LABORATORI NAZIONALI DI FRASCATI
SIS – Pubblicazioni

LNF-94/040(P)
5 September 1994

UAB–FT–335/94
UG–FT–41/94

Effective Chiral Lagrangians with an SU(3) –Broken Vector–Meson Sector

A Bramon [1]
Grup de Física Teòrica, Universitat Autonòma de Barcelona,
08193 Bellaterra (Barcelona), Spain

A. Grau[2]
Departamento de Física Teòrica y del Cosmos, Universidad de Granada,
18071 Granada, Spain

and

G. Pancheri[3]
INFN – Laboratori Nazionali di Frascati, P.O. Box 13, I–00044 Frascati, Italy

Abstract

The accuracy of effective chiral lagrangians including vector-mesons as (hidden symmetry) gauge fields is shown to be improved by including SU(3)-breaking in the vector-meson sector. The masses (M_V), couplings to the photon field ($f_{V\gamma}$) and pseudoscalar pairs (g_{VPP}) are all consistently described in terms of two parameters, which also fit the values for the pseudoscalar decay-constants, f_P, and charge radii, $< r_P^2 >$. The description of the latter is further improved when working in the context of chiral perturbation theory.

PACS.: 14.80.–j ; 12.40.Qq

(Submitted to Phys. Lett.)

1 16414::BRAMON
2 16487::IGRAU
3 PANCHERI@IRMLNF

The question of including spin-1 mesons in effective chiral lagrangians has been frequently discussed in the past and has enjoyed considerable renewed interest during the last few years. This is partly due to the various proposals for new low-energy, high-luminosity e^+e^- colliders which have been put forward and approved, e.g.the DAΦNE ϕ-factory, presently under construction in Frascati. DAΦNE is expected to provide very accurate and exhaustive experimental information on ϕ-decays, as well as on a number of low-energy parameters, both in the kaon system and in other pseudoscalar and vector meson decays. This recent revival, spurred by the expectation for new experimental data, has had its counterpart on the theoretical front. Indeed, traditional ideas concerned with Vector Meson Dominance, somehow associating spin-1 mesons with gauge bosons of local symmetries, have been further investigated and theoretically developed. The so-called "massive Yang-Mills approach" and "hidden symmetry scheme" recently reviewed by Meissner [1] and Bando [2] are two excellent examples. The parallel developement of Chiral Perturbation Theory (ChPT) [3] and its successful description of the low-energy interactions of the pseudoscalar-meson octet – a success considerably enlarged when spin-1 mesons are appropriately taken into account [4] – also points in the same direction. Our purpose in the present note is to go one step further by refining the predictions of the above schemes through the introduction of a well-known effect in hadron physics, namely, SU(3)-breaking in the vector-meson sector, a refinement that forthcoming data will certainly require.

Conventional ChPT accounts for the electro-weak and strong interactions of the pseudoscalar meson octet, P, in a perturbative series expansion in terms of their masses or four-momenta [3]. At lowest order in this expansion, the chiral lagrangian starts with the term

$$\mathcal{L}^{(2)} = \frac{f^2}{8} Tr(\mathcal{D}_\mu \Sigma \mathcal{D}^\mu \Sigma^\dagger) \tag{1}$$

where $\Sigma = \xi\xi = exp(2iP/f)$ and f is the pion decay constant $f = 132\ MeV = f_\pi = f_K$, at this lowest-order level. Electromagnetic interactions are contained in the covariant derivative

$$\mathcal{D}_\mu \Sigma = \partial_\mu \Sigma + ieA_\mu [Q, \Sigma] \tag{2}$$

where A_μ is the photon field and Q the quark-charge matrix (the extension to weak interactions is trivial). The mass degeneracy is broken via the additional mass term

$$\mathcal{L}^{(2)}_m = \frac{f^2}{8} Tr(\chi \Sigma^\dagger + \Sigma \chi^\dagger) \tag{3}$$

with χ containing the quark-mass matrix $M = diag(m_u, m_d, m_s)$ and transforming as a $(3, 3^*) + (3^*, 3)$ representation of $SU(3)_L \times SU(3)_R$. At this lowest order, ChPT essentially coincides with Current Algebra.

