Laboratori Nazionali di Frascati

LNF-93/064 (P)
11 Ottobre 1993

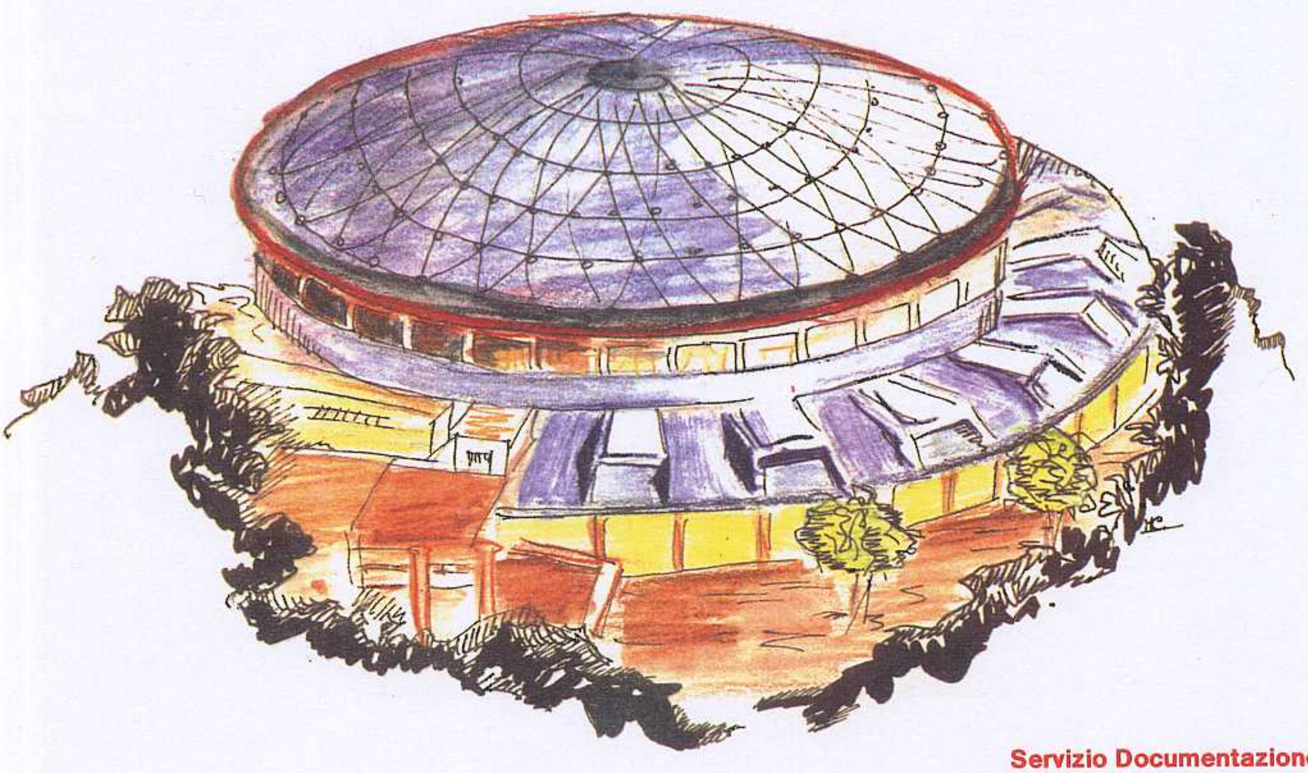
ENSLAPP-A-438/93

J.Lee–Franzini, Paula J. Franzini:

FINE STRUCTURE OF THE P STATES IN QUARKONIUM AND THE SPIN-DEPENDENT POTENTIAL

PACS.: 14.40.Gx; 13.25.+m; 13.40.Hq

Presented and included in the Proceedings of the
Third Workshop on the Tau–Charm Factory
1–6 June, Marbella, Spain



Servizio Documentazione
dei Laboratori Nazionali di Frascati
P.O. Box, 13 - 00044 Frascati (Italy)

INFN - Laboratori Nazionali di Frascati
Servizio Documentazione

LNF-93/064 (P)
11 Ottobre 1993

ENSLAPP-A-438/93

FINE STRUCTURE OF THE P STATES IN QUARKONIUM AND THE SPIN-DEPENDENT POTENTIAL

JULIET LEE-FRANZINI
Laboratori Nazionali di Frascati dell'INFN
Physics Department, SUNY at Stony Brook, Stony Brook, NY 11794, USA
PAULA J. FRANZINI
Laboratoire de Physique Théorique ENSLAPP,[⋆]
B.P. 110, F-74941 Annecy-Le-Vieux, Cedex, France

