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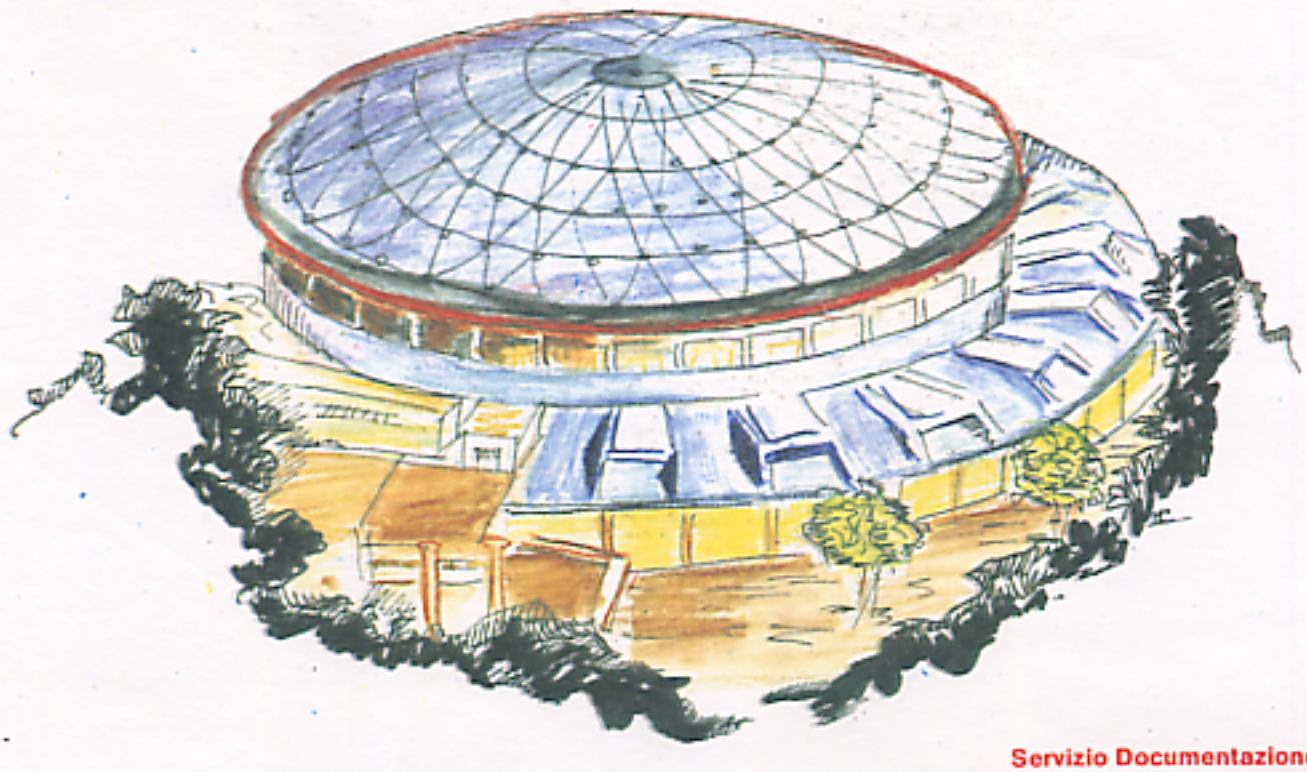

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THE SCHWINGER MODEL ON THE LATTICE IN THE MICROCANONICAL FERMIONIC AVERAGE APPROACH.

V. Azcoiti, V. Laliena,
Departamento de Física Teórica, Facultad de Ciencias, Universidad de Zaragoza, 50009 Zaragoza (Spain)

G. Di Carlo, A. Galante[1] and A.F. Grillo
Istituto Nazionale di Fisica Nucleare, Laboratori Nazionali di Frascati, P.O.B. 13 - Frascati (Italy)

ABSTRACT. The Microcanonical Fermionic Average method has been used so far in the context of lattice models with phase transitions at finite coupling. To test its applicability to Asymptotically Free theories, we have implemented it in QED_2, *i.e.* the Schwinger Model. We exploit the possibility, intrinsic to this method, of studying the whole β, m plane at negligible computer cost, to follow constant physics trajectories and measure the $m \to 0$ limit of the chiral condensate. We recover the continuum result within 3 decimal places.

