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Radiative Vector–Meson Decays in SU(3) Broken Effective Chiral Lagrangians

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Abstract

Conventional SU(3)-breaking effects are shown to be easily introduced in the sector of effective chiral lagrangians incorporating vector mesons and containing the VVP vertices related to the anomaly. An improved description of $VP\gamma$ and $P\gamma\gamma$ transitions is obtained.

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New e^+e^- -machines in project or under construction will certainly improve our knowledge on vector-meson decays and properties. In particular, the DAPHNE ϕ -factory is expected to provide some 10^{10} ϕ -meson decays per year[1], and, if the energy is lowered, good decay data for its SU(3)-partners ρ^0 and ω too. Therefore, the traditional arena of radiative $V \rightarrow P\gamma$ transitions for the low-mass vector-mesons seems to deserve further reconsideration.

Our purpose is to study the effects of SU(3) symmetry breaking for radiative decays of vector mesons, in the framework of effective chiral lagrangians including vector fields. There are several models dealing with the incorporation of vector mesons in effective lagrangians. Special mention has to be paid to two recent approaches: the massive Yang-Mills approach [2], and the “hidden symmetry” scheme [3]. Although both are equivalently successful in reproducing some low energy relations, the different way of incorporating the vector mesons in the low-energy lagrangians can lead to different predictions in some specific cases [4].

Here we shall follow the “hidden symmetry” scheme (see ref.[3] for details), in which vector mesons are incorporated as the gauge fields of a hidden local symmetry. Electromagnetic interactions can be further introduced in the usual way through the covariant derivative, which finally acquires the form

$$D_\mu \xi_{L(R)} = (\partial_\mu - igV_\mu) \xi_{L(R)} + ie\xi_{L(R)} A_\mu \cdot Q \quad (1)$$

where V_μ stands for the vector meson nonet

$$V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix} \quad (2)$$

and A_μ and Q are the photon field and the quark charge matrix. In eq.(1), the matrices ξ_L, ξ_R , contain exponentiated fields

$$\xi_{L,R} = \exp(i\sigma/f) \cdot \exp(\mp iP/f) \quad (3)$$

where P is the pseudoscalar matrix,

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} \end{pmatrix} \quad (4)$$

and σ are unphysical scalar fields that will be absorbed to give mass to the vector mesons. $f = f_\pi = 132 \text{ MeV}$ stands for the pion decay constant and a small SU(3)-singlet component has been added for the η through an $\eta - \eta'$ mixing angle $\theta = -\arcsin(1/3) = -19.5^\circ$.

In this approach [3], the most general linear Lagrangian with the smallest number of derivatives is given by the linear combination $\mathcal{L}_A + a\mathcal{L}_V$, a being an arbitrary parameter usually fixed to be $a = 2$ to reproduce VMD [5], with

$$\begin{aligned}\mathcal{L}_A &= \frac{-f_\pi^2}{8} \text{Tr} \left(D_\mu \xi_L \cdot \xi_L^\dagger - D_\mu \xi_R \cdot \xi_R^\dagger \right)^2 \\ \mathcal{L}_V &= \frac{-f_\pi^2}{8} \text{Tr} \left(D_\mu \xi_L \cdot \xi_L^\dagger + D_\mu \xi_R \cdot \xi_R^\dagger \right)^2\end{aligned}\quad (5)$$

In addition, there is an anomalous part of the lagrangian (the Wess-Zumino term, WZ) describing $P\gamma\gamma$ interactions. The incorporation of vector-mesons in the part of the lagrangian related to this WZ-anomaly has been studied by Fujiwara et al.[6]. It contains the vector-vector-pseudoscalar meson interactions VVP whose behaviour under $SU(3)$ symmetry breaking will be the central issue of our discussion.

