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Models and Mathematical Tools in Meson Spectroscopy

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Abstract

One of the most important tasks in modern spectroscopy is to identify and characterise different types of non-$(q\bar{q})$ mesons. Since a non-$(q\bar{q})$ state,if it exists,appears in an 'environment of many other 'normal' $(q\bar{q})$ states,and since the actual strategy is to observe similar meson states in a variety of processes (that the new generation of experiment are providing with good accuracy and high statistics) we need to develop methods and models which are able to disentangle from data the exotic information. In this paper we intend to make the point on the present status of modern analysis used specifically in meson spectroscopy from low energy nucleon-antinucleon annihilation putting in evidence the present uncertain points and the used approximations which still need a confirmation.

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1. Introduction

The main information on the meson-meson scattering amplitude has been obtained up to now from the peripheral di-meson production where the dominant contribution comes from one pion exchange.

The methods used for extracting data on meson-meson interaction are based on

- - extrapolating data to the pion pole,
- - amplitude analysis and extrapolation to the pole,
- - fitting different models in the physical region.

Apart from the One-Pion-Exchange mechanism - basic for reactions of the type

- $\pi\pi \rightarrow \pi\pi$ or $(\bar{K}K)$

or strangeness exchange , basic for

- $K\bar{K}\ (\ \pi) \rightarrow K\bar{K}\ (\pi)$

there are also other different mechanisms which contribute to multi-meson system production in reactions as :

- - diffractive reactions (meso/photo production)
- - $e^+\ e^-$ collisions
- - $\gamma\ \gamma$ collision
- - central production ,
- - nucleon antinucleon annihilation

the last one is a very important source of new spectroscopic information, a real, a 'meson-resonance factory'.

With the new generation of $\bar{N}N$ annihilation data, obtained in the last years at LEAR-CERN by experiments like Asterix, Crystal Barrel and Obelix, we have now a large collection of different mesonic final states.

If in 1976 'studies of $\bar{N}N$ annihilation in 3 mesons have proved stimulating but *ambivalent* ' [1] now the 3-meson systems from $\bar{N}N$ ($3\ \pi$, $K\bar{K}\ \pi$, $\eta\pi\pi$, $\eta\eta\pi$ ...) seems to be a very good tool for meson spectroscopy studies where the analysis can be already done in a more sofisticated and complex way, taking the advantage of high statistics and good quality of the data.

2. The general lines of the amplitude analysis

The formalism used to extract meson spectroscopy information from $\bar{N}N$ annihilation is based on

- - isobar model (see for example [2,3]) , and
- - two body dynamics

The initial $\bar{N}N$ state is given by the normal selection rules (as P and G parity, C charge conjugation, etc) and fix the chain of possible J^{PC} of the final meson state.

If $\bar{p}p$ annihilate at rest , due to the initial $\bar{p}p$ atomic state ,it is accepted that mainly initial S and P wave can contribute to the annihilation. Depending on some 'macroscopic' conditions (annihilation in liquid, gas at NTP or at low pressure) [4] the contribution of the initial S and P wave can be modified (Day-Snow-Sucher mechanism [5]).

The basic hypothesis in the isobar model is that the multibody final state can be

- - organized in 'pairs'
- - each pair does not interact with the 'spectator'

In fig.1 there are two examples of 'isobar decomposition' of 3 and 5 mesons in the final state.

A given final state is described by a general function named *probability density function* which is used directly in the fitting procedure:

$$I = \sum_i fr_i |M_i|^2 \tag{1}$$

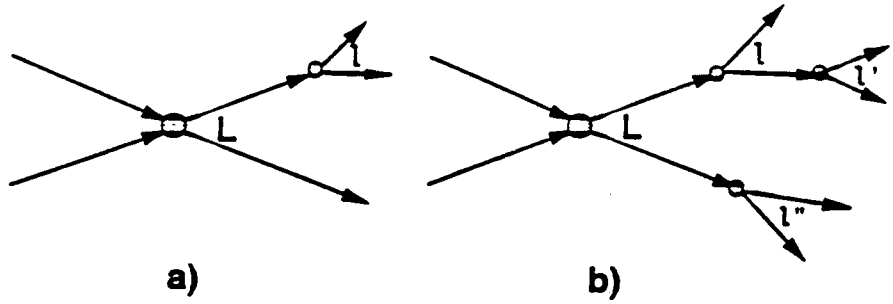

Figure 1: Isobar decomposition for annihilation in 2 (a) and 5 mesons (b)

with normalization over the Lorentz Invariant Phase Space:

$$\int IdLIPS = 1 \tag{2}$$

and where fr_i is the fraction of the initial $\bar{p}p$ (J^{PC}) state and M_i is the transition amplitude.