The Microcanonical Fermionic Average (M.F.A.) method for performing Lattice simulations with dynamical fermions [1] is ideally suited for discussing the phase structure of theories with phase transitions at finite couplings, and it has been applied so far in this context [2, 3].

The conventional wisdom, however, requires that physically interesting theories are

[1] and Dipartimento di Fisica dell'Universitá dell'Aquila, L'Aquila 67100, Italy

Asymptotically Free like QCD. It is then interesting to test the applicability of the M.F.A. method to a theory without phase transitions at finite coupling [4]. In this paper we present an analysis of the Schwinger Model on the lattice. Strictly speaking, the Schwinger Model in the continuum is not Asymptotically Free, since it is superrenormalizable and the Callan-Symanzik β function vanishes. However, in the lattice version, since the continuum coupling is dimensionful, the continuum theory is reached at infinite lattice coupling, much in the same way as four dimensional Asymptotically Free theories like QCD.

The continuum model is confining; it is exactly solvable at zero fermionic mass, so that we can compare the results of our simulations with exact ones.

We have simulated the (unquenched) model in lattices ranging from 16^2 to 100^2; we present here results for the average plaquette and for the chiral condensate, in the non symmetric ($\theta = 0$) vacuum of the model.

The evaluation of the chiral condensate has been made easier by the fact that, in the M.F.A. approach, the main computer cost resides in the evaluation of an effective fermionic action at fixed pure gauge energy by evaluating *all* the eigenvalues of the fermionic matrix at $m = 0$. It is then essentially possible, at no extra cost, to move in the plane β, m to follow constant physics trajectories in approaching the correct continuum limit. This is easier in this model since here the Renormalization Group amounts to simple dimensional analysis.

The M.F.A. method is fully described in [1]. Starting from the partition function in terms of the total Action $S = S_F + S_G$, sum of the fermionic and pure gauge contributions, we define the density of states at fixed pure gauge Action (*i.e.* Euclidean Energy)

$$N(E) = \int DU \delta(S_G(U) - VE) \tag{1}$$

and an effective fermionic action through

$$e^{-S^F_{eff}(m,n_f,E)} = \langle \det \Delta^{\frac{n_f}{2}} \rangle_E = \frac{\int DU \det \Delta^{\frac{n_f}{2}} \delta(S_G(U) - VE)}{N(E)} \tag{2}$$

which is the microcanonical average of the fermionic determinant.

In terms of the effective Action the partition function can thus be rewritten as

$$\mathcal{Z} = \int dE N(E) e^{-\beta VE - S^F_{eff}(m,n_f,E)} \tag{3}$$

Massless electrodynamics in $1+1$ dimensions is confining, superrenormalizable and exactly solvable.

Its partition function is

$$\mathcal{Z} = \int DA_\mu D\bar{\psi} D\psi e^{\int d^2x [\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \bar{\psi} \not{D} \psi]} \tag{4}$$

with the usual definitions of $F_{\mu\nu}$ and $D\!\!\!/$. The electric charge is dimensionful in this model.

The partition function (in the photonic sector) can be rewritten as [5]

$$\mathcal{Z} = \int DA_\mu e^{\int d^2x[\frac{1}{4}F_{\mu\nu}F_{\mu\nu}+\frac{e^2}{2\pi}A_\mu A_\mu]} \tag{5}$$

i.e. as that of a theory of free massive vector bosons of mass $M = \frac{e}{\sqrt{\pi}}$. In particular the Green's functions of purely bosonic operators are the same in both theories. This fact can be exploited for obtaining the average plaquette in the lattice (see later).