We start considering $SU(3)$ breaking in the normal (non-anomalous) part of the Lagrangian. As we have mentioned, the expression for the unbroken Lagrangian at lowest order is $\mathcal{L} = \mathcal{L}_A + a\mathcal{L}_V$ with \mathcal{L}_A and \mathcal{L}_V given by eq.(5). Symmetry breaking in the pseudoscalar mass sector is introduced through the quark-mass matrix $M = \text{diag}(m_u, m_d, m_s)$ appearing in an additional term of the lagrangian

$$\mathcal{L}_M = \frac{f_\pi^2 \mu}{2} \text{Tr}(MU + U^\dagger M) = \frac{f_\pi^2 \mu}{2} \text{Tr}(\xi_R M \xi_L^\dagger + \xi_L M \xi_R^\dagger) \quad (6)$$

and transforming as a $(3, 3^*) + (3^*, 3)$ representation of chiral $SU(3)$, with μM directly related to the physical masses m_π and m_K .

Quite analogously, we introduce $SU(3)$ breaking in the Lagrangian \mathcal{L}_A via a similar combination $(\xi_L \epsilon_A \xi_R^\dagger + \xi_R \epsilon_A \xi_L^\dagger)$ in the following additional $\Delta\mathcal{L}_A$ term

$$\mathcal{L}_A + \Delta\mathcal{L}_A = \frac{-f_\pi^2}{8} \text{Tr} \left\{ \left(D_\mu \xi_L \cdot \xi_L^\dagger - D_\mu \xi_R \cdot \xi_R^\dagger \right)^2 \left[1 + (\xi_L \epsilon_A \xi_R^\dagger + \xi_R \epsilon_A \xi_L^\dagger) \right] \right\} \quad (7)$$

and similarly for \mathcal{L}_V

$$\mathcal{L}_V + \Delta\mathcal{L}_V = \frac{-f_\pi^2}{8} \text{Tr} \left\{ \left(D_\mu \xi_L \cdot \xi_L^\dagger + D_\mu \xi_R \cdot \xi_R^\dagger \right)^2 \left[1 + (\xi_L \epsilon_V \xi_R^\dagger + \xi_R \epsilon_V \xi_L^\dagger) \right] \right\} \quad (8)$$

The matrix $\epsilon_{A(V)}$ in the breaking term is taken to be $\epsilon_{A(V)} = \text{diag}(0, 0, c_{A(V)})$, where $c_{A(V)}$ is a constant. Eqs.(7) and (8) are the simplest hermitian expressions incorporating $SU(3)$ -breaking in a canonical way to the unbroken "hidden symmetry lagrangian" (5). They differ from those discussed in refs.[3], [7] in that our expressions (7) and (8) are hermitian; they are similar in simplicity and number of derivatives.

Fixing the gauge $\xi_L^\dagger = \xi_R = \xi = \exp(iP/f)$ to eliminate the unphysical scalar fields and expanding in terms of the pseudoscalars, one observes that the kinetic terms have to be renormalized. This is achieved by rescaling the pseudoscalar fields

and this redefinition leads to the following relations for the pseudoscalar meson decay constants [7]:

$$\begin{aligned} f_K &= \sqrt{1 + c_A} f_\pi \\ f_\eta &= \sqrt{1 + 2c_A/3} f_\pi \end{aligned} \quad (9)$$

From these equations and the experimental results [8]

$$\frac{f_K}{f_\pi} = 1.22 \pm 0.02, \quad \frac{f_\eta}{f_\pi} = 1.06 \pm 0.08, \quad (10)$$

the later coming from the ratio between the η and π decay widths into two photons, one gets

$$c_A = 0.44 \pm 0.04. \quad (11)$$

Since our purpose is to calculate the effects of SU(3) symmetry breaking in radiative decays of vector mesons, we have to consider the part of the Lagrangian related to the WZ anomaly. In particular we concentrate on vector-vector-pseudoscalar meson (VVP) interactions as contained in the SU(3)-symmetric VVP lagrangian introduced in ref. [3]. Inserting as before the additional, symmetry breaking term ($\xi_L \epsilon \xi_R^\dagger + \xi_R \epsilon \xi_L^\dagger$) in an appropriate way to get an hermitian Lagrangian, the total broken Lagrangian can now be written as