Abstract

The CUSB-II detector at CESR has yielded precision measurements of branching ratios for electric dipole transitions in the Υ system, and of the masses and fine structure splittings of the $\chi_b(2P)$ states. Comparison of these measurements with those from the charmonium system indicate that problems exist with the present descriptions of the spin-dependent potential, and that more precision measurements are sorely needed.

⋆ *URA 14-36 du CNRS, associée à l'E.N.S. de Lyon, et au L.A.P.P. d'Annecy-le-Vieux*

1. THE SPIN-INDEPENDENT POTENTIAL

Using data collected in the CUSB-II detector[1] at the Cornell Electron Storage Ring (CESR), the CUSB-II collaboration performed many precision measurements of the Υ system which confirmed the validity of the spin-independent part of the interquark potential.[2] The latest of these are measurements of the masses of the P-states and electric dipole (E1) transitions in the Υ system.[3] [4] From the total width of the $\Upsilon(3S)$[5] and the branching ratios the rates (Γ_{E1}) of the transitions $\Upsilon(3S)\rightarrow\chi_b(2P_J)\gamma$ are computed. In table 1 the rates are compared with predictions of potential models.[6–11] The excellent agreement between experiment and theory indicates that potential models describe the SPIN-INDEPENDENT features of the Υ system well. In addition, as the E1 rates are proportional to $(2J+1)$ this agreement confirms the spin assignment for the $\chi_b(2P_J)$ states.

Table 1. E1 rates for the transitions $\Upsilon(3S)\rightarrow\chi_b(2P_J)\gamma$ in keV.

J	Γ_{E1} (keV)	GRR	MR	MB	KR	PF	LF
2	2.7±0.1±0.3	2.6	3.0	2.8	2.8	2.8	2.7
1	2.8±0.1±0.4	2.4	2.6	2.2	2.6	2.6	2.5
0	1.5±0.1±0.2	1.5	1.5	1.0	1.6	1.6	1.6

2. FINE STRUCTURE PARAMETERS

Using the photon energies and the mass of the $\Upsilon(3S)$[5] CUSB obtains the center of gravity of the $\chi_b(2P)$ states, $\overline{M} = (10259.5 \pm 0.4 \pm 1.0)$ MeV, and the fine structure splittings $M_2 - M_1 = (13.5 \pm 0.4 \pm 0.5)$ MeV, and $M_1 - M_0 = (23.5 \pm 0.7 \pm 0.7)$ MeV. M_J is the mass of the $\chi_b(2P_J)$ state.

In our analyses, we make pivotal use of the fact that the spin-independent potential, V_0, unlike the spin-dependent part, V_{sd}, is very well determined in the region where the χ_b and $\chi_b{}'$ wavefunctions are large. This has not been sufficiently exploited previously. Since it is V_0 that yields the quarkonium wavefunctions, we can calculate expectation values such as $\langle\chi_b|\, r^n\, |\chi_b\rangle$ and $\langle\chi_b{}'|\, r^n\, |\chi_b{}'\rangle$ with reasonable, and estimatable, accuracy.[10] Thus, we can study the effects of various functional forms for V_{sd}, the expectation values of which yield the fine structure splittings.

The general form for the spin-dependent potential V_{sd} in the equal mass case is:[12]

$$V_{sd}(r) = \left[(\vec{S}_1 + \vec{S}_2)\cdot\vec{L}\right]\left\{\left(\frac{-\mathrm{d}V_0(r)}{2r\mathrm{d}r} + 2\frac{\mathrm{d}V_2(r)}{r\mathrm{d}r}\right)\frac{1}{m_q^2}\right\} + S_{12}\left\{V_3(r)/12m_q^2\right\} + [\vec{S}_1\cdot\vec{S}_2]\left\{2V_4(r)/3m_q^2\right\}. \tag{2.1}$$

$\vec{S}_{1,2}$ are the total spin operators of the quark and antiquark, $S_{12} = 12\vec{S}_1\cdot\hat{r}\vec{S}_2\cdot\hat{r} - 4\vec{S}_1\cdot\vec{S}_2$, and $\vec{L}$ is the relative orbital angular momentum. The spin-dependent potentials V_2, V_3 and V_4 originate in expectation values of color electric and magnetic fields different from those in V_0 and are in general not related to it.

