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Gravitational Wave Observatory Based on Solid Elastic Spheres

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Abstract. Spherical GW detectors offer a wealth of so far unexplored possibilities to detect gravitational radiation. We find that a sphere can be used as a powerful testbed for *any* metric theory of gravity, not only GR as considered so far, by making use of a deconvolution procedure for all the “electric” components of the Riemann tensor. We also find that the sphere’s cross section is large at two frequencies, and advantageous at higher frequencies in the sense that a single antenna constitutes a real xylophone in its own. Proposed GW networks will greatly benefit from this. The main features of a two large sphere observatory are reported.

The view that direct observation of Gravitational Waves (GW) is going to open a new era for the observation of the Universe is widely shared within the international community. GW physics is also meant to be a powerful tool to select amongst competing theories of gravity on the basis of analysis of the multipole structure, polarisation states and propagation speed of the waves [1] inferred from detector data. Resonant antennas (cryogenic cylindrical bars) have been experimentally demonstrated to be capable of *long term operation*, and the antenna *EXPLORER*, for example, has reached a sustained strain sensitivity $h = 6 \times 10^{-19}$ for millisecond bursts over a several consecutive months period [2]. New generation ultracryogenic bars are now ready to begin taking data with an expected sensitivity in the range of one order of magnitude better than the above [3, 4].

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Negative detection results so far are a clear indication that improved sensitivities are necessary; the question of *how much* improvement is required is however difficult to answer, due to the large uncertainties in our current understanding of supernova physics. Supernova explosions typically happen a few times per century and galaxy [5], so a minimum coverage out to, say, Virgo cluster seems desirable. Event rates within this range would rise to several *per month*, though intensities would obviously be smaller on average. A rather optimistic $h \simeq 10^{-21}$ for Virgo cluster events is sometimes quoted in the literature [6], a figure well beyond present capabilities, anyway; one may not strongly stick to this, but the figure appears an interesting objective to beat, at least in principle.

A cylinder has only one quadrupole mode interacting strongly with GWs, and presents a markedly directional detection pattern: amplitude sensitivity drops as $\sin^2 \theta$ (θ = angle of wave incidence). It would take *six* such bars to determine with isotropic sensitivity the two GW amplitudes h_+ , h_\times and the two incidence direction angles (θ, φ) [7]. It has been recognised that a single spherical antenna, with its five degenerate quadrupole modes, also allows an isotropic sky coverage and determination of the desired parameters [8, 9, 10]. In addition, the sphere has a number of other features which make of this shape the natural next step towards a resonant antenna GW observatory. We have investigated these matters, and give in this report a synthesis of our new results.

A convenient way to characterise a resonant detector sensitivity is through its GW energy absorption cross section, defined as

$$\sigma_{abs}(\omega) = \frac{\Delta E_a(\omega)}{\Phi(\omega)} \quad (1)$$

where $\Delta E_a(\omega)$ is the energy absorbed by the detector at frequency ω , and $\Phi(\omega)$ is the incident flux density expressed e.g. in watt/m² Hz. Estimation of $\sigma_{abs}(\omega)$ requires a hypothesis about the underlying gravitation theory to calculate $\Phi(\omega)$, and specification of the antenna's shape to calculate $\Delta E_a(\omega)$. As shown by Weinberg [11], if General Relativity (GR) is assumed then $\sigma_{abs}(\omega)$ can be calculated *independently* of the details of the interaction antenna-GW by use of an *optical theorem*. Applying that theory to a spherical detector, we find that

$$\sigma_{abs}(\omega) = F_{ln} \frac{GMv_s^2}{c^3} \frac{\Gamma_{ln}}{(\omega - \omega_{ln})^2 + \Gamma_{ln}^2/4} \quad (2)$$

where G and c are the usual fundamental constants, M is the sphere's mass and v_s the speed of sound in the material the sphere is made of; ω_{ln} is the n -th l -pole $(2l+1)$ -degenerate resonance frequency of the sphere, and Γ_{ln} is the linewidth of the mode. Finally, F_{ln} is a dimensionless coefficient which is characteristic of each mode, whose calculation requires to solve the sphere's vibration eigenmode problem, and is rather lengthy. We find that F_{ln} is zero unless $l=2$, i.e., only *quadrupole* modes can possibly be excited—a reassuring result since we are assuming GR.