The next order piece of the chiral expansion (fourth order) contains one-loop corrections with vertices from (1) to (3) and a series of ten counterterms required to cancel loop divergencies [3]. Some of them, e.g.,

$$\mathcal{L}^{(4)}_9 = -ieL_9 F_{\mu\nu} Tr(Q\ \mathcal{D}^\mu \Sigma\ \mathcal{D}^\nu \Sigma^\dagger + Q\ \mathcal{D}^\mu \Sigma^\dagger\ \mathcal{D}^\nu \Sigma) \tag{4}$$

are chirally SU(3)-symmetric, whereas others, e.g.,

$$\mathcal{L}_5^{(4)} = L_5 Tr\left(\mathcal{D}_\mu\Sigma\mathcal{D}^\mu\Sigma^\dagger\left(\chi\Sigma^\dagger + \Sigma\chi^\dagger\right)\right) \tag{5}$$

break the symmetry as $\mathcal{L}_m^{(2)}$ in eq.(3). At this one-loop level, one obtains [3]

$$\frac{f_K}{f_\pi} = 1 + \frac{5}{4}\mu_\pi - \frac{1}{2}\mu_K - \frac{3}{4}\mu_\eta + \frac{8}{f^2}(m_K^2 - m_\pi^2)L_5(\mu) \tag{6}$$

where loop effects appear through the so-called chiral-logs, $\mu_P = \frac{m_P^2}{16\pi^2 f^2}\; ln\frac{m_P^2}{\mu^2}$. Similarly, the pseudoscalar electromagnetic charge-radii are found to be [3] .

$$\begin{aligned}
< r^2_{\pi^+} > &= \frac{-1}{16\pi^2 f^2}\left(3 + 2\; ln\frac{m_\pi^2}{\mu^2} + ln\frac{m_K^2}{\mu^2}\right) + \frac{24}{f^2}L_9(\mu) \\
< r^2_{K^+} > &= \frac{-1}{16\pi^2 f^2}\left(3 + \; ln\frac{m_\pi^2}{\mu^2} + 2\; ln\frac{m_K^2}{\mu^2}\right) + \frac{24}{f^2}L_9(\mu) \\
-< r^2_{K^0} > &= < r^2_{\pi^+} > - < r^2_{K^+} > = \frac{1}{16\pi^2 f^2}\; ln\frac{m_K^2}{m_\pi^2}
\end{aligned} \tag{7}$$

where, again, the divergencies accompanying the chiral-logs are absorbed in one of the counterterms (or low-energy constants), L_9, appearing in $\mathcal{L}^{(4)}$.

The large number of low-energy constants, which need to be determined along the lines just discussed, reduces considerably the predictive power of ChPT, unless one adopts the so called resonance saturation hypothesis[4]. According to this suggestion, the values of these constants are related to the exchange of the known meson resonances, so far ignored in the original ChPT. This is a very attractive possibility which has been successfully verified in many cases. In most of them, particularly in processes involving the Wess-Zumino anomalous action [5], vector-mesons turn out to play the dominant role. Their couplings to pseudoscalar plus photon states, as extracted from experiments, saturate an important part of the above counterterms. Accurate effective lagrangians incorporating additional properties of vector mesons and further details on their dynamics could therefore be extremely useful not only as a self-contained effective theory but also as an auxiliary lagrangian fixing the counterterms in the ChPT context.

