Laboratori Nazionali di Frascati

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FOUR-QUARK MESONS IN $\overline{p}p$ ANNIHILATION

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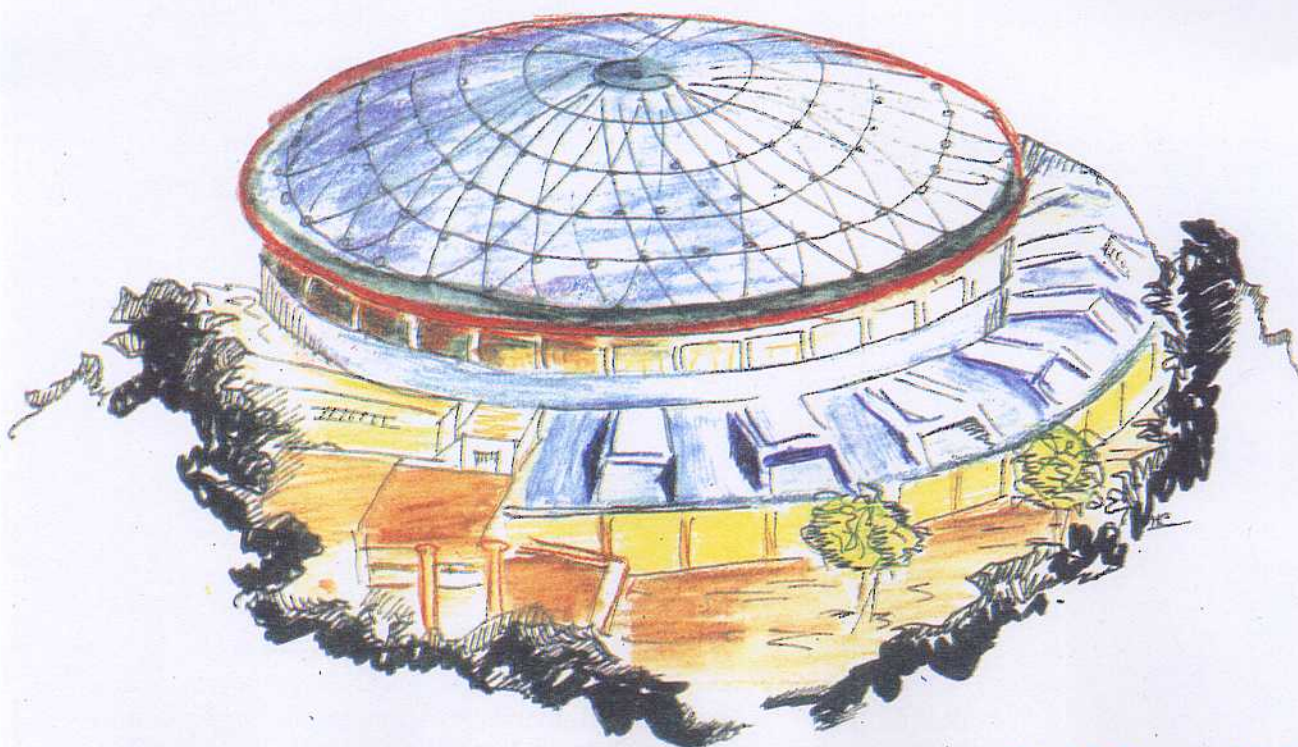
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Abstract

We discuss here a possible four-quark meson spectrum in the region of meson masses 1.2 − 1.8 GeV and a mechanism of these states production in $\bar{p}p$ annihilation. In correspondence with our previous paper we consider f_2/AX resonance, recently seen in $\bar{p}p$ annihilation by several groups, as a four-quark state. We show that experimental data in the frame of a simple model for four-quark meson production support the assumption that f_2/AX is the four-quark system of two scalar-isoscalar diquarks in D-wave.

Using this explanation of f_2/AX we construct the spectrum of non-strange four-quark mesons with masses around 1.5 GeV and discuss the possible channels to detect exotic four-quark states in $\bar{p}p$ annihilation.

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1. Introduction.

QCD, being a hadron theory, up to now did not reach a real success in description of hadron spectra, scattering amplitudes and strong decay widths. The calculations in different non-perturbative QCD approaches[1] and lattice QCD simulations (see review and conference proceedings[2]) have large theoretical uncertainties. By this reason model calculations still can compete with the more solid theoretical treatment.

One of the most striking features of hadron spectra is that they fit, at least approximately, in both Regge and non-relativistic quark model classifications.