The P state masses are given by

$$M(^3P_0) = \overline{M} - 2a - 4b \qquad M(^3P_1) = \overline{M} - a + 2b \qquad M(^3P_2) = \overline{M} + a - 2b/5, \quad (2.2)$$

where $\overline{M}$ is the c.o.g. of the triplet (weighted by $2J+1$) and

$$a = \frac{1}{m_q^2}\left\langle -\frac{\mathrm{d}V_0}{2r\mathrm{d}r} + 2\frac{\mathrm{d}V_2}{r\mathrm{d}r}\right\rangle \qquad b = \frac{1}{12m_q^2}\langle V_3\rangle \quad (2.3)$$

where $\langle x\rangle$ is the expectation value $\langle {}^3P_J | x | {}^3P_J\rangle$. The singlet P state mass is given by

$$M(^1P_1) = \overline{M}(^3P_3) - c \qquad c = \frac{2}{3m_q^2}\langle V_4\rangle . \quad (2.4)$$

We have estimated the error in our calculations of expectation values by using several potentials for V_0: the QCD inspired Cornell[13] and Richardson[14] potentials, and the phenomenologically inspired Kwong-Rosner[15] potential. We consider also modified versions of the Cornell* and Richardson† potentials that better fit the Upsilon spectrum. Generally, when we mention V_0 without specifying which potential we are using, we have considered this set of five representative potentials. V_0 is not as well-determined in the charmonium regime; we take as representative the Cornell (with $m_c = 1.84$ GeV) and Richardson (with $m_c = 1.49$ GeV) potentials.

$\langle \frac{dV_0}{rdr}\rangle$, which appears in a, is particularly well determined: for the potentials we have been considering, we have

$$\left\langle \frac{dV_0}{rdr}\right\rangle_{\chi_b} = 0.355 \pm 0.015 \text{ GeV}^3 \qquad \left\langle \frac{dV_0}{rdr}\right\rangle_{\chi_b'} = 0.261 \pm 0.007 \text{ GeV}^3 \quad (2.5)$$

For charmonium, using the Cornell and Richardson potentials, we get $\langle \frac{dV_0}{rdr}\rangle_{\chi_c} = 0.136$ and 0.113 GeV^3 respectively. The more general $\langle r^n\rangle$ varies by at most 10-20 percent in bottomonium, and by 15-30 percent in charmonium.

The recent CUSB measurements[2] give

$$R_{\chi_b'} = 0.584 \pm 0.024 \pm 0.02 \qquad a_{\chi_b'} = 9.5 \pm 0.2 \text{ MeV} \qquad b_{\chi_b'} = 2.3 \pm 0.1 \text{ MeV} . \quad (2.6)$$

The new CLEO measurements[16] give

$$R_{\chi_b'} = 0.574 \pm 0.013 \pm 0.009 \qquad a_{\chi_b'} = 9.4 \pm 0.2 \text{ MeV} \qquad b_{\chi_b'} = 2.3 \pm 0.1 \text{ MeV} , \quad (2.7)$$

in good agreement. The combined data give R, a and b to very high accuracy:

$$R_{\chi_b'} = 0.576 \pm 0.014 \qquad a_{\chi_b'} = 9.43 \pm 0.17 \text{ MeV} \qquad b_{\chi_b'} = 2.3 \pm 0.08 \text{ MeV}. \quad (2.8)$$

---

* We use $V(r) = r/(2.34 \text{ GeV}^{-1})^2 - 0.47/r$ and $m_q = 4.758$ GeV, giving 9.4599, 10.006 and 10.346 GeV for the 1S, 2S and 3S masses, and 9.903 and 10.254 GeV for the 1P and 2P masses. The Richardson potential fits the spectrum better than either version of the Cornell potential.

† For bottomonium, we have modified the potential slightly to fit the 1S, 2S and 3S measured values. We use $\Lambda = 0.392$ GeV, $m_b = 4.898$ GeV, and $33 - 2n_f = 26.26$ (phenomenologically slightly adjusting n_f, the number of families used in calculating α_s, from 3 in order to get the best agreement with the masses).