We follow the "hidden symmetry approach" of Bando et al.[2] to write the most general lagrangian containing pseudoscalar, vector and (external) electroweak gauge fields with the smallest number of derivatives. At this lowest-order, it is given by the linear combination $\mathcal{L}_A + a\mathcal{L}_V$, a being an arbitrary parameter, of the two lagrangians

$$\begin{aligned}
\mathcal{L}_A &= \frac{-f^2}{8} Tr\left(D_\mu\xi_L \cdot \xi_L^\dagger - D_\mu\xi_R \cdot \xi_R^\dagger\right)^2 \\
\mathcal{L}_V &= \frac{-f^2}{8} Tr\left(D_\mu\xi_L \cdot \xi_L^\dagger + D_\mu\xi_R \cdot \xi_R^\dagger\right)^2
\end{aligned} \tag{8}$$

both possessing a global (chiral) $SU(3)_L \times SU(3)_R$ and a local (hidden) $SU(3)_V$ symmetry. The $SU(3)$-valued matrices ξ_L and ξ_R contain the pseudoscalar fields, P, and the unphysical (or compensator) scalar fields, σ, that will be absorbed to give a mass to the vector mesons

$$\xi_{L,R} = exp(i\sigma/f) \cdot exp(\mp iP/f) \tag{9}$$

The full covariant derivative is

$$D_\mu \xi_{L(R)} = (\partial_\mu - igV_\mu)\,\xi_{L(R)} + ie\xi_{L(R)}A_\mu \cdot Q \tag{10}$$

where, as before, only the photon field, A_μ, has been explicitly shown, and P and V stand for the SU(3) octet and nonet matrices

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix} \qquad V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix} \tag{11}$$

Under $SU(3)_L \times SU(3)_R \times SU(3)_V$, the transformation properties of $\xi_{L,R}(x)$ and the $SU(3)_V$ gauge-fields $V_\mu(x)$ are $\xi_{L,R}(x) \to h(x)\xi_{L,R}(x)g^\dagger_{L,R}$ and $V_\mu(x) \to ih(x)\partial_\mu h^\dagger(x) + h(x)V_\mu(x)h^\dagger(x)$, where $g_{L,R} \in SU(3)_{L,R}$ and $h(x)$ parametrizes local, hidden $SU(3)_V$-transformations. The lagrangian $\mathcal{L}_A + a\mathcal{L}_V$ can be reduced to the chiral lagrangian (1) for any value of the parameter a [2]. In fact, working in the unitary gauge, *i.e.* fixing the local $SU(3)_V$ gauge by $\xi^\dagger_L = \xi_R = \xi = exp(iP/f)$ to eliminate the unphysical scalar fields, and substituting the solution of the equation of motion for V_μ, the $\mathcal{L}_V$ part vanishes and $\mathcal{L}_A$ becomes identical to the non linear chiral lagrangian (1).

The "hidden symmetry" lagrangian $\mathcal{L}_A + a\mathcal{L}_V$ (8) can be easily seen to contain, among other things, a vector meson mass term, the pseudoscalar weak decay constants, the vector-photon conversion factor and the couplings of both vectors and photons to pseudoscalar pairs. The latter can be eliminated fixing $a = 2$, thus incorporating conventional vector-dominance in the electromagnetic form-factors of pseudoscalars. With this lagrangian one can therefore attempt to describe the following sets of data (that we take from [6] neglecting error bars when negligible):

the vector meson mass spectrum

$$M^2_{\rho,\omega},\ M^2_{K^*},\ M^2_\phi = 0.60,\ 0.80,\ 1.04\ GeV^2; \tag{12}$$

the weak decay constants

$$f_\pi = 132\ MeV,\quad f_K = 160\ MeV; \tag{13}$$

the $\rho - \gamma, \omega - \gamma$ and $\phi - \gamma$ conversion factors, $f_{V\gamma}$. This fixes the gauge coupling g, once the respective quark charges in $V \to e^+e^-$ have been taken into account, to

$$g = 3.6 \pm 0.1,\ 4.0 \pm 0.1,\ 4.0 \pm 0.1; \tag{14}$$

the $g_{\rho\pi\pi} = \sqrt{2}\ g$, $g_{K^*K\pi} = \sqrt{3}\ g/\sqrt{2}$ and $g_{\phi KK} = \sqrt{2}\ g$ decay constants, which lead similarly to

$$g = 4.3,\ 4.5,\ 4.6 \tag{15}$$

with negligible errors. From [7] we take the following experimental values for the electromagnetic (e.m.) charge radii

