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The Geometrical Picture and Bremsstrahlung Analogy in Diffraction of High Energy Hadrons

E. Etim, Małecki*, M. Pallotta
INFN – Laboratori Nazionali di Frascati, P.O. Box 13, I–00044 Frascati, Italy

Abstract

The common probabilistic roots of the geometrical models of hadron diffraction and of the Bloch-Nordsieck theory of soft Bremsstrahlung are discussed. Their close relationship is best visible in the Chou-Yang model generalized to four dimensions. An interesting complementarity of the two descriptions in reproducing elastic scattering of high energy hadrons is pointed out.

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* From K E N Pedagogical University, Institut of Physics and Informatics, Kraców, Poland

High-energy collisions of hadrons are supposedly characterized by the two features: (i) the incident hadron is way out of the target before the effects which it induces in the latter take place; (ii) in its encounter with a target, an incident high energy hadron suffers very little deflection of its path.

The first assumption means that the incident hadron sees the target, essentially as a geometrical obstacle of given shape. The approximation is taken so much for granted that it is often non stated explicitly. It gives rise to a geometrical picture [1,2] of scattering where colliding hadrons are drawn as spatially extended objects of finite size. The geometrical models impose thus on themselves the condition that their S-matrix should tend to scattering by a black disc at large distances. In consequence, according to typical optical diffraction patterns, all these models predict multiple dips and peaks in elastic scattering of hadrons.

The second assumption allows to adopt the eikonal approximation borrowed from geometrical optics. It implies that single scattering events produce only very small momentum transfers. In other words, large momentum transfer $\sqrt{|t|}$ is obtained as the sum of small momentum transfers accumulated in a large number n of multiple scatterings. Hadron scattering is therefore similar to the process of energy loss by a charged particle through the emission of soft radiation. This analogy has long been noted and has given rise to the Bremsstrahlung picture of hadron scattering [3]. But there is more to it than a phenomenologically useful analogy. It will be shown in this paper that the two processes are exactly related. They are both probabilistic point processes governed by the Poisson distribution. In consequence, the geometrical models may be formulated exactly as the Bloch-Nordsieck theory [4] which describes the emission of low energy photons by a fast moving charged source.

The basic ingredient of geometrical models is the real, dimensionless opacity function $\Omega(b)$ which depends on a relative impact parameter b of interacting hadrons and appears in the eikonalized scattering amplitude , written as the integral over the impact plane:

$$T(q) = \frac{i}{(2\pi)^2} \int d^2b e^{i\mathbf{q}\cdot\mathbf{b}}(1 - exp[-\Omega(b)]), \tag{1}$$

$q = \sqrt{|t|}$ being the momentum transfer in the centre-of-mass system.

The dynamical conjecture is that the opacity can be expressed in terms of hadronic shapes known from other experiments, e.g. in the Chou-Yang model [2] it is assumed that the Fourier transform of $\Omega(b)$ is proportional to the product of the electromagnetic form factors of the colliding hadrons:

$$\Omega(b) = g\frac{h(b)}{h(0)}; \;\; h(b) = \int d^2q e^{-i\mathbf{q}\cdot\mathbf{b}} F_A(q^2) F_B(q^2) \tag{2}$$

where g is a dimensionless coupling parameter. It is equal to the value of opacity $\Omega(0)$ which corresponds to head-on collisions.

Rather than from experiment, for simplicity of illustration, we take these form factors from the quark model :

$$F_H(q^2) = \frac{1}{(1+\frac{q^2}{m^2})^{\nu_H - 1}} \tag{3}$$

where ν_H is the number of valence quarks in the hadron H. For the pion ($\nu_H = 2$) this gives a single pole form factor and for the proton ($\nu_H = 3$), a dipole. For simplicity, the mass scale parameter m will be taken the same in the two hadronic form factors. The opacity function then reads

$$\Omega(b) = g h_\nu(mb); \;\; h_\nu(u) = \frac{2}{\Gamma(\nu - 3)}(\frac{u}{2})^{\nu-3} K_{\nu-3}(u) \tag{4}$$

where $\nu \equiv \nu_A + \nu_B$, $u \equiv mb$ and $K_\nu(u)$ is the modified Bessel function. For proton-(anti)proton scattering when $\nu = 6$ one obtains the familiar K_3-behaviour of opacity in the Chou-Yang model [2].