$$\mathcal{L}_{VVP} + \Delta \mathcal{L}_{VVP} = \frac{G_P}{\sqrt{2}} \epsilon^{\mu\nu\rho\sigma} T_\tau [\partial_\mu V_\nu (1 + 2\epsilon_{WZ}) \partial_\rho V_\sigma P] \quad (12)$$

where $\epsilon_{WZ} = \text{diag}(0, 0, c_{WZ})$, c_{WZ} is the breaking parameter in the anomalous sector, and $G_P = \frac{3\sqrt{2}g^2}{4\pi^2 f_P}$ is the strong VVP coupling constant, where the c_A -dependent f_P factor (see eq.(9)) already includes the part of the effects of SU(3) breaking coming from the renormalization of the pseudoscalars fields in the \mathcal{L}_A lagrangian.

With this conventions, $G_\pi = \frac{3\sqrt{2}g^2}{4\pi^2 f_\pi}$ is the $\rho^0 \omega \pi^0$ coupling constant which contains no SU(3)-breaking and whose value can be obtained from the experimental radiative decay width [8] $\Gamma(\omega \rightarrow \pi^0 \gamma) = 0.72 \pm 0.05$ MeV. Alternatively, for $g = 4.1 \pm 0.2$ (coming from $\rho \rightarrow \pi\pi$ and $\rho, \omega \rightarrow e^+ e^-$ decay data [8]) and $f_\pi = 132$ MeV, we also obtain $\Gamma(\omega \rightarrow \pi^0 \gamma) = 0.74$ MeV in good agreement with the experimental result. In our normalization, the radiative decay widths for vector mesons are given by

$$\Gamma(V \rightarrow P\gamma) = \frac{1}{3} \alpha g_{VP\gamma}^2 q_\gamma^3, \quad q_\gamma = \frac{M_V^2 - M_P^2}{2M_V} \quad (13)$$

where the relevant coupling constants can be written as

$$g_{VP\gamma} = \sum_{V'} \frac{g_{VV'P} g_{V'\gamma}}{M_{V'}^2} \quad (14)$$

taking into account that these decays proceed via intermediate vector mesons V' . Thanks to this, one immediately recovers the successful relation $g_{\pi\gamma\gamma} = \sqrt{2} \alpha/\pi f_\pi$ coming from the WZ anomaly and fully satisfied by the experimental $\pi^0 \rightarrow \gamma\gamma$ decay rate.

The coupling constants $g_{V\gamma}$ are easily obtained from the lagrangian $a(\mathcal{L}_V + \Delta\mathcal{L}_V)$ (eq.(8)). An expansion in terms of the pseudoscalar fields shows that these couplings are essentially unmodified by SU(3) breaking. In fact, the corresponding part of the Lagrangian is given by:

$$\begin{aligned} \mathcal{L}_{V\gamma} &= -egf^2 A_\mu \text{Tr} \{ \{Q, V^\mu\} (1 + 2\epsilon_V) \} \\ &= -egf^2 A_\mu \left[\rho^{0\mu} + \frac{\omega^\mu}{3} - (1 + 2c_V) \frac{\sqrt{2}}{3} \phi^\mu \right] \end{aligned} \quad (15)$$

where c_V is the SU(3) breaking parameter in \mathcal{L}_V . However, the additional terms $a\Delta\mathcal{L}_V$ in eq.(8), also induce an SU(3) breaking on the vector meson masses in such a way that $2g^2 f^2 (1 + 2c_V) = M_\phi^2$ while $2g^2 f^2 = M_{\rho,\omega}^2$ [3, 7]. Then, the V- γ couplings, are simply given by the following expressions:

$$g_{\rho\gamma} = 3 g_{\omega\gamma} = M_{\rho,\omega}^2 / \sqrt{2}g \quad g_{\phi\gamma} = -M_\phi^2 / 3g \quad (16)$$

which coincide with those from the unbroken case provided the value of c_V is exclusively fixed to fit the V-mass spectrum (as we have chosen to do), i.e. $c_V = 0.36$.