$$\begin{aligned} < r^2_{\pi^+} > &= 0.44 \pm 0.03\ fm^2 \\ < r^2_{K^+} > &= 0.31 \pm 0.05\ fm^2 \\ - < r^2_{K^0} > &= 0.054 \pm 0.026\ fm^2 \end{aligned} \tag{16}$$

and the combined result, free from most systematic errors (see Dally et al. and Amendolia et al. in ref.[7])

$$< r^2_{\pi^+} > - < r^2_{K^+} > = 0.13 \pm 0.04\ fm^2 \tag{17}$$

The most immediate possibility to account for the above sets of data consists in using the "hidden symmetry lagrangian" (8) as a self contained effective theory in the exact SU(3) limit. In this case, the SU(3)-breaking effects shown by some of the above data remain unexplained, but two successful relations can be obtained for the non-strange sector. The lagrangian (8) predicts $M^2_{\rho,\omega} = 2g^2f^2$ in the $0.50 - 0.60\ GeV^2$ range when the values (13, 14, 15) for f and g are used. This agrees with the value obtained from the direct measurement of the ρ, ω mass (12). Moreover, it also agrees with that coming from the pion charge radius, leading in our approach to $M^2_\rho = 6/ < r^2_\pi >= 0.51\ GeV^2$. Alternatively, one can regard the lagrangian (8) in a ChPT context, include the chiral-logs from the loop corrections and saturate the finite part of the required counterterms with (8). This maintains the successful relation $M^2_{\rho,\omega} = 2g^2f^2$ and gives $L_9 = f^2/4M^2_\rho$, with a vanishing value for the SU(3)-breaking low-energy counterterm L_5. With this value for L_9, and evaluating the chiral-log correction at the conventional value $\mu = M_\rho$, (7) predicts $< r^2_\pi > = 0.46\ fm^2$ in complete agreement with experiment (16).

The next step is to include SU(3)-breaking effects. Since the pseudoscalar sector is known to break the symmetry as in eq.(3), *i.e.*, proportionally to $Tr(\xi_R M \xi_L^\dagger + \xi_L M \xi_R^\dagger)$, we incorporate SU(3) symmetry breaking, as already attempted in ref.[2], via a similar hermitian combination $(\xi_L \epsilon \xi_R^\dagger + \xi_R \epsilon \xi_L^\dagger)$ in both $\mathcal{L}_A$ and $\mathcal{L}_V$ terms, *i.e.*,

$$\mathcal{L}_A + \Delta\mathcal{L}_A = \frac{-f^2}{8} Tr \left\{ \left(D_\mu \xi_L \cdot \xi_L^\dagger - D_\mu \xi_R \cdot \xi_R^\dagger \right)^2 \left[1 + \left(\xi_L \epsilon_A \xi_R^\dagger + \xi_R \epsilon_A \xi_L^\dagger \right) \right] \right\} \tag{18}$$

and

$$\mathcal{L}_V + \Delta\mathcal{L}_V = \frac{-f^2}{8} Tr \left\{ \left(D_\mu \xi_L \cdot \xi_L^\dagger + D_\mu \xi_R \cdot \xi_R^\dagger \right)^2 \left[1 + \left(\xi_L \epsilon_V \xi_R^\dagger + \xi_R \epsilon_V \xi_L^\dagger \right) \right] \right\} \tag{19}$$

The matrix $\epsilon_{A(V)}$ is taken to be $\epsilon_{A(V)} = diag(0, 0, c_{A(V)})$, where $c_{A,V}$ are the SU(3) -breaking real parameters to be determined. Notice that the SU(3)-breaking terms,

$\Delta\mathcal{L}_{A,V}$, are hermitian, unlike those in ref.[2]. Working in the unitary gauge $\xi_L^\dagger = \xi_R = \xi = exp(iP/f)$, $(\sigma = 0)$, and expanding in terms of the pseudoscalar fields, one observes that the kinetic terms in $\mathcal{L}_A$ have to be renormalized. This is simply achieved by rescaling the pseudoscalar fields [2]

$$\begin{aligned} \sqrt{1+c_A}\, K &\rightarrow K \\ \sqrt{1+2c_A/3}\, \eta &\rightarrow \eta \end{aligned} \tag{20}$$

where an η-η' mixing angle of -19.5^0 has been used for the η case.