For the χ_b [5] and χ_c [17] states, the corresponding values are

$$R_{\chi_b} = 0.664 \pm 0.038 \qquad a_{\chi_b} = 14.23 \pm 0.30 \text{ MeV} \qquad b_{\chi_b} = 2.98 \pm 0.17 \text{ MeV} \tag{2.9}$$

$$R_{\chi_c} = 0.478 \pm 0.01 \qquad a_{\chi_c} = 34.96 \pm 0.27 \text{ MeV} \qquad b_{\chi_c} = 10.09 \pm 0.18 \text{ MeV}. \tag{2.10}$$

Finally, evidence for the observation of the ${}^1P_1$ state of charmonium, of 3.7σ of significance, has recently been presented by E760 at Fermilab,[18] giving

$$c_{\chi_c} = -0.7 \pm\ 0.2 \text{ MeV}. \tag{2.11}$$

The data confirms dominance of the spin-orbit, a, over the tensor term, b, a common feature of all models.

3. THE NATURE OF THE SPIN-DEPENDENT POTENTIAL

A standard ansatz is to assume that the $q\overline{q}$ interaction has effective vector (v) and scalar (s) contributions *only*, i.e., that the interaction Lagrangian has the form $\mathcal{L} = \tilde{s}(q^2)\overline{u}u\overline{v}v + \tilde{v}(q^2)\overline{u}\gamma_\mu u\overline{v}\gamma^\mu v$. In this case, we have $V_0 = v(r) + s(r)$, and

$$V_2 = v(r) \qquad V_3 = \frac{dv(r)}{rdr} - \frac{d^2v(r)}{dr^2} \qquad V_4 = \nabla^2 v(r), \tag{3.1}$$

The scalar term is generally understood as coming from quark confinement and therefore being long-range in nature. It enters only in a, through V_0. $v(r)$ is short-range if it comes from single gluon exchange and its associated $1/r$ behavior. In this ansatz, c can be written entirely in terms of a, b, and the expectation value of V_0:

$$c \equiv \frac{2}{3m_q^2}\left\langle \nabla^2 v(r)\right\rangle = \frac{2}{3m_q^2}\left\langle \frac{d^2v}{dr^2} + \frac{2dv}{rdr}\right\rangle = a - 8b + \frac{1}{2m^2}\left\langle \frac{dV_0}{rdr}\right\rangle \tag{3.2}$$

since the $\delta^3(r)$ term that may be present in $\nabla^2 v$ does not contribute to the expectation value for P states and above.[19]

One simple way to begin to explore expectations for R is to take the natural ansatz

$$s(r) = kr \qquad v(r) = -\frac{4}{3}\frac{\alpha_s}{r}. \tag{3.3}$$

Then

$$R = 0.8\frac{1 - 5\lambda/16}{1 - \lambda/8} \qquad \text{where} \qquad \lambda = \frac{k}{\alpha_s}\frac{\langle 1/r\rangle}{\langle 1/r^3\rangle}. \tag{3.4}$$

Thus, $R = 0.8$ for a purely Coulombic potential, and R decreases as we turn on the long-range linear piece (λ increases as k increases, even though $\langle 1/r\rangle / \langle 1/r^3\rangle$ decreases).

As we discussed in a previous paper,[20] eq. (3.3) leads, despite a naive expectation to the contrary, to the result $R_{\chi_b'} > R_{\chi_b} > R_{\chi_c}$ (explicitly, $R_{\chi_b'} = 0.73$, $R_{\chi_b} = 0.72$, $R_{\chi_c} = 0.54$ when k and α_s take on their Cornell potential values). In table 1 we show ('Naive (a)') values for the a's and b's using the modified Cornell potential. Another approach that has been used[15] is to take the $s(r) = kr$, $v(r) = -\frac{4}{3}\frac{\alpha_s}{r}$ ansatz while using another potential for V_0, obtaining values for $\langle 1/r\rangle$ and $\langle 1/r^3\rangle$. This leaves open the

Table 1. Current theoretical predictions for a and b, with measured values for comparison.

