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Photon-Photon Scattering to two loops in ChPT[#]

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Abstract

We analyze the Chiral Perturbation Theory predictions for $\gamma\gamma \rightarrow \pi^0\pi^0$ to two loops. The amplitude to this order depends on three new counterterms which can be estimated using resonance exchange. The low-energy cross section is in good agreement with the present data and with calculations based on dispersion relations. We predict the Compton cross section and the neutral pion polarizabilities to the two-loop order. The unitarization of the one-loop cross section is also discussed.

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[#] Or: "We should never underestimate the pleasure we feel when we listen to something we already know" (E. Fermi). Work supported in part by the EEC Human Capital and Mobility Program. Presented at the 28th International Symposium on the Theory of Elementary Particles, Wendisch-Rietz, Deutschland, August 30 – September 3, 1994.

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1 Introduction

The tree-level amplitude for $\gamma\gamma \rightarrow \pi^0\pi^0$ in Chiral Perturbation Theory (CHPT) low-energy expansion [1]-[5] starts only with the $O(E^6)$ contribution [6, 7]. Hence the cross section predicted to the one-loop order is independent of the free parameters of the chiral lagrangian. It should not come as a big surprise that this prediction does not agree with the measurements at Crystal Ball [8]. In fact, it is necessary to include in any CHPT prediction the first correction to the lowest nonvanishing order, in order to compare successfully with the experiments. As we show below, this case is no exception.

Indeed, CHPT yields an expansion of the matrix elements in powers of the momentum squared and the quark mass. One needs the next order, to know how good is the leading-order prediction. For instance, for $\gamma\gamma \rightarrow \pi^+\pi^-$, whose amplitude starts at order $O(E^2)$, one needs to calculate also $\pi\pi$ rescattering and resonance exchange, which are contributions of order $O(E^4)$. Several transitions, such as $K_s \rightarrow \gamma\gamma$, $\gamma\gamma \rightarrow \pi^0\pi^0$, $K_L \rightarrow \pi^0\gamma\gamma$, as well as physical quantities such as the π , K , η polarizabilities, begin to order $O(E^4)$. Therefore, to know how good is the lowest order cross-section for $\gamma\gamma \rightarrow \pi^0\pi^0$, one needs to carry out a two-loop calculation of the amplitude.

The $\gamma\gamma \rightarrow \pi^+\pi^-$ cross section has been calculated to the next-to-leading order in CHPT [6] and the result agrees with the low-energy experimental data from Mark II [9]. The one-loop prediction for $\gamma\gamma \rightarrow \pi^0\pi^0$ in the low-energy region disagrees also with calculations based on dispersion relations [10]-[17]. The two-loop amplitude calculated recently in the framework of CHPT [18] agrees at low energy with the Crystal Ball data and with the dispersive analysis of $\gamma\gamma \rightarrow \pi^0\pi^0$.

The electromagnetic pion polarizabilities are among the low-energy parameters describing the inner structure of the pion composite system [19]. The lowest order prediction of CHPT for the sum of the π^0 electric and magnetic polarizabilities is zero [20]. A sum rule estimate shows that this relation is violated, i.e. the result is nonvanishing [19]. A vector dominance model preserving the low-energy chiral symmetry of QCD yields a result compatible with the sum rule [21].

Analyticity and crossing relate the Compton scattering on neutral pions with the $\gamma\gamma \rightarrow \pi^0\pi^0$ amplitude. Hence, knowing the two-loop amplitude allows us to evaluate the polarizabilities at next-to-leading order in the quark mass expansion. The omega resonance exchange, which accounts for a substantial contribution to

the π^0 sum rule [21], yields the largest correction to the polarizabilities, whereas the modification due to the chiral logarithms is small [18].

In a generalized approach (GCHPT) [22] CHPT has been reformulated, in order to include into each order of the effective lagrangian additional terms, which in the standard expansion are considered as higher order. The GCHPT approach is described in [23]. Within the framework of GCHPT the cross section for $\gamma\gamma \rightarrow \pi^0\pi^0$ has been calculated up to $O(E^5)$ in Ref. [25].

This work is organized as follows. In section 2, we outline the procedure for the two-loop calculation. In section 3 various determinations of the low-energy constants which occur in the amplitude for $\gamma\gamma \rightarrow \pi^0\pi^0$ to two-loop order are discussed. In section 4 we touch briefly upon the pion polarizabilities and the possibility of matching the chiral expansion with the dispersive calculation by Donoghue and Holstein [14]. Finally, we offer some concluding remarks in section 5.

2 The two-loop calculation

The two-loop calculation of [18] entails the following steps:

1. Drawing of many diagrams.
2. Integrating over the internal momenta in d -dimensions and using Feynman parameters.
3. Checking Ward identities in $d \neq 4$ in the unphysical region.
4. Adding the low-energy constants, after letting $d \rightarrow 4$.
5. Continuation to the unphysical region.
6. Calculation of the cross section.

