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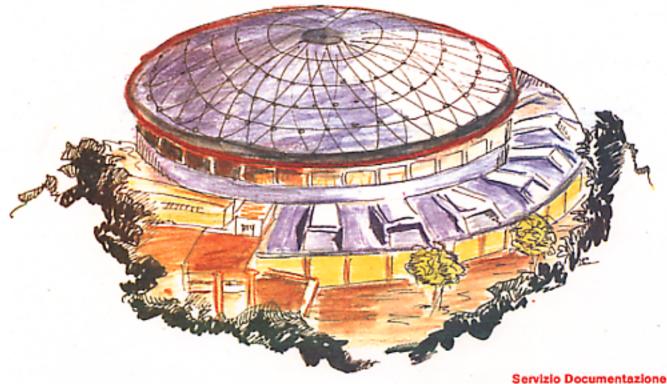
LNF-94/017 (P) 29 Marzo 1994

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PACS.: 13.60.Le

To appear in the Proc. of the Meeting on Two-Photon Physics from DAΦNE to LEP 200 and beyond 2-4 February (1994), Paris



Servizio Documentazione dei Laboratori Nazionali di Frascati P.O. Box, 13 - 00044 Frascati (Italy) Servizio Documentazione

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TWO-PHOTON REACTIONS BEYOND ONE-LOOP

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Abstract

We review the recent progress in the calculation of the amplitude for $\gamma\gamma \to \pi^0\pi^0$ to two loops in chiral perturbation theory. We match the low-energy amplitude in chiral perturbation theory with the result of the dispersion theoretic analysis. The neutral pion polarizabilities are also given to two-loop accuracy. Then, the results are compared with the dispersion relation calculation of the pion polarizabilities.

[#] Work supported in part by the EEC Human Capital and Mobility Program. To appear in the proceedings of the meeting on Two-Photon Physics from DAΦNE to LEP200 and beyond, 2-4 February 1994, Paris.

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The process producing a pion pair in the fusion of two photons has received a lot of attention in the recent years. It provides a very important test of chiral perturbation theory (CHPT) [1]-[5]. In this sense, the production of neutral pion pairs is the most interesting channel. This is because the Born amplitude vanishes in this case. Hence the one-loop scattering amplitude is finite and does not depend on the free parameters of the chiral lagrangian [6, 7]. It is by now a well-known fact that the one-loop cross section for $\gamma\gamma \to \pi^0\pi^0$ in CHPT [6, 7] does not agree with the experimental measurements at Crystal Ball [8], as well as with calculations based on dispersion relations [9]-[15], even at low-energy. We calculated recently the low-energy $\gamma\gamma \to \pi^0\pi^0$ amplitude to two-loops in CHPT [16] and obtained a prediction that agrees with the Crystal Ball data. Also, the low-energy CHPT amplitude compares very well with the dispersive analysis of $\gamma\gamma \to \pi^0\pi^0$ by Donoghue and Holstein [12].

The sum of the electric and magnetic polarizabilities of the neutral pion vanishes to lowest order in CHPT [17]. The value of this sum was estimated also by a sum rule, and it turned out to be different from zero [18]. We showed some time ago [19] that a vector dominance model preserving the chiral symmetry of QCD at low energy yields a value close to the one obtained from the sum rule. We obtained recently a refinement of this prediction, by calculating the two-loop Compton scattering amplitude in CHPT [16]. The largest modification in the polarizabilities, with respect to the one-loop order value, is given by the omega resonance exchange which accounts for a large fraction of the neutral pion sum rule [19], and the contribution from the chiral logarithms is small [16].

The $\gamma\gamma \to \pi^0\pi^0$ cross section for off-shell photons has been calculated recently [20] in the framework of CHPT. There, it has been shown that the measurement of the azymuthal correlations in the process $e^+e^- \to e^+e^-\pi^0\pi^0$ allows to test the higher order CHPT corrections independently from the measurement of the cross-section.

Recently, CHPT has been reformulated to include [21] into each order additional terms which in the standard CHPT are of higher order. Within this generalization of the chiral expansion of the amplitude, the process $\gamma\gamma \to \pi^0\pi^0$ has been analyzed [22].