The physical content of this new, SU(3)-broken "hidden symmetry" lagrangian (18) and (19) can now be easily worked out. From $a(\mathcal{L}_V + \Delta\mathcal{L}_V)$, we obtain the conventional SU(3)-splitting for the vector meson masses ($a = 2$).

$$M_\rho^2 = M_\omega^2 = 2g^2 f^2, \qquad M_{K^*}^2 = M_\rho^2(1+c_V), \qquad M_\phi^2 = M_\rho^2(1+2c_V) \tag{21}$$

For the V-γ couplings, the corresponding part of the lagrangian is explicitly given by:

$$\begin{aligned} \mathcal{L}_{V\gamma} &= -egf^2 A_\mu\, Tr\left[\{Q, V^\mu\}\ (1+2\epsilon_V)\right] \\ &= -\frac{eM_{\rho,\omega}^2}{\sqrt{2}g} A_\mu \left[\rho^{0\mu} + \frac{\omega^\mu}{3} - (1+2c_V)\frac{\sqrt{2}}{3}\phi^\mu\right] \end{aligned} \tag{22}$$

The new terms in the lagrangian (18,19) also induce an SU(3) symmetry breaking in the g_{VPP} coupling constants. One obtains

$$\begin{aligned} g_{\rho\pi\pi} &= \sqrt{2}\, g \\ g_{\rho KK} &= g_{\omega KK} = \frac{g}{\sqrt{2}} \frac{1}{1+c_A} \\ g_{\phi KK} &= \sqrt{2}\, g\, \frac{1+2c_V}{1+c_A} \\ g_{K^*K\pi} &= \frac{\sqrt{3}}{\sqrt{2}}\, g\, \frac{(1+c_V)}{\sqrt{1+c_A}} \end{aligned} \tag{23}$$

where the c_A-dependence comes from the symmetry breaking in the $\mathcal{L}_A + \Delta\mathcal{L}_A$ lagrangian due to the renormalization of the pseudoscalar fields (see eq.(20)). This redefinition of the pseudoscalar fields also implies symmetry breaking in the pseudoscalar meson decay constants, namely,

$$\begin{aligned} f_K &= \sqrt{1+c_A}\, f_\pi \\ f_\eta &= \sqrt{1+2c_A/3}\, f_\pi \end{aligned} \tag{24}$$

One can now attempt a description of the whole set of data (12-16) solely in terms of the SU(3)-broken lagrangian (18) and (19). This fixes the values of the two new free parameters to $c_V = 0.30$ and $c_A = 0.45$. The fit is completely satisfactory for the four sets of data quoted in eqs.(12) to (15). For the pseudoscalar charge radii, one gets from eqs.(22, 23, 24)

$$\begin{aligned} < r^2_{\pi^+} > &= \frac{6}{M^2_\rho} \\ < r^2_{K^+} > &= \frac{< r^2_{\pi^+} >}{(1+c_A)} \frac{1}{3} \left(2 + (1+2c_V)\frac{M^2_{\rho,\omega}}{M^2_\phi}\right) \\ - < r^2_{K^0} > &= \frac{< r^2_{\pi^+} >}{(1+c_A)} \frac{1}{3} \left(1 - (1+2c_V)\frac{M^2_{\rho,\omega}}{M^2_\phi}\right) \end{aligned} \quad (25)$$

and the above values of c_V and c_A imply

$$< r^2_{\pi^+} >= 0.39\ fm^2, \qquad < r^2_{K^+} >= 0.26\ fm^2, \qquad - < r^2_{K^0} >= 0.01\ fm^2 \quad (26)$$

