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QED Two Photon Width of η_c

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Abstract

We discuss the recently measured partial width of the pseudoscalar charmonium, η_c , into two photons. Predictions from potential models are examined and compared with experimental values. Including radiative corrections, it is found that present measurements are compatible both with a QCD type potential and with a static Coulomb potential, with α_s evaluated at two loops.

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Abstract

We discuss the recently measured partial width of the pseudoscalar charmonium, η_c , into two photons. Predictions from potential models are examined and compared with experimental values. Including radiative corrections, it is found that present measurements are compatible both with a QCD type potential and with a static Coulomb potential, with α_s evaluated at two loops.

The charmonium spectrum has been the basic testing grounds for a variety of models for the interquark potential, ever since the discovery of the J/ψ in 1974 [1]. The experimental scenario describing the $c\bar{c}$ bound states is now close to completion, with the recently observed higher excitation states $^3P_1$, 0 and spin 2 states [2]. Decay width into various leptonic and hadronic states have been measured and compared with potential models [3, 4]. In this note, we examine the theoretical predictions for the electromagnetic decay of the simplest and lowest lying of all the charmonium states, i.e. the pseudoscalar η_c. We shall compare the two photon decay width with the leptonic width of the J/ψ, which has been recently measured with higher precision [5] and found to be 20% higher than in previous measurements [6]. This implies that a number of potential models whose parameters had been determined by the leptonic width of the J/ψ may need some updating, and so do some predictions from these models. The decay width of η_c into two photons is one of them, and we shall limit our attention to this case. This decay width has been recently measured in different experiments, with preliminary results which are consistent with each other within one standard deviation, but are

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however affected by rather large errors. The most recently reported values for the radiative decay width are the following :

ARGUS:	$\Gamma_{\gamma\gamma} = 11.3 \pm 4.2 \ \ KeV$	[7]
E760:	$\Gamma_{\gamma\gamma} = 7.0^{+2.9}_{-2.0} \pm 2.3 \ \ KeV$	[8]
L3:	$\Gamma_{\gamma\gamma} = 8.0 \pm 2.3 \pm 2.4 \ \ KeV$	[9]
CLEO:	$\Gamma_{\gamma\gamma} = 5.9^{+2.1}_{-1.8} \pm 1.9 \ \ KeV$	[10]
TPC/2γ:	$\Gamma_{\gamma\gamma} = 6.4^{+5.0}_{-3.4} \ \ KeV$	[11]

where, for the last two measurements, the first error is statistical, and the second systematic.

At the Born level, the two photon decay width of a pseudoscalar quark-antiquark bound state can be written as [12]

$$\Gamma_B(\eta_c \to \gamma\gamma) = 12 e_q^4 \alpha_{QED}^2 4\pi \frac{|\psi(0)|^2}{M^2} \equiv \Gamma_B^P, \tag{1}$$

where $\psi(0)$ is the wavefunction of the interquark potential evaluated at the origin, while the first order QCD correction [13] reads as

$$\Gamma(\eta_c \to \gamma\gamma) = \Gamma_B^P \left[1 + \frac{\alpha_s}{\pi}\left(\frac{\pi^2 - 20}{3}\right)\right] \approx \Gamma_B^P (1 - \alpha_s). \tag{2}$$

It is useful to compare eqs. (1) and (2) with the expressions for the vector state J/ψ, i.e.

$$\Gamma_B(J/\psi \to ee) = 4 e_q^2 \alpha_{QED}^2 4\pi \frac{|\psi(0)|^2}{M^2} \equiv \Gamma_B^V \tag{3}$$

in the tree approximation, and

$$\Gamma(J/\psi \to ee) = \Gamma_B^V \left(1 - \frac{16}{3}\frac{\alpha_s}{\pi}\right) \approx \Gamma_B^V (1 - 1.698\alpha_s) \tag{4}$$

for the first order corrected one [14]. From (2) and (4) we have the relation

$$\Gamma(\eta_c \to \gamma\gamma) = \frac{4}{3}\,\Gamma_1(J/\psi \to ee)\,\frac{1 + \frac{\alpha_s}{\pi}\frac{\pi^2 - 20}{3}}{1 - \frac{16}{3}\frac{\alpha_s}{\pi}}. \tag{5}$$