Numerical analysis yields $F_{21}=2.98$, about 17% better than the cylinder's $8/\pi$ in its fundamental mode: *on equality of mass, the sphere is a slightly better GW antenna than an optimally oriented cylinder*; if we average over directions and polarisations, the sphere's cross section becomes a factor 4.4 better for equal masses. This result confirms that obtained by Wagoner and Paik using a different approach [9], and has also been recently discussed in numerical detail by Zhou and Michelson [10].

We have gone beyond this, however, to assess the higher mode cross section values. We find $F_{22}=1.14$, a remarkably high value, which means that the sphere has a good sensitivity also in its second quadrupole mode. Higher mode values are $F_{23}=0.107$, $F_{24}=0.039$, $F_{25}=0.115$, $F_{26}=0.044$, etc. A plot of the first three of these coefficients is shown in Figure 1 as a function of the eigenvalue frequency, along with the corresponding figures for a cylinder of the same fundamental frequency for comparison. Interesting consequences can be derived from these numbers, as we now discuss.

Consider the following: let there be several solid spheres made of the same material, such that sphere 2 has its fundamental frequency at the *second* frequency of sphere 1, sphere 3 has its fundamental frequency at the *third* frequency of sphere 1, etc. This array constitutes a *xylophone* of frequencies precisely defined once the first is known. Table I shows the frequencies and diameters of these spheres and also, in the third column, the ratio of the cross section of the largest sphere at each frequency to that of the respective smaller ones in their first mode. These numbers indicate that, save for the non-significant exception of the fourth mode, *the single larger sphere has better sensitivity than the xylophone*. The situation is graphically displayed in Figure 2 for the first two spheres; it is based on the model of reference

Table 1: A xylophone of spheres of Al5056 whose fundamental frequencies are equal to the successive harmonics of a larger one. In the third column, the ratio of the cross section of the larger sphere to that of the smaller one at the corresponding frequency.

Frequency (Hz)	Diameter (metres)	CS ratio
910	3.10	1
1747	1.61	2.72
2959	0.95	1.23
3750	0.75	0.92
4217	0.67	3.81
5271	0.54	2.84

[12] for the strain noise spectrum $\tilde{h}(\omega)$, and figures correspond to 3.1 and 1.6 metre diameter spheres of Al5056 operated at quantum limited noise level. The just described “single detector xylophone” has the limitation that its frequencies are fixed and too widely spaced; still, xylophone proposals [12] should benefit from the above considerations in the sense of reducing the number of required elements in them. Clearly, in order to make full use of these properties without severely complicating the detector readout, a set of non-resonant wideband transducers should be employed.

An also important result is the *integrated sensitivity* for broadband, short duration bursts. This is shown in Figure 3 for the same material as in Figure 2. The tails in the sensitivity curves tend to overlap in the higher frequency region. This emphasises the convenience of going for larger spheres in future design work, as smaller ones do not offer better *burst* sensitivity, even in their fundamental mode. Integrated cross section figures are given in Table II for projected spheres and existing cylinders. Inspection of the numbers reveals that the sphere can absorb over 20 times more energy than the cylinder for the materials and dimensions quoted, while cross section for the sphere second mode is still high. The quoted numbers correspond to the detectors *EXPLORER* and *NAUTILUS* at INFN Frascati, *AURIGA* at INFN Legnaro, and *ALLEGRO* at LSU. Cross sections for more massive bars having the same fundamental frequency scale as D^2 , where D is the

Table 2: Integrated cross sections for a typical Al5056 cylinder in its first longitudinal mode with *optimal* orientation with respect to the incoming radiation, and for a sphere of the same material and fundamental frequency in its first two quadrupole modes. Antenna dimensions are also specified.

Cylinder	Sphere
$\nu_{1c} = 910 \text{ Hz}$	$\nu_{1s} = 910 \text{ Hz}$ $\nu_{2s} = 1747 \text{ Hz}$
$L = 3.0 \text{ metres}$ $D = 0.6 \text{ metres}$	$\phi = 3.1 \text{ metres}$
$M_c = 2.3 \text{ tons}$	$M_s = 42 \text{ tons}$
$\sigma_{1c} = 4.3 \times 10^{-21} \text{ cm}^2 \text{ Hz}$ (Optimal orientation)	$\sigma_{1s} = 9.2 \times 10^{-20} \text{ cm}^2 \text{ Hz}$ $\sigma_{2s} = 3.5 \times 10^{-20} \text{ cm}^2 \text{ Hz}$ (Omnidirectional)

cylinder's diameter. The figures quoted in Table II are similar to those of Zhou and Michelson [10], but we also include sphere's *second* mode data.