As for the chiral properties of the model, the chiral current is anomalous. If the chiral limit is obtained from $m \neq 0$, then the $\theta = 0$ vacuum is selected. In this vacuum the chiral condensate is (with one flavour)

$$\frac{1}{e}\langle\bar{\psi}\psi\rangle_c = \frac{e^{\gamma_e}}{2\pi\sqrt{\pi}} = 0.15995 \tag{6}$$

while it diverges at zero flavour (*i.e.* the quenched limit) and is zero with two flavours. This is the value of the chiral condensate to be compared with the results of lattice simulations, where its chiral limit is obtained from $m \neq 0$

In the present simulation the pure gauge part is described in terms of *non compact* fields, while for the fermionic - gauge term we use the standard staggered formulation with n_f species.

Since the photonic sector of the continuum theory is equivalent to a theory of a free, massive vector boson, the average plaquette of the Schwinger model can be compared with that of the vector boson, which can be exactly computed on a finite lattice:

$$\langle E\rangle_L = \frac{1}{2V}\sum_{p_1 p_4}\frac{2-\cos p_1-\cos p_4}{2\beta\sum_\gamma(1-\cos p_\gamma)+M^2} \tag{7}$$

($p_\mu = \frac{2\pi}{Na}k_\mu$) and for $V \to \infty$

$$E = \frac{1}{2}\int\frac{d^2p}{(2\pi)^2}\frac{2(1-\cos p_1)+2(1-\cos p_4)}{M^2+2\beta\sum_\gamma(1-\cos p_\gamma)} \tag{8}$$

The value $M = \frac{1}{\sqrt{\pi}}$ corresponds to the continuum Schwinger model, while the quenched value is

$$E(M=0) = \frac{1}{2\beta} \tag{9}$$

Since e_c is dimensionful, β explicitely contains the lattice spacing: $\beta = \frac{1}{a^2e_c^2}$ so that the continuum limit of the theory is approached at $\beta \to \infty$. The limit must be reached keeping fixed the dimensionless ratio $\frac{m_c}{e_c} = \sqrt{\beta}m$. This ratio defines constant physics trajectories.

We have performed simulations in lattices up to 100^2. We present here the results for the 64^2 lattice, where we have the best statistics (for a total of 70 Cray-equivalent hours) [8]. We will mainly discuss the 1−flavour case.

As stated before, we compute all the eigenvalues of the fermionic matrix. This allows us to compute the Effective Action for all values of the mass, including $m = 0$. We have done so for 20 values of the energy, from 0.08 to 1.3.