The expression in eq. (5) can be used to estimate the radiative width of η_c from the measured values of the leptonic decay width of J/ψ. Notice that in these expressions we have tacitly assumed that the $\psi(0)$ values for both the pseudoscalar and the vector state should be the same. Actually this turns out not to be the case, since there is a dependence on the angular momentum and on the spin as well. However, since we are dealing with the $L=0$ state, corrections of the kind $S \cdot L$ are absent, and we are left only with the $S \cdot S$ interaction ("hyperfine splitting" in atomic physics). Here is an example of the perturbation for a Cornell type potential [15, 16]:

$$\frac{2}{3m_c^2} S_1 \cdot S_2 \frac{4\alpha_s}{3} 4\pi\delta(r) + \frac{1}{m_c^2} \left(3S_1 \cdot R S_2 \cdot R - S_1 \cdot S_2\right) \frac{4\alpha_s}{3} \frac{1}{r^3} \tag{6}$$

In any case this contribution to the interquark potential is very small and is masked by the first order correction to the width. We shall therefore take the wavefunction of the two cases to be equal.

Equation (5) can be now used for a first (rather rough) estimate of the width for process $\eta_c \to \gamma\gamma$. From PDG data [17] the current value is

$$\Gamma_{exp}(J/\psi \to ee) = 5.36 \pm 0.29 \ KeV. \tag{7}$$

Expanding eq. (5) in α_s one has

$$\Gamma(\eta_c \to \gamma\gamma) = \frac{4}{3} \, \Gamma_{exp}(J/\psi \to ee) \left(1 + 1.96 \, \frac{\alpha_s}{\pi}\right), \tag{8}$$

Using the two loop expression for α_s, i.e.

$$\alpha_s^{(2)}(Q) = \frac{4\pi}{b_0 f(Q)} \left\{1 + \frac{c}{f(Q)} - \frac{b_1}{b_0^2} \frac{\log[f(Q)]}{f(Q)}\right\} \tag{9}$$

with

$$b_0 = \frac{33 - 2N_f}{3}, \quad b_1 = \frac{306 - 38N_f}{3}, \quad c = \frac{1}{b_0}\left(\frac{93 - 10N_f}{9}\right) + 2\gamma_E$$

$$f(Q) = \log\left[\left(\frac{Q}{\Lambda_{\overline{MS}}}\right)^2\right]. \tag{10}$$

and N_f the number of flavours, one obtains $\alpha_s = 0.276$ with $Q = m_c$ and $\Lambda_{\overline{MS}} = 0.2$, and we are led to

$$\Gamma(\eta_c \to \gamma\gamma) = 8.38 \pm 0.45\ KeV, \tag{11}$$

i.e. a value consistent with the most recent measurement from the L3 Collaboration [9]. On the other hand, using the *full* formula for the width (5) and the same value of α_s as above, we would arrive at:

$$\Gamma(\eta_c \to \gamma\gamma) = 9.46 \pm 0.51\ KeV. \tag{12}$$

Thus the estimated values depend upon using either of the two approximations from eq.(5) or (8). Notice that the approximation of eq.(5) is in general much more scale dependent than the first order expansion, eq.(8), as shown in fig. 1, where the width has been calculated using both approximations, as a function of the scale of α_s. As it stands from the error of the data listed before, this is not enough for a definite choice of scale. It is clear that since we are in a low energy region the Born approximation simply cannot be correct. At the same time the application of a non-negligible perturbative correction calls for a rather precise knowledge for α_s in this region. We should start with the recent estimate [18]

$$\alpha_s(35\ GeV) = 0.146 \pm 0.030\ \ (\Lambda_{\overline{MS}}^{N_f=5} \approx 306\ MeV) \tag{13}$$

and by evolving the two–loops formula (9) down to the scale of the charm mass, taken to be 1.5 GeV, we have

$$\alpha_s(Q = 1.5\ GeV) = 0.405^{+0.516}_{-0.176}. \tag{14}$$

It is clear that in this way we have an enormous error on the value of α_s . To circumvent this problem we have to choose a value obtained at a scale closer to the charm mass, like

$$\alpha_s(Q = 1.78\ GeV) = 0.36^{+0.05}_{-0.05}\ \ (\Lambda_{\overline{MS}}^{N_f=4} \approx 386\ MeV) \tag{15}$$