Gauge symmetry and Lorentz invariance can be used to write the scattering matrix element

$$<\pi^{0}(p_{1})\pi^{0}(p_{2})\text{out} \mid \gamma(q_{1})\gamma(q_{2})\text{in}> = i(2\pi)^{4}\delta^{4}(P_{f}-P_{i})T^{N}$$
, (0.1)

with

$$T^{N} = e^{2} \epsilon_{1}^{\mu} \epsilon_{2}^{\nu} V_{\mu\nu} ,$$

$$V_{\mu\nu} = i \int dx e^{-i(q_{1}x+q_{2}y)} < \pi^{0}(p_{1})\pi^{0}(p_{2}) \text{out} \mid T j_{\mu}(x) j_{\nu}(y) \mid 0 >, \qquad (0.2)$$

where j_{μ} is the electromagnetic current, and $\alpha = e^2/4\pi \simeq 1/137$, as follows:

$$V_{\mu\nu} = A(s,t,u)T_{1\mu\nu} + B(s,t,u)T_{2\mu\nu} + ,$$

$$T_{1\mu\nu} = \frac{s}{2}g_{\mu\nu} - q_{1\nu}q_{2\mu} ,$$

$$T_{2\mu\nu} = 2s\Delta_{\mu}\Delta_{\nu} - \nu^{2}g_{\mu\nu} - 2\nu(q_{1\nu}\Delta_{\mu} - q_{2\mu}\Delta_{\nu}) ,$$

$$\Delta_{\mu} = (p_{1} - p_{2})_{\mu} ,$$

$$(0.3)$$

in terms of the standard Mandelstam variables

$$s = (q_1 + q_2)^2, t = (p_1 - q_1)^2, u = (p_2 - q_1)^2,$$

 $\nu = t - u.$ (0.4)

One can go from the analytic functions A and B of the variables s,t and u, symmetric under crossing $(t,u) \to (u,t)$, to the helicity amplitudes in the following way:

$$H_{++} = A + 2(4M_{\pi}^{2} - s)B ,$$

$$H_{+-} = \frac{8(M_{\pi}^{4} - tu)}{s}B . \qquad (0.5)$$

In Ref. [16] the renormalization procedure is formulated in the minimal subtraction scheme, and the expressions of the order E^6 renormalized amplitudes involve three parameters, i.e. a_1^r , a_2^r and b^r

$$A_6 = \frac{a_1^r M^2 + a_2^r s}{(16\pi^2 F^2)^2} + \cdots, \qquad (0.6)$$

$$B_6 = \frac{b^r}{(16\pi^2 F^2)^2} + \cdots , \qquad (0.7)$$

where the ellipses stand for finite contributions from the loop-integrals. Here F is the pion decay constant in the chiral limit, $F_{\pi} = F(1 + O(\hat{m})), F_{\pi} \simeq 93$ MeV, and the physical pion mass is

$$M_{\pi}^2 = M^2(1 + O(\hat{m})),$$

 $M^2 = 2\hat{m}B,$ (0.8)

Table 1: Resonance contributions to the coupling constants a_1^r, a_2^r and b^r . Column 6 contains the sums of those contributions which have a definite sign.

	I^R					I^R	
I^r	ω	$ ho^{0}$	φ	$A(1^{+-})$	$\sum_{R} I^{R}$	$S(0^{++})$	f_2
a_1^r	-33.2	-6.1	-0.1	0.0	-39	±0.8	∓4.1
$\mid a_2^r \mid$	12.5	2.3	$\simeq 0$	-1.3	13	± 1.3	± 1.0
b^r	2.1	0.4	$\simeq 0$	0.7	3	0.0	± 0.5

where B is related to the order parameter $< 0 \mid \bar{q}q \mid 0 >$, and in the isospin symmetry limit $m_u = m_d = \hat{m}$.

The three unknown parameters a_1^r , a_2^r and b^r are estimated by resonance saturation [16] and the results are displayed in table 1. The ω exchange yields the dominant contribution.

