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Large p_T Hadroproduction of Heavy Quarks and Quarkonia

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Abstract

The production of heavy quarks at large p_T ($p_T \gg m$) in hadronic collisions is considered in the framework of perturbative fragmentation functions, allowing a resummation at the NLO level of final state large mass logarithms of the kind $\log(p_T/m)$. The resulting theoretical uncertainty from factorization/renormalization scales at large p_T is shown to be much smaller than that shown by the full $O(\alpha_s^3)$ perturbative calculation. Then the production of heavy quarkonia at large tranverse momenta is discussed by including the mechanism of fragmentation, in particular the direct fragmentation to J/ψ and the fragmentation to χ states followed by radiative decay to J/ψ. The overall theoretical estimate is shown to be nearly consistent with the experimental observation for J/ψ. On the contrary the situation is quite unsatisfactory for the ψ', demanding for a new mechanism dominating the production process.

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The hadroproduction of heavy quarks has recently been a subject of intense studies both experimentally and theoretically, in particular as an important testing ground for QCD. Also the study of the production properties of bound states of heavy quarks plays a central role in our understanding of the theory on the very border between the perturbative and non perturbative domains. A large amount of experimental data on the hadroproduction of b and c quarks and their bound states has been accumulated so far, to be compared with next-to-leading order (NLO) calculations recently available on the market. In this talk I will not attempt to review the full range of theoretical predictions on the production processes, the general properties having been already discussed nicely by Berger [1]. Rather I will concentrate on some new results on the production of heavy quarks and quarkonia at collider energies and large transverse momenta, which go beyond the strict realm of fixed order perturbation theory, giving more precise predictions and also some new ideas for the production mechanisms.

We consider first the production of heavy quarks in hadronic collisions. On the theoretical side the calculation in perturbative QCD of the differential and total cross sections to order α_s^3 has been performed [2–6], providing a firm basis for a detailed study of the properties of the bottom and charm quarks, and leading to reliable predictions for the production rate of the top quark [8–10].

These results do however present a non-negligible residual renormalization/factorization scale dependence, particularly at large p_T. Furthermore, the validity of this NLO $O(\alpha_s^3)$ calculation is limited when $p_T \gg m$, m being the large quark mass, by the appearance of potentially large logarithms of the type $\log(p_T/m)$, which have to be resummed to all orders. The physical reason for that is quite clear. For example, terms of order $(\alpha_s^3)\log(p_T/m)$ or $(\alpha_s^4)\log(p_T/m)^2$ are simply related to the mass singularities originating from collinear configurations when $m \to 0$ for fixed p_T. The theoretical uncertainty associated to those corrections has been roughly estimated in [3]. Whereas for top quark production this uncertainty is irrelevant, this is not the case for the production of bottom and charm quarks at large p_T, leading to relevant phenomenological consequences.

A solution to this problem has recently been considered [7], following an approach based on the properties of fragmentation of a generic parton p ($p = q, g, Q$) in the heavy quark Q, after the parton has been produced inclusively in the hard collision of the two initial hadrons. The basic formula is represented by eq. (1), where the partonic cross sections $\hat{\sigma}_{ij\to kX}$ at $O(\alpha_s^3)$ have been given in ref. [11] in the massless quark limit. $\hat{\sigma}_{ij\to kX}$ introduces an explicit dependence on p_T and on renormalization/factorization mass scales. The dependence on the heavy quark mass is then obtained through the fragmentation function of the parton $p \to Q + X$, evolved at NLO accuracy from an initial scale $\mu_0 \sim m$ (see below) to $\mu \sim p_T$. This approach explicitly resums potentially large terms ot the kind $[\alpha_s \log(p_T/m)]^n$, giving a better description of the theoretical predictions at large p_T. Indeed the corresponding uncertainty is quite reduced in this region with respect to the fixed order result, due to a significantly smaller sensitivity to the relevant scales. On the other hand, because of the massless limit used for the $O(\alpha_s^3)$ kernel cross sections $\hat{\sigma}_{ij\to kX}$, this approach does not allow to recover in a simple way the limit $p_T \lesssim m$ of the perturbative calculation.

I will briefly review the main ideas of this analysis. According to factorization theorems [12] the cross section for the inclusive hadroproduction of a hadron at high transverse momentum, i.e. for the process

$$H_1 + H_2 \to H_3 + X$$

can be written as

$$\mathrm{d}\sigma = \sum_{i,j,k} \int \mathrm{d}x_1\, \mathrm{d}x_2\, \mathrm{d}x_3\, F^i_{H_1}(x_1, \mu_F) F^j_{H_2}(x_2, \mu_F) \mathrm{d}\hat{\sigma}_{ij\to kX}(x_1, x_2, x_3, \mu_R, \mu_F) D^{H_3}_k(x_3, \mu_F) \qquad (1)$$

As usual, the F's are the distribution functions of the partons in the colliding hadrons, $\hat{\sigma}$ is the kernel cross section and D is the fragmentation function of the observed hadron. The factorization mass scales μ_F of the structure and fragmentation functions are assumed to be equal for the sake of simplicity. μ_R is the renormalization scale.

Due to the presence of collinear singularities both in the initial and final state this process is not fully predictable by QCD itself. We can actually calculate the kernel cross section and the evolution of the structure and fragmentation functions, but we have to rely on some phenomenological input to obtain the latter at some given initial scale.

This situation changes drastically when we come to consider the inclusive production of a heavy quark. In this case its mass, being finite and considerably greater than Λ, makes the perturbative expansion feasible and prevents collinear singularities from appearing in the splitting vertices which involve the heavy quark. Having this in mind two approaches can be pursued in the calculation of heavy quark production.

The first one is to directly calculate in perturbation theory the process $\mathrm{d}\hat{\sigma}_{ij\to QX}$, Q being the heavy quark and i, j the initial state *light* partons (i.e. light quarks and gluons). This kernel cross section will then be convoluted with initial state structure functions only, the final state showing no singularities of any kind. This approach has been followed in the past [2–5], providing a full perturbative $O(\alpha_s^3)$ calculation. In the following we shall use for comparisons the results of Nason, Dawson and Ellis, and refer to them as NDE. In this fixed order approach, as stated earlier, terms of the kind $\alpha_s \log(p_T/m)$ will appear. They are remnant of the collinear singularity screened by the finite quark mass. As noted in ref. [3] they can grow quite large at high tranverse momenta, thereby spoiling the validity of the expansion in α_s. Therefore they have to be summed to all orders.

The alternative way is to consider that when a quark, of whichever flavour, is produced at very high transverse momentum $p_T \gg m$ its mass plays almost no role at all in the scattering process. This is to say that mass effects in the kernel cross section are suppressed as power ratios of mass over the scale of the process. We can therefore devise a picture in which all quarks are produced in a massless fashion at the high scale $\mu_F \sim p_T \gg m$ and only successively, as their virtuality decreases, they can *fragment* into a massive heavy quark. The cross section can therefore be described by a formula analogous to eq. (1), with $H_3 = Q$. The key difference to the hadron production case considered in eq. (1) is that initial state conditions for the heavy quark fragmentation functions are now calculable from first principles in QCD (hence the definition of "perturbative" fragmentation functions, PFF) and do not have to be taken from experiment.

Actually, the following set of next-to-leading initial state conditions can be obtained [13] in the $\overline{MS}$ scheme for the fragmentation function of a heavy quark, gluon and light quark respectively, in the heavy quark Q

$$D_Q^Q(x,\mu_0) = \delta(1-x) + \frac{\alpha_s(\mu_0)C_F}{2\pi}\left[\frac{1+x^2}{1-x}\left(\log\frac{\mu_0^2}{m^2} - 2\log(1-x) - 1\right)\right]_+ \qquad (2)$$

$$D_g^Q(x,\mu_0) = \frac{\alpha_s(\mu_0)T_f}{2\pi}(x^2+(1-x)^2)\log\frac{\mu_0^2}{m^2} \qquad (3)$$

$$D^Q_{q,\bar{q},\bar{Q}}(x,\mu_0) = 0 \qquad (4)$$

where μ_0 must be taken of the order of the heavy quark mass.

Then using the usual Altarelli-Parisi evolution equations at NLO accuracy one finds the fragmentation functions set at any desired factorization scale μ_F. An important feature of this approach can now be appreciated. The "almost-singular" logarithmic term $\log(p_T/m)$ splits into two, as follows. A $\log(p_T/\mu_F)$ will be found in the kernel cross section $\hat{\sigma}$ which has no dependence on the heavy quark mass, according to the assumption that it is produced in a *massless* way). Moreover, by choosing $\mu_F \sim p_T$ it will not contain large logarithms and its perturbative expansion will behave correctly. The remaining part of the log will instead be lurked into the fragmentation function $D(x_3,\mu_F)$. The large $\log(\mu_F/\mu_0)$ is resummed to all orders by the evolution equations, and only the small $\log(\mu_0/m)$ provided by the initial state condition is treated at fixed order in perturbation theory. Therefore one expects a better control of the theoretical uncertainty at large p_T. On the other hand, for $p_T \lesssim m$ the fragmentation approach does not allow to recover easily the $O(\alpha_s^3)$ result, which, of course, holds exactly.