a value obtained from data on τ decay [18]. Evolving downwards as before, we obtain

$$\alpha_s(Q = 1.5\ GeV) = 0.399^{+0.062}_{-0.061}. \tag{16}$$

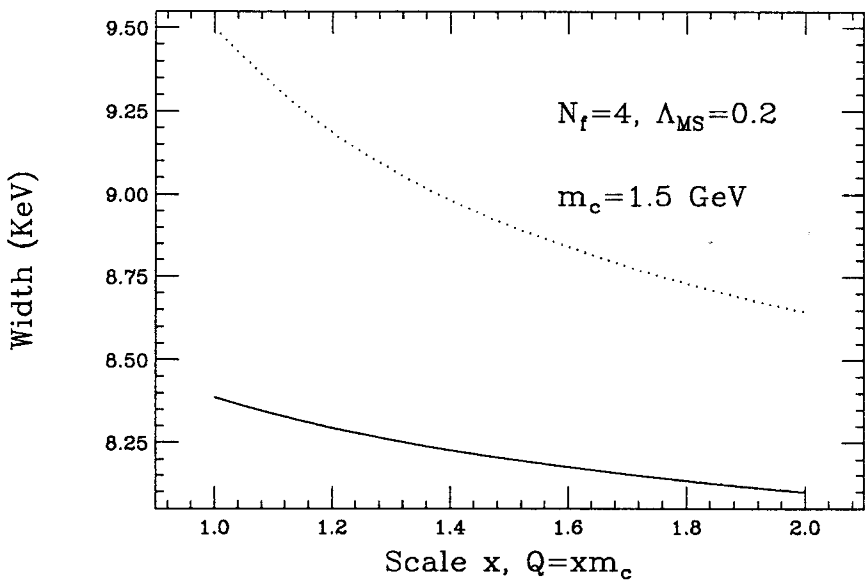

Figure 1: η_c decay width to $\gamma\gamma$ in KeV, with respect to the scale chosen for α_s.

which gives, apart from a much better error, a good value for $\alpha_s(M_Z)$, equal to $0.123^{+0.006}_{-0.006}$, consistent with the most recent LEP estimates[19]. An independent check of these results can be obtained from recent data on $^3P_{0,1,2}$ states χ_{2c} [2], a system close to η_c in mass scale (for a more thorough discussion about the charmonium system, see for instance [20]). The reported values for χ_2 partial widths are:

$$\Gamma(\chi_2 \to \gamma\gamma) = 0.321 \pm 0.095 \quad KeV \tag{17}$$

and [17]

$$\Gamma(\chi_2 \to gg) = 1.730 \pm 0.180 \quad MeV \quad . \tag{18}$$

Since the ratio of the two widths does not depend on the wavefunction at the origin [21], we can write

$$\frac{\Gamma(\chi_2 \to gg)}{\Gamma(\chi_2 \to \gamma\gamma)} = \frac{9}{8}\left(\frac{\alpha_s}{\alpha}\right)^2 \left[\frac{1 - 2.2\,\alpha_s/\pi}{1 - 16/3\,\alpha_s/\pi}\right] \tag{19}$$

or

$$\frac{\Gamma(\chi_2 \to gg)}{\Gamma(\chi_2 \to \gamma\gamma)} = \frac{9}{8}\left(\frac{\alpha_s}{\alpha}\right)^2 \left[1 + 3.13\,\frac{\alpha_s}{\pi}\right] \tag{20}$$

to order α_s^3 . One obtains: $\alpha_s = 0.37 \pm 0.04$ and $\alpha_s = 0.42 \pm 0.06$ from eqs. (19) and (20) respectively. We see that in both cases there is a good agreement with the evolved value (16) for α_s at $Q = 1.5\,GeV$.