The straight-line Regge trajectories with the same slope for mesons and baryons have the natural explanation in string model (see[3] for a modern approach to the problem), where meson with high spin is a rotating quark-antiquark pair coupled with a string, while baryon with high spin is a rotating quark-diquark configuration with the same string between quark and diquark. In the frame of string model it seems rather natural to substitute antidiquark instead of quark into baryon and thus to obtain a four-quark meson. Moreover, in this approach it seems impossible to avoid the existence of such type of objects. Glueballs in string model are closed strings and hybrids should be represented as a string with a kink or as a folded string and because of this glueballs and hybrids are objects of different type with respect to mesons, baryons and four-quark mesons.

On the other hand, non-relativistic constituent quark model (see review[4] and references therein), being built on the very different ideas with respect to the string model, describes rather successfully the same meson and baryon spectra. Again the description of four-quark states in constituent quark model does not require an introduction of additional objects, like constituent gluons, while description of hybrids and glueballs does.

We can conclude that the existence of four-quark mesons has a more solid basis than that of hybrids and glueballs.

It is obvious to look for glueballs in so called gluon-rich channels, like J/ψ radiative decays and central meson production in double-Pomeron exchange reactions. From this point of view it is reasonable to look for four-quark mesons in diquark rich channels. The $\bar{p}p$ annihilation is just this type of reaction because there are diquarks and antidiquarks in the initial state and the simple rearrangement model leads to four-quark meson production. Taking all these considerations into account we discuss here the spectra of four-quark mesons in the frame of a simple string model and the production of these states in $\bar{p}p$ annihilation in a naive rearrangement model.

2. Four-quark interpretation of $f/AX(1520)$ meson.

Recently discovered by several groups in $\bar{p}p$ annihilation $I^G, J^{PC} = 0^+, 2^{++}$ $f/AX(1520)$ meson[5] has no room in $q\bar{q}$ meson classification and has to be described either as quasinuclear $N\bar{N}$ bound state[6] or as four-quark meson[7]. In what follows we will develope the four-quark treatment of $f/AX(1520)$ in accordance with[7]. In the frame of four-quark description[8], there are two possibilities to have low-lying $I^G, J^{PC} = 0^+, 2^{++}$ states: a meson made from vector-isovector diquarks in S-wave and a meson made from scalar-isoscalar diquarks in D-wave. Below we consider these two possibilities in more detailes and will show that the simple model for four-quark meson production in $\bar{p}p$ annihilation supports the second possibility and eliminates the first one. Than, based on this interpretation of $f/AX(1520)$ meson we will construct the spectrum of non-strange four-quark mesons with masses in the region 1.2 - 1.8 GeV and will discuss four-quark mesons with exotic quantum numbers and their decay modes.

3. Model for four-quark meson production.

We will consider the production of four-quark mesons in $\bar{N}N$ annihilation accompanied by one pseudoscalar meson. For the production amplitude we will use a very simple rearramgement model shown in Fig. 1.
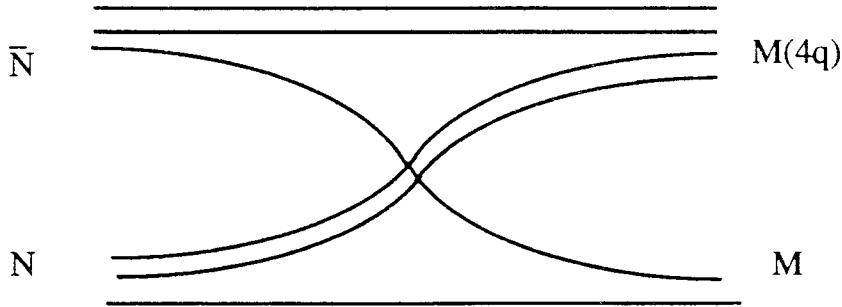
Fig.1

In this model diquark and antidiquark from initial baryon and antibaryon form a four-quark meson, while the rest of $q\bar{q}$ pair transforms into accompanying pseudoscalar meson. We will consider the decays of four-quark mesons into two pseudoscalar mesons and compare the result of our calculations with experimental data on $\bar{p}p$ annihilation into three pseudoscalar mesons. We will use SU(3) symmetric wave functions for initial nucleons and will consider η meson as unitary octet η_8 and

η' as unitary singlet η_0. Corresponding mixing angle could be taken into account without any difficulty. In SU(3) limit nucleon wave functions can be written in terms of vector (scalar) diquark and spectator quark:

$$|p\rangle = \frac{1}{\sqrt{3}}|d\rangle|(uu)\rangle + \frac{1}{\sqrt{2}}|u\rangle|(ud)_0\rangle - \frac{1}{\sqrt{6}}|u\rangle|(ud)_1\rangle$$

$$|n\rangle = \frac{1}{\sqrt{3}}|u\rangle|(dd)\rangle + \frac{1}{\sqrt{2}}|d\rangle|(ud)_0\rangle + \frac{1}{\sqrt{6}}|d\rangle|(ud)_1\rangle \tag{1}$$

Here $(ud)_0$ is a scalar diquark and $(ud)_1$ is a vector diquark. Diquark formed by two identical quarks in S-wave is a vector diquark.