The results quoted so far rely on the hypothesis that GR is correct³. As it turns out, however, a *spherical GW antenna is a unique laboratory to probe that very hypothesis*, too. This is so thanks to the possibility of using the sphere as a multimode detector. The potentialities hidden in such capability have not been fully realised so far, so let us describe how they can be accomplished. Here, we shall give a sketch of the procedure; full details will be found in [13].

In a metric theory, the GW driving term is given by the tidal force density

$$f_i = \rho c^2 R_{0i0j} x_j \quad (3)$$

where the “electric” components R_{0i0j} of the Riemann tensor are themselves the components of a symmetric 3-tensor; like all such tensors, it can be split as

³Except the discussion on the “single sphere xylophone” above, which is in fact valid under any metric theory of gravity [13].

$$R_{0i0j} = S_{ij} + \frac{1}{3} T \delta_{ij} \quad (4)$$

where S_{ij} is traceless and $T=R_{0i0i}$ is the trace; S_{ij} contains the five quadrupole contributions to the driving force, and T contains the monopole one. This means that measurement of the sphere's monopole *and* quadrupole mode amplitudes enables *full* reconstruction of the Riemann tensor components R_{0i0j} *regardless of any assumption about any specific theory of gravity*. The following precision must however be made in this respect: since actual detector bandwidth is severely limited by noise, in order to accurately reconstruct the R_{0i0j} it is necessary that its spectrum be sufficiently broadband to overlap with the antenna bandwidths at its resonance frequencies. For a source with a sufficient dependence of strength on frequency may well emit monopole (quadrupole) radiation and not excite the antenna's monopole (quadrupole) modes, since the sphere quadrupole and monopole lowest frequencies differ by a factor of almost 2.

Now, if the GW incidence direction is known ahead of time (from whichever astrophysical evidence) then the possibility arises to assess which is the theory—or, indeed, class of theories—compatible with the *measured* R_{0i0j} as follows: pick a rotation taking the laboratory vertical axis into coincidence with the GW wave vector, and apply it to the measured matrix R_{0i0j} ; then apply a classification algorithm to the so transformed Riemann tensor components in order to decide in which class the tensor belongs, thence in which GW theory. The classification scheme has been described in detail in [14]. If, on the contrary, the incidence direction is *not* known prior to detection, then the above test cannot be performed; if, however, a specific theory is assumed *a priori* then this assumption can be used to sort out the unknown incidence direction. Such is the basis of Wagoner and Paik's proposal [9], and can be extended to theories other than scalar-tensor [13].

Clearly then, if the GW incidence direction is unknown then a GW theory cannot be *probed*—since it has to be *assumed*. However, vetoes can be established on the *tracelessness* and *transversality* properties of GWs as a result of the full reconstruction of the R_{0i0j} in the laboratory frame. If, for instance, six identical resonant transducers arranged with dodecahedral symmetry and tuned to the fundamental quadrupole frequency—as proposed in [12]—, plus one additional transducer, tuned to the monopole frequency (which is about a factor of two higher than the quadrupole one) are attached to the sphere's

surface, then evidence of excitation of the monopole modes is straightaway a veto on tracelessness, while evidence of excitation of quadrupole modes other than those with $m=\pm 2$ implies some degree of non-transversality in the incident GW. Let us emphasise that *these vetoes can be excersised with no SNR penalty*, as they are obtained by monitoring mode amplitudes with the sensitivity reported in Figures 2 and 3. We underline that unprecedented limits on scalar radiation could be set up by monitoring the sphere's monopole mode.

A single sphere, however, will not suffice to autonomously identify a GW event: at least *two* antennas are always necessary for minimum coincidence analysis. Furthermore, if timing of signal arrival time is sufficiently precise, two detectors can remove the *antipodal* ambiguity in the source direction, i.e., discern whether its position in the sky is (θ, φ) or $(\pi - \theta, \varphi + \pi)$, and be used to calculate the GW propagation speed, thus completing the features of the observatory [15]. It has been shown [7] that a timing accuracy of a small fraction of a millisecond is attainable in a network of resonant detectors with a post-detection bandwidth of about 50 Hz, provided *SNR* is about 10 or more. Such time resolution is sufficient to resolve differences in signal arrival times at two detectors an earth surface distance apart.