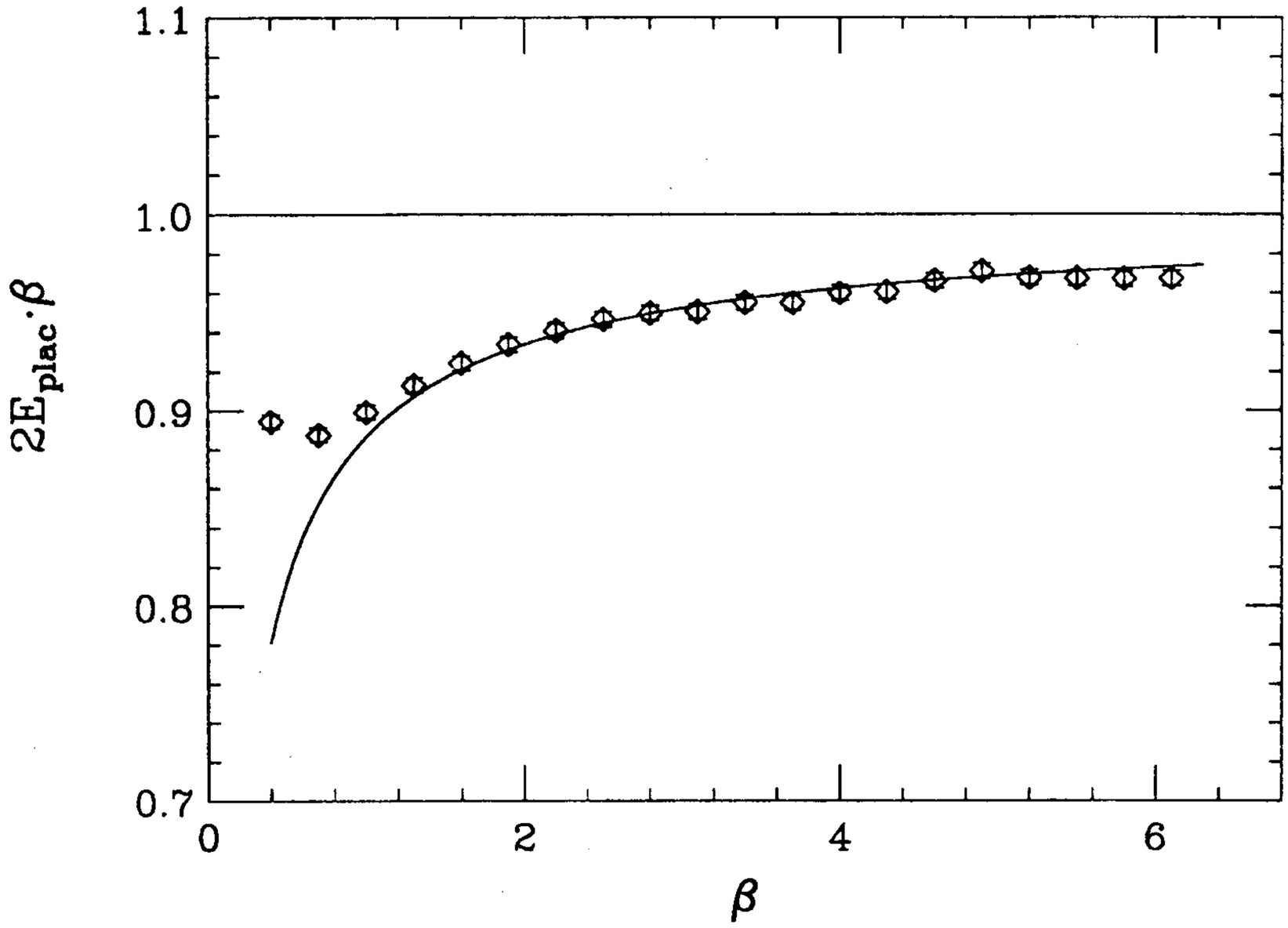

Figure 1: Average plaquette $64^2, m = 0, n_f = 1$.

One advantage of the MFA method is that the phase structure of the theory can be inspected directly from the fermionic effective action, whose derivatives must be discontinuous in order to generate a phase transition, at least for small n_f, if the underlying pure gauge theory has no transition [2]. In the case of QED_2 the continuum theory is obtained as $\beta \to \infty$ and one does not expects finite β transitions. The effective fermionic action numerically evaluated for the model does not show any sign of non analyticity and hence of phase transition [8].

The average plaquette is obtained as

$$\langle E \rangle_L = \frac{\int dE N(E) E e^{-\beta V E} e^{-S^F_{eff}(m,n_f,E)}}{\mathcal{Z}} \tag{10}$$

and can be directly computed at $m = 0$. Since the underlying pure gauge theory is quadratic the density of states is known analytically

$$N(E) = C_G E^{\frac{1}{2}V - \frac{3}{2}} \tag{11}$$

so that the integrals in (10) are simple one-dimensional integrals.