In order to implement the "perturbative fragmentation function (PFF) approach" at a numerical level we need three ingredients, which are all available at the next-to-leading level:

i) the distribution functions of any parton (including the heavy flavour in question) in the hadrons (proton or antiproton), evolved at NLO accuracy. All modern sets satisfy this requirement. An important point must however be made clear. A heavy quark present in the initial state can be directly brought to the final state where it is fragmented to the detected heavy flavour through the D_Q^Q, and therefore with high probability. This means that the resulting cross section is particularly sensitive to the overall heavy flavour content of the colliding hadrons. In the Parton Distribution Functions (PDF) sets available this content is generated through perturbative gluon splitting above a given threshold. The total yield will therefore depend on the choice made. For instance, the HMRS-B set [14] ($\Lambda_5^{\overline{MS}} = 123$) takes $F_b(x,2m_b) = 0$ as initial condition, whereas the MT-B2 [15] ($\Lambda_5^{\overline{MS}} = 124$) and the CTEQ1M [16] ($\Lambda_5^{\overline{MS}} = 152$) ones choose, according to ref. [17], $F_b(x,m_b) = 0$.

ii) the kernel cross section for the scattering of any two massless partons into another massless parton. This calculation is provided at the NLO in various renormalization/factorization schemes in ref. [11].

iii) the fragmentation functions of any parton into the heavy flavour. They are obtained by evolving the initial conditions given above (eqs. 2,3,4) with NLO accuracy [18]. This is done through numerical inversion of the Mellin moments of the evolved distributions.

With these ingredients at our disposal we can now evaluate the cross section for the high p_T inclusive hadroproduction of a bottom quark. Figure 1 shows, from ref. [7] and for the b-quark

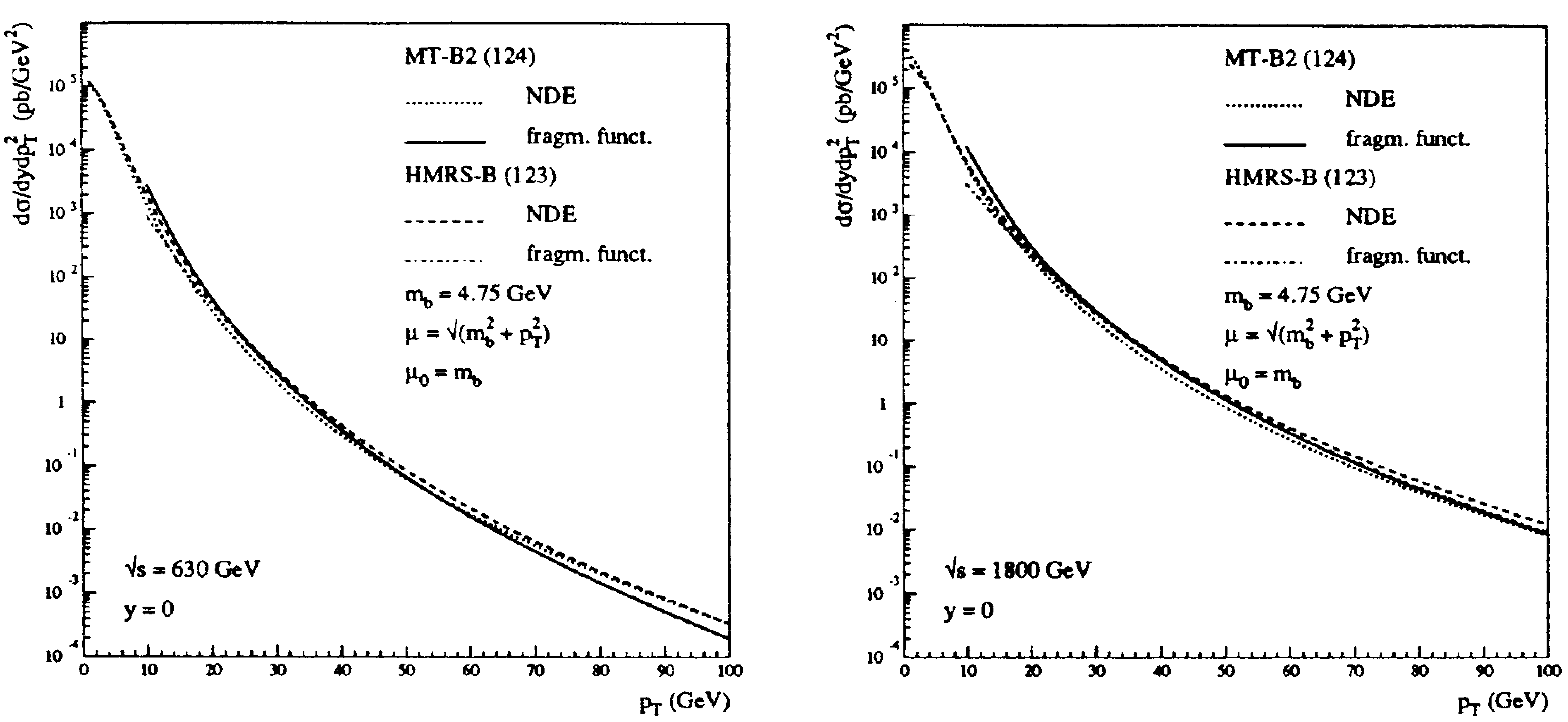

Figure 1: Results from the PFF approach compared to the fixed order prediction of NDE, with the MT-B2 and the HMRS-B sets, at 1800 and 630 GeV for $p\bar{p}$ collisions.

production, the comparison of the result of ref. [3] with our calculations for two different PDF sets, for $p\bar{p}$ collisions at the Cern and Fermilab colliders energies of 630 Gev and 1800 GeV respectively, for $\mu = \sqrt{m_b^2 + p_T^2}$. We can see that in the high p_T region the fixed order cross section is quite sensitive to the structure function set choice, the MT one giving a markedly lower result. The opposite happens in the PFF approach, which becomes very sensitive in the low p_T region to the neglect of the heavy quark mass terms in the partonic cross sections and to the threashold behaviour of the PDF sets in the heavy quark content. Our calculation, on the other hand, leads to substantially identical predictions obtained in the high p_T region with the two PDF sets.

Next we consider the dependence on the choice of the renormalization/factorization mass scale μ. Figure 2 shows, for $p\bar{p}$ collisions at 1800 GeV and with the MT-B2 and HMRS-B sets, the theoretical uncertainty resulting from the variation of the factorization and renormalization mass scales between $\mu_{ref}/2$ and $2\mu_{ref}$, where μ_{ref} is defined as $\sqrt{m_b^2 + p_T^2}$ and we have taken $\mu = \mu_F = \mu_R$. As expected the band of the NDE calculation is sensibly larger than ours, showing the improvement brought by the resummation of the large logarithms of p_T/m_b. These features can be better appreciated in figure 3, where the cross section at 1800 GeV with the MT-B2 set is plotted, at fixed y and p_T, as a function of $\mu = \xi\mu_{ref}$, for ξ varying between 0.25 and 4. This figure also shows a comparison with the factorized calculation with a Born (i.e. LO) cross section kernel (but with two-loop α_s and NLO structure and fragmentation functions). As expected, the lack of the next-to-leading terms strongly enhances the scale dependence. The similarity between the NDE result and the fragmentation function approach with Born kernel cross section is striking. The small scale sensitivity of our full NLO calculation shows that the factorization/renormalization scale dependence is a real $O(\alpha_s^4)$ effect, whereas in the fixed order approach (e.g. NDE) the presence of large $\log(p_T/m)$ results in an effective $O(\alpha_s^3)$ dependence.

