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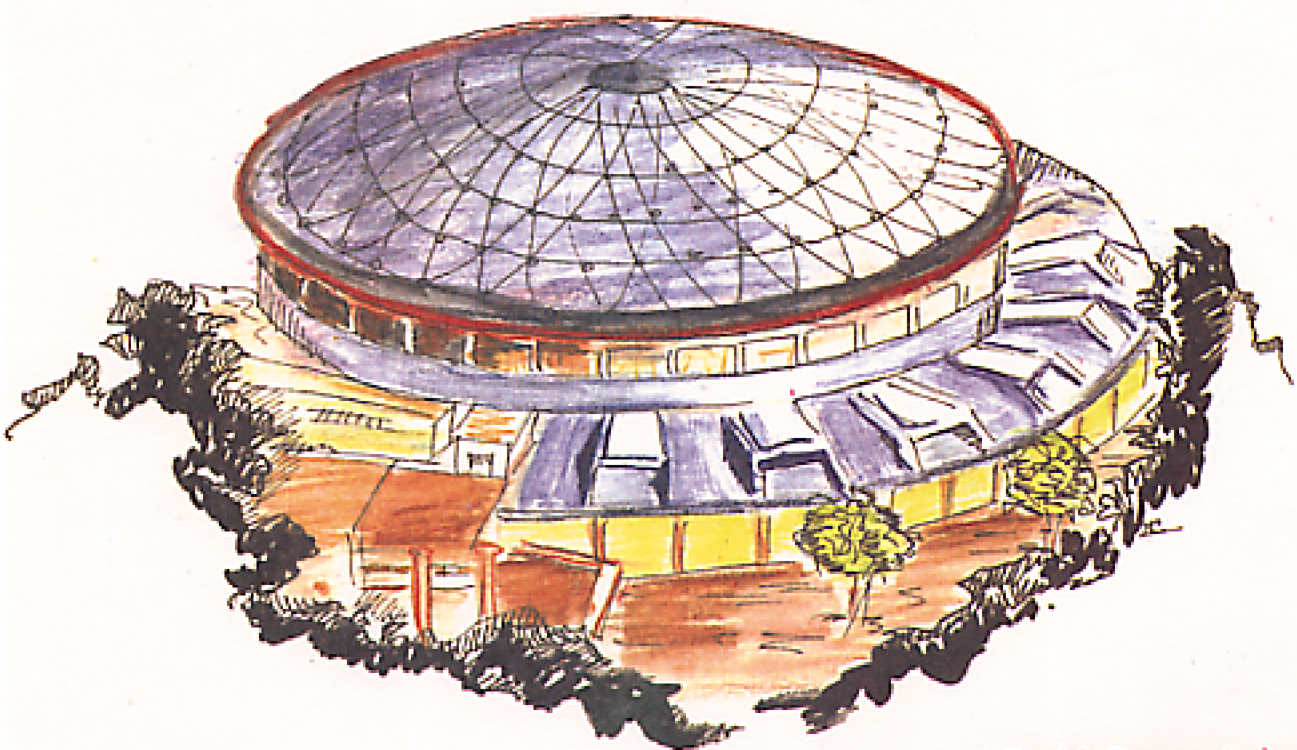

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N = 2 SUPER–$W_3^{(2)}$ ALGEBRA

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Abstract

We construct an $N = 2$ superextension of Polyakov-Bershadsky $W_3^{(2)}$ algebra with an arbitrary central charge in the framework of Polyakov "soldering" procedure. It contains as non-intersecting subalgebras $N = 2$ superconformal algebra and $W_3^{(2)}$ and can be regarded as a nonlinear closure of these two. Besides the currents generating these subalgebras, it involves two pairs of fermionic currents with spins 1 and 2. A hybrid fields-currents realization of this $N = 2$ super-$W_3^{(2)}$ is presented.

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1. Introduction

During last few years, an impressive progress has been achieved in understanding the procedure of supersymmetrizing W-type nonlinear algebras and a number of nonlinear superalgebras have been explicitly constructed, both at the classical and quantum levels (see, e.g., Ref.[1] and references therein). Among nonlinear bosonic algebras there exist special algebras which contain the bosonic currents with non-canonical half-integer spins [2-6]. Until present, no their supersymmetric extensions were known in an explicit form.

The $W_3^{(2)}$ algebra [2, 3] provides the simplest nontrivial example of such an algebra. It is a bosonic analog of the linear $N = 2$ superconformal algebra(SCA) [7] and contains, besides two currents with the spins 2 and 1 (conformal stress-tensor and $U(1)$-Kac-Moody current), also two currents with non-canonical spins 3/2. It is interesting to look for its supersymmetric extensions. Clearly, if existing, these should involve fermionic currents with non-canonical integer spins. In the present letter we construct an $N = 2$ supersymmetric extension of the $W_3^{(2)}$ algebra and its realization on a set of fields and currents.

