsic transverse momentum" behaviour, as we discussed above. In this region, we now have

$$h_{\text{intrinsic}} \approx \tfrac{1}{4} b^2 \langle k_t^2 \rangle_{\text{int}} \tag{22}$$

with

$$\langle k_t^2 \rangle_{\text{int}} = \frac{32}{33 - 2N_f} \frac{1}{(1-p)} \times \left(\frac{1}{2(1-p)} + \ln \frac{2M}{\Lambda} \right) \Lambda^2 \tag{23}$$

By comparison with Eq. (8), we see that $A = \langle k_t^2 \rangle_{\text{int}}/4$, which corresponds to an intrinsic transverse momentum of a few hundred MeV for Λ in the 100 MeV range and $M \le 1$ GeV. For $N_f = 3$, we show in Fig. 1 the variation of $\sqrt{\langle k_t^2 \rangle_{\text{int}}}$ as a function of M for a range of values of Λ.

The "intrinsic" behaviour just discussed appears in the very small $K_\perp$ region, i.e. $K_\perp \le \Lambda$. In general, the full expression (20) should be used.

Having set up our formalism, we shall now examine its implications. The distribution $A(b)$ depends upon the hadronic scale M in the function $h(b)$. This scale depends upon the energy of the specific subprocess and, through this, upon the hadron scattering energy. In the calculation of the transverse momentum distribution of a lepton pair produced in quark-antiquark annihilation [17], the function $e^{-h(b)}$ was the Fourier-transform of such distribution and the scale M was obtained as the maximum transverse momentum allowed by kinematics to a single gluon emitted by the initial $q\bar{q}$ pair of c.m. energy $\sqrt{\hat{s}}$ in the process

$$q\bar{q} \to g + \gamma(Q^2) \tag{24}$$

Following Ref. [17], for a lepton pair of mass squared Q^2 and rapidity y, this quantity is given by

$$q_{\max}(\hat{s}, y) = \frac{\sqrt{\hat{s}}}{2}(1 - \frac{Q^2}{\hat{s}}) \frac{1}{\sqrt{1 + z \sinh^2 y}} \tag{25}$$

with $z = Q^2/\hat{s}$. This would be the upper limit of integration in the function $h(b)$ for each particular subenergy $\hat{s}$ of the quark-antiquark pair. The transverse momentum distribution can then be factorized from the overall hadronic cross-section, by substituting M in $h(b)$ with the average of $q_{\max}$ over all quark energies and configurations, with a weight $1/\hat{s}$, proportional to the cross-section. For a fixed value of the Feynman variable x_F, one obtains

$$M(s, x_F) = \frac{\int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \delta(x_1 - x_2 - x_F) q_{\max} f(x_1) f(x_2)}{\int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \delta(x_1 - x_2 - x_F) f(x_1) f(x_2)} \tag{26}$$

for given quark densities $f(x)$. To understand the energy dependence of this scale, one can use a simple toy model, in which one approximates the valence quark densities with $1/\sqrt{x}$ and obtain from, Eq. (26) at $x_F = 0$,

$$M(s, 0) \approx \frac{2Q}{3} \left(1 - 3\frac{Q}{2\sqrt{s}}\right), \quad Q \ll \sqrt{s} \tag{27}$$

which shows that M increases with energy towards the asymptotic value $2Q/3$. This tendency was confirmed by numerical estimates with realistic quark densities [24]. An asymptotic increase of M with energy is also obtained for the case of interest here, i.e. parton-parton scattering contribution to the total cross-section. In the Drell-Yan case, one needed $h(b)$ to calculate the transverse momentum distribution of the lepton pair, here we use it to evaluate the average number of partons in the overlap region of two colliding hadrons. In this case $e^{-h(b)}$ is the F-transform of the transverse momentum distribution induced by initial state radiation in the process

$$q\bar{q} \to \text{ jet jet} + X \tag{28}$$

where X can also include the quark-antiquark pair which continues undetected after emission of a gluon pair which stimulated the initial state bremsstrahlung. The jet pair in process (28) is the one produced through gluon-gluon or other parton-parton scattering with total jet-cross-section $\sigma_{\rm jet}$. We work in a no-recoil approximation, where the transverse momentum of the jet pair is balanced by the emitted soft gluons. Then the maximum transverse momentum allowed to a single gluon is still given by an expression similar to Eq. (25), i.e.

$$q_{\rm max}(\hat{s}) = \frac{\sqrt{\hat{s}}}{2}(1 - \frac{\hat{s}_{\rm jet}}{\hat{s}}) \tag{29}$$

except that now $\sqrt{Q^2}$ has been replaced by the jet-jet invariant mass $\sqrt{\hat{s}_{\rm jet}}$, over which one needs to perform further integrations. In other words, this kinematic limit is the same as the one obtained for the gluon accompanying the Drell-Yan process mentioned above: there the quark-antiquark pair annihilates into a dilepton pair and a soft gluon, here it may continue on its way – undetected – after semi-hard emission of a parton pair which stimulated the soft gluon initial state bremsstrahlung.