Much CPU time was needed at step 2, whereas more CPU time is required at step 4.

3 The low-energy constants to $O(p^6)$

In our calculation we considered the effective lagrangian with two flavours in the isospin symmetry limit $m_u = m_d$. The Feynman graphs of order $O(E^6)$ depend on

three constants $h_{+,-,s}$ - related to the coefficients of the $O(E^6)$ counterterms in the effective lagrangian - which have been estimated in [18] by considering the exchange of vector, axial-vector, scalar and spin-2 resonances

$$h_i^r(\mu) = \sum_R h_i^R + \hat{h}_i^r(\mu), \quad i = +, -, s, \quad R = V, A, S, f_2. \quad (3.1)$$

Here, following [2, 26], we assume $\hat{h}^r(M_\rho) = 0$, which works for the $O(E^4)$ constants l_j^r , $j = 1, \dots, 6$. It has been noted by several authors [27, 28, 29, 31] that the constants h_i^r do not affect the cross section for $\gamma\gamma \rightarrow \pi^0\pi^0$ at center of mass energies $\sqrt{s} \leq 0.4$ GeV.

In [25, 32] it is shown how to estimate these couplings from sum rules. Here we follow the presentation offered in [33]. Let us consider the vector-vector two-point function

$$i \int d^4x e^{iqx} \langle 0 | T(V_\mu^a(x) V_\nu^b(0)) | 0 \rangle = \delta^{ab} (q_\mu q_\nu - g_{\mu\nu} q^2) \Pi^a(q^2), \quad a = 3, 8. \quad (3.2)$$

Following [1]-[3] and [14] we write a dispersion relation for $\Pi^a(q^2)$

$$\Pi^a(q^2) = \frac{1}{\pi} \int ds \frac{Im \Pi^a(s)}{s - q^2 - i\epsilon} + \text{subtractions}, \quad a = 3, 8. \quad (3.3)$$

The high-energy behaviour of the theory is obtained from the perturbative QCD sum rules

$$\begin{aligned} \rho_V^a(s) &= \frac{1}{\pi} Im \Pi^a(s), \\ \lim_{s \rightarrow \infty} \rho_V^a(s) &= \frac{1}{8\pi^2} + O\left(\frac{1}{s}\right), \quad a = 3, 8. \end{aligned} \quad (3.4)$$

Thus, the following sum rule is readily obtained:

$$\Pi^3(0) - \Pi^8(0) = \int ds \frac{\rho_V^3 - \rho_V^8}{s}. \quad (3.5)$$

Making use of CHPT one can calculate $\Pi^a(q^2)$ for small q^2 values [32]. This calculation is carried out in $SU(3) \times SU(3)$ to the two-loop order, and the result depends on the $O(p^4)$ and $O(p^6)$ low-energy constants L_i and, respectively, d_j [32]. The integral $\int ds \frac{\rho_V^3 - \rho_V^8}{s}$ can be evaluated using the e^+e^- data between 0.4 GeV and 2 GeV. The result of [25, 32] can be compared with the Extended Nambu Jona-Lasinio Model prediction which has become recently available [34]. For all cases the results are compatible, within the uncertainties, with the constants estimated by resonance saturation in [18].

4 Pion polarizabilities

The electric and magnetic pion polarizabilities enter the low-energy limit of the coupling of the pion with the photon. The charged pion Compton amplitude

$$\gamma(q_1)\pi^+(p_1) \rightarrow \gamma(q_2)\pi^+(p_2) , \quad (4.1)$$

admits an expansion near threshold

$$T^C = 2 \left[\vec{\epsilon}_1 \cdot \vec{\epsilon}_2^* \left(\frac{\alpha}{M_\pi} - \bar{\alpha}_\pi \omega_1 \omega_2 \right) - \bar{\beta}_\pi (\vec{q}_1 \times \vec{\epsilon}_1) \cdot (\vec{q}_2 \times \vec{\epsilon}_2^*) + \dots \right] \quad (4.2)$$

with $q_i^\mu = (\omega_i, \vec{q}_i)$. Below we denote

$$\begin{aligned} (\alpha \pm \beta)^C &= \bar{\alpha}_\pi \pm \bar{\beta}_\pi , \\ (\alpha \pm \beta)^N &= \bar{\alpha}_{\pi^0} \pm \bar{\beta}_{\pi^0} , \end{aligned} \quad (4.3)$$

for charged and neutral pions, respectively.