2. Preliminaries

To simplify our task, we assume as a starting point that the $N = 2$ super-$W_3^{(2)}$ algebra we are looking for contains both $W_3^{(2)}$ and $N = 2$ SCA as subalgebras and does not include additional bosonic currents besides those present in these subalgebras. Let us stress that here we are interested only in those $N = 2$ superextensions of $W_3^{(2)}$ which contain it as a genuine subalgebra. A more general problem of classifying all superalgebras which combine the fractional spin 3/2 currents with $N = 2$ supersymmetry is beyond the scope of the present letter.

To understand, how $W_3^{(2)}$ and $N = 2$ SCA could be embedded in $N = 2$ super-$W_3^{(2)}$, let us firstly discuss some analogies between these algebras, which will allow us to make some suggestive conjectures.

The algebra $W_3^{(2)} \propto \{J_w, G^+, G^-, T_w\}$ and $N = 2$ SCA $\propto \left\{J_s, S, \bar{S}, T_s\right\}$ have the same spin content $\{1, 3/2, 3/2, 2\}$, but while the currents $S, \bar{S}$ are fermionic, their $W_3^{(2)}$ counterparts G^+, G^- are bosonic. The defining operator-product-expansions (OPE) for these algebras at the classical level have the following structure [2, 3, 7] [1]

$\underline{W_3^{(2)}}$

$$
\begin{aligned}
J_w(z_1)J_w(z_2) &= \frac{\frac{c}{6}}{z_{12}^2} \quad , \quad J_w(z_1)T_w(z_2) = \frac{J_w}{z_{12}^2}, \\
J_w(z_1)G^{\pm}(z_2) &= \mp\frac{1}{2}\frac{G^{\pm}}{z_{12}} \quad , \quad T_w(z_1)G^{\pm}(z_2) = \frac{3}{2}\frac{G^{\pm}}{z_{12}^2} + \frac{G^{\pm\prime}}{z_{12}}, \\
T_w(z_1)T_w(z_2) &= -\frac{3c}{z_{12}^4} + \frac{2T_w}{z_{12}^2} + \frac{T_w'}{z_{12}}, \\
G^+(z_1)G^-(z_2) &= \frac{2c}{z_{12}^3} - \frac{6J_w}{z_{12}^2} - (T_w - \frac{12}{c}J_w^2 + 3J_w')\frac{1}{z_{12}},
\end{aligned} \qquad (1)
$$

[1]Hereafter, we explicitly write down only singular terms in OPE's. All the currents appearing in the right-hand sides of the OPE's are evaluated at point z_2. We strictly fix the relation between the central charges of $W_3^{(2)}$ and $N = 2$ SCA. The meaning of this restriction will be clear later.

<u>$N = 2$ SCA</u>

$$\begin{aligned}
J_s(z_1)J_s(z_2) &= \frac{\frac{c}{2}}{z_{12}^2} \quad , \quad J_s(z_1)T_s(z_2) = \frac{J_s}{z_{12}^2}, \\
J_s(z_1)S(z_2) &= \frac{1}{2}\frac{S}{z_{12}} \quad , \quad J_s(z_1)\bar{S}(z_2) = -\frac{1}{2}\frac{\bar{S}}{z_{12}}, \\
S(z_1)\bar{S}(z_2) &= \frac{2c}{z_{12}^3} + \frac{2J_s}{z_{12}^2} + \frac{T_s + J_s'}{z_{12}}, \\
T_s(z_1)S(z_2) &= \frac{3}{2}\frac{S}{z_{12}^2} + \frac{S'}{z_{12}} \quad , \quad T_s(z_1)\bar{S}(z_2) = \frac{3}{2}\frac{\bar{S}}{z_{12}^2} + \frac{\bar{S}'}{z_{12}}, \\
T_s(z_1)T_s(z_2) &= \frac{3c}{z_{12}^4} + \frac{2T_s}{z_{12}^2} + \frac{T_s'}{z_{12}}.
\end{aligned} \tag{2}$$

where $z_{12} = z_1 - z_2$.