Now we can write the amplitudes of $\bar{p}p$ and $\bar{p}n$ annihilation for production of four-quark states in accordance with diagram in Fig.1.

$$\bar{p}p \rightarrow \frac{1}{3}[(\overline{uu}) - (uu)](\overline{dd}) + \frac{1}{6}[(\overline{ud})_1 - (ud)_1](\overline{uu}) + \frac{1}{2}[(\overline{ud})_0 - (ud)_0](\overline{uu}) +$$

$$+ \frac{1}{3\sqrt{2}}[(\overline{uu}) - (ud)_1](\overline{du}) + \frac{1}{3\sqrt{2}}[(\overline{ud})_1 - (uu)](\overline{ud}) \tag{2}$$

$$\bar{p}n \rightarrow \frac{1}{3}[(\overline{uu}) - (dd)](\overline{du}) - \frac{1}{6}[(\overline{ud})_1 - (ud)_1](\overline{ud}) + \frac{1}{2}[(\overline{ud})_0 - (ud)_0](\overline{ud}) +$$

$$+ \frac{1}{3\sqrt{2}}[(\overline{uu}) - (ud)_1](\overline{dd}) - \frac{1}{3\sqrt{2}}[(\overline{ud})_1 - (dd)](\overline{uu}) \tag{3}$$

Four-quark meson states formed by vector diquarks and antidiquarks can be written in terms of states with isospin $I = 0,1,2$. We will denote isoscalar 2^{++} meson as f_2, isovector meson as a_2 and will introduce notation c_2 for isotensor 2^{++} meson.

Using standard Clebsh-Gordan coefficients we can write fot neutral components of diquark-antidiquark states:

$$[(\overline{uu}) - (uu)] = \frac{1}{\sqrt{6}}c_2^0 + \frac{1}{\sqrt{2}}a_2^0 + \frac{1}{\sqrt{3}}f_2$$

$$[(\overline{dd}) - (dd)] = \frac{1}{\sqrt{6}}c_2^0 - \frac{1}{\sqrt{2}}a_2^0 + \frac{1}{\sqrt{3}}f_2, \tag{4}$$

$$[(\overline{ud})_1 - (ud)_1] = \frac{\sqrt{2}}{\sqrt{3}}c_2^0 - \frac{1}{\sqrt{3}}f_2$$

and for charged components:

$$[(\overline{ud})_1 - (ud)_1] = \frac{\sqrt{2}}{\sqrt{3}} c_2^0 - \frac{1}{\sqrt{3}} f_2$$

$$[(\overline{uu}) - (ud)_1] = \frac{1}{\sqrt{2}} c_2^- + \frac{1}{\sqrt{2}} a_2^-$$

$$[(\overline{ud})_1 - (dd)] = \frac{1}{\sqrt{2}} c_2^- - \frac{1}{\sqrt{2}} a_2^- \tag{5}$$

$$[(\overline{dd}) - (ud)_1] = \frac{1}{\sqrt{2}} c_2^+ - \frac{1}{\sqrt{2}} a_2^+$$

$$[(\overline{uu}) - (dd)] = c_2^{--}$$

The isoscalar diquarks form isoscalar meson $f_{2,}^s$.

$$[(\overline{ud})_0 - (ud)_0] = f_2^s \tag{6}$$

We will assume that $q\overline{q}$ pair can be decomposed over pseudoscalar mesons with the weights coming from quark model:

$$\overline{uu} = \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 + \frac{1}{\sqrt{3}} \eta_0$$

$$\overline{dd} = -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 + \frac{1}{\sqrt{3}} \eta_0 \tag{7}$$