At one-loop in CHPT one has [20, 37]

$$\bar{\alpha}_{\pi^0} = -\bar{\beta}_{\pi^0} = -\frac{\alpha}{96\pi^2 M_\pi F^2} = -0.50 . \quad (4.4)$$

At order $O(p^6)$ it was calculated in Ref. [18]

$$\begin{aligned} \bar{\alpha}_{\pi^0} &= -0.35 \pm 0.10 , \\ \bar{\beta}_{\pi^0} &= 1.50 \pm 0.20. \end{aligned} \quad (4.5)$$

The charged pion polarizabilities have been determined in an experiment on the radiative pion-nucleus scattering $\pi^- A \rightarrow \pi^- \gamma A$ [35] and in the pion photoproduction process $\gamma p \rightarrow \gamma \pi^+ n$ [36]. In addition, the low-energy $\gamma\gamma \rightarrow \pi^+ \pi^-$ data [9] have been used in [37] to obtain the numerical value for the leading-order $\bar{\alpha}_\pi = 2.7 \pm 0.4$, plus systematic uncertainties due to the $O(p^6)$ corrections.

In [38] the validity of the errors quoted in a recent estimate of $(\alpha + \beta)^{C,N}$ by Kaloshin and collaborators [39] is discussed. Here the polarizabilities appear as adjustable parameters in the unitarized D-wave amplitudes, hence the values of $(\alpha + \beta)^{C,N}$ can be determined from the data with the result [39]

$$\begin{aligned} (\alpha + \beta)^C &= 0.22 \pm 0.06 [9] , \\ (\alpha + \beta)^N &= 1.00 \pm 0.05 [8] . \end{aligned} \quad (4.6)$$

The authors of [38], arguing on the partial wave analysis of the data that shows large uncertainties even at the $f_2(1270)$ mass, concluded that the errors quoted in (4.6) for $(\alpha + \beta)^N$ are unbelievably small.

It is interesting to match the chiral expansion [18] and the dispersive calculation of [14]. The two representations of the S-wave amplitude agree very well below $E = 0.4$ GeV [18]. In the dispersive method the higher orders are partially summed up. This agreement therefore indicates that orders higher than $O(p^6)$ in the chiral expansion do not affect much the threshold amplitude. The pion loops in CHPT generate chiral logarithms in the amplitude calculated in [18] that are not fully taken into account in the dispersive analysis. This fact is reflected in the difference between the dispersive result and the corresponding two-loop result (quoted in square brackets)

$$\begin{aligned} (\alpha - \beta)^N &= -1.76 \text{ [two loop: } -1.90] \text{ ,} \\ \bar{\beta}_{\pi^0} &= 1.26 \text{ [two loop: } 1.50] \text{ .} \end{aligned} \tag{4.7}$$

5 Conclusions

The cross section for $\gamma\gamma \rightarrow \pi^0\pi^0$ depends, to lowest order, on F_π , M_π only. This prediction is as good as any leading order prediction in $SU(2)_L \times SU(2)_R$. The two-loop corrections have a reasonable size, i.e. about 30% in the amplitude. The $O(p^6)$ parameters h_\pm^r , h_s are unfluent for $\sqrt{s} \leq 0.4$ GeV.

There is agreement of the two-loop cross section with both experiments and dispersion relation calculations, for $\sqrt{s} \leq 0.45$ GeV. We need to wait for higher precision data from DAΦNE, the Frascati Φ -factory [30, 13, 24], in order to have a more accurate test of the predicted cross section.

The π^0 polarizabilities $\bar{\alpha}_{\pi^0} \pm \bar{\beta}_{\pi^0}$ depend on the low-energy constants h_\pm^r , whose estimated values are dominated by the ω resonance exchange [21]. From precise data on $\gamma\gamma \rightarrow \pi^0\pi^0$ it may become possible to extract the value of h_s , since the cross section in Fig. 9 of [18] shows a sizeable dependence on this low-energy constant for energies near 600 MeV. It will be useful to carry out a unitarization of the 2-loop result, using a procedure analogous to [14] and matching the dispersive calculation for the all-order amplitude with the 2-loop amplitudes for $\gamma\gamma \rightarrow \pi^0\pi^0$ and $\gamma\gamma \rightarrow \pi^+\pi^-$, once the latter will have been calculated. The consideration of this improved, unitarized amplitude will justify the inclusion in the analysis of experimental data

up to 600 MeV.

Another source of information on h_s and h_{\pm} is the decay $\eta \rightarrow \pi^0 \gamma \gamma$, whose width depends in CHPT [40] on these same low-energy constants, just as the process $\gamma \gamma \rightarrow \pi^0 \pi^0$. A complete treatment for these two transitions is necessary, as it is described in [34].

Finally we remark that the calculation of [18], which was motivated by noises on a supposed failure of CHPT in the prediction of the $\gamma \gamma \rightarrow \pi^0 \pi^0$ amplitude, not only proved that there is no chiral mystery in this transition, but opened the way to a clear understanding of the neutral pion polarizability issue. Having computed a definite and reliable CHPT prediction yields the possibility to determine in future experiments for the first time the coefficient of some $O(p^6)$ counterterms in the chiral expansion.

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