One could wonder whether these algebras, while treated as subalgebras of the sought $N = 2$ super-$W_3^{(2)}$, may intersect over some set of their bosonic currents. In other words, may we from the beginning identify, say, J_w with J_s or T_w with T_s ? It turns out that such an identification does not match our assumption about the bosonic currents content of $N = 2$ $W_3^{(2)}$. An inspection of eqs. (1) and (2) shows that both $W_3^{(2)}$ and $N = 2$ SCA are Z_2-graded and the currents with half-integer and integer conformal spins belong to two different Z_2-grading classes. This means, in particular, that all the currents with integer spins appear in the right hand-side of OPE's between the currents with half-integer spins. Therefore, any ad hoc identification of J_s or T_s with J_w or T_w puts some constraints on the fermionic and half-integer spin bosonic currents. We have checked that these constraints contradict Jacobi identities until one introduces some extra currents with half-integer spins, including the bosonic ones. So it is reasonable to assume that under our starting simplifying assumption all the currents of $W_3^{(2)}$ and $N = 2$ SCA are independent, i.e. there is no overlapping between these subalgebras when they are embedded in the $N = 2$ super-$W_3^{(2)}$ algebra. This assumption together with the initial one entirely fix the bosonic currents content of the $N = 2$ $W_3^{(2)}$ algebra we are searching for. It should comprise the $W_3^{(2)}$ currents as well as the currents generating the Virasoro and $U(1)$ subalgebras of $N = 2$ SCA.

3. The $N = 2$ super $W_3^{(2)}$ algebra.

Surprisingly, the conjectured algebraic structure naturally comes out in the framework of Polyakov "soldering" procedure [2]. In this approach one writes down a gauge potential $\mathcal{A}$ valued in the (super)algebra of appropriate (super)group G and performs a "soldering" by putting some components of $\mathcal{A}$ equal to constants. From the residual gauge transformations of the remaining components of $\mathcal{A}$ one can immediately read off the OPE's of some W-algebra, while these components themselves become the currents generating this algebra. As the residual gauge transformations clearly form a closed set, the Jacobi identities of the resulting W algebra prove to be automatically satisfied. Here we apply the "soldering" procedure to the case of supergroup $SL(3|2)$[2].

[2]An analogous approach with the same gauge supergroup $SL(3|2)$ has been used in ref. [8] under a different choice of soldering to derive the classical $N = 2$ super-W_3 algebra.

Let us start with the following "soldering" choice for the $sl(3|2)$-valued gauge potential $\mathcal{A}$:

$$\mathcal{A}=\frac{1}{c}\begin{pmatrix} 2J_s-3J_w & G^+ & T_1 & S_1 & S_2 \\ 0 & 2J_s-6J_w & G^- & 0 & S \\ 1 & 0 & 2J_s-3J_w & 0 & S_1 \\ \bar{S}_1 & \bar{S} & \bar{S}_2 & 3J_s-6J_w & T_2 \\ 0 & 0 & \bar{S}_1 & 1 & 3J_s-6J_w \end{pmatrix}, \quad (3)$$

where $\{J_s, J_w, G^+, G^-, T_1, T_2\}$ and $\left\{S_1, \bar{S}_1, S, \bar{S}, S_2, \bar{S}_2\right\}$ are bosonic and fermionic currents, respectively.

The potential $\mathcal{A}$ possesses the standard infinitesimal gauge transformation law

$$\delta\mathcal{A} = \partial\Lambda + [\mathcal{A}, \Lambda] \quad (4)$$

with the $sl(3|2)$-valued matrix of the parameters Λ

$$\Lambda=\begin{pmatrix} 2l_1+l_2+l_3 & a_1 & a_2 & b_1 & b_2 \\ a_3 & 2l_1-2l_3 & a_4 & b_3 & b_4 \\ a_5 & a_6 & 2l_1-l_2+l_3 & b_5 & b_6 \\ c_1 & c_2 & c_3 & 3l_1+l_4 & a_7 \\ c_4 & c_5 & c_6 & a_8 & 3l_1-l_4 \end{pmatrix}. \quad (5)$$

One can easily find the residual gauge transformations preserving the form (3) of the gauge potential. They correspond to the following parameters

$$l_1, l_3, a_3, a_5, a_6, a_8, b_3, b_5, (b_1+b_6), c_4, c_5, (c_1+c_6). \quad (6)$$

The remaining twelve combinations of the parameters are expressed through these ones and the currents. Then one can obtain the transformations of all the currents in (3) with respect to this restricted class of gauge transformations. We have checked that after representing these transformations in the form

$$\begin{aligned}\delta\phi(z_1) &= c\oint dz_2\left[-6l_1J_s+18l_3J_w+a_3G^++a_6G^-+3a_5T_1-3a_8T_2+b_3\bar{S}+b_5\bar{S}_2\right.\\ &+ \left.(b_1+b_6)\bar{S}_1-c_4S_2-c_5S-(c_1+c_6)S_1\right]\phi(z_2), \end{aligned} \quad (7)$$

where $\phi(z)$ is any current, the self-consistent set of OPE's for the currents can be obtained.