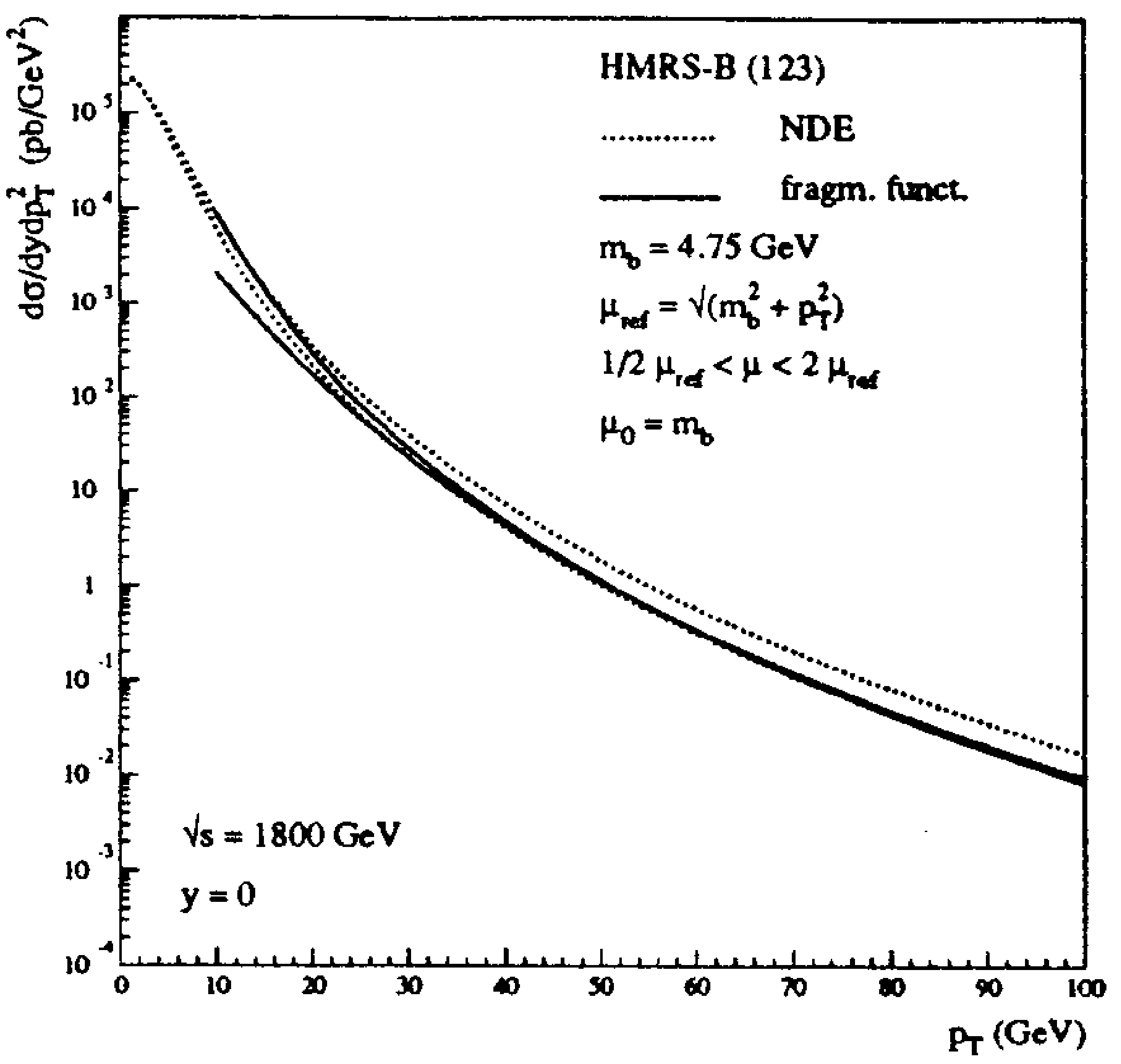

Figure 2: Scale dependence at 1800 GeV, with MT-B2 and HMRS-B structure functions sets.

We can now turn to quantities of more direct experimental interest and see how the advantages of our approach are reflected onto them. Namely, we will consider the total cross section for one particle inclusive heavy quark production, integrated above a given p_T^{min} and within a rapidity region $|y| < y_{max}$. Only the variation of the theoretical prediction due to changes in the factorization/renormalization scale and in the PDF set used will be studied. Other possible sources of uncertainties, aside the change of μ_0 which has been shown to be almost neglibible, are the value of the QCD scale Λ and of the bottom mass m_b. They should however be common to both the fixed order and the fragmentation function approach, and have been studied in detail in ref. [3].

The overall smaller theoretical uncertainty of the PFF result can be appreciated in fig. 4, where the highest and lowest predictions of the two approaches, out of the six curves previously considered, have been plotted. Note that we have not considered in detail the uncertainties for $p_T \lesssim 10$ GeV. The CDF experimental data [19] are also shown. Similarly in fig. 5 the comparison is shown with UA1 data [20] at 630 GeV.

The same kind of result is obtained for charm production. Figure 6 compares compares our predictions to NDE and also in this case we find a sizeable reduction in the uncertainty of the theoretical prediction. An independent analysis has been carried out [21] for the hadronic production of the charmed mesons D and D^*, using phenomenological inputs for the fragmentation functions in e^+e^- annihilation and NLO evolution equations. The predicted tranverse momentum distributions behave similarly to the open charm results.

We consider now the production of the bound states of heavy quarks which plays a central role in our understanding of QCD on the very border between the perturbative and non-perturbative domain. In particular it is of key importance to have accurate estimates of the production cross sections at large tranverse momenta for precision tests of the theory and possible evidence of new phenomena.

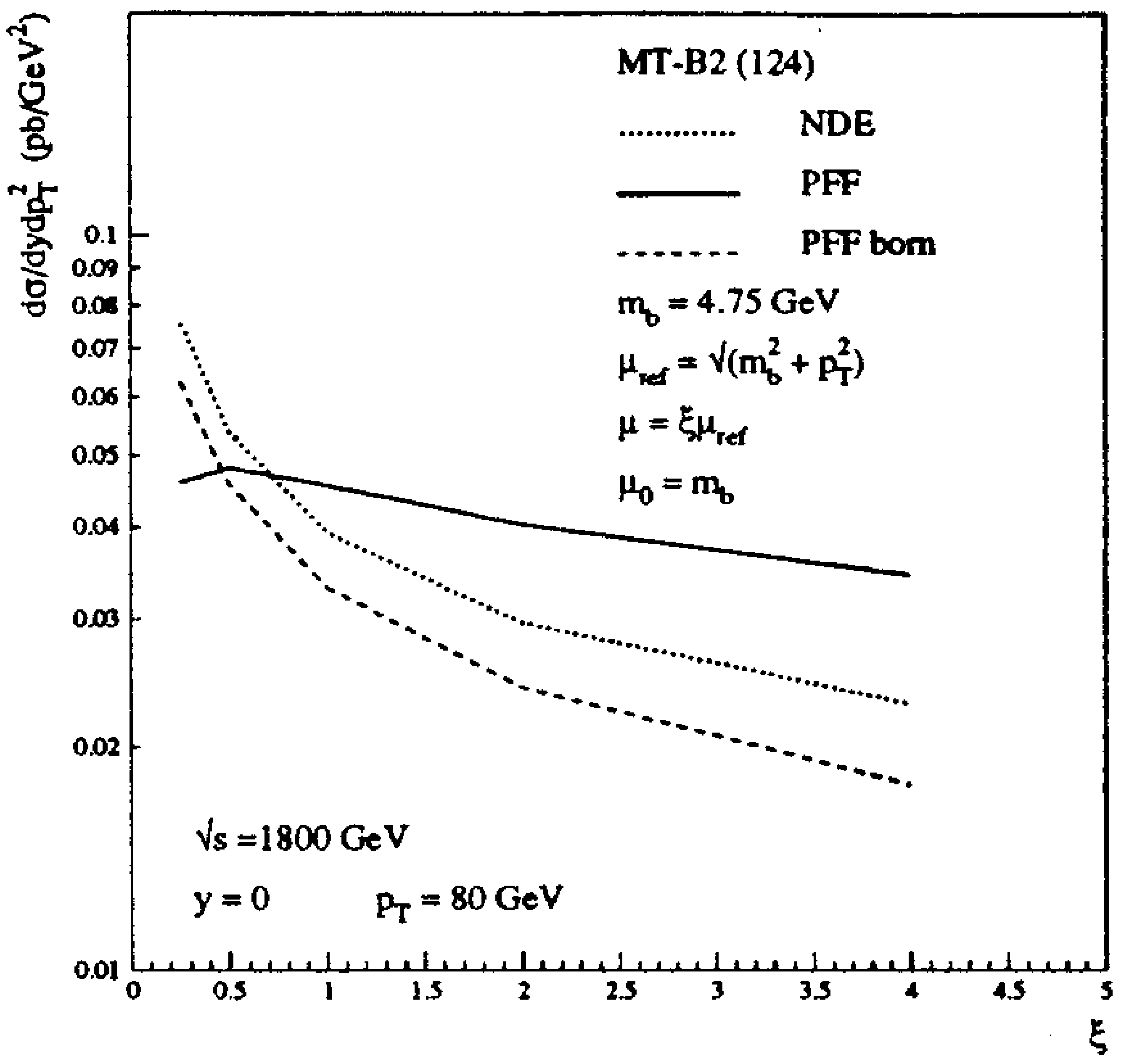

Figure 3: Scale dependence of the cross section as a function of $\mu = \xi\mu_{ref}$, at $y = 0$ and $p_T = 80$ GeV. "NDE" and "PFF" refer to the full NLO calculations, whereas "PFF born" is the result of the fragmentation function approach with LO kernel cross section.

So far there has been an intensive experimental study of the $c\bar{c}$ $1S$ vector bound state, namely J/ψ and ψ', both at UA1 [22] and CDF [23]. The results have been compared with theoretical calculations [24, 2, 25] which take into account two different mechanisms for J/ψ production: direct charmonium production, including the contribution from the χ states, i.e.

$$gg \to J/\psi\, g \qquad \begin{array}{l} gg, q\bar{q} \;\to\; \chi g \\ \qquad\qquad\; \hookrightarrow J/\psi\, \gamma \end{array} \qquad \begin{array}{l} qg \;\to\; \chi q \\ \qquad\;\; \hookrightarrow J/\psi\, \gamma \end{array} \tag{5}$$

and the production resulting from B mesons decay

$$\begin{array}{l} p\bar{p} \;\to\; bX \\ \qquad\;\; \hookrightarrow B \to J/\psi\, X \end{array} \tag{6}$$

These calculations are in disagreement with the results from CDF, the J/ψ rate observed being actually higher, by a factor of two or more, than the predicted one [23,25,26].