According to the last two paragraphs, SU(3)-breaking effects in $V \rightarrow P\gamma$ decays proceed exclusively through the $g_{VV'P}$ factor in (14), which depends on c_A and c_{WZ} as indicated in eq.(12) and the subsequent discussion. Using eqs.(14), (16) and the broken VVP Lagrangian (12) to extract the $g_{VV'P}$ couplings, we obtain the following expressions for the coupling constants $g_{VP\gamma}$

$$\begin{aligned} g_{\omega\pi^0\gamma} &= \frac{G_\pi}{\sqrt{2}g} & g_{\rho^0\pi^0\gamma} &= \frac{1}{3}g_{\omega\pi^0\gamma} \\ g_{\omega\pi\pi} &= \frac{\sqrt{2}}{3\sqrt{3}} \frac{f_\pi}{f_\eta} (1 - \sqrt{2}\epsilon') g_{\omega\pi^0\gamma} & g_{\rho\pi\pi} &= \frac{\sqrt{2}}{\sqrt{3}} \frac{f_\pi}{f_\eta} g_{\omega\pi^0\gamma} \\ g_{\phi\pi\pi} &= \frac{2}{3\sqrt{3}} \frac{f_\pi}{f_\eta} \left(1 + 2c_{WZ} + \frac{\epsilon'}{\sqrt{2}} \right) g_{\omega\pi^0\gamma} & g_{\phi\pi^0\gamma} &= \epsilon' g_{\omega\pi^0\gamma} \\ g_{K^0\pi^0\gamma} &= -\frac{2}{3} \frac{f_\pi}{f_K} (1 + c_{WZ}) g_{\omega\pi^0\gamma} & g_{K^0\pi^\pm K^\pm\gamma} &= \frac{1}{3} \frac{f_\pi}{f_K} (1 - 2c_{WZ}) g_{\omega\pi^0\gamma} \end{aligned} \quad (17)$$

The parameter ϵ' appearing (at leading order) in the expressions of $g_{\phi\pi^0\gamma}$, $g_{\phi\pi\pi}$ and $g_{\omega\pi\pi}$ stands for the small contamination of non-strange (strange) quarks in the ϕ (ω) meson. Assuming that $\epsilon' = 0.058 \pm 0.004$, we reproduce the experimental value for $\phi \rightarrow \rho\pi \rightarrow \pi^+\pi^-\pi^0$ and the observed $\omega - \phi$ interference effects (thus fixing ϵ' sign) in

$e^+e^- \rightarrow \pi^+\pi^-\pi^0$ [8]. Similarly, the value of the anomalous breaking parameter c_{WZ} can be directly obtained from the ratio between the experimental decay widths [8] $K^{*0} \rightarrow K^0\gamma$ and $K^{*\pm} \rightarrow K^\pm\gamma$. The ratio depends only of c_{WZ} and leads immediately to $c_{WZ} = -0.10 \pm 0.03$.