In Figure 1 we report the value of the average plaquette energy (diamonds), multiplied by 2β to improve the visibility, compared with the exact result for a Massive Vector Model on the lattice. It is important to notice that the Schwinger Model is equivalent to a Vector Model in the continuum. On the lattice, there is no guarantee that the two models are

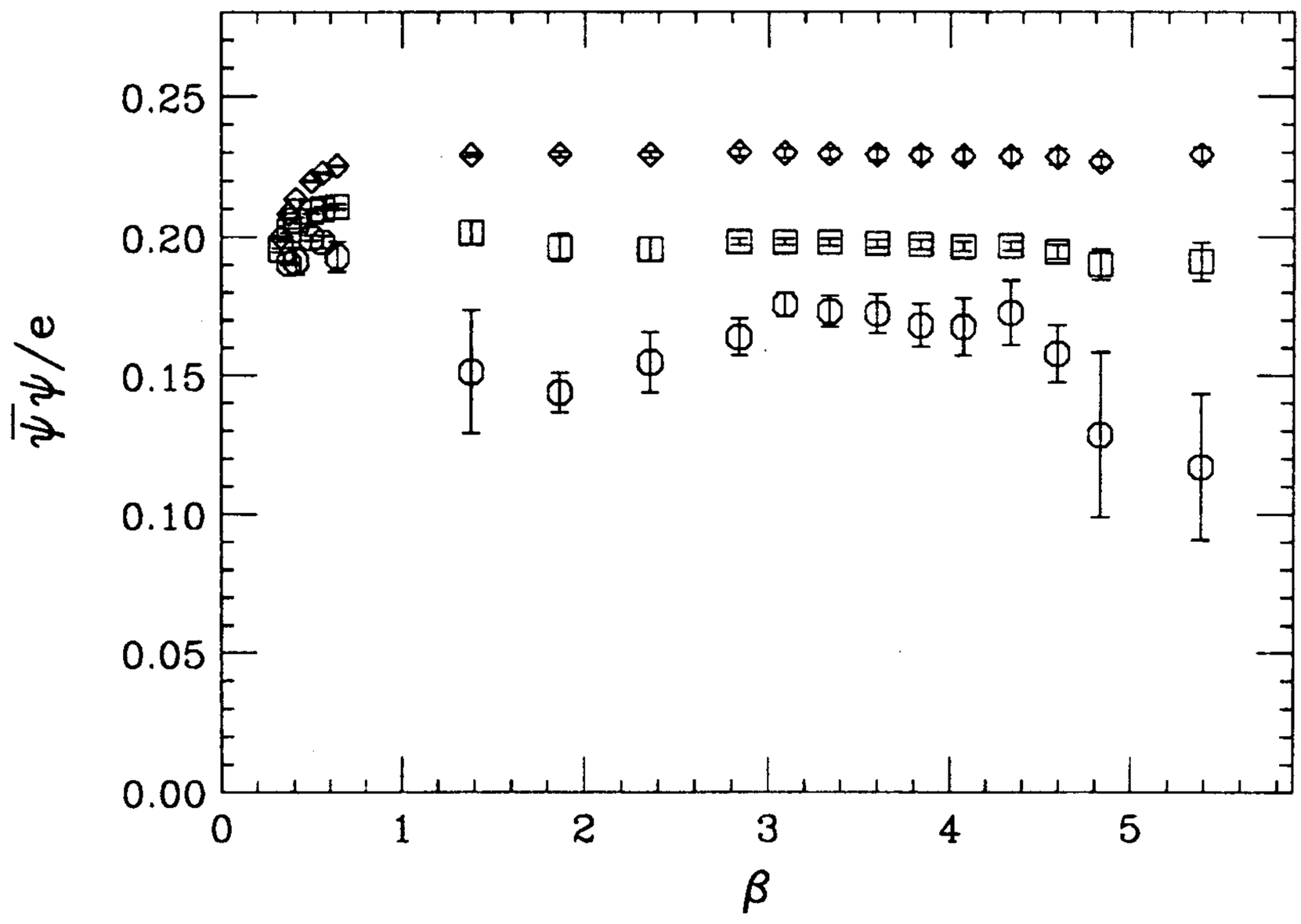

Figure 2: Chiral condensate, 64^2, $\frac{m}{e} = 0.01, 0.04, 0.08$

related. From Figure 1 one can see that at small β, where presumably we are far from the continuum, there is disagreement between the numerical results and the analytical ones. However, already at $\beta \sim 1$ the agreement becomes excellent, showing that, at least for this operator, the continuum physics is reached fastly. The straight line is the quenched value $2\beta\langle E\rangle = 1$, and one can see that, as m increases, the asymptotic value of $\langle E\rangle$ moves towards it. The chiral condensate