To establish a link with the algebras (1) and (2), one should redefine the currents in the following way:

$$T_1 = T_w + \frac{1}{c}S_1\bar{S}_1 - \frac{3}{c}J_w^2 \quad , \quad T_2 = -T_s - \frac{1}{c}S_1\bar{S}_1 + \frac{1}{c}J_s^2. \quad (8)$$

Then the currents $\{J_w, G^+, G^-, T_w\}$ and $\left\{J_s, S, \bar{S}, T_s\right\}$ can be shown to obey just OPE's (1),(2) and so they form $W_3^{(2)}$ and $N=2$ SCA with the related central charges. Thus we are eventually left with the set of currents consisting of those of $W_3^{(2)}$, $N=2$ SCA and four extra fermionic currents $\left\{S_1, \bar{S}_1, S_2, \bar{S}_2\right\}$ with the integer spins $\{1,1,2,2\}$.

All the currents with the aforementioned spins are primary with respect to the following Virasoro stress-tensor T with a zero central charge :

$$T = T_s + T_w + \frac{4}{c} S_1 \bar{S}_1 - \frac{4}{c} J_s^2 + \frac{12}{c} J_w J_s - \frac{12}{c} J_w^2 \quad , \tag{9}$$

except for the currents T_s and T_w which are quasi primary with the central charges $3c$ and $-3c$ respectively. We have checked that there is no basis in the $N = 2$ super-$W_3^{(2)}$ algebra such that all currents are primary with respect to some Virasoro stress-tensor.

In terms of these currents the whole set of OPE's of our $N = 2$ super-$W_3^{(2)}$ algebra contains, besides the OPE's of the subalgebras $W_3^{(2)}$ and $N = 2$ SCA (1),(2), the following non-trivial relations:

$$S_1(z_1)\bar{S}_1(z_2) = -\frac{\frac{c}{2}}{z_{12}^2} + \frac{\frac{3}{2}J_w - \frac{1}{2}J_s}{z_{12}} \quad , \quad J_s(z_1)S_1(z_2) = -\frac{\frac{1}{2}S_1}{z_{12}},$$

$$J_w(z_1)S_1(z_2) = -\frac{\frac{1}{6}S_1}{z_{12}} \quad , \quad J_s(z_1)J_w(z_2) = \frac{\frac{c}{3}}{z_{12}^2},$$

$$J_s(z_1)T_w(z_2) = \frac{2J_w}{z_{12}^2} \quad , \quad J_s(z_1)G^+(z_2) = -\frac{G^+}{z_{12}},$$

$$J_s(z_1)S_2(z_2) = -\frac{\frac{1}{2}S_2}{z_{12}} \quad , \quad J_w(z_1)T_s(z_2) = \frac{\frac{2}{3}J_s}{z_{12}^2} \quad , \quad J_w(z_1)S_2(z_2) = -\frac{\frac{1}{6}S_2}{z_{12}},$$

$$T_s(z_1)T_w(z_2) = \frac{\frac{4}{c}\left(S_1\bar{S}_1 + J_wJ_s\right)}{z_{12}^2} + \frac{\frac{2}{c}\left(S_1\bar{S}_2 + S_1\bar{S}_1' - S_2\bar{S}_1 + S_1'\bar{S}_1 + 2J_wJ_s'\right)}{z_{12}},$$

$$T_s(z_1)G^+(z_2) = -\frac{\frac{2}{c}\left(G^+J_s - S_1\bar{S}\right)}{z_{12}}, \quad T_s(z_1)S_1(z_2) = -\frac{\frac{1}{2}S_1}{z_{12}^2} + \frac{S_2 - S_1' - \frac{1}{c}\left(S_1J_s + 3J_wS_1\right)}{2z_{12}},$$

$$T_s(z_1)S_2(z_2) = -\frac{3S_1}{z_{12}^3} + \frac{2S_2 - 3S_1' + \frac{1}{c}\left(3S_1J_s - 9J_wS_1\right)}{2z_{12}^2} + \frac{\frac{2}{c}G^+S - \frac{4}{c}S_1T_s + \frac{3}{c^2}S_1J_s^2 + \frac{1}{c}S_1J_s'}{2z_{12}}$$
$$-\frac{\frac{3}{c}S_2J_s - \frac{3}{c}J_wS_2 - \frac{6}{c^2}J_wJ_sS_1 + \frac{9}{c^2}J_w^2S_1 + \frac{6}{c}J_wS_1' - \frac{2}{c}S_1'J_s + \frac{3}{c}J_w'S_1 - S_2' + S_1''}{2z_{12}},$$

$$T_w(z_1)S_1(z_2) = -\frac{\frac{1}{2}S_1}{z_{12}^2} - \frac{S_2 - \frac{1}{c}S_1J_s + \frac{5}{c}J_wS_1 + S_1'}{2z_{12}} \quad , \quad T_w(z_1)S(z_2) = -\frac{\frac{2}{c}G^-S_1 - \frac{2}{c}J_wS}{z_{12}},$$