This probability density function must be corrected for the detector efficiency and resolution via Monte Carlo simulation before fitting the data by :

- max-likelihood method (multivariate fit in the full phase space)
- χ^2 minimization (as in direct Dalitz plot fit)

Each transition amplitude M_i from the general expression of the probability density function (for a given initial J^{PC}) is a sum of different partial waves which depend on some angular momentum from the isobar decomposition (see fig.1) :

$$M_i = \sum_{L,l...} T_{(L,l...)} \tag{3}$$

For each (L,l,...) T amplitude there is a possible sum over amplitudes describing identical particle combinations and entering with a proper isospin Clebsh-Gordan coefficient. The result of such a sum is an amplitude with correct symmetrisation properties:

$$T = \sum C_k A_k^{(L,l..)} \tag{4}$$

Two examples of true identical particles in the final state which introduce combinatorial arrangement (distributions with few entries) are given below

- $\bar{p}p \rightarrow \pi^0\pi^0\pi^0$
- $\bar{p}p \rightarrow K^q K^0 \pi^{-q} \pi^{-q} \pi^q$

The amplitude A_k is a sum of terms which describes the decay chain:

$$A_k^{(L,l..)} = \sum_m D_m^{(L,l..)} \tag{5}$$

where m represents the decay channel, (i.e. E/ι decaying K^*K and/or in $a_0\pi$) . Each partial amplitude $D_m^{(L,l..)}$ is a product of factors :

- the angular part of decay chain (using helicity or Zemach tensors), (Z),
- the energy dependence of the amplitude, (F)

$$D = F * Z \tag{6}$$

The part of the amplitude which describes the energy dependence is in fact also a product of two factors:

- production amplitude (production of the isobar from $p\bar{p}$)
- decay part (energy dependence of the isobar decay)

Usually the production amplitude is taken as a complex constant for a given isobar and is in fact the parameter in the fit giving also the relative branching ratio for isobar production. The decay part is also a product of factors which describe the two body decays involved at the different levels of the isobar decay chain.

$$F = \alpha e^{i\varphi} * f_1 * f_2... \tag{7}$$

For example for $p\bar{p} \rightarrow (\pi\pi\pi)$there is only one 'isobar' and the amplitude for a given l is given by:

$$F = \alpha e^{i\varphi} * f(\pi\pi) \tag{8}$$

For $p\bar{p} \rightarrow K^0K(\pi\pi\pi)$there are two levels (see fig.1(b)) and F is given by:

$$F = \alpha e^{i\varphi} * f(E/\iota \rightarrow a_0\pi) * f(a_0 \rightarrow K\bar{K}) \tag{9}$$

and/or (dependig on the decay mode) by:

$$F = \alpha e^{i\varphi} * f(E/\iota \rightarrow K^*\bar{K} + cc) * f(K^* \rightarrow \bar{K}\pi) \tag{10}$$

The two body scattering amplitude is modified in order to take into account:

- the correct threshold behaviour of the total amplitude (so called centrifugal barrier factors) ($\sim P^L q^l$), where P and q are the isobar CMS momenta.
- the decay character of this part of the amplitude which means to take only 'half' of the two body scattering amplitude. In the usual analyses this last requirement is fulfilled by using the Breit-Wigner propagator or 'half' of two body amplitude [6,7].

3. Unitarity in P and Q formalisms

In the above formalism the amplitude which describes the production of the final multi meson system does not fulfill any constraint imposed by the unitarity. A method in which the unitarity can be imposed is based on the generalized Watson theorem given by Aitchison [8]. If in the K matrix approach the two body amplitudes are given by

$$T = (I - iK\rho)^{-1}K \tag{11}$$

where ρ is the channel momenta or the two body phase space at a power depending on the orbital angular momentum [9,10,11] (which for our discussion is not important to be specified) then the production amplitudes are given (F from eq.6) by:

$$F = (I - iK\rho)^{-1}P \tag{12}$$

where $(I - iK\rho)^{-1}$describes the propagation of two-body intermediate states, while the P term describes the formation and decay of these states. It is shown that P vector has the same poles as K matrix [8]. From eq.(11) the production amplitude can be written in the so called Q formalism as:

$$F = T * Q \tag{13}$$

where $Q = K^{-1}P$.