The coupling parameter g governs the decomposition of scattering process into a series of multiple scatterings. To see how this comes about consider the S-matrix $S = 1 + iT$ corresponding to the scattering amplitude (1):

$$S(q) = \frac{i}{(2\pi)^2} \int d^2b e^{i\mathbf{q}\cdot\mathbf{b}} exp[-\Omega(b)]. \tag{5}$$

Expanding the exponential and making use of (2) one obtains upon integration over the impact plane:

$$S(q) = e^{\langle n \rangle} \sum_{n=0}^{\infty} (-1)^n P_n \delta^{(2)}(\mathbf{q} - \mathbf{q}_n). \tag{6}$$

This formula describes the distribution of all possible partitions of the momentum transfer $\mathbf{q}$. The partition of the momentum $\mathbf{q}_n$ and of the number n is into cells labelled by the index j, with momentum $\mathbf{k}_j$ and number n_j:

$$\mathbf{q}_n = \sum_j n_j \mathbf{k}_j; \;\; n = \sum_j n_j \tag{7}$$

and the partitions are governed by the Poisson distribution :

$$P_n = e^{-\langle n \rangle} \frac{\langle n \rangle^n}{n!} \tag{8}$$

with the mean value $< n >= g \equiv \Omega(0)$.

It is instructive to consider the opacity $\bar{\Omega}(b)$, associated to the given opacity Ω through the relation:

$$\Omega(b) \equiv \Omega(0) - \bar{\Omega}(b). \tag{9}$$

Since the physical opacity $\Omega(b)$ attains its maximum value at $b = 0$, the associated opacity $\bar{\Omega}(b)$ is a real, positive function. Its peculiarity is that at large impact parameters $\bar{\Omega}(b)$ approaches a constant value $\neq 0$. The corresponding S-matrix is :

$$\bar{S}(q) = \sum_{n=0}^{\infty} P_n \delta^{(2)}(\mathbf{q} - \mathbf{q}_n) \tag{10}$$

which is precisely, adopted to the two-dimensional space, the famous Bloch-Nordsieck theorem [4], describing the probability density of registering a total momentum $\mathbf{q}$ carried by all photons emitted by a moving charged source. Therefore the opacity $\bar{\Omega}(b)$ can be referred to as the Bloch-Nordsieck opacity in two dimensions.

With this analogy in mind Eq.(10) can be interpreted as describing the distribution $\varrho(k_\perp) \equiv \bar{S}(q = k_\perp)$ of transverse momentum $k_\perp$ acquired in a process of 'hadronic Bremsstrahlung' [3]. Though the integrals like (5) are Cauchy divergent, the distributions (6) and (10) may be regularized by means of Cesàro summability [5]. A numerical verification then yields a convenient expression (for $k_\perp \neq 0$) of $\varrho(k_\perp)$ in terms of the summable Cauchy integral:

$$\varrho(k_\perp) = \frac{1}{2\pi} exp[-\Omega(0)] \int_0^\infty dbbJ_0(k_\perp b)(exp[+\Omega(b)] - 1). \tag{11}$$

In Fig.1 we give the plot of $\varrho(k_\perp)$ in the case of proton-proton scattering ($\nu = 6$) for various values of the coupling g and the fixed value of $m^2 = 0.71\, GeV^2$.

The analogy with Bremsstrahlung can be pursued even further by reformulating the Chou-Yang model with the form factors (3) in 4 dimensions. To this aim introduce the 4-dimensional coordinate vector $x_\mu \equiv (t, \mathbf{x})$, where $\mathbf{x} \equiv (x_{\|}, \mathbf{x}_\perp = \mathbf{b})$, and the 4-momentum vector $k_\mu \equiv (\omega, \mathbf{k})$, where $\mathbf{k} \equiv (k_{\|}, \mathbf{k}_\perp)$, $\omega \equiv \sqrt{(\mathbf{k}^2 + m^2)}$. Next we use the identity:

$$\frac{1}{(1 + \frac{q^2}{m^2})^{\nu-2}} = \frac{\Gamma(\nu - 3/2)}{\Gamma(1/2)\Gamma(\nu - 2)} \int_{-\infty}^{+\infty} \frac{dq_{\|}}{m} \frac{1}{(1 + \frac{q^2+q_{\|}^2}{m^2})^{\nu-3/2}}. \tag{12}$$

to rewrite Eq.(2) as

$$\Omega(b) \equiv \Omega(t = x_{\|} = 0, \mathbf{x}_\perp = \mathbf{b}) \tag{13}$$

where

$$\Omega(x) = g\frac{h_\nu(x)}{h_\nu(0)};\ h_\nu(x) = \int \frac{d^3k}{2\omega} |j_\mu^{(\nu)}(k)|^2 e^{-ik\cdot x}, \tag{14}$$

$j_\mu^{(\nu)}$ being the four-dimensional current:

$$j_\mu^{(\nu)}(k) \equiv [\frac{2\Gamma(\nu - 3/2)}{\Gamma(1/2)\Gamma(\nu - 2)}]^{1/2} (m/\omega)^{(\nu-3)} j_\mu(k); j_\mu(k) \equiv \frac{p_\mu}{p \cdot k}, \tag{15}$$

where p_μ is a fixed four-momentum which may be attributed to a charged emitting source of mass M coupled to a vector field of mass m and four-momentum k_μ. $j_\mu(k)$

has the form of the classical (i.e. non quantised) current describing the motion of this source. The coupling constant is $\sqrt{g}$. In the p_μ rest frame, with $|j_\mu(k)|^2 = 1/\omega^2$, one re-obtains Eqs (2) and (4).

It should be pointed out that in passing to 4 dimensions the form factors of the Chou-Yang model are replaced by the current-current interaction. Now introducing a scale operator $L_\nu(m^2)$ which is expressible in terms of the operator $m^2 d/dm^2$ and serves to increase the index ν of the current (above the value $\nu = 3$ for which the two currents in (15) coincide) through the relation:

$$L_{\nu-3}(m^2)\int \frac{d^3k}{2\omega}|j_\mu(k)|^2 = \int \frac{d^3k}{2\omega}|j_\mu^{(\nu)}(k)|^2 = \frac{\pi}{\nu-3}, \tag{16}$$

one rewrites Eq.(14) as

$$\Omega(x;m) = g - \frac{\nu-3}{\pi}L_{\nu-3}(m^2)\Omega_{BN}(x;m) \tag{17}$$

where the function

$$\Omega_{BN}(x;m) = g\int \frac{d^3k}{2\omega}|j_\mu(k)|^2(1-e^{-ik\cdot x}) \tag{18}$$

is the classical Bloch-Nordsieck opacity. It gives rise to the following momentum distribution:

$$S_{BN}(q) = \sum_{n=0}^{\infty} P_n \delta^{(4)}(q_\mu - \sum_j n_j k_{\mu j}). \tag{19}$$

Equations (17) and (19) , formulated in 4 dimensions, are to be compared with Eqs (9) and (10) which are their two-dimensional analogues.

The probability assumptions underlying the treatment of Bremsstrahlung are, essentially, the same as those which go into the construction of geometrical models. But there is an important difference. In the Bloch-Nordsieck theory each term of the perturbative expansion is positive - see Eqs. (10) and (19). This fact guarantees, via Böchner's theorem of probability theory [6], that for all values of the coupling constant g there are no zeros in the scattering amplitude corresponding to the Bremsstrahlung-like opacity $\tilde{\Omega}(b)$ defined in (9). Böchner's theorem rules out the possibility of such zeros if certain conditions of positive-definitness are satisfied. On the contrary, the corresponding perturbative expansion in geometrical models is the multiple scattering series (6) , the terms of which alternate in sign. Consequently, the multiple scattering expansion satisfies the condition of Böchner's theorem only for very small values of the coupling constant $g \ll 1$. Thus in geometrical models for values of g close to unity and larger, the scattering amplitude being no longer a positive function of the momentum transfer $\sqrt{t}$, necessarily has zeros which give rise to the dips in the differential cross-section.