Thus, both the Renormalization Group (RG) equation from tau-decay data and radiative corrections for the higher excitations of the charmonium system, χ_2, indicate a value for α_s in the 0.4 range. Yet this value may lead to a problem : using $\alpha_s \approx 0.4$, and computing back $\Gamma(\eta_c \to gg)$ from $\Gamma(\eta_c \to \gamma\gamma)$ with the aid of the relation

$$\frac{\Gamma(\eta_c \to gg)}{\Gamma(\eta_c \to \gamma\gamma)} = \frac{9}{8}\left(\frac{\alpha_s}{\alpha}\right)^2 \left[\frac{1 - 3.4\,\alpha_s/\pi}{1 + 4.8\,\alpha_s/\pi}\right] \tag{21}$$

one obtains $\Gamma_{gg}(\eta_c) = 43 \div 58\ MeV$ with $\Gamma_B(\eta_c \to \gamma\gamma) = 8 \div 12\ KeV$. Comparing this gluonic width with the measured total width of the charmonium, $\Gamma_{tot}(\eta_c) = 10.3^{+3.8}_{-3.4}\ MeV$ [17], we see there is a potential problem. Other contradictions have been pointed out in [22], where the authors reach the conclusion that effects like the gluon mass may produce an α_s somewhat different from what expected and propose the value [22]

$$\alpha_s(m_c) = 0.191 \pm 0.006\ . \tag{22}$$

quite lower than the one discussed above and hard to reconcile with the usual two loops Renormalization Group evolution. It should be pointed out that the authors of ref.[22] do not include the above recent χ_2 data in their discussion. In any event, using (22) we obtain respectively from eqs. (5) and (8):

$$\Gamma(\eta_c \to \gamma\gamma) \pm \Delta\Gamma(\eta_c \to \gamma\gamma) = 8.41 \pm 0.46 \pm 0.06\quad KeV \tag{23}$$

and

$$\Gamma(\eta_c \to \gamma\gamma) \pm \Delta\Gamma(\eta_c \to \gamma\gamma) = 8.00 \pm 0.43 \pm 0.03\quad KeV\ . \tag{24}$$

where the first error comes from the J/ψ measurement, eq.(7), and the other from α_s as determined in [22]. We notice that both estimates fairly agree with the most recent result from the L3 Collaboration[9].

So far, we have discussed the expectation from the already known J/Ψ data. We present now the results one can obtain for the absolute width, through the extraction of the wave function at the origin from potential models. For the calculation of the wavefunction we have used four differ-