It has however recently been pointed out by E. Braaten and T.C. Yuan [27] that at large p_T an additional production mechanism comes into play, namely the fragmentation of a gluon or a charm quark into a charmonium state. While being of higher order with respect to direct production by a power of the running coupling constant α_s, this mechanism becomes dominant at large p_T because of a factor $O(p_T^2/m_c^2)$ which overcomes the extra power of α_s. The fragmentation functions describing these processes can be calculated perturbatively. Indeed it has been argued in [27] and subsequently shown at LO in [28] that J/ψ production via fragmentation will overcome the direct one (i.e. $gg \to J/\psi g$) at $p_T \sim$ 6-8 GeV. A similar exercise for the χ production, when the total fragmentation probability $\int D_g^\chi$ (see below) times the $gg \to$

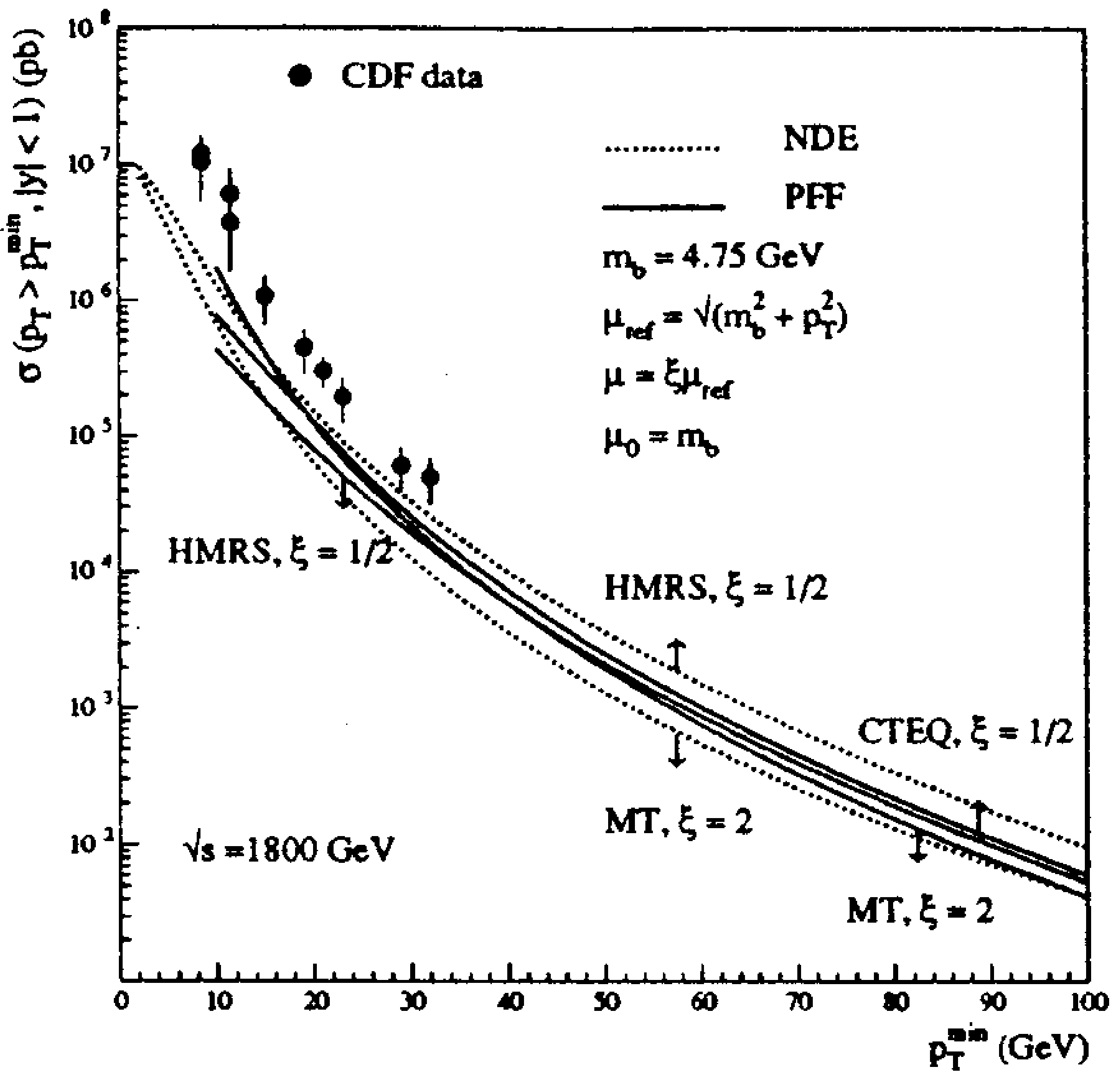

Figure 4: Comparison of the overall theoretical uncertainty of inclusive high p_T heavy quark production at 1800 GeV in the PFF and fixed order approaches. CDF experimental data are also shown.

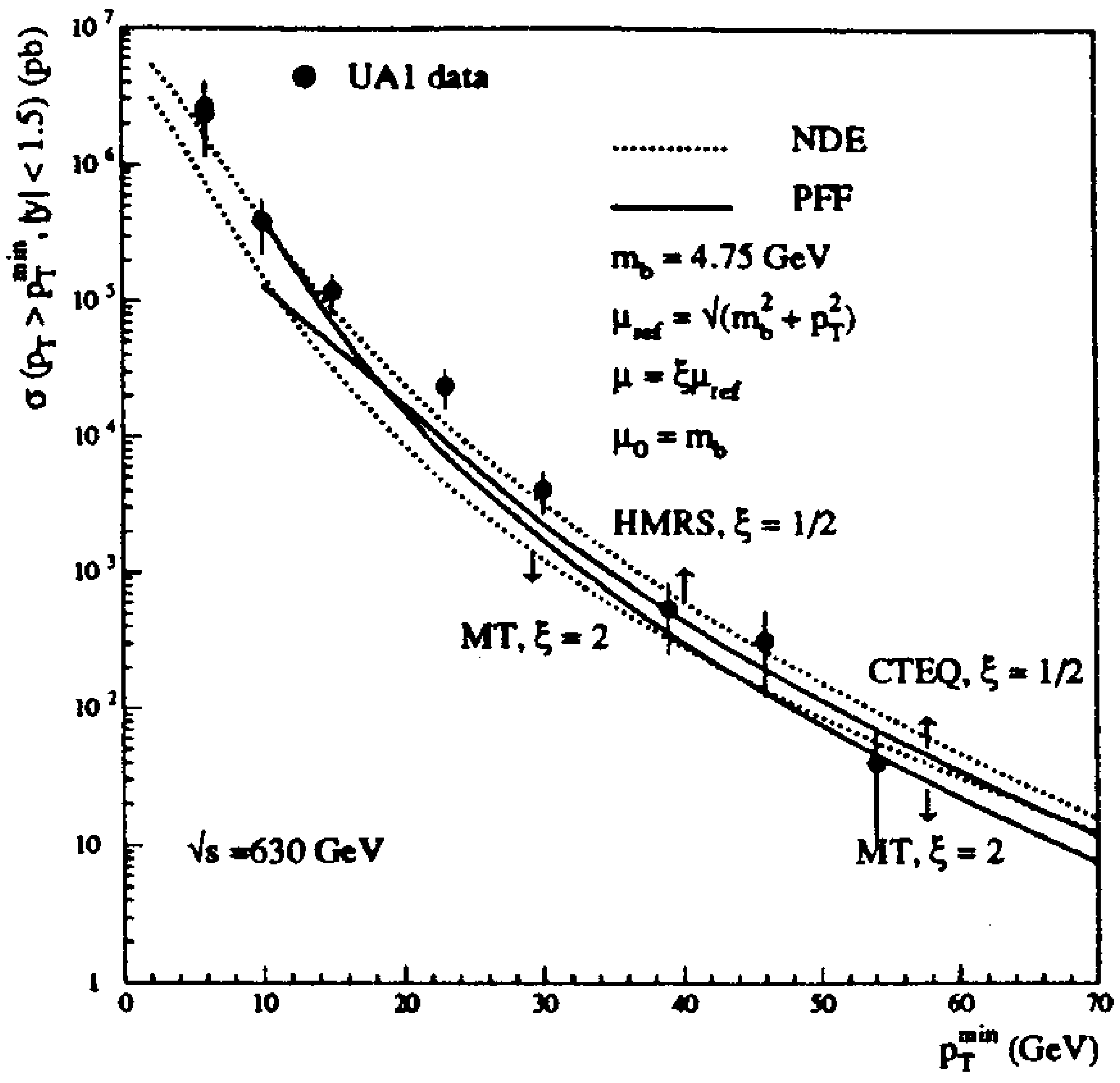

Figure 5: Comparison of the overall theoretical uncertainty of inclusive high p_T heavy quark production at 630 GeV in the PFF and fixed order approaches. UA1 experimental data are also shown.

gg cross sections is compared to the direct production $gg \to \chi\, g$, reveals that fragmentation should dominate for p_T already at ~ 2 GeV. Since this result is at the limit of validity of the fragmentation function approach, we can however still expect that the fragmentation mechanism will dominate over the direct one at p_T values as low as 5-6 GeV.