	$a(\chi_b)$ (MeV)	$b(\chi_b)$ (MeV)	$a(\chi_b')$	$b(\chi_b')$	$a(\chi_c)$	$b(\chi_c)$
Experiment	14.2 ± 0.3	3.0 ± 0.2	9.4 ± 0.2	2.3 ± 0.1	35.0 ± 0.3	10.1 ± 0.2
GRR 1982[22]	11.3	2.3	9.2	1.8	35.7	10.6
GRR 1986[22]	10.8	2.4	9.3	1.9	36.0	10.0
MR[23]	8.9	2.8	6.5	2.7	26.7	8.5
Fulcher 1988[24]	12.	2.5	9.9	2.0	—	—
Fulcher 1989[24]	9.0	2.0	7.7	1.6	—	—
Fulcher 1990[24]	9.1	2.1	8.0	1.7	—	—
Naive (a)	12.3	2.4	10.6	2.1	19.4	5.1
Naive (b)	15.0	2.9	11.6	2.2	18.6	5.6

uncertainty of what to use for k/α_s, which can be eliminated if desired by fitting R_{χ_b} and then predicting $R_{\chi_b'}$. If we do this, it turns out not to make much difference which of the V_0 we choose. Fitting to $R_{\chi_b} = 0.66$, we get $R_{\chi_b'} \approx 0.67$. In table 1 ('Naive (b)') we show values for the a's and b's using the modified Richardson potential to calculate $\langle 1/r \rangle$ and $\langle 1/r^3 \rangle$, s and v as in Eq. (3.3), and k and α_s as used in the unmodified Cornell potential.

In fact, all published quarkonium models that give values for the R's predict $R_{\chi_b} > R_{\chi_c}$ and $R_{\chi_b'}$ slightly larger than, or approximately equal to, R_{χ_b}.[21–24] The fully spin-dependent QCD potentials give results consist with the above naive estimations: for example Gupta *et al.* 1986[22] find $R_{\chi_b} = 0.64$ and $R_{\chi_b'} = 0.67$ and Fulcher 1990[24] finds $R_{\chi_b} = 0.67$ and $R_{\chi_b'} = 0.70$. The lack of agreement between experiment and theory is even more striking if we look at a and b, as shown in table 1. While ratios such as $a(\chi_b')/a(\chi_b)$ and $a(\chi_b)/b(\chi_b)$ are more meaningful than the absolute numbers for a and b, given the uncertainty of what to use for m_q, it is impressive that with such variation in theoretical predictions, not one of these theories agrees with more than one out of the four $b\bar{b}$ system measurements.

3.1 Fractions of $s(r)$ and $v(r)$

We can use the data to solve for relative fractions of the two interactions by making the assignments of $v(r)$ and $s(r)$. After all, there is no guarantee that the linear term should contribute only to the scalar part of the potential. Maintaining $s(r) + v(r)$ unchanged, we can try

$$s(r) = f_1 kr \qquad v(r) = (1 - f_1)kr - \frac{4\alpha_s}{3r} \tag{3.5}$$

reducing to Eq. (3.3) for $f_1 = 1$. If we then solve for f_1 and $\frac{k}{\alpha_s}$, we find very large and *negative* values for k/α_s (*quarks are not confined!*) for $\frac{\langle 1/r \rangle}{\langle 1/r^3 \rangle}$ in the range predicted by the usual spin-independent potential models. We note, however, that this problem arises only when we try to fit simultaneously the data for the χ_b and the χ_b'. If we calculate the value of f_1 indicated by each triplet individually, using values of $\frac{k}{\alpha_s}$ consistent with potential models, we DO obtain values consistent with 1, that is, the confining potential transforms as a Lorentz scalar.

If, for the sake of argument, we also vary the assignment of the single gluon exchange term to the vector part of the potential, i.e.,

$$s(r) = f_1 kr + (1 - f_2)\left(-\frac{4\alpha_s}{3r}\right) \qquad v(r) = (1 - f_1)kr + f_2\left(-\frac{4\alpha_s}{3r}\right) \tag{3.6}$$

we then have sufficient freedom to input values for $\frac{k}{\alpha_s}$ and $\frac{\langle 1/r \rangle}{\langle 1/r^3 \rangle}$ suggested by potential models and solve for f_1 and f_2. Variations of potentials and choices for k/α_s all give f_1 and f_2 both near zero, in other words $s(r)$ and $v(r)$ are essentially switched around from what we expect. However this scenario gives values for R_{χ_c} above one, to be compared with the experimental value 0.48 ± 0.01, and very small values for a and b, and therefore can be immediately discarded.