An improved Eq. (2) now reads

$$\begin{aligned} n(b,s) &= n_{\rm soft}(b,s) \\ &+ \sum_{i,j,} \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} f_i(x_1) f_j(x_2) \\ &\times \int dz \int dp_t^2 A_{\rm BN}(b, q_{\rm max}) \frac{d\sigma}{dp_t^2 dz} \end{aligned} \tag{30}$$

where f_i are the quark densities in the colliding hadrons, $z = \hat{s}_{\rm jet}/(sx_1x_2)$, and $\frac{d\sigma}{dp_t^2 dz}$ is the differential cross-section for process (28) for a given p_t of the produced jets. In absence of a precise prescription of how to deal with the non-perturbative contribution to $n(b,s)$, it is customary in these eikonal models to introduce a lower cut-off in the jet transverse momentum, usually indicated as $p_{t\,\rm min}$ to separate the mini-jet contribution from the soft part of the cross-section.

Unlike the usual expressions for $n(b,s)$, Eq. (30) does not exhibit factorization between the longitudinal and transverse degrees of freedom since the distribution $A_{\rm BN}$ depends upon the quark subenergies. Factorization can be obtained however, through an averaging process similar to the one described above for the transverse momentum distribution of Drell-Yan pairs: one can factorize the b-distribution in Eq. (30), by evaluating $A_{\rm BN}$ with $q_{\rm max}$ at its mean value, i.e. write

$$n(b,s) = n_{\rm soft}(b,s) + A_{\rm BN}(b, \langle q_{\rm max}(s)\rangle)\sigma_{\rm jet} \tag{31}$$

with

$$\begin{aligned} \sigma_{\rm jet} &= \sum_{i,j,} \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} f_i(x_1) f_j(x_2) \\ &\times \int dz \int dp_t^2 \frac{d\sigma}{dz dp_t^2} \end{aligned} \tag{32}$$

and

$$\begin{aligned} \langle q_{\rm max}(s)\rangle &= \tfrac{1}{2}\sqrt{s} \\ &\times \frac{\sum_{i,j} \int \frac{dx_1}{x_1} f_{i/a}(x_1) \int \frac{dx_2}{x_2} f_{j/b}(x_2) \sqrt{x_1 x_2} \int dz (1-z)}{\sum_{i,j} \int \frac{dx_1}{x_1} f_{i/a}(x_1) \int \frac{dx_2}{x_2} f_{j/b}(x_2) \int (dz)} \end{aligned} \tag{33}$$

with the lower limit of integration in the variable z given by $z_{\rm min} = 4p_{t\,\rm min}^2/(sx_1x_2)$. Using for the valence quarks the same approximation as in the Drell-Yan case, we obtain to leading order

$$\langle q_{\rm max}(s)\rangle \sim \tfrac{3}{8} p_{t\,\rm min} \ln \frac{\sqrt{s}}{2p_{t\,\rm min}} \tag{34}$$

for $2p_{t\,\rm min} \ll \sqrt{s}$. For $p_{t\,\rm min} = 1.4$ GeV, as in typical eikonal mini-jet models for proton-proton scattering [6], one obtains values of $\langle q_{\rm max}(s)\rangle$ which range from 0.5 to 5 GeV for $\sqrt{s}$ between 10 GeV and 14 TeV respectively. A more precise evaluation of the above quantities depends upon the type of parton densities one uses, and will be discussed in a forthcoming paper.

From the discussion about the large b-behaviour of the function $h(b)$, we then expect $A_{\rm BN}(b,s)$ to fall