This is illustrated in Fig.2 by comparing the elastic p-p differential cross-section of the Chou-Yang model with the corresponding cross-section in the Bremsstrahlung picture. The confrontation with the experimental data at 52.8 GeV [7] reveals an astonishing complimentarity of the two models. The geometrical model reproduces perfectly the forward diffraction peak but fails in predicting two dips while only one shallow minimum is observed experimentally. On the other hand, the Bremsstrahlung model satisfactorily approximates the elastic differential cross-section outside the minimum, being especially correct at large momentum transfers.

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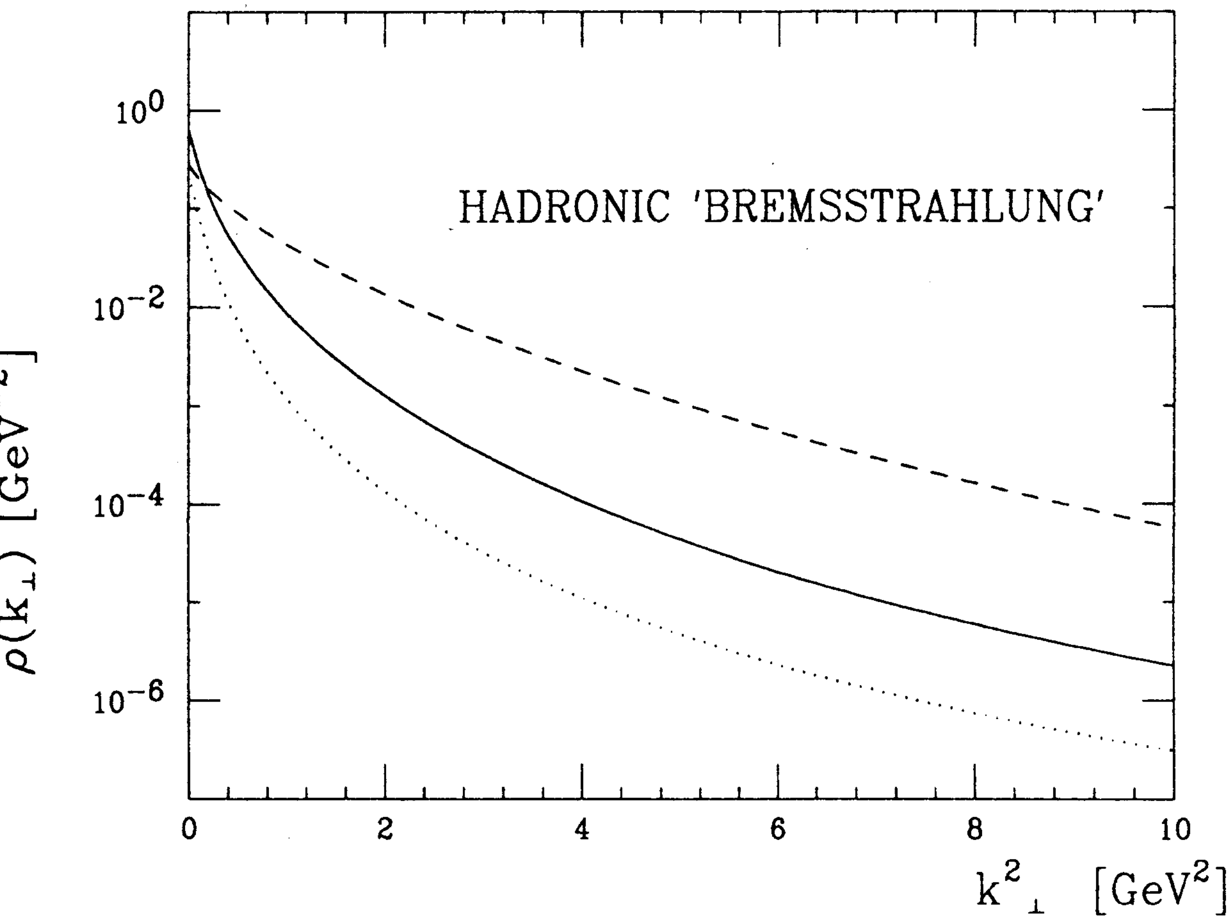

Fig. 1. The distribution of transverse momentum in the hadronic Bremsstrahlung, with the coupling parameters g = 1.05, 0.2 and 5.0 for the solid, dotted and dashed curves, respectively.