If we consider the simple case of one resonance in one channel, the K matrix element is given by

$$K = \alpha^2/(s_0 - s) \tag{14}$$

which results in the familiar Breit-Wigner form (neglecting the mass-dependence of the width). The P matrix element, is given by:

$$P = \gamma\alpha/(s_0 - s) \tag{15}$$

and the production amplitude by:

$$F = \gamma\alpha/(s_0 - s - i\alpha^2) \tag{16}$$

The unitarity specifies the form of the imaginary part of the amplitude as a sum over all available intermediate states ; which in this simple case of two body channel is :

$$ImT = |T|^2 \tag{17}$$

and for the production amplitude is :

$$ImF = F * T \tag{18}$$

If we accept this generalisation of the unitarity [12,13], the constant γ , which represents the 'production strength', is a real constant or a smooth real function of energy. The unitarity relations for production in the case of two channels problem :

$$ImF_1 = \rho_1 F_1 T_{11} + \rho_2 F_2 T_{21} \tag{19}$$

$$ImF_2 = \rho_1 F_1 T_{12} + \rho_2 F_2 T_{22} \tag{20}$$

are satisfied by:

$$F_1 = a_1(s)T_{11} + a_2(s)T_{21} \tag{21}$$

$$F_2 = a_1(s)T_{21} + a_2(s)T_{22} \tag{22}$$

where $a_1(s)$ and $a_2(s)$ are two real functions of energy and are in fact the elements of the Q vector.

In the case of one resonance decaying in two channels we can arrive simply to the so called Flatte [14] formula (created for $a_0(980)$), which from time to time become popular. The K matrix elements are : $K_{11} = \alpha^2/(s_0 - s)$ $K_{22} = \beta^2/(s_0 - s)$ $K_{12} = \alpha\beta/(s_0 - s)$ and the P vector elements are $P(1) = \gamma\alpha/(s_0 - s)$ $P(2) = \gamma\beta/(s_0 - s)$ where α and β are the coupling-channel constants. The two-body scattering and production amplitudes 'a la Flattè' are

$$T_{11} = \alpha^2/\Delta \tag{23}$$

where $\Delta = (s - s_0 - i(\rho_1\alpha^2 + \rho_2\beta^2))$; in the Q formalism:

$$F_1 = (\alpha^2 Q_1 + \alpha\beta Q_2)/\Delta \tag{24}$$

while in the P formalism:

$$F_1 = \gamma\alpha/\Delta \tag{25}$$

The relation between the P and Q vectors elements in this case is given by

$$\gamma = \alpha Q_1 + \beta Q_2 \tag{26}$$

For two poles in one channel the result is similar:

$$F = \gamma(s)(bw_1 + bw_2) \tag{27}$$

where bw_i are the elementary Breit-Wigner amplitudes.

An important remark here is that in the above formulas for the production amplitudes there are no 'supplementary' relative phase between different two-body amplitudes. In other words, the two-body scattering amplitude, which appears in any part of the total amplitude in the isobar decomposition, is 'modified' by the production process only by a multiplicative real function .

Strictly speaking, the unitarity relations for production amplitudes, derived in the framework of generalized Watson theorem given by Aitchison, are applied to non hadronically production processes, as in $\gamma\gamma \to \pi\pi/K\bar{K}$; however they probably could be extended in the isobar model to the multimesons production, when the 'isobar' does not interact with the 'spectator'[13] , as for example in J/Ψ decay in $\phi(\pi\pi)/(K\bar{K})$

The open problem for us is how far we can extend to use the *production unitarity relation for processes as nucleon antinucleon annihilation* .

4. N/D method in $N\bar{N}$ anihilation

Motivated by the general considerations on rescattering diagrams [15], the production amplitude in N/D method [11] applicated to annihilation processes is written in a general way as :

$$F = N(s)/D(s) \tag{28}$$

where D(s) has only the right-hand singularities of the two-body subprocesses ($a+b \to a+b$ from $N\bar{N} \to a+b+c$ for example) and therefore F has the whole set of poles corresponding to all the resonances which can be formed in this subprocess. The N(s) function, which contains left-hand singularities related to any exchange diagram, it is of course, not calculated and therefore parametrized as a complex function.

Depending on the formalism from which the information on the two-body amplitude has been obtained (simple Breit-Wigner as in $(\pi\pi)_{(l=1)}$ in the ρ region, or many channels K-matrix as in $(\pi\pi)_{(l=0)}$ up to 1.2 GeV/c^2), the D(s) function can be fixed, but N(s) remains somewhat arbitrary. The analysis of the reaction $\bar{p}p \to \pi^0\pi^0\pi^0$ with this approach [15,16] has been done by using different forms for N(s) and *empirically has been found* that a good χ^2 and a more rapid convergence is achieved by

$$N(s) = \lambda_1(s)K_{11} + i\rho_2\lambda_2(s)\Delta \tag{29}$$

where $\Delta = K_{11}K_{22} - K_{12}$ and λ_1 and λ_2 are complex linear functions of dipion mass.