somewhat below , by one or two σ's, the experimental data (16). As previously discussed, an alternative, more sophisticated possibility is to use our SU(3)-broken lagrangian (18,19) in conjunction with ChPT. This can only modify the predictions for f_P (24) and $< r^2_P >$ (25) related to processes without vector-mesons in the external legs. The chiral-logs in eq.(6), evaluated at the conventional value $\mu = M_\rho$, account now for some 35% of the observed difference between f_K and f_π, thus requiring a smaller contribution from the L_5 counterterm and, hence, a smaller value for c_A. Accordingly, the best global fit is now achieved by the slightly modified values

$$c_V = 0.28 \qquad\qquad c_A = 0.36 \quad (27)$$

which preserve the goodness of the preceding fit for M_V, $f_{V\gamma}$, g_{VPP} and f_P, while improving the agreement in the $< r^2_P >$ sector. Indeed, adding the chiral loop contributions to the charge radii (25), one obtains

$$\begin{aligned} < r^2_{\pi^+} > &= \frac{-1}{16\pi^2 f^2}\left(3 + 2\ ln\frac{m^2_\pi}{\mu^2} + ln\frac{m^2_K}{\mu^2}\right) + \frac{6}{M^2_\rho} \\ - < r^2_{K^0} > &= \frac{1}{16\pi^2 f^2}\ ln\frac{m^2_K}{m^2_\pi} + \frac{2}{1+c_A}\left[\frac{1}{M^2_\rho} - \frac{(1+2c_V)}{M^2_\phi}\right] \\ < r^2_{\pi^+} > - < r^2_{K^+} > &= \frac{1}{16\pi^2 f^2}\ ln\frac{m^2_K}{m^2_\pi} + \frac{6}{M^2_\rho} - \frac{2}{1+c_A}\left[\frac{2}{M^2_\rho} + \frac{(1+2c_V)}{M^2_\phi}\right] \end{aligned} \quad (28)$$

The chiral-logs enhance the previous predictions for $< r^2_P >$ and we find

$$< r^2_{\pi^+} >= 0.46 fm^2, \qquad < r^2_{K^+} >= 0.32 fm^2, \qquad - < r^2_{K^0} >= 0.043 fm^2 \quad (29)$$

in better agreement with the data (16).

Notice that the last two expressions in eq.(28), which explicitly contain SU(3) breaking effects, considerably improve the one-loop ChPT results [3] quoted in the last line of eq.(7), namely $-<r^2_{K^0}>=<r^2_{\pi^+}>-<r^2_{K^+}>=0.036\ fm^2$, which was significantly below the data (17). This improvement stems from the fact that in saturating the ChPT counterterms with our SU(3)-broken lagrangian one goes beyond the fourth order counterterms in ChPT, such as the SU(3)-symmetric counterterm L_9, and introduces corrections from its SU(3)-breaking analogues belonging to sixth order counterterm(s) in $\mathcal{L}^{(6)}$. This is explicitly seen in eqs.(28), which reduce to the conventional ChPT result (7) only in the exact SU(3) limit $c_A = c_V = 0$ and $M^2_{\rho,\omega} = M^2_\phi$. We summarize the above in Table I, where we indicate the results obtained for the charge radii in the three models so far discussed, *i.e.*, Chiral Perturbation Theory (ChPT) and SU(3) broken "Hidden Symmetry" scheme (HSS) with and without chiral loops.

Table I (all data are in fm^2)

	exp.[7]	*ChPT*[3]	*SU(3)broken HSS*	*SU(3)broken HSS + Ch loops*
$<r^2_{\pi^+}>$	0.44 ± 0.03	0.44	0.39	0.46
$<r^2_{K^+}>$	0.31 ± 0.05	0.40	0.26	0.32
$-<r^2_{K^0}>$	0.054 ± 0.026	0.036	0.01	0.043
$<r^2_{\pi^+}>-<r^2_{K^+}>$	0.13 ± 0.04	0.036	0.13	0.14