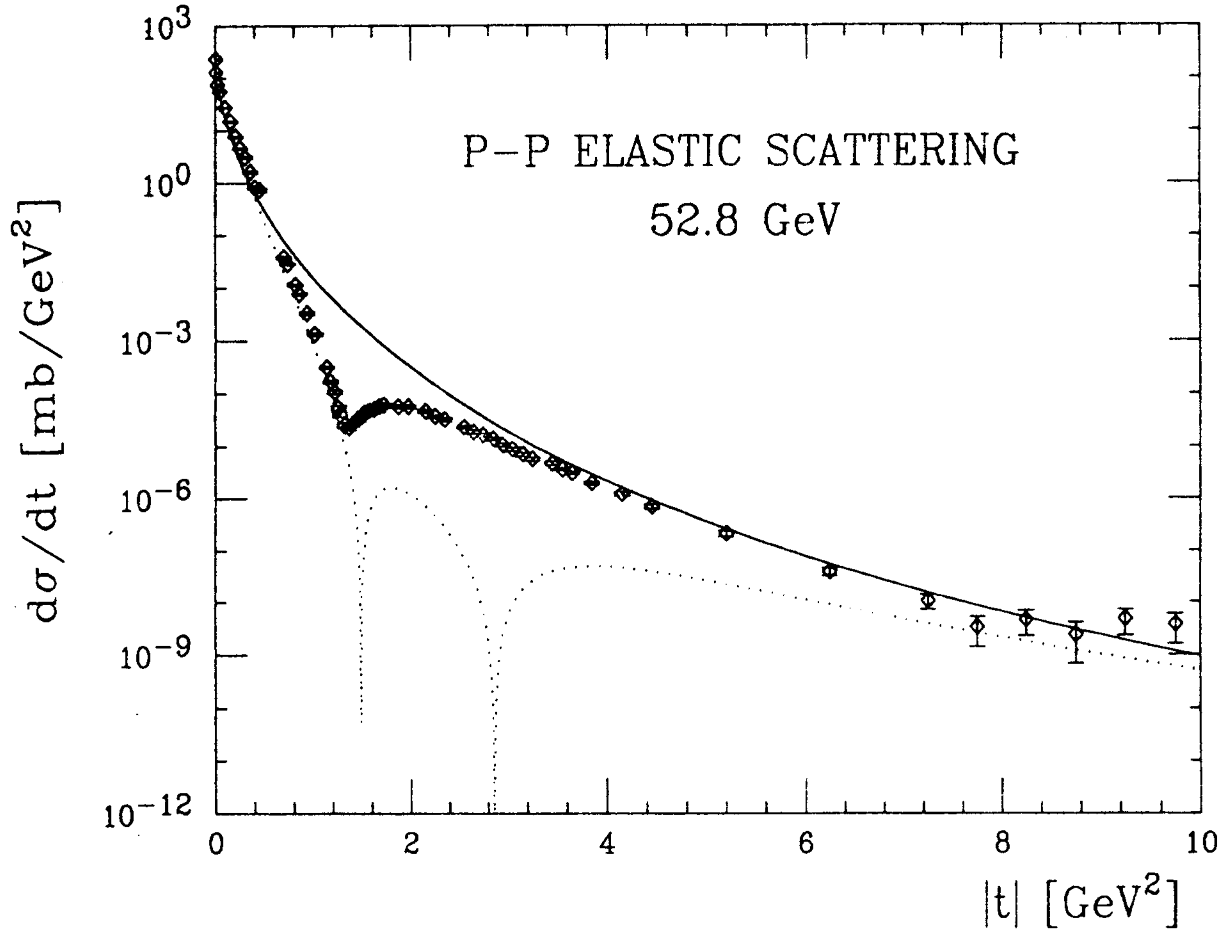

Fig. 2. The elastic differential cross-section corresponding to geometrical diffraction (solid curve) and to hadronic Bremsstrahlung (dotted curve) with the value of the total cross-section σ_{tot} = 42.7 mb and g = 1.05.