These ideas have been recently applied [36, 37, 39] to a more quantitative determination of the J/ψ production rate in hadron collisions, taking also into account the production via fragmentation processes of the χ states and subsequent radiative decays to J/ψ.

To this aim the following fragmentation functions play a major role: the gluon fragmentation to J/ψ [27], $D_g^{J/\psi}$ (see fig. 7); the charm (or anticharm) fragmentation to J/ψ [29], $D_c^{J/\psi}$ (see fig. 8); the charm fragmentation to χ states [30, 31], D_c^{χ}; and finally the gluon fragmentation to χ states [32], D_g^{χ} (see fig. 9).

They have been all calculated by perturbative techniques at an initial scale of the order of the mass of the J/ψ. Of course in the evaluation of the actual cross sections they must be evolved to the appropriate scale, and one gets to the usual expression

$$d\sigma(p\bar{p} \to J/\psi(p_T) + X) = \sum_i \int_{z_{min}}^{1} dz\, d\sigma(p\bar{p} \to i(p_T/z) + X, \mu) D_i^{J/\psi}(z, \mu) \qquad (7)$$

for the J/ψ production, the sum running over g, c and $\bar{c}$. A similar formula does hold for χ production. The cross section on the right hand side corresponds to the inclusive production of the parton i, convoluted with the appropriate structure functions and summed over all relevant parton-parton scattering processes. μ is the factorization scale, which is taken of order $\mu_0 = \sqrt{p_T^2 + M_{J/\psi}^2}$.

The evolution of the fragmentation functions given above obeys the usual Altarelli-Parisi

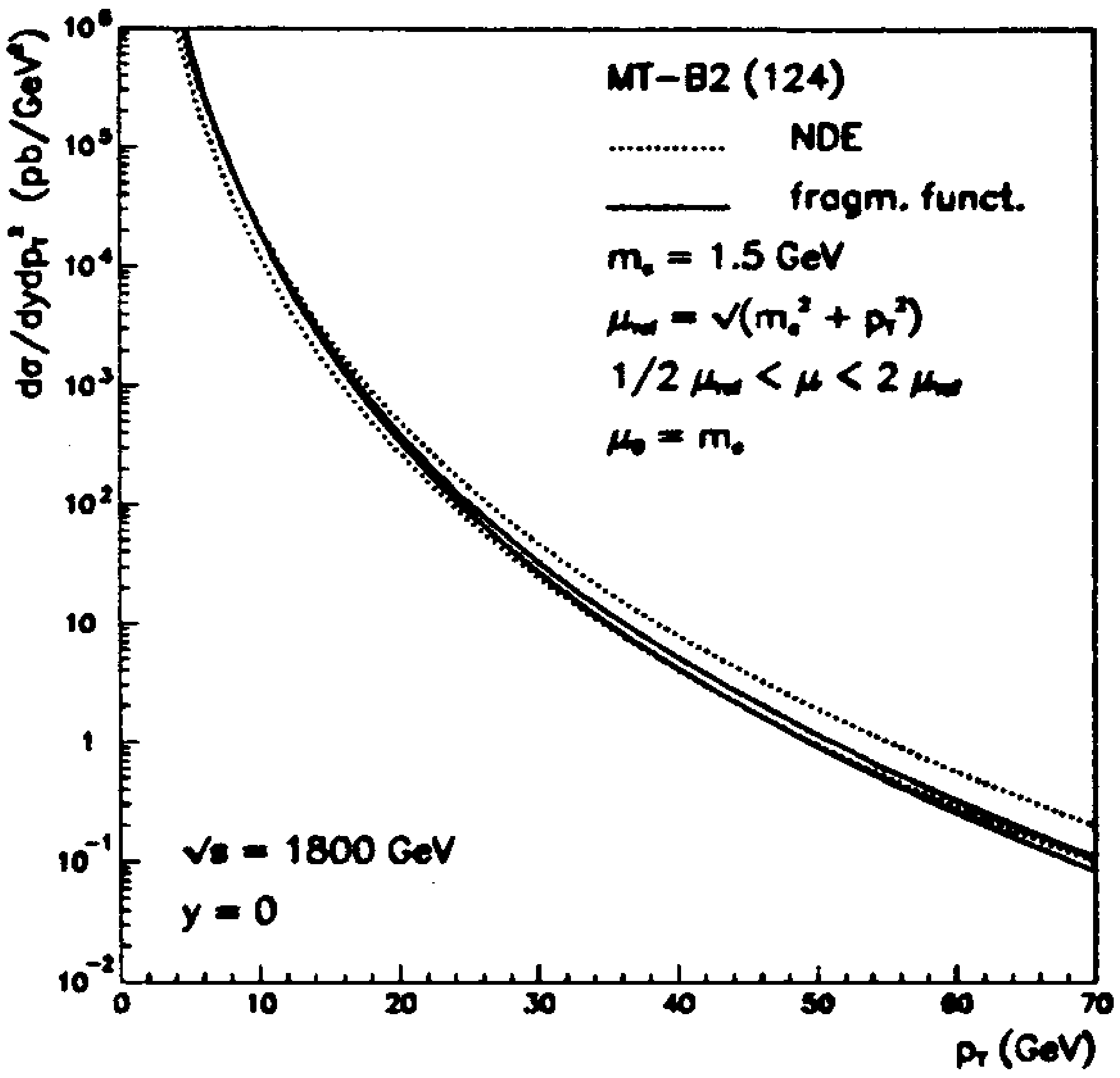

Figure 6: Results from the PFF approach to c quark production compared to the fixed order prediction of NDE, with the MT-B2 set, at 1800 GeV for $p\bar{p}$ collisions.

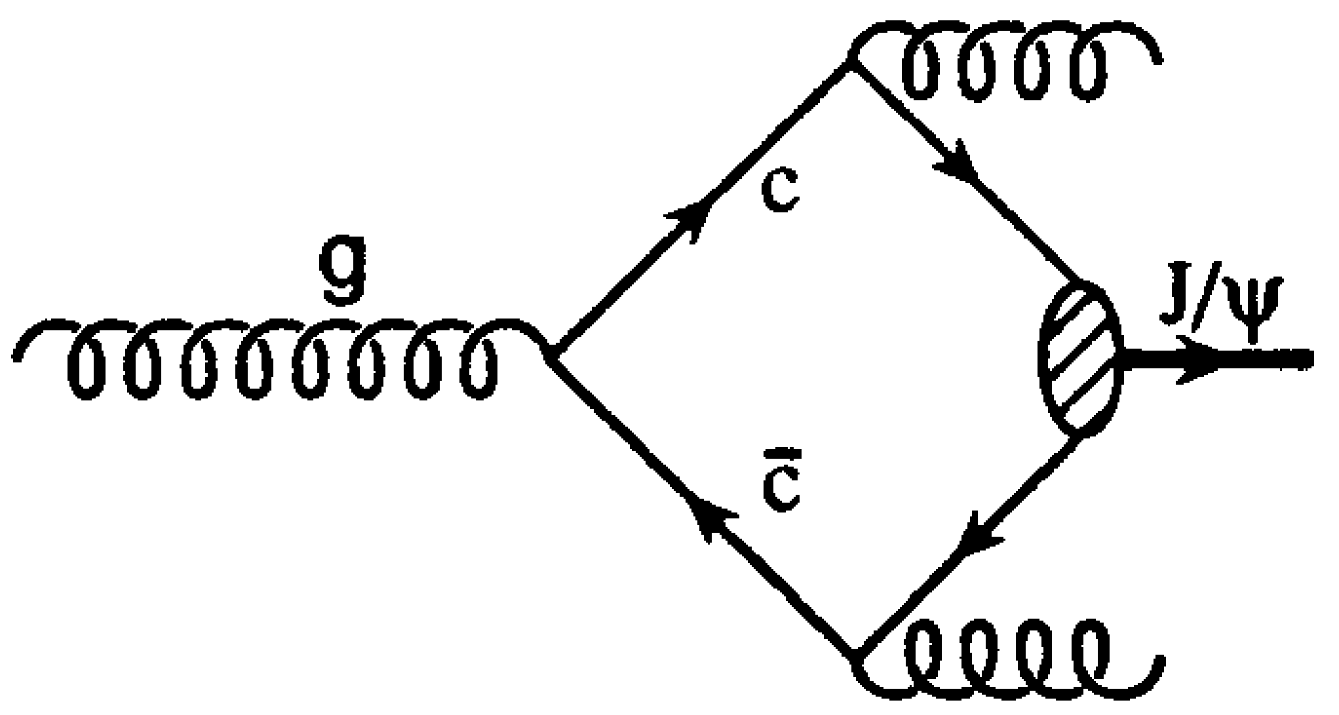

Figure 7: One of the diagrams for the gluon fragmentation function at the scale $\mu = 2m_c$.