A minimal observatory constituted by two spheres has additional advantages over a network of several directional antennas with different orientations. For example, coincidence analysis between spheres is greatly simplified, since the *same* amount of GW energy will be deposited in each detector, while in an array of directional antennas everyone will absorb according to their orientations, thus complicating that analysis. (The reader is referred to [10] for a discussion of coincidence analysis between spherical detectors).

An additional point we want to stress is this: when accidental events from cosmic rays are considered (over 10^4 are expected per day in a several ton resonant detector operated at the quantum limit [16]), it is enough to place *just one* detector in an underground laboratory to reduce the number of random coincidences to about one in 3 years. A system of two large Al 5056 spheres resonating at 800 Hz (3.5 metre diameter, 60 tons of weight) would reach an omnidirectional burst sensitivity ($SNR=1$) of about $h_{min} \simeq 3 \times 10^{-22}$. Feasibility studies on the fabrication technique and cooling of such large detectors are currently underway [17].

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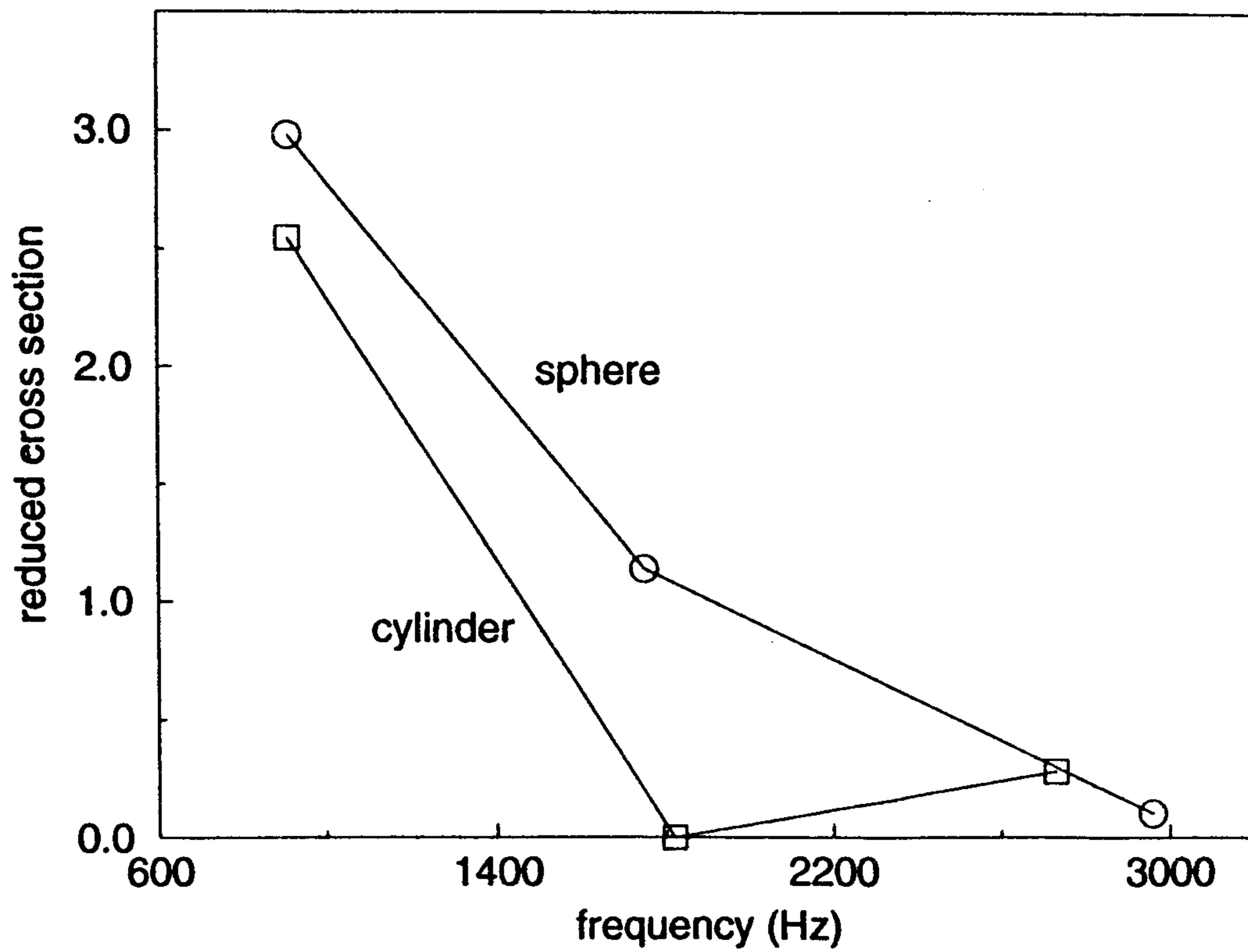


Figure 1 Reduced cross section (i.e., per unit mass) for a sphere and a cylinder having the same fundamental frequency. Note that the second sphere mode still shows a remarkably high cross section, while the third and subsequent (not shown) decay sharply.