$$T_w(z_1)S_2(z_2) = \frac{3S_1}{z_{12}^3} + \frac{2S_2 - \frac{3}{c}S_1J_s + \frac{9}{c}J_wS_1 + 3S_1'}{2z_{12}^2} - \frac{\frac{2}{c}G^+S + \frac{4}{c}S_1T_w - \frac{1}{c^2}S_1J_s^2 + \frac{1}{c}S_1J_s'}{2z_{12}}$$
$$-\frac{\frac{1}{c}S_2J_s - \frac{1}{c}J_wS_2 + \frac{6}{c^2}J_wJ_sS_1 - \frac{21}{c^2}J_w^2S_1 - \frac{6}{c}J_wS_1' + \frac{2}{c}S_1'J_s - \frac{3}{c}J_w'S_1 - S_2' - S_1''}{2z_{12}},$$

$$G^+(z_1)S(z_2) = -\frac{2S_1}{z_{12}^2} - \frac{S_2 - \frac{1}{c}S_1J_s - \frac{3}{c}J_wS_1 + S_1'}{z_{12}} \quad , \quad G^+(z_1)S_2(z_2) = -\frac{\frac{3}{2c}G^+S_1}{z_{12}},$$

$$G^-(z_1)S_1(z_2) = -\frac{\frac{1}{2}S}{z_{12}} \quad , \quad G^-(z_1)S_2(z_2) = \frac{\frac{3}{2}S}{z_{12}^2} - \frac{\frac{1}{c}G^-S_1 + \frac{1}{c}J_sS - \frac{9}{c}J_wS - S'}{2z_{12}},$$

$$S_1(z_1)\bar{S}(z_2) = \frac{G^+}{2z_{12}} \quad , \quad S_1(z_1)\bar{S}_2(z_2) = \frac{T_s + T_w + \frac{2}{c}S_1\bar{S}_1 - \frac{1}{c}J_s^2 - \frac{3}{c}J_w^2}{2z_{12}},$$

$$S(z_1)S_2(z_2) = \frac{3S_1S}{2cz_{12}} \quad , \quad S(z_1)\bar{S}_2(z_2) = -\frac{3G^-}{2z_{12}^2} - \frac{\frac{3}{c}G^-J_s + \frac{1}{c}\bar{S}_1S - \frac{3}{c}J_wG^- + G^{-\prime}}{2z_{12}},$$

$$S_2(z_1)S_2(z_2) = \frac{2S_1S_2}{cz_{12}},$$
$$S_2(z_1)\bar{S}_2(z_2) = \frac{3c}{z_{12}^4} + \frac{3J_s - 9J_w}{z_{12}^3} + \frac{2T_s - 2T_w + \frac{1}{c}J_s^2 - \frac{18}{c}J_wJ_s + \frac{33}{c}J_w^2 + 3J_s' - 9J_w'}{2z_{12}^2}$$
$$+ \frac{\frac{1}{c}G^+G^- + \frac{1}{c}S_1\bar{S}_2 + \frac{1}{c}S\bar{S} + \frac{1}{c}S_2\bar{S}_1 - \frac{1}{2c^2}J_s^3 + \frac{1}{c}T_sJ_s - \frac{1}{c}T_wJ_s - \frac{3}{c}J_wT_s + \frac{3}{c}J_wT_w - \frac{3}{2c^2}J_wJ_s^2}{z_{12}}$$
$$+ \frac{\frac{33}{2c^2}J_w^2J_s - \frac{45}{2c^2}J_w^3 - \frac{9}{2c}J_wJ_s' + \frac{1}{2c}J_s'J_s - \frac{9}{2c}J_w'J_s + \frac{33}{2c}J_w'J_w + \frac{1}{2}T_s' - \frac{1}{2}T_w' + \frac{1}{2}J_s'' - \frac{3}{2}J_w''}{z_{12}}. \quad (10)$$

Here we omitted the OPE's which can be obtained from (10) via the discrete automorphisms: $J_{w,s} \to -J_{w,s}$, $G^\pm \to \pm G^\mp$, $S \to \bar{S}$, $\bar{S} \to S$, $S_1 \to \bar{S}_1$, $\bar{S}_1 \to -S_1$,$S_2 \to -\bar{S}_2$, $\bar{S}_2 \to S_2$.

All these OPE's are guaranteed to define a closed nonlinear algebra (with all the Jacobi identities satisfied) because they have been deduced directly from the gauge transformations algebra.

Besides the $W_3^{(2)}$ and $N = 2$ SCA sub-algebras, the obtained $N = 2$ super-$W_3^{(2)}$ algebra contains an affine Kac-Moody supersubalgebra formed by the currents $S_1, \bar{S}_1$ and $3J_w - J_s$, with the local part given by the anticommutator

$$\left\{(S_1)_0, \left(\bar{S}_1\right)_0\right\} = \frac{1}{2}\left(3J_w - J_s\right)_0, \quad (11)$$

all other (anti)commutators vanishing. This algebra is a contraction of $sl(1|1)$.