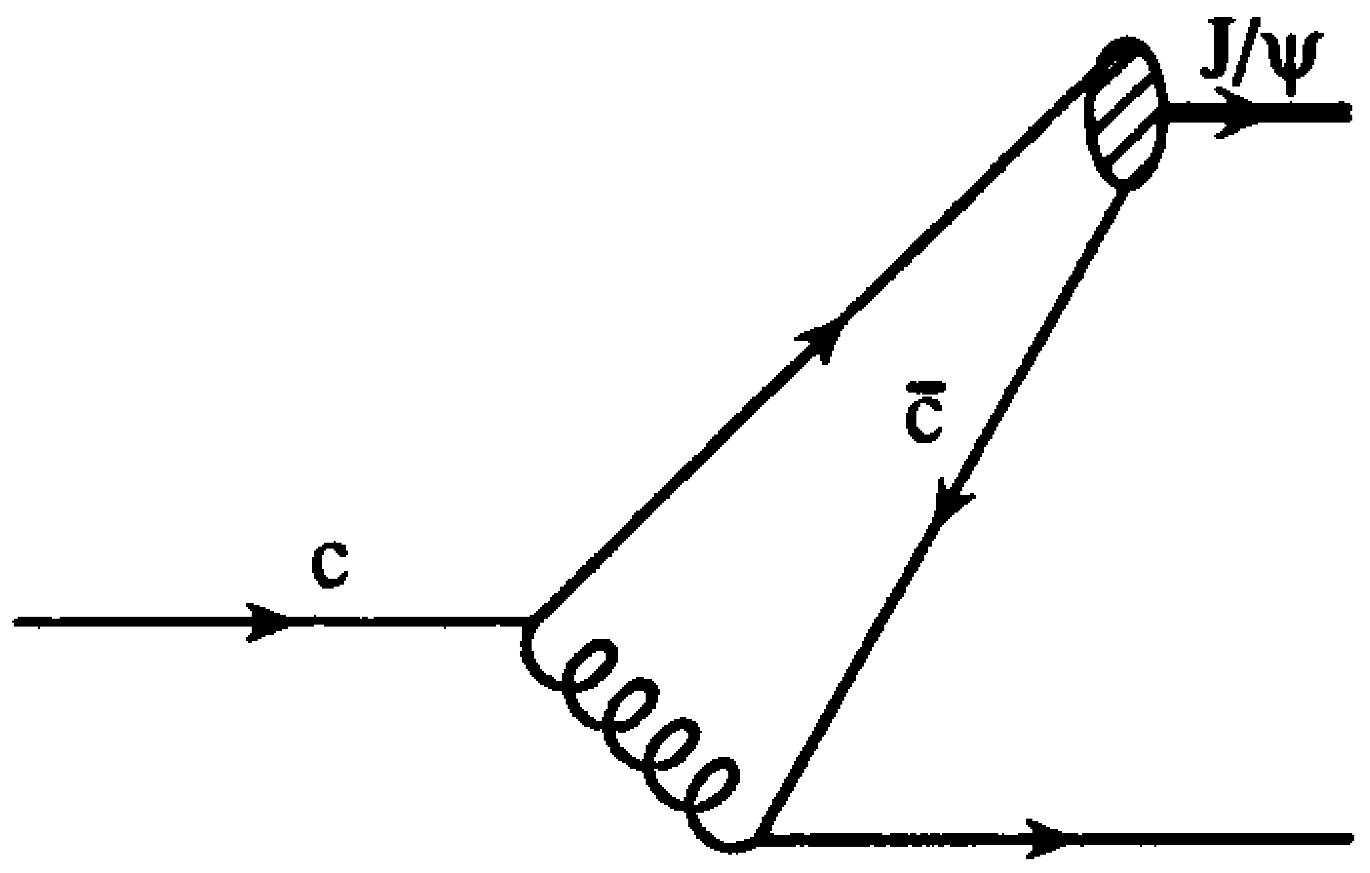

Figure 8: One of the diagrams for the charm fragmentation function, at the scale $\mu = 3m_c$.

(AP) equations

$$\mu\frac{\partial}{\partial\mu}D_i^{J/\psi}(z,\mu) = \sum_j \int_z^1 \frac{dy}{y}\, P_{i\to j}(z/y,\mu)\; D_j^{J/\psi}(y,\mu) \tag{8}$$

Furthermore it has been pointed out in ref. [33] that when one considers the whole set of the AP equations, with the appropriate mixings taken into account, the evolution of the $D_c^{J/\psi}$ will induce a gluon fragmentation function through the splitting $g \to c\bar{c}$ and subsequent fragmentation of one of the quarks into a J/ψ (see fig. 10). This process is of order α_s^3 but, being enhanced by a factor $\log(\mu/M_{J/\psi})$, will dominate over the the contribution from $D_g^{J/\psi}$ at large p_T.

We present first the leading order (LO) results using the Born partonic cross sections and then, to reduce the theoretical uncertainty, by taking into account the full NLO [11] information on the partonic scattering processes.

From ref. [36] we plot in fig. 11 the LO cross sections, differential in p_T and integrated over

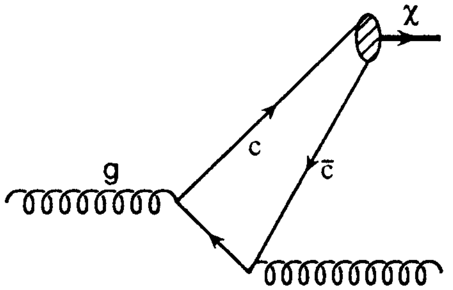

Figure 9: One of the diagrams for the gluon fragmentation function to the χ states, at the scale $\mu = 2m_c$.

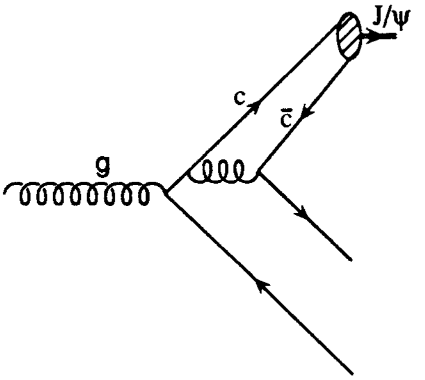

Figure 10: One of the "perturbative" contributions to the induced gluon fragmentation function, at the scale $\mu = 4m_c$.

the $\eta < |0.5|$ range, for producing a J/ψ via fragmentation, either directly or after radiative decay of a χ state. The values of the various parameters entering into the calculation are reported in ref. [36], and we are using $\mu = \mu_0$ for the factorization/renormalization (f/r) scales. We also use the MRS-D0 set of structure functions. The curves labeled by χ are due to gluon fragmentation only. We have not included $c \rightarrow \chi$ fragmentation contributions since, from the total fragmentation probabilities listed in [30], they can be predicted to lie two orders of magnitude below the $c \rightarrow J/\psi$ curve and be therefore surely negligible.

From inspection of fig. 11 the contributions from χ_1 and χ_2 states can be clearly seen to dominate all over the p_T range considered.

Next we compare, in fig. 12, the results obtained for the dominant $\chi_1 + \chi_2$ contribution in the LO approach with those obtained by inserting also the next-to-leading (NLO) partonic cross sections, to order α_s^3, with α_s evaluated to two loop accuracy. Figure 12 clearly shows that the higher order terms enhance the cross section by a factor about 1.5. This is consistent with previous studies of higher order corrections in heavy quark [2–5] and inclusive jets [11,34] production in hadron collisions. The effect of variations of the f/r scales μ between 0.6 μ_0 and $2\mu_0$ is also shown. As expected, the inclusion of the NLO terms reduces the sensitivity to scale variations.

Finally we show, in fig. 13 and from ref. [36] prediction for J/ψ production by adding the mechanism of fragmentation to the direct one [35] and to the production from B decays as taken from ref. [25], together with the reported theoretical uncertainty. The bands are made by choosing the highest and lowest curve which could be obtained by varying some of the parameters. The total result is obtained by adding togheter the two highest and the two lowest curves respectively. The size of the the fragmentation contribution is seen to be comparable with the previous estimate for the sum of the two mechanisms considered up to now, leading therefore to a sizeable enhancement of the predicted overall production rate, which we also show in the figure. Similar results have been obtained in ref. [37].