3.2 An F-state

Another possible way to explain $R_{\chi_b'} < R_{\chi_b}$, or $R_{\chi_b'}$ smaller than theoretical expectations, independently of the value of R_{χ_b}, is via mixing of the $J = 2$ states of the P and F triplets. Since from potential models the $1F$ is expected to be slightly above the $2P$, this mixing would depress the $J = 2$ state of the $2P$ triplet, therefore decreasing $R_{\chi_b'}$. It would also raise $b_{\chi_b'}$ and lower $a_{\chi_b'}$, improving agreement with experiment. Since the $1F$ would be much further from the $1P$, the depression of the $J = 2$ state of the $1P$ triplet, and therefore of R_{χ_b}, would tend to be much smaller. The shift in the $J = 2$ line required to change R from 0.584 to 0.66 is 1.75 MeV.

The off-diagonal term δm of the $P - F$ mass matrix is derived completely analogously to the diagonal mass terms. From Eq. (2.1) we get

$$\delta m = \langle 1F | V | 2P \rangle = b_{P/F} \langle 1F | S_{12} | 2P \rangle \qquad b_{P/F} = \frac{1}{12m_q^2} \langle 1F | V_3 | 2P \rangle \tag{3.7}$$

since only the tensor interaction can mediate a $\Delta L = 2$ transition. The mixed and unmixed (subscript 0) masses are related by:

$$\delta m = \sqrt{(M_{P_0} - M_P)^2 + (M_{P_0} - M_P)(M_{F_0} - M_{P_0})}\ . \tag{3.8}$$

$\langle {}^3F_2 | S_{12} | {}^3P_2 \rangle$ is a purely numerical factor, found to be $\frac{6\sqrt{6}}{5}$. To evaluate $b_{P/F}$, however, we must choose a form for V_3, and for the $1F$ and $2P$ wavefunctions. In the ansatz of Eq. (3.3), we have $V_3 \propto \frac{1}{r^3}$, and

$$\delta m = \frac{6\sqrt{6}}{5} b_{meas} \frac{\langle 1F | 1/r^3 | 2P \rangle}{\langle 2P | 1/r^3 | 2P \rangle} = \frac{6\sqrt{6}}{5}\frac{2}{13}\sqrt{\frac{2}{7}} b_{meas} \approx 0.2 \text{ MeV} \tag{3.9}$$

scaling to the measured value of b for $\chi_b{}'$, to eliminate various uncertainties of the calculation, and evaluating the expectation values of $1/r^3$ using our standard set for V_0. Strictly speaking we can't just use b_{meas} here since b is also affected by this hypothetical mixing, but since the possible mixing turns out to be small this is not a problem.

In an attempt to get a larger δm we can try the more general ansatz for $v(r)$ and $s(r)$, which we shall write in the form

$$v(r) = w_1 r - w_2/r \qquad s(r) = V_0(r) - v(r), \tag{3.10}$$

to take advantage of $\langle V_0 \rangle$ being well-determined. We find, for all $1F$, $2P$ wavefunctions tried, $\langle 1F | 1/r | 2P \rangle / \langle 2P | 1/r | 2P \rangle >> \langle 1F | 1/r^3 | 2P \rangle / \langle 2P | 1/r^3 | 2P \rangle$. If w_1 and w_2 are both positive coefficients, the maximal δm found with this variation is

$$\delta m = \frac{6\sqrt{6}}{5} b_{meas} \frac{\langle 1F|\,1/r\,|2P\rangle}{\langle 2P|\,1/r\,|2P\rangle} = \frac{6\sqrt{6}}{5}\frac{4}{9}\sqrt{\frac{2}{7}} b_{meas} \approx 2.7 \text{ MeV} \tag{3.11}$$

giving a 2.7 MeV shift if P and F are degenerate, 7 MeV divided by the $P - F$ difference in MeV if they are far apart.

Thus, we find, a large enough shift is barely achievable if the states are degenerate. If they are about 70-95 MeV apart, as predicted by potential models, δm is an order of magnitude too small.

3.3 A Pseudoscalar Interaction

In view of these failures, we would like to consider a naive extension of the scalar + vector interaction picture to include a pseudoscalar field in the interaction,[25] that fits the available data extremely — and unexpectedly — well, in comparison with other models.