We close this Section with several comments.

First, despite the fact that the $N = 2$ super-$W_3^{(2)}$ algebra constructed has equal numbers of bosonic and fermionic currents, it seems unlikely that they can be arranged into $N = 2$ supermultiplets. The main obstruction against existence of a superfield description is the fact that in our superalgebra numbers of the currents with integer and half-integer spins do not match with each other while any $N = 2$ superfield clearly contains equal number of components with integer and half-integer spins.

Secondly, a generalization of the proposed construction to the case of $N = 2$ super-$W_n^{(l)}$ is straightforward like in the case of $N = 2$ super-W_n algebras[8]. Starting with the gauge potential valued in the superalgebra $sl(n|n-1)$ and choosing the "soldering" that gives rise to the algebra $W_n^{(l)} \times W_{n-1}^{(l-1)}$ in the bosonic part of $sl(n|n-1)$, one can deduce the relevant OPE's. Of course, there is a lot of other options for "soldering" in the bosonic part of $sl(n|n-1)$. A detailed consideration of these cases is beyond the scope of this letter (a discussions of this subject within the general Drinfeld-Sokolov scheme can be found in [9]).

4. Hybrid fields-currents realization.

The $N = 2$ super-$W_3^{(2)}$ algebra (1),(2),(10) has the same feature as its $W_3^{(2)}$ and $N = 2$ SCA subalgebras, namely, all the currents with integer spins appear in the right hand-side of OPE's between the currents with half-integer spins. So, to construct some realization of $N = 2$ super-$W_3^{(2)}$, it suffices to specify only four spin 3/2 currents G^+, G^-, S and $\bar{S}$. This also implies that the $N = 2$ super-$W_3^{(2)}$ algebra is a closure of its $W_3^{(2)}$ and $N = 2$ SCA subalgebras, providing central charges are related in an appropriate way. Keeping in mind these useful properties, let us construct realizations for $N = 2$ super-$W_3^{(2)}$ algebra on set of the spin 1/2 fields and spin 1 currents.

The minimal realizations of $W_3^{(2)}$ and $N=2$ SCA algebras include the spins $\left\{\frac{1}{2}^B, \frac{1}{2}^B, 1^B, 1^B\right\}$ [3] and $\left\{\frac{1}{2}^F, \frac{1}{2}^F, 1^B, 1^B\right\}$[10], respectively. These multiplets are not large enough to form a realization of $N=2$ super-$W_3^{(2)}$ algebra in their own right. Let us remind that $N=2$ super-$W_3^{(2)}$ is generated by twelve independent currents which are those independent components of the gauge potential $\mathcal{A}$ (3) which transform inhomogeneously (at $c\neq 0$) with respect to the residual gauge transformations (7). So, to reproduce these inhomogeneous transformations, it is necessary to introduce an independent basic field for each current. For the remaining four fermionic currents $S_1, \bar{S}_1, S_2$ and $\bar{S}_2$ of $N=2$ super-$W_3^{(2)}$ algebra we are led to introduce four fermionic spin 1 basic fields (alias currents). Thus, the whole multiplet of the basic fields contains six bosonic fields - $\left\{U_1, U_2, V_1, V_2, \xi, \bar{\xi}\right\}$ and six fermionic ones - $\left\{\lambda_1, \bar{\lambda}_1, \lambda_2, \bar{\lambda}_2, \psi, \bar{\psi}\right\}$ with the spins $\left\{1,1,1,1,\frac{1}{2},\frac{1}{2}\right\}$, respectively, and with the J_s- and J_w- $U(1)$ charges equal to the charges of the currents with which these fields are associated.

Now we are ready to construct the realization of $N=2$ super-$W_3^{(2)}$. Taking the most general Ansatz for the currents (in terms of the introduced basic fields) as well as for the OPE's between basic fields, and requiring the latter to be consistent with the OPE's (1),(2),(10) we obtain the following realization