We can now calculate the V radiative decay widths following eq.(13) with the coupling constants given by eq.(17). Our results, obtained using $g = 4.1 \pm 0.2$ and $\epsilon' = 0.058 \pm 0.004$, as well as $c_A = 0.44 \pm 0.04$ and $c_{WZ} = -0.10 \pm 0.03$ for the symmetry breaking parameters, are shown in Table 1. For comparison we also include the corresponding experimental decay widths as taken from ref.[8] (in the $\rho \rightarrow \pi\gamma$ case we have averaged for neutral and charged decays). The description of all these data turns out to be quite satisfactory, with SU(3)-breaking effects playing a central role in some cases. As already noted by Hajuj [7], a non-vanishing value for c_A (thus achieving $f_\pi < f_\eta < f_K$) is essential to reduce the predicted $\phi \rightarrow \eta\gamma$ and $K^{*0} \rightarrow K^0\gamma$ decay rates to their experimental values. Our value $c_{WZ} = -0.10 \pm 0.03$ is also crucial to improve the results of ref.[7] (particularly, for the K^* radiative decays) where such a source of SU(3)-breaking has been neglected. As mentioned, the other SU(3)-breaking parameter c_V is fixed here so as to satisfy the relation $M_\phi^2 = (1 + 2c_V)M_\rho^2$.

Our final comment concerns an alternative use of the above SU(3)-broken lagrangians incorporating vector mesons. Up to this point they have been used as effective low-energy lagrangians accounting for the whole relevant dynamics. This is a reasonable and economic attitude but, certainly, not the only possible one. Due to the successes of Chiral Perturbation Theory [9] (ChPT) in describing low-energy interactions of the pseudoscalar octet, one can adopt the alternative attitude of using our lagrangians in conjunction with the ChPT approach. In this case one has to assume that the (finite part of the) low-energy constants or counterterms required in the full ChPT lagrangian [9] turn out to be dominated by the exchange of meson resonances (mainly, V-mesons) as described by our effective lagrangians. Such a resonance saturation hypothesis was already introduced by Gasser and Leutwyler in their pioneering work in ChPT and has been extensively confirmed (for a review in the anomalous sector, see [10]). More precisely, the present authors [11] have recently achieved a reasonable and accurate description of a substantial part of the low-energy phenomenology (not related to the anomalous sector) for vector and pseudoscalar mesons along these lines. Such a description lead to fit values for the SU(3)-breaking parameters in reasonable agreement with those used here, namely, $c_A = 0.36$ and $c_V = 0.28$, which are also in line with the results of refs. [7] and [3]. For completeness, we extend our previous results to the anomalous sector showing also in Table 1 our predictions for the above values of $c_{A,V}$ and the slightly different preferred value $c_{WZ} = -0.05$. Again, the agreement is quite good and the decision of how optimizing the use of our lagrangians remains open waiting for improved data and analyses.

In summary, well-known SU(3)-breaking effects has been easily shown to be introduced in effective lagrangians incorporating vector-mesons. In particular, the VVP interactions, related to radiative vector-meson decays – for which accurate new data

are expected – and to the anomalous π , $\eta \rightarrow \gamma\gamma$ decays, are accurately described, improving the results of previous related work [7].

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Table I

<i>Decay</i>	$\Gamma(\text{keV})$	$\Gamma(\text{keV})$	$\Gamma(\text{keV})$
$V \rightarrow P\gamma$	exp. (ref.[8])	$c_A = 0.44 \pm 0.04$ $c_{WZ} = -0.1 \pm 0.03$	$c_A = 0.36$ $c_V = 0.28$ $c_{WZ} = -0.05$
$\omega \rightarrow \pi^0\gamma$	716 ± 43	740 ± 70	740
$\rho \rightarrow \pi\gamma$	76 ± 10	67 ± 6	67
$\omega \rightarrow \eta\gamma$	4 ± 2	5.2 ± 0.6	5.4
$\rho \rightarrow \eta\gamma$	58 ± 11	47 ± 6	49
$\phi \rightarrow \eta\gamma$	56.7 ± 2.8	53 ± 8	55
$\phi \rightarrow \pi^0\gamma$	5.8 ± 0.6	5.7 ± 0.7	5.7
$K^{*0} \rightarrow K^0\gamma$	117 ± 10	101 ± 14	107
$K^{*\pm} \rightarrow K^\pm\gamma$	50 ± 5	44 ± 6	47