In higher order perturbation theory, all five covariants (scalar (s), vector (v), pseudoscalar (p), axial vector (av), and tensor (t)) will arise in the effective kernel of a Bethe-Salpeter equation for the $q\bar{q}$ system.[26] The av and t contributions, since their zeroth order term ($\propto 1/m^0$) is a spin-spin term (V_4), must be highly suppressed. Since the spin-orbit and tensor terms (V_2 and V_3) are higher order ($\propto 1/m^2$), av and t might be of some interest in adjusting c, but are useless in adjusting a and b. The p term, however, vanishes in the static limit, and, at the $1/m^2$ level, gives the following contributions:

$$V_0 = V_2 = 0 \qquad V_3 = -\frac{dp(r)}{rdr} + \frac{d^2p(r)}{dr^2} \qquad V_4 = \nabla^2 p(r)/2. \tag{3.12}$$

Thus, if we can adjust the a inside the $s + v$ ansatz, we may attempt to use p to fix b and c.

We construct our model in three stages. In each stage we manage to agree with more measurements than free parameters are fixed in the model. In the first stage, we consider the $s + v$ ansatz and attempt to fit the three a values. We parametrize s and v as follows:

$$v = \alpha r^m \qquad s = V_0 - v. \tag{3.13}$$

V_0 comes into our calculations twice: in thus determining s and in calculating the expectation values need to determine a, b and c. For this analysis we have selected the potential that best fits (among those that we considered) the quarkonium c.o.g.'s, and, coincidentally or not, best allows us to fit the a values in this model. This is the modified Richardson potential, described in Sec. 2. The ratio $a(\chi_b{}')/a(\chi_b)$ has generally been particularly ill-fit by proposed models (too large theoretically), so we make this the criteria for determining the exponent m. We find $m = -0.4$; rather small in magnitude compared to the expected Coulomb behavior, $m = -1$, but still appropriately short-range. We then adjust the coefficient α to optimize the magnitudes of $a(\chi_b)$ and $a(\chi_b{}')$, and get $a(\chi_c)$ for free. In table 3 we show the pseudoscalar-less model, with $m = -1$ and $m = -0.4$, as models (A) and (B) respectively. All the parameters for our models are given in table 4.

In the second stage, we introduce the pseudoscalar term, of the form

$$p = \beta r^n. \tag{3.14}$$

It turns out that for essentially any n, the pseudoscalar term, for appropriate β, satisfactorily adjusts b_{χ_b} and $b_{\chi_b'}$, *simultaneously*. See Models (C) and (D) for examples.

Finally, setting the exponent $n = 0.24$, we obtain agreement with the two remaining parameters, b_{χ_c} and c_{χ_c}, again *simultaneously* (Model E).

We conclude by giving our predictions for c in the bottomonium system: we find a splitting of about 1 MeV for both the χ_b and $\chi_b{}'$ systems. The $^1P_1$ states in bottomonium (the h_b and h_b') have not been found in the current generation of experiments[27] but will be searched for at B–factories.

Table 3. a, b and c (in MeV) in the $b\bar{b}$ and $c\bar{c}$ systems, as measured, and in various spin-dependent quarkonium models.

	$a(\chi_b)$	$b(\chi_b)$	$a(\chi_b{}')$	$b(\chi_b{}')$	$a(\chi_c)$	$b(\chi_c)$	$c(\chi_c)$
Exp.	14.2 ± 0.3	3.0 ± 0.2	9.4 ± 0.2	2.3 ± 0.1	35.0 ± 0.3	10.1 ± 0.2	-0.7 ± 0.2
GRR 1986[22]	10.8	2.4	9.3	1.9	36.0	10.0	−2
Naive (a)	12.3	2.4	10.6	2.1	19.4	5.1	0
(A)	12.9	2.6	10.3	2.0	14.6	5.0	0
(B)	14.1	2.2	9.5	1.5	35.0	6.0	12
(C)	14.1	3.02	9.5	2.28	35.0	6.9	13
(D)	14.1	3.22	9.5	2.18	35.0	11.7	−22
(E)	14.1	3.23	9.5	2.18	35.0	10.3	−0.7

Table 4. Our models (coefficients in units of GeV to the appropriate powers).