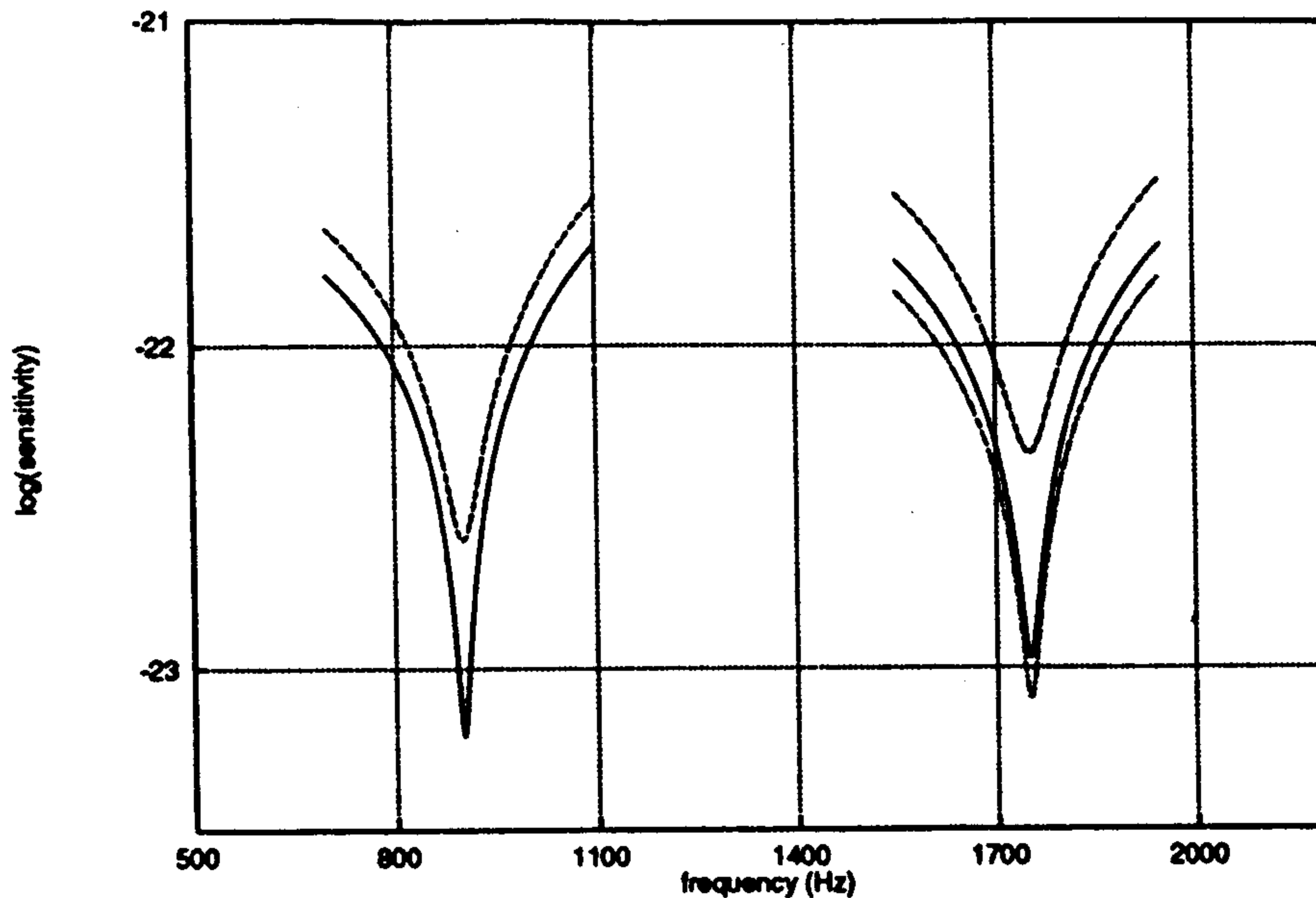


Figure 2 Calculated strain noise spectrum \tilde{h} (in $1/\sqrt{Hz}$) for various detectors at the quantum limit: *solid* lines for the lowest quadrupole mode of Al 5056 spherical detectors 3.1 and 1.6 metres diameter, respectively; *dot-dashed* line for the second quadrupole mode of the 3.1 metre sphere, and *dashed* lines for the equivalent cylindrical bar optimally oriented.