The low-energy constants h^r_{\pm} and h^r_s corresponding to the helicity amplitudes H_{++} and H_{+-} read

$$H_{++}^{2loops} = \frac{1}{(16\pi^2 F^2)^2} \left\{ h_+^r M^2 + h_s^r s \right\} + \cdots ,$$

$$H_{+-}^{2loops} = \frac{8(M^4 - tu)}{s(16\pi^2 F^2)^2} h_-^r + \cdots ,$$

$$h_+^r = a_1^r + 8b^r , h_s^r = a_2^r - 2b^r , h_-^r = b^r . \tag{0.9}$$

From column 6 in table 1 the central values of these couplings are obtained in Ref. [16], where a 30% uncertainty is associated to the contributions generated by the vector and axial-vector exchange, and a 100% error to the contributions from scalars and from f_2 . Adding these errors in quadrature, one finds [16]

$$h_{+}^{r}(M_{\rho}) = -14 \pm 5$$
,
 $h_{s}^{r}(M_{\rho}) = 7 \pm 3$, (0.10)
 $h_{-}^{r}(M_{\rho}) = 3 \pm 1$.

The values of h_+^r and h_s^r are not affected by the tensor exchange, since the corresponding coupling is purely D-wave. Scalars do not contribute to the value of h_-^r .

In Ref. [23], these couplings have been determined i) from vector-meson exchange

and using nonet-symmetry, and ii) from the chiral quark model, with the result

$$(h_{+}^{r}, h_{s}^{r}, h_{-}^{r})|_{\mu=M_{\rho}} = \begin{cases} (-18, 9, 2) & \text{vector-mesons (nonet)} \\ (-12, 6, 2) & \text{chiral quark model} \end{cases}$$
(0.11)

which agrees within the uncertainties with the values in (0.10).

The following analytic results are obtained in Ref. [16] for the amplitude A to two loops:

$$A = \frac{4\bar{G}_{\pi}(s)}{sF_{\pi}^{2}}(s - M_{\pi}^{2}) + U_{A} + P_{A} + O(E^{4}). \tag{0.12}$$

The unitary part U_A contains s, t and u-channel cuts, and P_A is a linear polynomial in s. Explicitly,

$$U_{A} = \frac{2}{sF_{\pi}^{4}}\bar{G}(s)\left[\left(s^{2}-M_{\pi}^{4}\right)\bar{J}(s)+C(s,\bar{l}_{i})\right] + \frac{\bar{l}_{\Delta}}{24\pi^{2}F_{\pi}^{4}}(s-M_{\pi}^{2})\bar{J}(s) + \frac{\left(\bar{l}_{2}-5/6\right)}{144\pi^{2}sF_{\pi}^{4}}(s-4M_{\pi}^{2})\left\{\bar{H}(s)+4\left[s\bar{G}(s)+2M_{\pi}^{2}(\bar{\bar{G}}(s)-3\bar{\bar{J}}(s))\right]d_{00}^{2}\right\} + \Delta_{A}(s,t,u) ,$$

$$(0.13)$$

with

$$C(s,\bar{l}_i) = \frac{1}{48\pi^2} \left\{ 2(\bar{l}_1 - 4/3)(s - 2M_\pi^2)^2 + (\bar{l}_2 - 5/6)(4s^2 - 8sM_\pi^2 + 16M_\pi^4)/3 - 3M_\pi^4 \bar{l}_3 + 12M_\pi^2(s - M_\pi^2)\bar{l}_4 - 12sM_\pi^2 + 15M_\pi^4 \right\} ,$$

$$d_{00}^2 = \frac{1}{2} (3\cos\theta^2 - 1) . \tag{0.14}$$

The loop-functions \bar{J} etc. are given in appendix C of Ref. [16].

The polynomial part is

$$P_{A} = \frac{1}{(16\pi^{2}F_{\pi}^{2})^{2}} [a_{1}M_{\pi}^{2} + a_{2}s] ,$$

$$a_{1} = a_{1}^{r} + \frac{1}{18} \left\{ 4l^{2} + l(8\bar{l}_{2} + 12\bar{l}_{\Delta} - \frac{4}{3}) - \frac{20}{3}\bar{l}_{2} + 12\bar{l}_{\Delta} + \frac{110}{9} \right\} ,$$

$$a_{2} = a_{2}^{r} - \frac{1}{18} \left\{ l^{2} + l(2\bar{l}_{2} + 12\bar{l}_{\Delta} + \frac{2}{3}) - \frac{5}{3}\bar{l}_{2} + 12\bar{l}_{\Delta} + \frac{697}{144} \right\} ,$$