One can easily extend the above results for the e.m. charge radii of pseudoscalars to include their weak analogue in K_{l3} decays. Sirlin's theorem [8], valid up to first order in SU(3)-breaking, and requiring

$$<r^2_{K\pi}>-<r^2_{K^0}>=\frac{1}{2}<r^2_{\pi^+}>+\frac{1}{2}<r^2_{K^+}> \tag{30}$$

provides the clue. The data (16) and the experimental value [6] $<r^2_{K\pi}>=0.36 \pm 0.02\ fm^2$ are fully compatible with Sirlin's theorem (30). On the other hand, one immediately sees that the predictions of our first-order SU(3)-breaking lagrangian verify the theorem, thus providing a check of our calculations and their automatic extension to the $K\pi$ case. From the expression

$$<r^2_{K\pi}>=\frac{1+c_V}{\sqrt{1+c_A}}\frac{6}{M^2_{K^*}}+\ chiral\ loop\ contributions \tag{31}$$

where the contribution from chiral loops is the one to be found in ref. [3], we obtain the very acceptable value $<r^2_{K\pi}>=0.33\ fm^2$. The above equation can be compared with the chiral perturbation theory result [3]

$$<r^2_{K\pi}>=\ chiral\ loop\ contributions\ +\frac{24}{f^2}L_9(\mu) \tag{32}$$

which gives $< r^2_{K\pi} >= 0.38\ fm^2$.

An extension of the present treatment to processes related to the anomaly or Wess-Zumino lagrangian is also possible and will be discussed elsewhere. Here we simply note that introducing our value $c_A = 0.36$, eq.(27), in eq.(24) for f_η, leads to $\Gamma(\eta \rightarrow \gamma\gamma) = 0.53$ KeV quite in line with the measured decay width [6], i.e. $\Gamma(\eta \rightarrow \gamma\gamma) = 0.46 \pm 0.05$ KeV.

In summary, we have presented a model for low energy hadronic interactions in which SU(3)-breaking effects in the vector-meson sector are introduced in chiral lagrangians including vector-mesons as (hidden symmetry) gauge fields. By comparing a set of low energy data with the model predictions, we have shown that it is possible to extract values for the SU(3) breaking parameters which are very reasonable and which can lead to predictions for the electromagnetic charge radii of the pseudoscalar mesons in good agreement with experimental data. Further applications, primarily to the anomalous sector of the lagrangian, are being studied and will be presented in a forthcoming paper.

Acknowledgements

Thanks are due to M. Lavelle for a critical reading of the manuscript. This work has been supported in part by the Human Capital and Mobility Programme, EEC Contract # CHRX-CT920026, and by CICYT, AEN 93-0520(A.B.) and AEN 93-0615(A.G.).

References

[1] U.-G. Meissner, Phys. Rep. 161 (1988) 213.

[2] M. Bando, T. Kugo and K. Yamawaki, Nucl. Phys. *B*259 (1985) 493, and Phys. Rep. 164 (1988) 217.

[3] J. Gasser and H. Leutwyler, Nucl. Phys. *B*250 (1985) 465, 517, 539.

[4] G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. *B*321 (1989) 311.
J.F. Donoghue, C. Ramirez and G. Valencia, Phys. Rev. *D*39 (1989) 1947.

[5] J. Bijnens, Int. J. Mod. Phys. *A*8 (1993) 3045 and references therein.

[6] *Particle Data Group*, Phys. Rev. *D*45 (1992) 1.

[7] A. Quenzer et al., Phys. Lett. *B*76 (1977) 512.
E.B. Dally et al., Phys. Rev. Lett. 39 (1977) 1176.
E.B. Dally et al., Phys. Rev. Lett. 48 (1982) 375.
S.R. Amendolia et al., Phys. Lett. *B*178 (1986) 435.
W.R. Molzon et al., Phys. Rev. Lett. 41 (1978) 1231.
C. Erkal and M.G. Olsson, J. Phys. G: Nucl. Phys. 13 (1987) 1355.

[8] A. Sirlin, Phys. Rev. Lett. 43 (1979) 904; Ann. Phys.(N.Y.) 61 (1970) 294.