$$
\begin{aligned}
S &= \sqrt{c}\left(\psi' - \frac{3}{2}V_1\psi\right) - \bar{\xi}\lambda_2 + \frac{1}{\sqrt{c}}\left(\xi\bar{\xi}\psi + U_1\psi + U_2\psi\right), \\
\bar{S} &= \sqrt{c}\left(\bar{\psi}' - \frac{1}{2}V_2\bar{\psi}\right) + \xi\lambda_1 - \frac{1}{\sqrt{c}}\left(\xi\bar{\xi}\bar{\psi} - U_1\bar{\psi}\right), \\
G^+ &= \sqrt{c}\left(V_1\xi - \frac{1}{2}V_2\xi + \xi'\right) + \bar{\lambda}_1\bar{\psi} + \frac{1}{\sqrt{c}}\left(U_1\xi - \xi\xi\bar{\xi} - \xi\psi\bar{\psi}\right), \\
G^- &= -\sqrt{c}\left(\frac{3}{2}V_1\bar{\xi} + V_2\bar{\xi} - \bar{\xi}'\right) + \lambda_2\psi + \frac{1}{\sqrt{c}}\left(\xi\bar{\xi}\bar{\xi} + \bar{\xi}\psi\bar{\psi} + U_1\bar{\xi} + U_2\bar{\xi}\right), \\
J_s &= \frac{c}{4}(3V_1 - V_2) - \frac{1}{2}U_2 - \xi\bar{\xi} - \frac{1}{2}\psi\bar{\psi}, \\
J_w &= \frac{c}{12}(5V_1 + V_2) - \frac{1}{6}U_2 - \frac{1}{2}\xi\bar{\xi} - \frac{1}{3}\psi\bar{\psi}, \\
S_1 &= \frac{\sqrt{c}}{2}\left(\bar{\lambda}_1 + \bar{\lambda}_2\right) - \frac{1}{2}\xi\psi, \\
\bar{S}_1 &= \frac{\sqrt{c}}{2}(\lambda_1 + \lambda_2) - \frac{1}{2}\bar{\xi}\bar{\psi}, \\
S_2 &= \frac{\sqrt{c}}{2}\left(V_2\bar{\lambda}_2 - V_1\bar{\lambda}_1 + \bar{\lambda}_1' - \bar{\lambda}_2'\right) + \frac{3}{2}V_1\xi\psi - \frac{1}{2}\xi\psi' - \frac{1}{2}V_2\xi\psi + \frac{1}{2}\xi'\psi \\
&- \frac{1}{\sqrt{c}}\left(\frac{1}{4}\xi\bar{\xi}\bar{\lambda}_1 - \frac{3}{4}\xi\bar{\xi}\bar{\lambda}_2 + \frac{3}{4}\psi\bar{\psi}\bar{\lambda}_1 - \frac{1}{4}\psi\bar{\psi}\bar{\lambda}_2 - U_1\bar{\lambda}_1 + U_1\bar{\lambda}_2 - \frac{1}{2}U_2\bar{\lambda}_1 + \frac{1}{2}U_2\bar{\lambda}_2\right) \\
&- \frac{1}{2c}\left(\frac{3}{2}\xi\xi\bar{\xi}\psi + U_2\xi\psi\right), \\
\bar{S}_2 &= -\sqrt{c}\left(\frac{1}{2}V_1\lambda_1 - V_1\lambda_2 + V_2\lambda_1 - \frac{1}{2}V_2\lambda_2 - \frac{1}{2}\lambda_1' + \frac{1}{2}\lambda_2'\right) + \frac{1}{2}\bar{\xi}\bar{\psi}' - \frac{1}{2}\bar{\xi}'\bar{\psi} + \frac{1}{2}V_1\bar{\xi}\bar{\psi} + \frac{1}{2}V_2\bar{\xi}\bar{\psi} \\
&+ \frac{1}{\sqrt{c}}\left(\frac{3}{4}\xi\bar{\xi}\lambda_1 - \frac{1}{4}\xi\bar{\xi}\lambda_2 + \frac{1}{4}\psi\bar{\psi}\lambda_1 - \frac{3}{4}\psi\bar{\psi}\lambda_2 + U_1\lambda_1 - U_1\lambda_2 + \frac{1}{2}U_2\lambda_1 - \frac{1}{2}U_2\lambda_2\right)
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{1}{c}\left(\frac{3}{4}\xi\bar{\xi}\xi\bar{\psi} + \frac{1}{2}U_2\xi\bar{\psi}\right), \\
T_s &= \frac{c}{4}(3V_1' + V_2' - 3V_1V_2) + \lambda_1\bar{\lambda}_2 + \frac{1}{2}\psi\bar{\psi}' - \frac{1}{2}\psi'\bar{\psi} - \frac{3}{2}V_1\xi\bar{\xi} + \frac{1}{2}V_2\xi\bar{\xi} + \frac{3}{2}U_1V_1 + \frac{1}{2}U_1V_2 \\
&+ \frac{1}{2}U_2V_2 - U_1' - \frac{1}{2}U_2' - \frac{1}{\sqrt{c}}\left(\xi\lambda_1\psi - \bar{\xi}\bar{\lambda}_2\bar{\psi}\right) + \frac{1}{c}\left(U_2\xi\bar{\xi} - U_1U_1 - U_1U_2 + \xi\xi\bar{\xi}\bar{\xi}\right), \\
T_w &= \frac{c}{4}\left(\frac{7}{3}V_1V_1 + \frac{7}{3}V_1V_2 + \frac{7}{3}V_2V_2 - V_1' - 3V_2'\right) - \bar{\lambda}_1\lambda_2 - \frac{1}{2}\xi\bar{\xi}' + \frac{1}{2}\xi'\bar{\xi} \\
&- \frac{5}{6}V_1\psi\bar{\psi} - \frac{1}{6}V_2\psi\bar{\psi} - \frac{1}{2}U_1V_1 - \frac{3}{2}U_1V_2 - \frac{2}{3}U_2V_1 - \frac{5}{6}U_2V_2 + U_1' + \frac{1}{2}U_2' \\
&- \frac{1}{\sqrt{c}}\left(\xi\lambda_2\psi - \bar{\xi}\bar{\lambda}_1\bar{\psi}\right) + \frac{1}{c}\left(U_1U_1 + U_1U_2 + \frac{1}{3}U_2U_2 + \frac{1}{3}U_2\psi\bar{\psi}\right). \qquad (12)
\end{aligned}
$$