$$l = \ln \frac{M^{2}}{\mu^{2}} . \qquad (0.15)$$

Here μ denotes the renormalization scale, the \bar{l}_i are the renormalized coupling constants of the $O(E^4)$ lagrangian [2], and $\bar{l}_{\Delta} = \bar{l}_6 - \bar{l}_5$. The values of these coupling

constants can be found in column 2 of table 1 of Ref. [16]. The result for B is

$$B = U_B + P_B + O(E^2) , \qquad (0.16)$$

with unitary part

$$U_B = \frac{(\bar{l}_2 - 5/6)\bar{H}(s)}{288\pi^2 F_-^4 s} + \Delta_B(s, t, u) . \qquad (0.17)$$

The polynomial is obtained in Ref. [16]

$$P_B = \frac{b}{(16\pi^2 F_{\pi}^2)^2} ,$$

$$b = b^r - \frac{1}{36} \left[l^2 + l(2\bar{l}_2 + \frac{2}{3}) - \frac{\bar{l}_2}{3} + \frac{393}{144} \right] . \tag{0.18}$$

The integrals $\Delta_{A,B}(s,t,u)$ contain contributions that very small for the cross sections below $\sqrt{s} \leq 400$ MeV, both for $\gamma\gamma \to \pi^0\pi^0$ (0.1% at 400 MeV) and for the crossed channel $\gamma\pi^0 \to \gamma\pi^0$ (1.5% at 400 MeV) [16].

The cross section $\gamma\gamma \to \pi^0\pi^0$ receives a substantial correction near threshold due to $\pi\pi$ final-state interactions – which are absent in Compton scattering. Yet, it is shown in Ref. [16] that the two-loop contributions are not small in this channel. Since in the one-loop approximation the amplitude H_{++} is one order of magnitude larger in the $\gamma\gamma \to \pi^0\pi^0$ channel than at Compton threshold, even very small corrections in $\gamma\gamma \to \pi^0\pi^0$ may appear large in Compton scattering [19]. The result of the two-loop calculation and the one-loop approximation differ by one order of magnitude already near threshold [16]. This is mainly due to the effect of the low-energy constant h_-^r in H_{+-} (omega-exchange in the language of resonance saturation [19]).

The low-energy limit of the coupling with the photon in the Compton amplitude for a composite system is characterized (among other parameters) by the electric and magnetic polarizabilities. One can test the hadron dynamics through experiments on the hadron polarizabilities [18]. The expansion of the amplitude for charged pion Compton scattering,

$$\gamma(q_1)\pi^+(p_1) \to \gamma(q_2)\pi^+(p_2)$$
 , (0.19)

near threshold reads

$$T^{C} = 2\left[\vec{\epsilon}_{1} \cdot \vec{\epsilon}_{2}^{\star} \left(\frac{\alpha}{M_{\pi}} - \bar{\alpha}_{\pi}\omega_{1}\omega_{2}\right) - \bar{\beta}_{\pi} \left(\vec{q}_{1} \times \vec{\epsilon}_{1}\right) \cdot \left(\vec{q}_{2} \times \vec{\epsilon}_{2}^{\star}\right) + \cdots\right] \quad (0.20)$$

with $q_i^{\mu} = (\omega_i, \vec{q}_i)$. For neutral pions, one has, in terms of A and B,

$$\bar{\alpha}_{\pi^0} = \frac{\alpha}{2M_{\pi}} (A + 16M_{\pi}^2 B)|_{s=0, t=M_{\pi}^2} ,$$

$$\bar{\beta}_{\pi^0} = -\frac{\alpha}{2M_{\pi}} A|_{s=0, t=M_{\pi}^2} . \qquad (0.21)$$

Below we denote

$$(\alpha \pm \beta)^{C} = \bar{\alpha}_{\pi} \pm \bar{\beta}_{\pi} ,$$

$$(\alpha \pm \beta)^{N} = \bar{\alpha}_{\pi^{0}} \pm \bar{\beta}_{\pi^{0}} . \qquad (0.22)$$

The pion polarizabilities have been estimated¹ through dispersion sum rules [18]

$$(\alpha + \beta)^C = 0.39 \pm 0.04$$
 ,
 $(\alpha - \beta)^C = 10 \pm 3$,
 $(\alpha + \beta)^N = 1.04 \pm 0.07$,
 $(\alpha - \beta)^N = -10 \pm 4$. (0.23)