$$\langle\bar{\psi}\psi\rangle = -\frac{1}{n_f V}\frac{\int dE e^{-S_{eff}}\frac{\partial}{\partial m}S^F_{eff}}{\int dE e^{-S_{eff}}} \tag{12}$$

cannot be directly computed at $m = 0$, where it vanishes on the lattice, so it must be obtained as the limit $m \to 0$. To reach the correct continuum value, this limit has to be taken simultaneously with the $\beta \to \infty$ one, keeping the product $\sqrt{\beta}m$ fixed.

This can be easily done with this method, which does not require a separate simulation of the fermionic contribution for each pair of parameters (β, m).

In Figure 2 we report the value of the chiral condensate for three values of the ratio $\frac{m_c}{e_c}$. For relatively large values of this ratio, scaling sets up already near $\beta \sim 1$, but even for a very small value (in this lattice) as $\frac{m_c}{e_c} = 0.01$, where finite spacing and volume effects appear in the small and large β regions, there is a clear scaling window. We have repeated this procedure for 12 values of $\frac{m_c}{e_c}$, and the values of the chiral condensate in the scaling window so obtained have been reported in Figure 3. The behaviour of the condensate is very clear towards the continuum value, indicated in the figure as a circle.

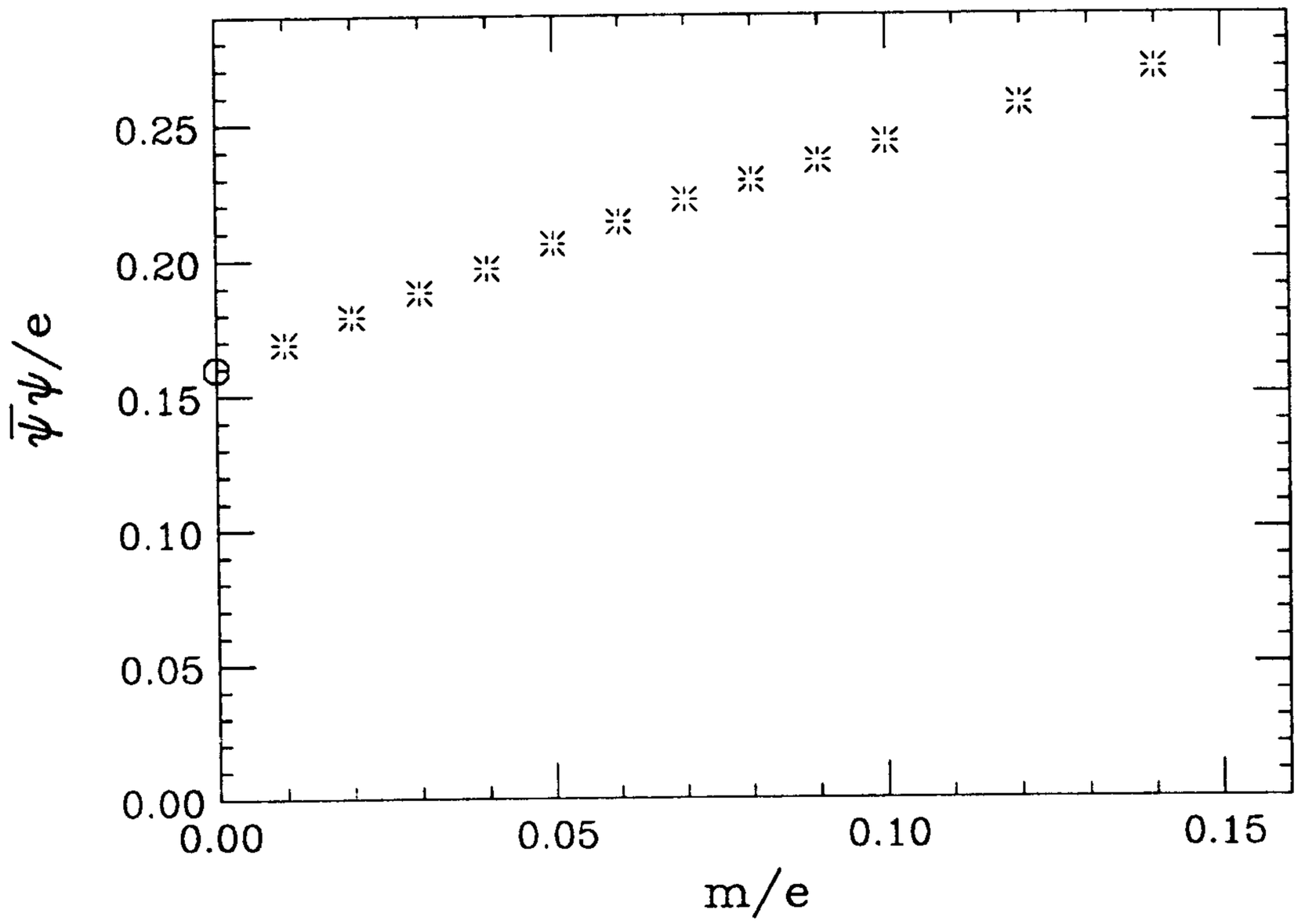

Figure 3: Chiral condensate vs $\frac{m}{e}, n_f = 1$, errors are smaller than symbols

By fitting the points at small $\frac{m_c}{e_c}$ with polynomials we have always obtained consistent results for the intercept:

$$\langle\bar{\psi}\psi\rangle = 0.160 \pm 0.002 \tag{13}$$

in perfect agreement with the theoretical value.

One can exploit the analogy between the fermionic system and a magnetic one to predict that the chiral condensate should behave as a power in $\frac{m_c}{e_c}$ in the scaling region, so allowing an unambiguous extrapolation at $m = 0$.

From a formal point of view this result is also important, since it shows that, even with staggered fermions, where it cannot be proven rigorously, the usual introduction of the flavour number through powers of the fermionic determinant is correct; in fact the numerical value for the chiral condensate, which exactly matches the continuum value, is obtained here by taking the square root of the determinant in the partition function.