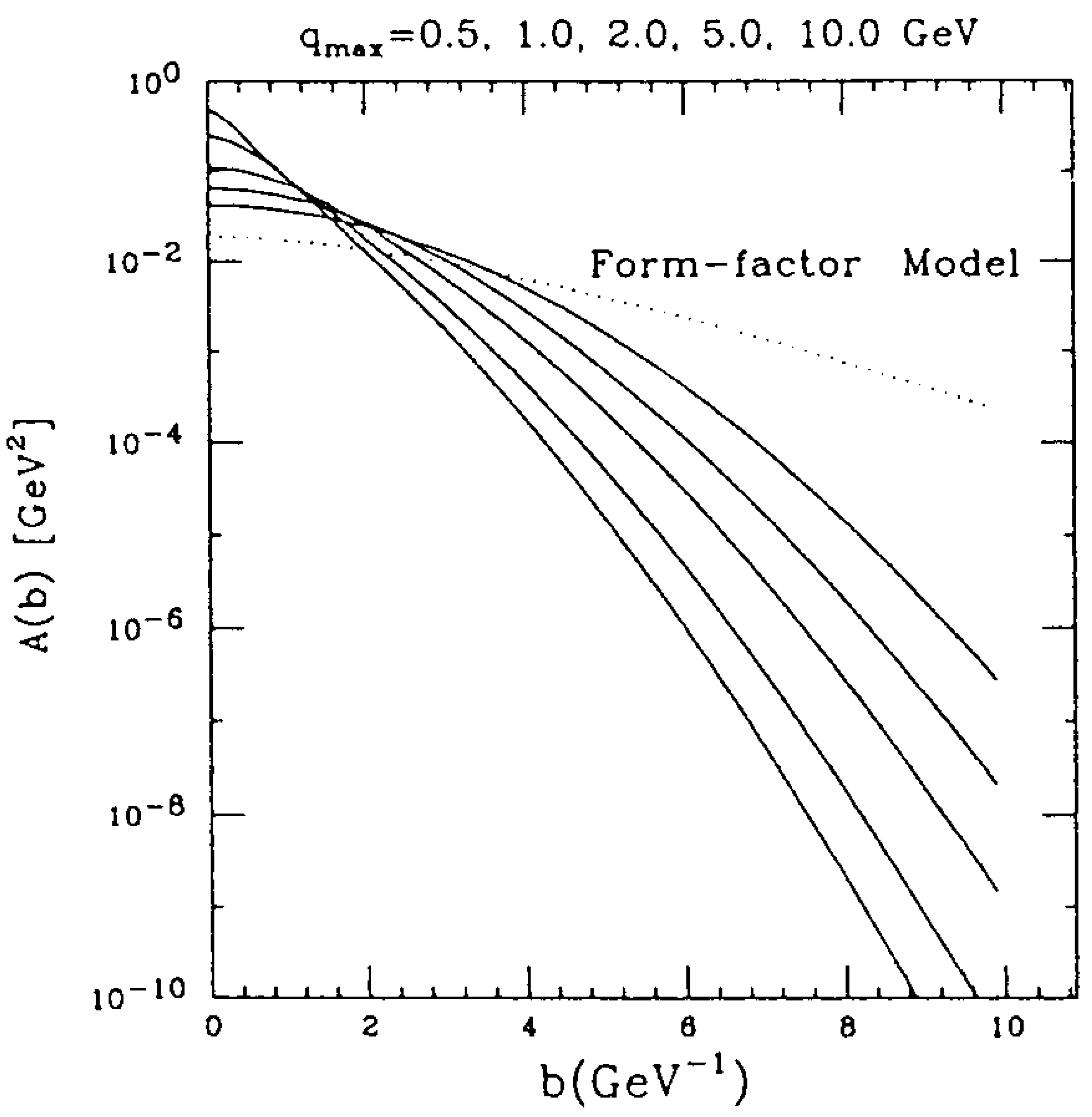

Fig. 2. Comparison between the b-distribution from the Bloch-Nordsieck model (full) and the form factor model (dots).

at large b more rapidly as the energy increases from $\sqrt{s}$ = 10 GeV into the TeV region. In Fig. 2, we compare this behaviour for the function $A(b)$ with the one obtained through the Fourier transform of the squared e.m. form factor of the proton, i.e.

$$A(b) = \int \frac{d^2Q}{(2\pi)^2} e^{iQ\cdot b} \left(\frac{\nu^2}{Q^2+\nu^2}\right)^4$$
$$= \frac{b\nu^2\sqrt{\nu^2}}{96\pi} \mathcal{K}_3(b\sqrt{\nu^2}), \quad \nu^2 = 0.71 \text{ GeV}^2 \qquad (35)$$

which is the one proposed by Durand et al. [5] in the first eikonal mini-jet model for proton-proton collisions. The function $A(b)$ from the Bloch-Nordsieck model is calculated for Λ = 0.1 GeV and values of $\langle q_{max}\rangle$ which include those obtainable from Eq. (34) in the energy range $\sqrt{s} \approx$ 10 GeV–14 TeV.

We notice that, as the energy increases, $A(b)$ from the form factor model remains substantially higher at large b than in the Bloch-Nordsieck case. As a result, for the same σ_{jet} the Bloch-Nordsieck model will give smaller $n(b,s)$ at large b than the form factor model and a softening effect of the total eikonal mini-jet cross-sections can be expected.

In summary, a soft gluon summation model allows one to obtain a value for the intrinsic transverse momentum cut-off required in the b-distribution of partons in a proton in agreement with current phenomenological estimates [12]. For such a description, an ansatz is necessary for the behaviour of the QCD coupling constant in the near zero region. By using suitable energy scale for the maximum energy allowed for emission of a single soft gluon by the valence quarks, and borrowing the expression for α_s used elsewhere which leads to calculable non-singular integrals, we have obtained a progressively softer distribution in the large b-region as energy is increased. The physical interpretation of this effect can be traced to the fact that, as the energy increases, partons undertake scattering at smaller and smaller b-distances. A comparison with the b-distribution from proton form-factor type models indicates a distinctly different behaviour in this large b-region suggesting a softening of the rise of the total cross-section in mini-jet models relative to the ones with the proton form-factor. Work is in progress to apply this model to hadron-hadron and photo-hadron collisions.

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