(A)	$v(r) = -0.46r^{-1}$	$p(r) = 0$
(B)	$v(r) = -1.33r^{-0.4}$	$p(r) = 0$
(C)	$v(r) = -1.33r^{-0.4}$	$p(r) = 0.03r^{-2}$
(D)	$v(r) = -1.33r^{-0.4}$	$p(r) = -0.52r^{0.8}$
(E)	$v(r) = -1.33r^{-0.4}$	$p(r) = -1.20r^{0.28}$

4. CONCLUSION

Current data pose intriguing questions about the nature of the spin-dependent potential. While we do find evidence for the confining potential transforming as a Lorentz scalar, conventional theories are unable to agree with the data in detail. It is essential that we find all the states in bottomonium and charmonium in their respective factories to confirm the possible existence of a pseudoscalar term in V_{sd} and to investigate its origin.

5. ACKNOWLEDGEMENTS

We would like to thank all members of the CUSB collaboration for their contributions to this work, Paolo Franzini for help in preparing this paper, and Rita Kirkby for her warm hospitality.

REFERENCES

1. R. D. Schamberger et al., *Nucl. Instr. Meth.* **A309** (1991) 450.
2. U. Heintz et al., Physics Review **D46** (1992) 1928.
3. M. Narain et al., Phys. Rev. Lett. **66** (1991) 3113.
4. U. Heintz et al., Phys. Rev. Lett. **66** (1991) 1563.
5. Particle Data Group, *Phys. Lett.* **239B** (1990).
6. S.N. Gupta, S.F. Radford and W.W. Repko (GRR), *Phys. Rev.* **D34** (1986) 201.
7. P. Moxhay and J. L. Rosner (MR), *Phys. Rev.* **D28** (1983) 1132.
8. R. McClary and N. Byers (MB), *Phys. Rev.* **D28** (1983) 1692.
9. W. Kwong and J. L. Rosner (KR), *Phys. Rev.* **D38** (1988) 279.
10. P. J. Franzini (PF), Physics Letters **B296** (1992) 199.
11. L. P. Fulcher (LF), *Phys. Rev.* **D39** (1989) 295.
12. E. Eichten and F. Feinberg, Phys. Rev. **D23** (1981) 2724; D. Gromes, Z. Phys. **C26** (1984) 401.
13. E. Eichten *et al.*, Phys. Rev. **D17** (1978) 3090; **D21** (1980) 203.
14. J. L. Richardson *et al.*, Phys. Lett. **82B** (1979) 272.
15. W. Kwong and J. L. Rosner, Phys. Rev. **D38** (1988) 279.
16. CLEO Collaboration, R. Morrison *et al.*, Phys. Rev. Lett. **67** (1991) 1696.
17. The numbers for the χ_c states are based on our combination of the measurements quoted by the Particle Data Group, Phys. Lett. **239B** (1990).
18. T. A. Armstrong et al., Phys. Rev. Lett. **69** (1992) 2337.
19. See for example V. Gupta and B. Ram, Phys. Rev. **D34** (1986) 3524; V. Gupta and R. Kögerler, Phys. Rev. **D37** (1988) 740.
20. C.O. Dib, F. J. Gilman, and P. J. Franzini, Phys. Rev. **D37** (1988) 735.
21. D. Beavis *et al.*, Phys. Rev. **D20** (1981) 2345; W. Buchmüller, Phys. Lett. **112B** (1982) 479; R. McClary and N. Byers, Phys. Rev. **D28** (1983) 1632; M. G. Olsson and C. J. Suchyta, Phys. Rev. **D36** (1987) 1459.
22. S. N. Gupta, S. F. Radford, and W. W. Repko, Phys. Rev. **D26** (1982) 3305; *ibid.* **D34** (1986) 201.
23. P. Moxhay and J. L. Rosner, Phys. Rev. **D28** (1983) 1132.
24. L. Fulcher, Phys. Rev. **D37** (1988) 1255; *ibid.* **D39** (1989) 295; *ibid.* **D42** (1990) 2337.
25. P. J. Franzini, Phys. Lett. **B296**, 195 (1992).
26. See D. Gromes, Ref. 12, and Nucl. Phys. **B131** (1977) 80, for discussions and formulae.
27. M. Narain, UMI-92-05244-mc (SUNY at Stony Brook Ph.D. thesis, May 1991, unpublished); CLEO Collaboration, I. C. Brock, Phys. Rev. **D43** (1991) 1448.