We have also analyzed the zero and two flavours cases [8]. In the zero flavour limit there is really no scaling region, with the chiral condensate increasing at large β, indicating that it diverges as expected [6]. On the contrary in the two flavour case, the behaviour of the chiral condensate at finite mass indicates a vanishing value in the chiral limit, again in agreement with expectations [7].

In conclusion, the results we find agree completely with the analytical expectations of the continuum theory. In this respect we believe that the MFA method can be applied to lattice models where the continuum limit is approached at infinite inverse coupling, like

QCD.

It is particularly interesting, in view of more ambitious applications, the ease with which constant physics trajectories can be followed in this approach: in particular, since the mass dependence of the lattice Dirac operator has become trivial, it is possible to move in the β, m parameter space at negligible computer cost. It is useful to remember that also the n_f dependence is trivial [2].

This potentiality has been fully exploited in the Schwinger Model, where Renormalization Group amounts to simple dimensional analysis and Constant Physics trajectories can be exactly defined through the whole parameter space; as a consequence our numerical results for the chiral condensate are quite independent from the extrapolation to zero fermion mass and (as shown in Figure 3) are by far the best available in the literature.

We believe that this potentiality can be used in more realistic theories like QCD.

All the above simulations have been performed on various Transputer networks at L'Aquila University, Zaragoza University (RTN), the bulk on the Transputer Networks of the Theory Group of the Frascati National Laboratories of the INFN.

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