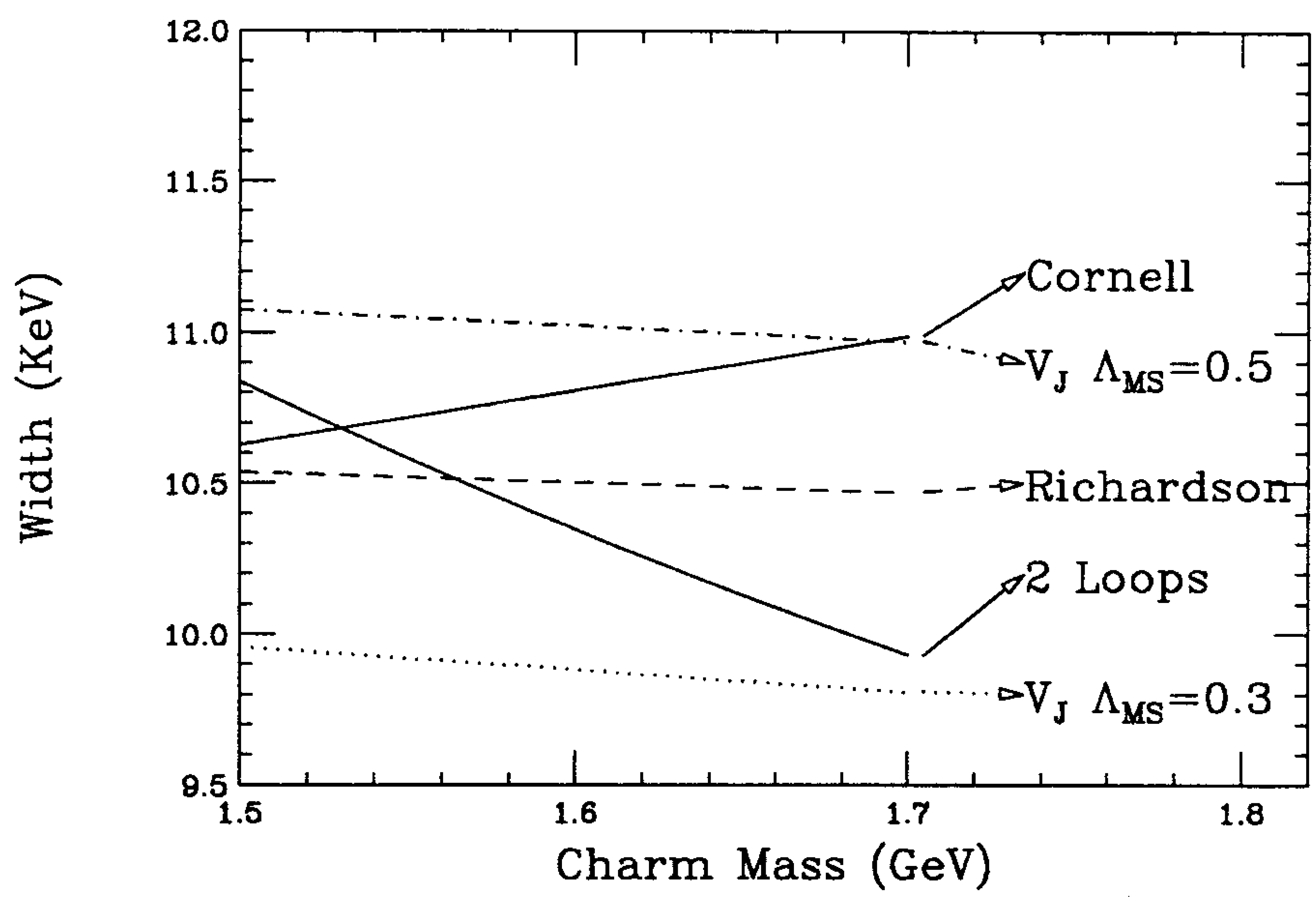

Figure 2: η_c decay width to $\gamma\gamma$ in KeV, with respect to the charm mass, at Born level from some models.

ent potential models, like the Cornell type potential [15] $V(r) = -\frac{k}{r} + \frac{r}{a^2}$ with parameters $a = 2.43$, $k = 0.52$, the Richardson potential [23] $V_R(r) = -\frac{4}{3}\frac{12\pi}{33-2N_f}\int\frac{d^3q}{(2\pi)^3}\frac{e^{iqr}}{q^2\log(1+q^2/\Lambda^2)}$ with $N_f = 3$, $\Lambda = 398\ MeV$, and the QCD inspired potential V_J of Igi-Ono [24, 25]

$$V_J(r) = V_{AR}(r) + dre^{-gr} + ar, \quad V_{AR}(r) = -\frac{4}{3}\frac{\alpha_s^{(2)}(r)}{r} \tag{25}$$

with two different parameter sets, corresponding to $\Lambda_{\overline{MS}} = 0.5\ GeV$ and $\Lambda_{\overline{MS}} = 0.3\ GeV$ respectively [24]. We also show the results from a Coulombic type potential with the QCD coupling α_s frozen to a value of r which corresponds to the Bohr radius of the quarkonium system (see for instance [26]).