The superalgebra of the basic fields is represented by the following OPE's

$$
\begin{aligned}
&\xi(z_1)\bar{\xi}(z_2) = -\frac{1}{z_{12}}, \quad \psi(z_1)\bar{\psi}(z_2) = -\frac{1}{z_{12}}, \quad \lambda_1(z_1)\bar{\lambda}_1(z_2) = \frac{1}{z_{12}^2} + \frac{V_1}{z_{12}}, \\
&\lambda_2(z_1)\bar{\lambda}_2(z_2) = \frac{1}{z_{12}^2} + \frac{V_2}{z_{12}}, \quad U_2(z_1)V_2(z_2) = -\frac{1}{z_{12}^2}, \quad U_2(z_1)V_1(z_2) = -\frac{1}{z_{12}^2}, \\
&U_1(z_1)V_1(z_2) = \frac{1}{z_{12}^2}, \quad U_1(z_1)\lambda_1(z_2) = \frac{\lambda_1}{z_{12}}, \quad U_1(z_1)\bar{\lambda}_1(z_2) = -\frac{\bar{\lambda}_1}{z_{12}}, \quad U_2(z_1)\lambda_1(z_2) = -\frac{\lambda_1}{z_{12}}, \\
&U_2(z_1)\bar{\lambda}_1(z_2) = \frac{\bar{\lambda}_1}{z_{12}}, \quad U_2(z_1)\lambda_2(z_2) = -\frac{\lambda_2}{z_{12}}, \quad U_2(z_1)\bar{\lambda}_2(z_2) = \frac{\bar{\lambda}_2}{z_{12}}. \qquad (13)
\end{aligned}
$$

It is instructive to examine the structure of the stress-tensor T (9) in this realization

$$
T = -\lambda_1\bar{\lambda}_1 - \lambda_2\bar{\lambda}_2 + \frac{1}{2}\xi'\bar{\xi} - \frac{1}{2}\xi\bar{\xi}' + \frac{1}{2}\psi\bar{\psi}' - \frac{1}{2}\psi'\bar{\psi} + U_1V_1 - U_1V_2 - U_2V_2 + \frac{c}{2}V_1' - \frac{c}{2}V_2' \,. \quad (14)
$$

As we could expect, it is bilinear in all basic fields unlike T_w and T_s in (12). The relations (12) are specified up to possible automorphisms of both the $N=2$ super-$W_3^{(2)}$ algebra and the superalgebra (13). In particular, the OPE's (13) possess the one-parameter automorphism

$$
U_1(x) \to \tilde{U}_1(x) = U_1(x) - \alpha V_2(x) \quad , \quad U_2(x) \to \tilde{U}_2(x) = U_2(x) + \alpha\left(V_2(x) - V_1(x)\right), \quad (15)
$$

with α beings an arbitrary constant.

5. Conclusion

To summarize, we have constructed the classical $N=2$ super-$W_3^{(2)}$ algebra and its realization on a set of the spin 1/2 fields and spin 1 currents.

While our soldering procedure corresponds to the non-principal embedding of $sl(2)$ in $sl(3|2)$, in principle, there is another possibility of combining the bosonic spin 3/2 currents with $N=2$ SCA. Namely, we could consider a non-principal embedding of $sl(2)$ in the $sl(3|1)$ superalgebra rather than in $sl(3|2)$. Simple calculations show that for such a choice the resulting $N=2$ superalgebra does not contain $W_3^{(2)}$ as a genuine bosonic subalgebra, despite the presence of bosonic spin 3/2 currents in it.

In a forthcoming publication [12] we will show how to describe $N=2$ super-$W_3^{(2)}$ constructed here in terms of constrainted superfields in the superspace with a non-standard dimension of Grassman coordinates $\theta, \bar{\theta}$ (0 and 1 instead 1/2 and 1/2). We will also consider the reduction of our $N = 2$ super-$W_3^{(2)}$ algebra to $N = 2$ super-W_3 [8], and extend our consideration to the full quantum $N = 2$ super-$W_3^{(2)}$ algebra [13].

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