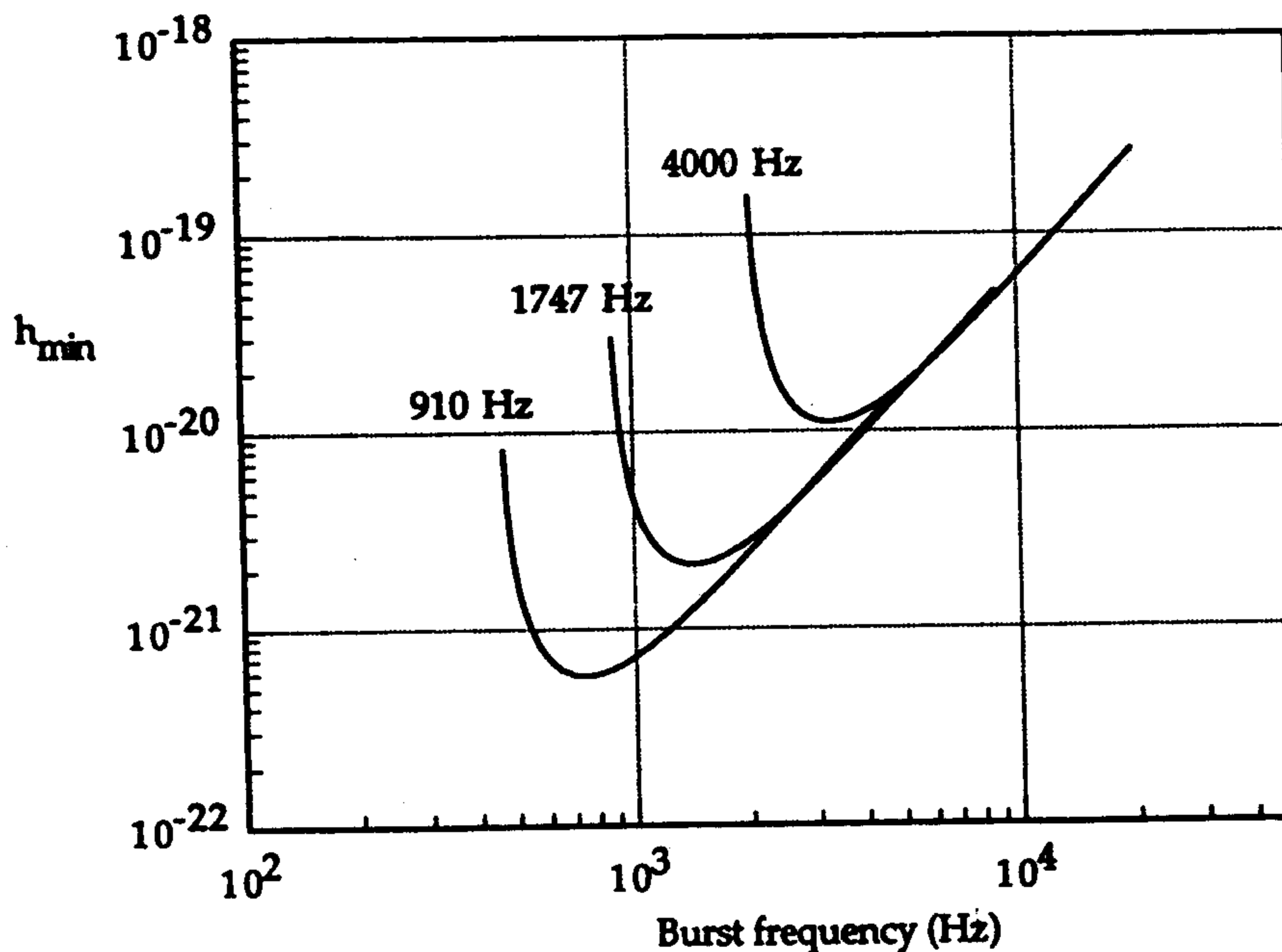


Figure 3 Burst sensitivity curves for Al 5056 spherical antennas of different sizes and quantum limited noise. Burst is one sinusoidal cycle of duration τ_g , and we represent in abscissas the frequencies $\nu = \tau_g^{-1}$.