Summarising, the production amplitude in N/D approach
- is quite general,
- can be a convenient parametrization,
- is flexible enough to account for the energy dependence of many hadron production, but in the same time there are:
- no transparent meaning of the parameters from N(s) function, and
- no constraints due to unitarity (see P matrix approach).

Moreover, due to the factorization of the production amplitude in two complex functions, it is possible, in some cases, (the annihilation in $3\pi^0$ from $p\bar{p}$ in 0^{-+} initial state for example) we have ambiguities just as in the case of the classical Barrelet ambiguity problem for two-body scattering amplitude[17,18,10].

The production amplitude which fit the Dalitz plot can be written in this case as a ratio of two complex functions represented by their complex zeros:

$$F = \prod_k Z_k / \prod_i P_i \tag{30}$$

with

$$Z_k = (s_{12} - sz_k) * (s_{23} - sz_k) * (s_{31} - sz_k) \tag{31}$$

$$P_i = (s_{12} - sp_i) * (s_{23} - sp_i) * (s_{31} - sp_i) \tag{32}$$

due to symmetry, where sp_k are the complex positions of the resonances formed in the two-body subprocess: $\pi^0\pi^0 \rightarrow \pi^0\pi^0$; sz_i are the complex zeros in s plane 'created' by the general complex function N(s). The experimental data are reproduced by the $|F|^2$, the fit will be equally good with very different forms for N(s) when one or more zeros sz_k are changed to their complex conjugated values. We have done such a test for $p\bar{p} \rightarrow 3\pi^0$ in liquid. Data obtained by Monte Carlo simulation with the formulas given in the original paper [16] have been refitted with a production amplitude as above. The whole Dalitz plot is well fitted by 6 complex zeros and 6 complex poles which can now 'produce' many different amplitudes.

Besides this *mathematical ambiguity* , there is also a physical ambiguity when a classical type of analysis is done (as in ref [19]). This second kind of ambiguity concerns the role of *the direct many body annihilation (or resonance decay)* which in our example of $3\pi^0$ produced from 0^{-+} $p\bar{p}$ have the same 'angular isotropy' as $(\pi\pi)_s\pi$ isobar decomposition. We have fitted the same $3\pi^0$ simulated data in the classical way using for $(\pi\pi)_s$ the AMP parametrization [12] and also using a simple constant. The main features of the data are well reproduced by both fits (of course not the small peak at 1. GeV/c^2 when $(\pi\pi)_s$ is constant), and, as expected, the annihilation is dominated by the 0^{-+} initial state. But the second resonance from $\pi\pi$ in S wave ($(\pi\pi)_s$) claimed in ref. [16] (M=1.367 GeV/c^2 and $\Gamma = 0.268 GeV/c^2$) change its relative contribution from 10 % (AMP parametrization) to 30 %, while the last 0^{++} (AX) remains 'insensitive' to changements in the energy behaviour of $\pi\pi$ S wave.

Therefore, depending on the formalism used to 'accomodate' the experimental production data, it is well possible to obtain different results concerning the observed structure . At least for $(\pi\pi)$ in S wave serious ambiguities in disentangling true physical states from the 'mathematical' accidents still remain.

5. Relative phases

From general Quantum Mechanics considerations each amplitude in a sum of different amplitudes that can interfere, has a relative phase. For example, for a given initial J^{PC} state of $p\bar{p}$ annihilating at rest, the total amplitude has the form:

$$M_{JPC} = \sum_k e^{i\varphi_k} F_k Z_k \tag{33}$$

where Z_k is the angular component (Zemach tensors etc...) and F_k is a product of quasi-two-body amplitudes, one for each permitted angular momentum and with its own dependence on the quasi-two-body total energy. If the energy (and of course the physics) permits to have, in the region of interest, more than one resonance, the classical way of analysis is to introduce different amplitudes for all resonances, each one having its own relative phase (actually unknown complex constants which must be fitted).

In the N/D method, because N(s) is a rather arbitrary complex function, the relative phase is also an arbitrary function of energy. When single channel exists, the N/D and classical methods are equivalent. Both introduce relative phases between different resonances which are present in the same amplitude of a given angular momentum.

If the unitarity (via P or Q matrix approach) is imposed, the relative phase between different resonances are fixed to be the same as in two-body scattering and the only phase which exists is a common one, that of the total amplitude which describes the whole chain of the quasi-two-body decomposition:

$$e^{i\varphi_k} F_k = e^{i\varphi_k} * (T_l^{11} Q_1 + T_l^{12} Q_2 + T_l^{13} Q_3 + ...) \tag{34}$$

where T_l^{ij} is two-body scattering amplitude of the channel $i \rightarrow j$ for a given l and Q_i are real functions of energy.