The charged pion polarizabilities have been determined in an experiment on the radiative pion-nucleus scattering $\pi^- A \to \pi^- \gamma A$ [24] and in the pion photoproduction process $\gamma p \to \gamma \pi^+ n$ [25]. Assuming the constraint $(\alpha + \beta)^C = 0$ the two experiments yield

$$(\alpha - \beta)^C = \begin{cases} 13.6 \pm 2.8 & [24] \\ 40 \pm 24 & [25] \end{cases}. \tag{0.24}$$

Relaxing the constraint $(\alpha + \beta)^C = 0$, one obtains from the Serpukhov data

$$(\alpha + \beta)^C = 1.4 \pm 3.1(\text{stat.}) \pm 2.5(\text{sys.})$$
 [26],
 $(\alpha - \beta)^C = 15.6 \pm 6.4(\text{stat.}) \pm 4.4(\text{sys.})$ [26]. (0.25)

At one-loop one has [17, 27]

$$\bar{\alpha}_{\pi^0} = -\bar{\beta}_{\pi^0} = -\frac{\alpha}{96\pi^2 M_{\pi} F^2} = -0.50$$
 (0.26)

At order $O(E^6)$ we found [16]

$$\bar{\alpha}_{\pi^0} = -0.50 + 0.21 - 0.07 \simeq -0.35$$
,
 $\bar{\beta}_{\pi^0} = 0.50 + 0.79 + 0.24 \simeq 1.50$ (0.27)

where the three contributions that add up to the final results on the r.h.s. are the one-loop, the resonance, and the two-loop contributions, respectively. There is a

¹The values of the polarizabilities are in units of 10⁻⁴fm³ in what follows.

large contribution from the resonance exchange. Our results saturates the forward sum rule $(\alpha + \beta)^N = 1.04 \pm 0.07$ in (0.23)

$$(\alpha + \beta)^N = 1.0 + 0.16 \simeq 1.15$$
 (0.28)

Information on the charged pion polarizabilities may be obtained from $\gamma\gamma \to \pi^+\pi^-$ data [27]. The low-energy constant \bar{l}_{\triangle} appears as the only free parameter order in the $O(E^4)$ amplitude, as well as in the leading-order expression for $\bar{\alpha}_{\pi}$ and $\bar{\beta}_{\pi}$. A fit to the cross section then determines $\bar{\alpha}_{\pi}$ and $\bar{\beta}_{\pi}$. The result [27] $\bar{l}_{\triangle} = 2.3 \pm 1.7$ corresponds to numerical value for the leading-order $\bar{\alpha}_{\pi} = 2.7 \pm 0.4$, plus systematic uncertainties due to the $O(E^6)$ corrections. For the charged pions, a two-loop calculation is not yet available. The charged pion polarizabilities are given beyond the one-loop order by including the meson resonance contribution in Refs. [28, 19]

The construction of unitarized S-wave amplitudes for $\gamma\gamma \to \pi\pi$ which contain $(\alpha - \beta)^{C,N}$ as adjustable parameters has been carried out in Ref. [15]. In this case, only $(\alpha - \beta)^{C,N}$ can be determined from the data [8, 29], with the result

$$(\alpha - \beta)^C = 4.8 \pm 1.0 [15],$$

 $(\alpha - \beta)^N = -1.1 \pm 1.7 [15].$ (0.29)

The value (0.29) for $(\alpha - \beta)^N$ is consistent with the two-loop result for the neutral pion, whereas the corresponding calculation for charged pions is not available and so it cannot be compared with the value (0.29) for $(\alpha - \beta)^C$.

Finally, it is interesting to compare the chiral expansion [16] with the dispersive calculation carried out by Donoghue and Holstein [12]. The two representations of the S-wave amplitude agree numerically very well below $E=0.4\ GeV$. In the dispersive method, higher order terms are partially summed up. The agreement indicates that yet higher orders in the chiral expansion do not affect much the threshold amplitude.

Acknowledgements

I am very grateful to the organizers of this meeting for creating such a pleasant working environment. I take this opportunity to thank warmly my collaborators in this reasearch D. Babusci, J. Gasser, G. Giordano, G. Matone and M. Sainio.

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