When we compare with CDF data points [23] we see that they are now more compatible with the theoretical band. This improves sensitivly the previous situation, where only by making very

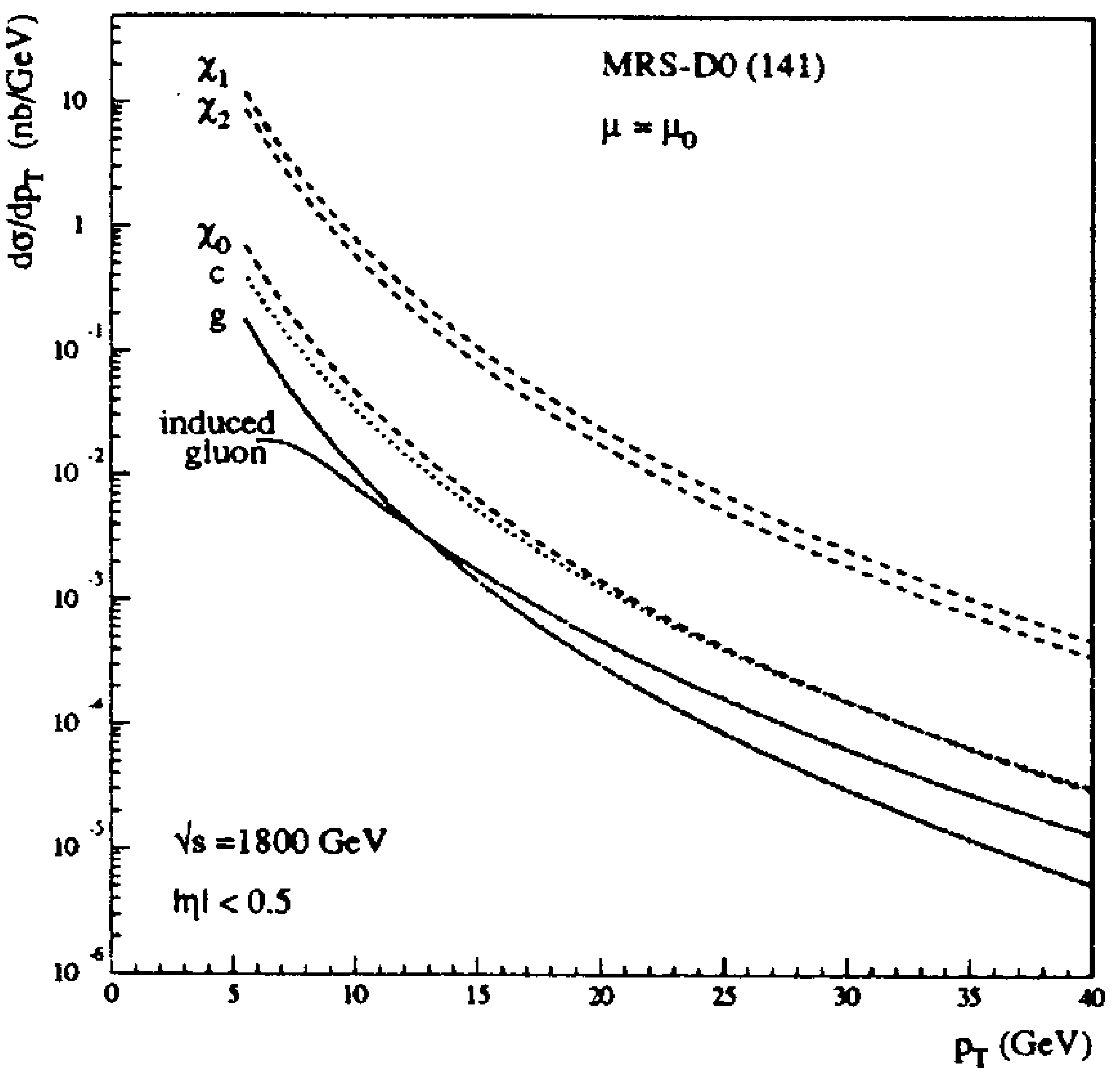

Figure 11: Leading order differential cross sections due to various fragmentation processes: c, g, induced gluon fragmentation to J/ψ and gluon fragmentation to χ followed by radiative decay to J/ψ are shown.

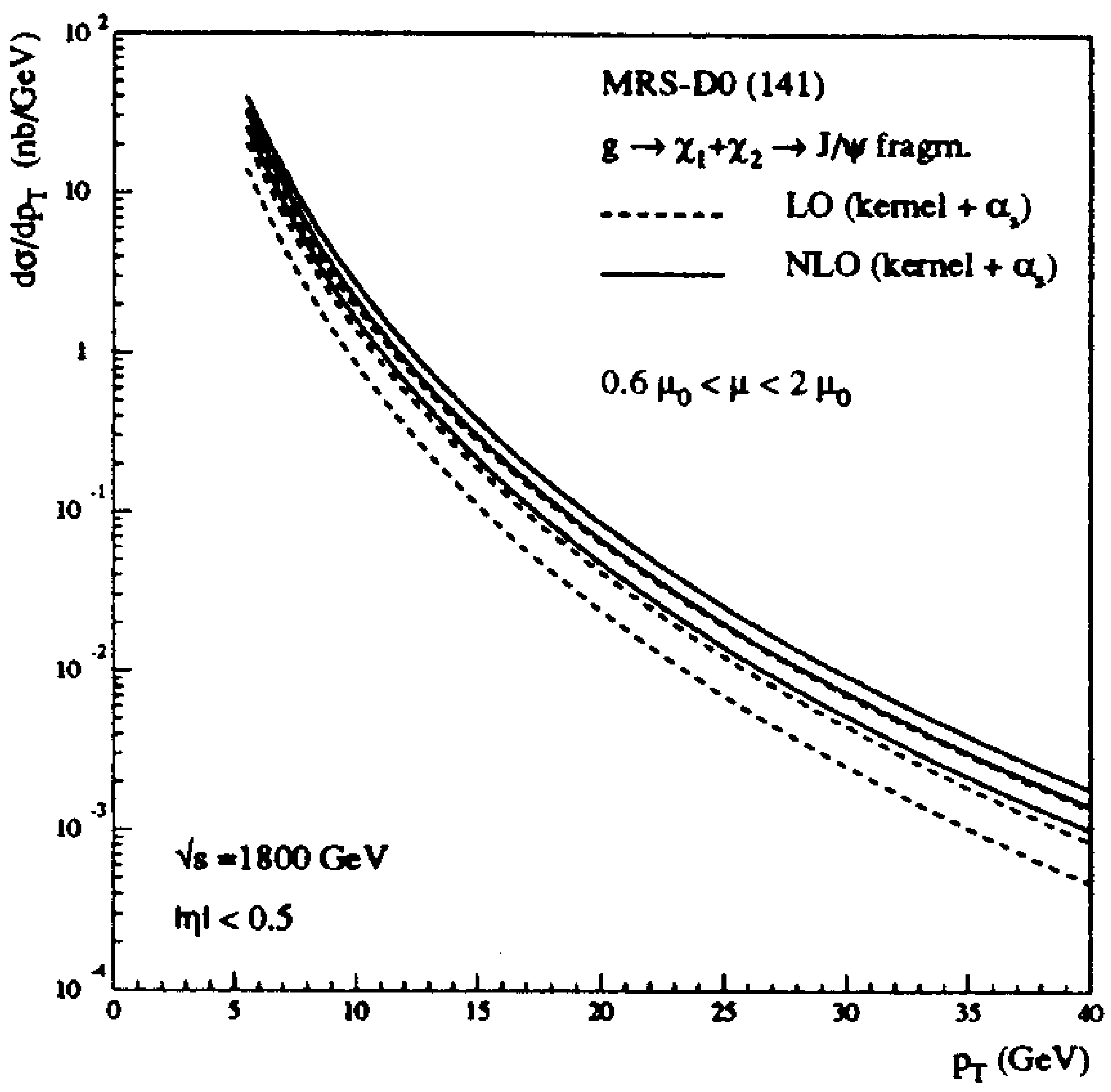

Figure 12: Leading vs. next-to-leading cross section for producing a J/ψ via fragmentation. Only the dominant χ_1 and χ_2 contributions are included.

extreme choices of the parameters one could get close to the experimental findings. In addition the new set of CDF data [38], which selects out via a microvertex detector the fraction of J/ψ coming from B decays, also agrees better with the theoretical predictions after fragmentation has been taken into account, as shown in fig. 14.

Whereas the fragmentation mechanism discussed above seems to give a better description of the J/ψ production, the situation concerning the ψ' on the contrary is still far from our present understanding. Indeed the the production rate of prompt ψ' at the Tevatron is larger than the theoretical prediction at large p_T by about a factor of thirty. This large discrepancy clearly demands for a new mechanism dominating the production process. Indeed very recently new proposals have been put forward as, for example, the existence of higher P-wave or D-wave states which decay into the ψ', similarly to the χ states decaying into the J/ψ [39,40]. Also the existence of new metastable charmonium states or hybrid charmonium states [41], and, more recently, the contribution of colour octet states [42] which subsequently evolve non perturbatevely into a ψ' or J/ψ plus light hadrons, have been advocated as possible candidates for solving this ψ' anomaly. A more accurate analysis of ψ' production data could certainly shade some light on the relative importance ot the above mentioned mechanisms in the production of S-wave states of heavy quarkonia.