But in some cases 'adding' a relative phase φ_k in eq.(33) seems to be unnecessary. When in a given final state isobars can be represented by very narrow states their decay length is greater than the effective annihilation range [20]. In this case there is no interference, and in principle, the relative phase is therefore unnecessary .

An example can be the reaction $\bar{p}p \rightarrow \pi^0 K_s K_L$ from annihilation in liquid at rest (only 1^{--} initial state permitted), described in terms of 3 isobars : $\rightarrow \phi\pi^0$ $\rightarrow K^*\bar{K}$ and $\rightarrow \bar{K}^*K$. The intensity over Dalitz plot has been written as [21]:

$$I = |\alpha_1 A(\phi\pi^0) + \alpha_2 e^{i\varphi}[A(K_L^* K_s) - A(K_s^* K_L)]|^2 \tag{35}$$

where A contains, as usual, the Zemach tensors to describe the angular distribution and Breit-Wigner terms for ϕ and K^* with normal $\Gamma(m)$, correct threshold behaviour and Blatt-Weisskoph damping factors. Since ϕ is a very narrow state, more probable physical expression of the intensity over the Dalitz plot might be:

$$I = |\alpha_1 A(\phi\pi^0)|^2 + |[A(K_L^* K_s) - A(K_s^* K_L)]|^2 \tag{36}$$

When a new resonance is suspected to exists, the common practice is to take it into account by adding to the old production amplitude a new Breit-Wigner term weighed by a complex constant :

$$F_{(L,l)}(new) = F_{(L,l)}(old) + \alpha e^{i\varphi} BW_l(new) \tag{37}$$

where $F_{(L,l)}(old)$ incorporates as usual information on two-body scattering amplitude obtained from other experiments. Adding the new state as a Breit-Wigner multiplicated by a complex constant, independent on the method used for production amplitude (P, Q or N/D) is in fact a mixture of all. We lose in this way the unitarity description of the two-body scattering, having only (anyway important) information on the existence of a new structure at $s = M^2$.

Following the ideas from the N/D method and supposing that the unitarity relations derived from P (Q) approach are really valid in nucleon-antinucleon annihilation, we can only conclude that if the complex constant $\alpha e^{i\varphi}$ is far to be a real constant, the BW(new) is, probably, 'produced' by a strong rescattering effect, or there is a new inelastic channel in this region which was not properly taken into account by $F_{(L,l)}(old)$ and the new BW can well simulate a 'cusp' [11] due to this new channel. Therefore a simple method can be imaginated :

- the new suspected resonance must be taken into account in the total two-body scattering amplitude using the same formalism (K matrix).

-the production amplitude for a given angular momentum l must contain the Q (or P) matrix elements as real functions on the isobar total energy, and only

- in the final part of the fitting procedure we can 'relax' the coefficients of Q (or P) to be complex and see the quantity :

$$R = T_l^{11} ImQ_1 + T_l^{21} ImQ_2 + T_3^{31} Q_3 + ... \tag{38}$$

which will be an approximation of the rescattering effect as in the N/D approach. An example of P matrix method with complex coefficients can be found in ref. [22,23].

But in order to go more deeply in understanding the structure of the amplitudes we must pass to the simultaneous analysis of many different annihilation channels.

6. More complex analyses

The simplest simultaneous fit of different $p\bar{p}$ data is the one of the same final state obtained in different initial conditions. Since the annihilation at rest in liquid is dominated by the initial S wave , in gas at NTP by about equal fractions of S and P waves, and in gas at very low preasure the annihilation in P wave dominates [4], an analysis of the same final state coming from different initial conditions will permit to separate the population probability of the initial $p\bar{p}$ atomic state from the dynamical branching ratios, which are specific to the given final state and its decomposition in different 'isobars'.

The next step in complexity is the isospin-coupling analysis, which means a simultaneous fit of annihilation data containing in the final states particles belonging

to the same isomultiplet, as for :

- $\bar{p}p \rightarrow \pi^0\pi^0\pi^0$
- $\bar{p}p \rightarrow \pi^+\pi^-\pi^0$
- $\bar{p}n \rightarrow \pi^-\pi^0\pi^0$

which introduces useful constraints between amplitudes due to the isospin weights but also, very important, due to different interference effects. For example, if we compare the $\pi^0\pi^0$ invariant mass distribution from the first annihilation channel, with the $\pi^+\pi^-$ spectrum from the second reaction will be easily to observe that the $f_2(1270)$ signal has a peak at $1.225\ GeV/c^2$ in $\pi^+\pi^-$ and at $1.304\ GeV/c^2$ in $\pi^0\pi^0$, and nobody claimed the existence of two different resonances (Fig.2). Using the simulated data we have tried to fit both final states simultaneously. A relatively good fit of both Dalitz plots is obtained only if the dipion from S wave is taken from the AMP parametrization. Both peaks due to $f_2(1270)$ are, of course, reproduced by using the same Breit-Wigner amplitude for it. But if $(\pi\pi)_s$ amplitude is taken from N/D fit of $3\pi^0$ final state, the second annihilation channel give a very bad fit. From this exercise we can conclude that:

- there is a strong and channel dependent (may be due to the presence of ρ in $\pi^+\pi^-$) rescattering effect , or

- there is an ambiguity (as discussed in sec.4) for N(s) from the N/D expression of the production amplitude.

Another type of simultaneous fit of different data is represented by the coupling channel analysis of 'coupled' production processes. Indeed this is not a fully coupling channel analysis, because only few partial wave are, as usually, coupled and therefore in common. An example is given by the reactions:

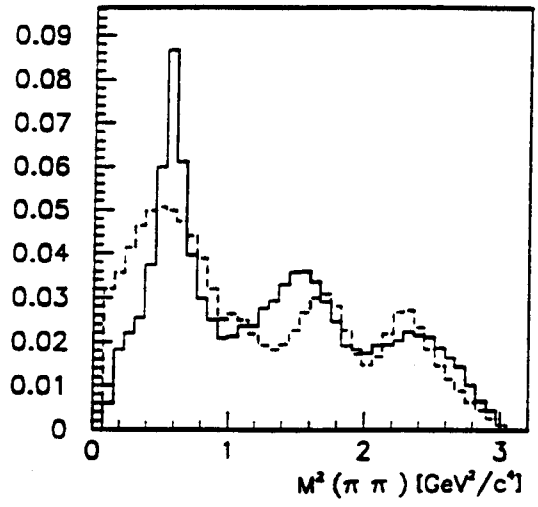

Figure 2: $\pi^+\pi^-$ (solid line) and $\pi^0\pi^0$ (broken line) invariant mass squared distribution from $\bar{p}p$ annihilation at rest in $\pi^+\pi^-\pi^0$ and $\pi^0\pi^0\pi^0$

- $\bar{p}p \rightarrow \pi^0\pi^0\pi^0/\pi^+\pi^-\pi^0$
- $\bar{p}p \rightarrow \pi^0\eta\eta$
- $\bar{p}p \rightarrow \pi K\bar{K}$

In such Production-Coupling-Channel (PCC) analysis the production amplitude, for example for two channels, is given in Q formalism by eq. (21) and (22). In N/D approach only the complex function N(s) must be different, due to possible different rescattering effects:

$$F_1 = N_1(s)/D(s) \tag{39}$$

$$F_2 = N_2(s)/D(s) \tag{40}$$

A PCC analysis of $3\pi^0$ and $\pi^0\eta\eta$ final states has been done by the Crystal Barrel collaboration [16] using N/D method. The interesting result which emerges from this analysis concerns the $AX(0^{++}, 1520)$ which seems to be coupled to $\pi\pi$ as well as to $\eta\eta$. This result does not contradict the previous 'classical' analysis of the $\pi^0\eta\eta$ channel [24] (here the 0^{++} has larger mass and larger width), and also is on the same line with the observation that a $f_0(1500)$ has a rather considerable coupling to $\eta\eta$ when three coupling channels are used to analyse the $\pi\pi$ data [25].

In a more complex analysis of the $(\pi\pi)$ in S wave will be necessary to take into account also the $\rho\rho$ channel, which incidentaly, has its nominal threshold just around the mass of $AX(0^{++})$. Is this resonance an 'unstable bound state' (see next section) of the $\rho\rho$ system ? The uncertain point in this type of the coupling channel analysis seems to be the way in which we must correctly take into account the *threshold created by the unstable particles (resonances)* .

A similar question arise in the analysis of the $(K\bar{K}\pi)$ system for the mass region 1.4 - 1.5 GeV/c^2. This is the E/ι region where in different production processes have been observed one or more overlapping 0^{-+} and/or 1^{++} states decaying in $(a_0\pi)$ and/or in $(K^*\bar{K} + cc)$. A classical type of analysis of this mass region in $p\bar{p} \rightarrow KK_{miss}3\pi$ has been presented at this conference [26]. Besides the problem of 'unstable threshold' created by the channel $E/\iota \rightarrow (K^*\bar{K} + cc)$ there is also the one, mentioned before (sect.4), of the possible direct E/ι decay in $(K\bar{K}\pi)$.