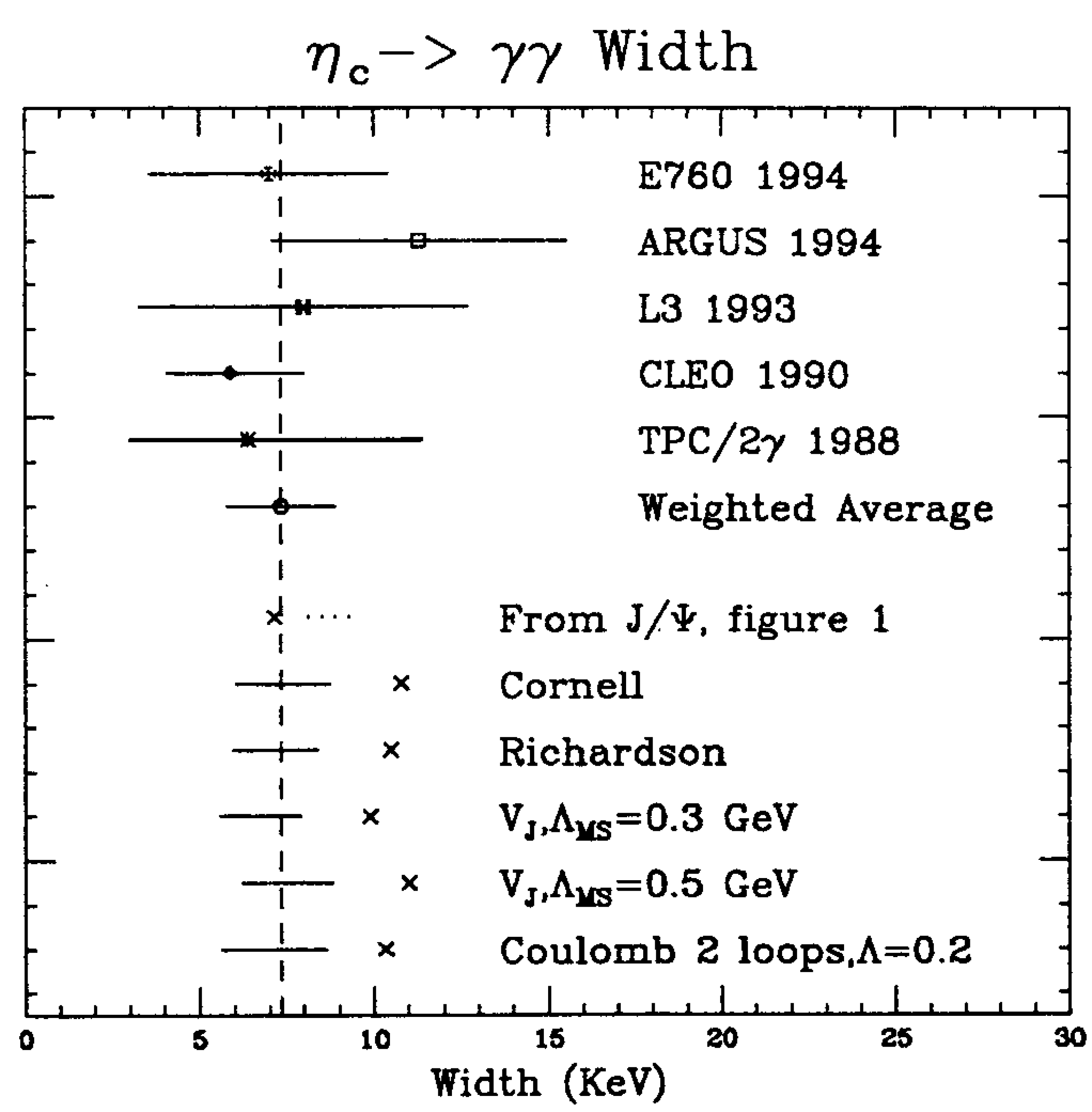

Figure 3: η_c decay width to $\gamma\gamma$ in KeV. The crosses are Born calculations, the first order calculations are for $\alpha_s = 0.191 \div 0.399$, the J/Ψ range is from figure 1. The vertical line passes through the central value of the weighted average.

We show in Fig. 2 the predictions of the decay width at Born level from these potential models. The figure has to be compared with the experimental data [7, 9, 10, 11], and it is clear than the predictions are above the allowed range of values. Nonetheless we still have to add the correction from eq.(2), and the result is shown in the next figure, where, to estimate the α_s correction, we have used both the value from Consoli and Field [22] as well as the RG two loops evolved expression, with an appropriate $\Lambda_{\overline{MS}} = 0.3$ and 0.5 for the two QCD-type potentials and $\Lambda = 0.398$ for Richardson. We see that for the case in which we have used for α_s the value of eq.(22), the calculated widths stay well within the central value of the measure of [9]. The associated error has several contributions: the value of Γ_B , the error in α_s , together with the choice of its evaluation scale and $\Lambda_{\overline{MS}}$. The contribution of α_s alone is of the order of 10% .

In conclusion, the calculations for the radiative decay of η_c for some potential models, as well as for a Coulombic type potential, give a value compatible with the most recent measures obtained [7, 9, 10, 11]. There are however some problems due to the correct value of α_s to be used, as we have shown that RG evolution equation is not feasible for estimates in the low Q^2 region, and on the (perhaps) nonperturbative terms left out from this kind of approach.

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