To conclude, we have discussed the production of heavy quarks at large p_T ($p_T \gg m$) in hadronic collisions. In order to make the theoretical predictions more reliable, we have studied this problem in the framework of the perturbative fragmentation functions, which allow a NLO evaluation of the potentially large logarithms of the kind $\log(p_T/m)$, which are resummed to all orders. Our analysis for the b and c quarks leads to much more stable results with respect to

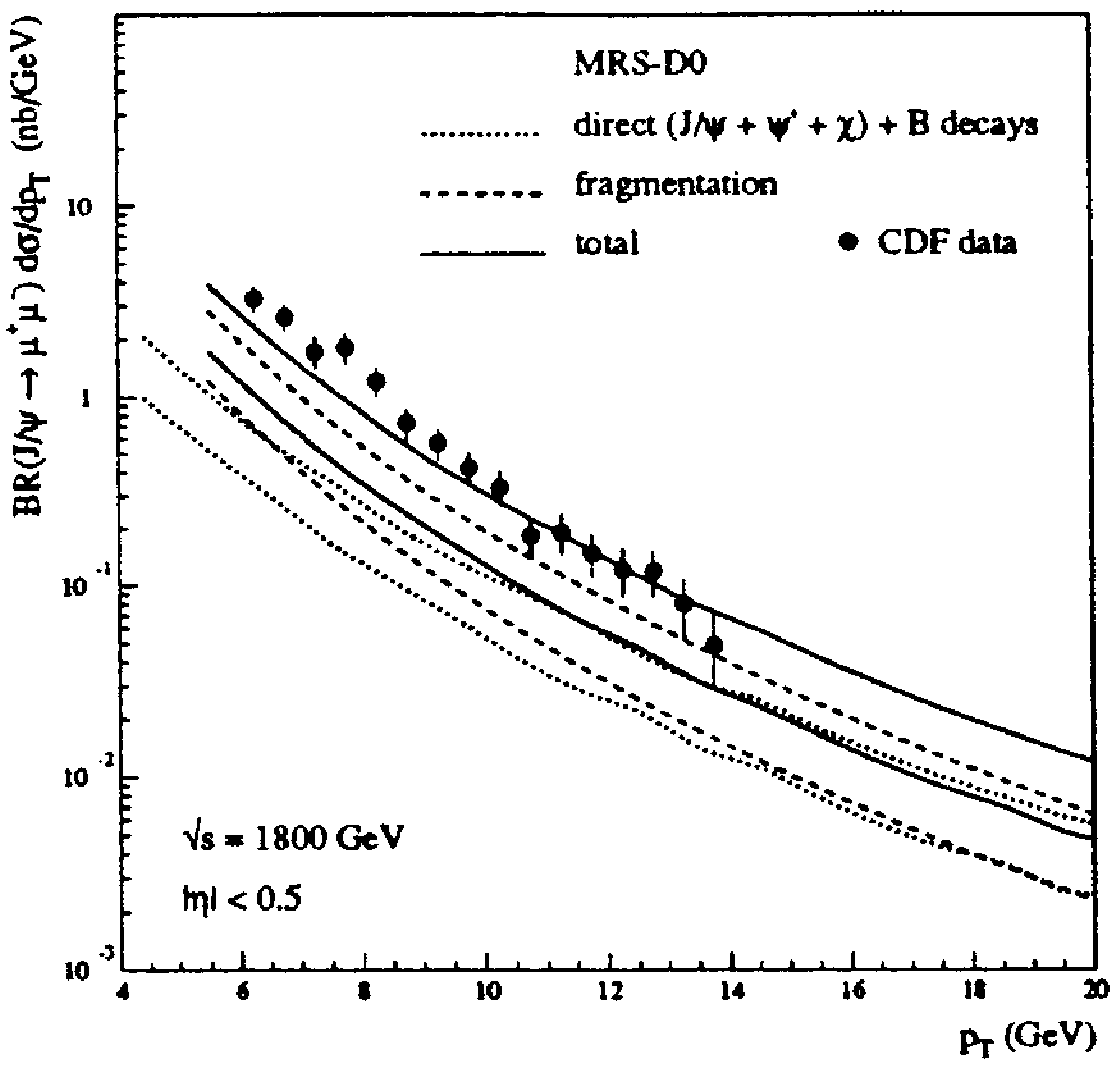

Figure 13: Theoretical prediction for J/ψ production at Tevatron, compared to experimental data from CDF [23]. Both the old result (dotted line) from ref. [25, 35] (upper curve: $\mu = m_T/4$, $\Lambda = 275$ MeV; lower curve $\mu = m_T$, $\Lambda = 215$ MeV) and the new fragmentation (dashed line) contribution (upper curve: $\mu = 0.6\mu_0$, $\mu^g_{ini} = 4$ GeV; lower curve: $\mu = 2\mu_0$, $\mu^g_{ini} = 2$ GeV) are included.

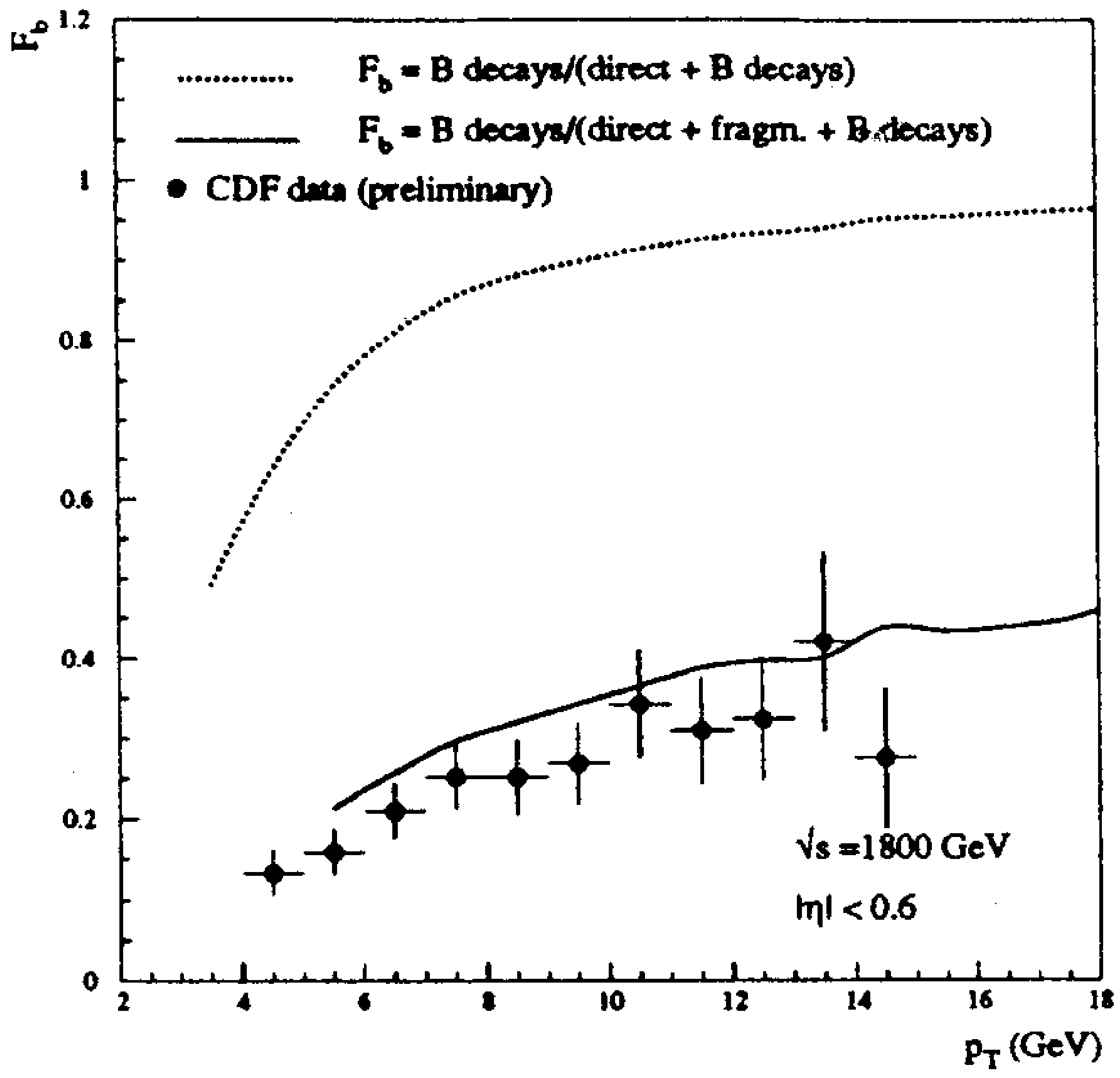

Figure 14: Fractional contribution to the J/ψ production cross section from B decays and prompt sources.

changes of the factorization/renormalization scales compared to what is obtained in the $O(\alpha_s^3)$ calculation. Also the theoretical uncertainty related to different choices of PDF sets is reduced. Other possible sources of uncertainties, like the scale μ_0 of the initial state condition in the fragmentation functions evolutions, are negligible.

We have also considered the inclusive production of J/ψ in hadron collisions in the framework of fragmentation functions. We have shown explicitly that the production and successive radiative decays of the χ states plays a dominant role. The overall theoretical estimate, including the contribution from B decays, is nearly consistent with the experimental observations. The situation for ψ', on the contrary, clearly demands for new mechanisms of production which might explain the large discrepancy with the experimental observation.

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