7. Model-independent anlayses

A model-independent way to characterize the resonance phenomena seems to be the position of the *pole* of the partial wave in the complex energy plane. This is an objective test which can be decisive to identify and characterize different types of possible non-$(q\bar{q})$ mesons.

The unstable particles correspond to poles of the S-matrix in the complex energy plane below the real axis at the same position in all the processes to which this resonance couples. Due to the existence of different channel thresholds, the complex energy plane is in fact a multi sheet complex plane and the resonance poles can be located on the different Riemann sheets, a fact which is crucial for dynamics of the resonance.

A classical example concerns the problem of 'elementarity' of the deuteron or of the $\Lambda(1405)$. In our field of meson spectroscopy, the 'molecule or elementary $q\bar{q}$ state' has been long discussed for f_0 and a_0, but probably, can be also very interesting to address the same question for Ax(1520-1550) or $E/\iota(1410-1450)$ structures.

Describing the amplitude pole topology we distinguish the following cases:

- one channel reactions,
- one inelastic channel ,
- two or more coupled channels.

Using the Jost functions for representing the amplitude as a real-analitic function of energy it is well known that :

- a bound state (virtual state) create a pole in momentum plane situated on the positive (negative) imaginary axis (therefore below the threshold), and
- a resonance creates two poles at $k_1 = k_R + ik_I$ and $k_2 = -k_R - ik_I$.

Due to the relation between momentum and energy, in the energy plane the resonance poles are on the same place but on different Riemann sheets.

The S-matrix has also zeros in the complex momentum plane located at the complex conjugated position of the poles. In the case of one inelastic channel the zeros of the S-matrix will be 'displaced' from pole complex-conjugated position, the 'distance' between them beeing proportional to the elasticity in that partial wave. This fact is very useful in describing 'strong inelastic' resonances.

Since in practice, only one resonance pole is nearby (near the region where are the experimental data), we speak about resonance as represeted by one pole on the second Riemann sheet (and one zero of the S-matrix on the first sheet).

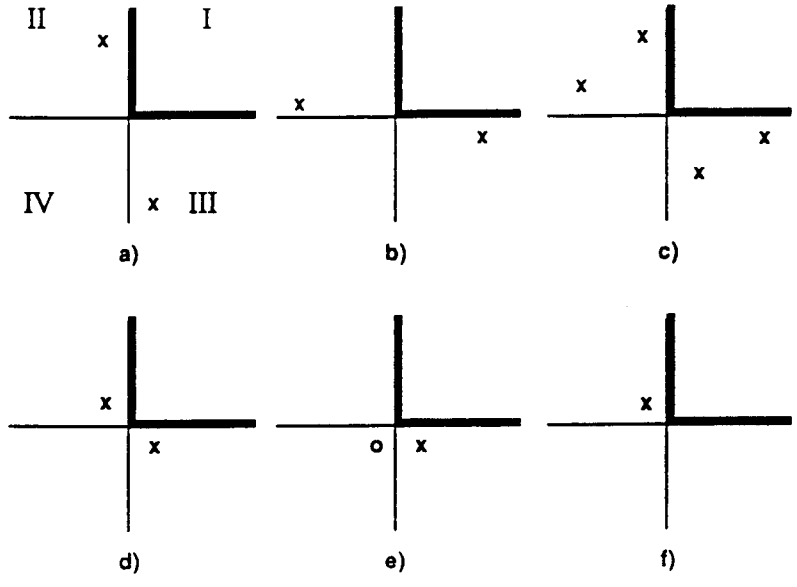

Figure 3: Pole topology in complex k_2 plane. Nearby poles for one resonance at $E < E_2$ (a) , $E > E_2$ (b) and two resonances (c). In (d) and (e) are shown one resonance near the threshold and in (f) a 'molecular' state. By x are denoted poles in the lower energy halfplane and by o in the upper one.

The nonrelativistic version of the Breit-Wigner denominator in the case of two coupling channels approach (we can say 'a la Flattè') and which creates the poles:

$$E - E_0 + i\gamma_1 k_1 + i\gamma_2 k_2 = 0 \tag{41}$$

can be reduced to a polynomial of degree four in momentum and therefore in this case the number of energy sheets and poles increases to four: two different poles in the lower energy halfplane ($ImE < 0$) and two at the complex conjugated positions. Following the standard notation for the energy sheets given by the signs of the imaginary parts of the channel momenta (Imk_1, Imk_2) , (sheet I : ++, sheet II: −+, III: −−, IV: +−) , it is possible to show that one pole (and its complex conjugated partner) is on the sheet III and the other one on sheets II or IV depending on the channel coupling strength : $\gamma_1 k_1 > (or <) \gamma_2 k_2$. The nearest regions to the physical one (where the experimental data lies) are sheet II ($ImE < 0$) for $E < E_2$ and sheet III ($ImE < 0$) for $E > E_2$ (here E_2 is the energy threshold of the second channel) and, as in the one channel case, we speak here about one resonance as represeted by two poles. Only in the vicinity of the threshold, sheet IV (but $ImE > 0$) can be also near the physical region (two times cross of the energy axis), in which case a complex conjugated pole can be obseved if $\gamma_1 k_1 < \gamma_2 k_2$.

Using the Jost functions representation and the unitarity, the S-matrix elements when $k_1 > 0$ may be treated as functions of momentum of the second channel k_2

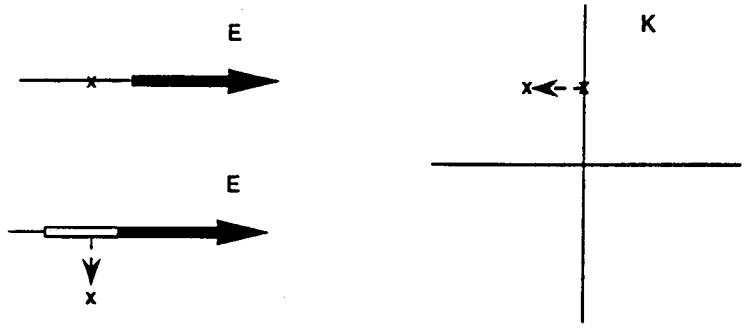

Figure 4: Origin of 'molecular' pole in complex k_2 plane.

[13] and the pole topology in two channels can be simply vizualized in k_2 plane as in Fig.3.

Far from the threshold one resonance has only one important pole (on sheet II or III) but near the threshold two poles must exist. When only one pole is seen near the threshold it will be a signal for the existence of a 'composite' system (molecular or unstable bound state). This 'molecular' picture has its origin in the fact that a bound state in a single channel situation is shifted from the imaginary axis to the left when a new channel (with lower threshold) is gradually open (see Fig.4).

Because the number and the nature (sheet location) of the resonances are determined by the poles of the S-matrix, we need, in this type of analysis, to know not only the modulus of the amplitude, but also its phase and, therefore, in the context of many interfering amplitudes, the constraints imposed by the general principles, as

analyticity and unitarity of the amplitudes which describe the data, become of a very great utility.

In order to find the number and location of the poles, a parametrization of the S-matrix provided by the Jost function is mostly used. The unitarity specifies the form of the S-matrix as a function of the appropriate channel momenta. The sheet location is easily specified (see for example [13]) by characterizing the resonance pole by its channel-momenta values or using a new uniformizing variable which maps all four Riemann sheets of the energy (as in the case of two channels problem) into this new complex variable (see for example [11,25]).

A rapid and stable method to find zeros and poles of the S-matrix (or T-matrix) is based on the use of the complex Pade approximants of the second type (PA II) and its application in χ^2 minimizations (PA III).

Given the complex function F(z) on the complex variable $z = x + iy$ and using its N values on the real axis $F(x_k)$, $k = 1 \ldots N$,there is an algorithm [27] which creates an approximant of F as a ratio of two complex polynomials of degree n and m :

$$F(z) \approx P_N^n(z, F_k(x_k))/Q_N^m(z, F_k(x_k)) \tag{42}$$

Moreover, the values of $F_k(x_k)$ can be used as free parameters in a fitting procedure which converges very rapidly and the zeros and poles of the approximant can be easily found using standard methods. This procedure may serve as a guide to distinguish the 'stable poles' (physical ones, connected with resonances) from the 'statistical poles' (produced by the experimental 'noise') by increasing polynomial degrees n and m.

Doing the *pole analysis* of the production amplitude in a coupling channel formalism we can hope to avoid the risk of confusing resonances with some other mechanisms or interference effects and even more, to obtain some hints concerning dynamical origin of them. Therefore the next generation of production amplitude analyses must probably do this.

8. Conclusions

The models and formalisms used presently for analysing the production processes are still 'unstable' and some urgent problems which must be workedout in the near future exist:

- a better understanding of the unitarity in many particles production reactions,
- quantitative calculations concerning rescattering,
- the role of the direct decay in more than two particles,
- multichannel formalism involving unstable (resonances) particles.

From the experimental point of view, now there is a variety of good and high statistic data in nucleon-antinucleon annihilation which can be analysed by 'even still not perfect' but more complicated formalisms like :

- coupling channel analysis,
- amplitude pole analysis , using together,
- different annihilation reactions.

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