Now we can write down the amplitudes of four-quark meson production accompanied by pseuduscalar meson:

$$\overline{p}p \rightarrow -\frac{1}{3\sqrt{3}} c_2^0 \pi^0 - \frac{1}{6} a_2^0 \pi^0 - \frac{1}{6\sqrt{6}} f_2 \pi^0 +$$

$$+ \frac{1}{6\sqrt{3}} a_2^0 \eta_8 + \frac{1}{6\sqrt{2}} f_2 \eta_8 + \frac{1}{3\sqrt{6}} a_2^0 \eta_0 + \frac{1}{6} f_2 \eta_0 +$$

$$+ \frac{1}{2\sqrt{2}} f_2^s \pi^0 + \frac{1}{2\sqrt{6}} f_2^s \eta_8 + \frac{1}{2\sqrt{3}} f_2^s \eta_0 - \tag{8}$$

$$- \frac{1}{6} c_2^- \pi^+ - \frac{1}{6} a_2^- \pi^+ - \frac{1}{6} c_2^+ \pi^- + \frac{1}{6} a_2^+ \pi^-$$

$$\overline{p}n \rightarrow -\frac{\sqrt{6}}{18} c_2^0 \pi^- + \frac{\sqrt{3}}{18} f_2 \pi^- + \frac{1}{2} f_2^s \pi^- - \frac{\sqrt{2}}{6} c_2^- \pi^0 + \frac{1}{3} c_2^{--} \pi^+ +$$

$$+ \frac{1}{3\sqrt{6}} a_2^- \eta_8 + \frac{1}{3\sqrt{3}} a_2^- \eta_0 \tag{9}$$

In this model a_2 four-quark meson cannot be produced with pions but only with η mesons: a_2^0 is not produced because symmetric $(ud)_1 - \overline{(ud)_1}$ state has no $I = 1$

component. The same symmetry considerations forbid the production of $a_2^- \pi^0$ pair. As far as $c_2 \eta$ and $f_2 \eta$ systems cannot be produced in $\overline{p}n$ annihilation because of isospin conservation we always have negative G-parity of final meson states and corresponding selection rule for initial states.

Now we can express all modes of $N\overline{N}$ annihilation into three pseudoscalar mesons when two of them are in the resonance in terms of production cross-sections of f_2^s, f_2, a_2, and c_2 four-quark resonances and their branching ratios into two pseudoscalar mesons.

The explicit formulas for production cross sections of three pseudoscalar mesons, where two of them come from resonance decay are given in Appendix 1.

If f_2 and c_2 four-quark mesons with the same masses are produced together in $N\overline{N}$ annihilation, the distribution of $\pi^0 \pi^0$ and $\pi^+ \pi^-$ mesons at the resonance peak is due to interference between isospin 0 and isospin 2 resonance amplitudes. In general case of this interference the three-pion cross-section cannot be expressed in terms of resonance production cross-sections and corresponding branching ratios. If to neglect with the phase difference between the amplitudes of production of resonances with isospin 0 and 2 and with the phase difference between decay amplitudes, the expression for three-pion production cross-section $\sigma_{3\pi}$ has the form:

$$\sigma_{3\pi} = \sigma_0 B_0 + \sigma_2 B_2 + \varsigma \sqrt{\sigma_0 B_0 \sigma_2 B_2} \left(\sqrt{\Gamma_2^{tot}/\Gamma_0^{tot}} + \sqrt{\Gamma_0^{tot}/\Gamma_2^{tot}} \right), \qquad (10)$$

where σ_0 is the cross-section of f_2 production, σ_2 is the cross-section of c_2 production and B_0 and B_2 are the branching ratios into two-pion channel. Γ_0^{tot} and Γ_2^{tot} are total widths of f_2 and c_2 mesons and ς is relative sign of the product of production and decay aplitudes for f_2 meson with respect to the same product for c_2 meson. If $\Gamma_2^{tot} \approx \Gamma_0^{tot}$, eq.(10) can be written as

$$\sigma_{3\pi} = \left(\sqrt{\sigma_0 B_0} + \varsigma \sqrt{\sigma_2 B_2} \right)^2 \qquad (11)$$

Let us consider the two possible explanations of $f/AX(1520)$ as a four-quark resonance, discussed above. If $f/AX(1520)$ is vector diquark-antidiquark pair in S-wave, there must be an interference between $I = 0$ and $I = 2$ amplitudes, which we can take into account with the help of eq.(11).

Let us denote

$$\sigma_p^{00} = \sigma[\bar{p}p \to (\pi^0\pi^0)\pi^0]$$

$$\sigma_p^{+-} = \sigma[\bar{p}p \to (\pi^+\pi^-)\pi^0]$$

$$\sigma_n^{00} = \sigma[\bar{p}n \to (\pi^0\pi^0)\pi^-]$$

$$\sigma_n^{+-} = \sigma[\bar{p}n \to (\pi^+\pi^-)\pi^-] \tag{12}$$

Eq's (7,8) together with eq.(11) and Clebsh-Gordan coefficients leads to relations between cross-sections of eq.(12).

$$\sigma_n^{00} = \left(\sqrt{9\sigma_p^{+-}} - \sqrt{8\sigma_p^{00}}\right)^2 \tag{13}$$

$$\sigma_n^{+-} = \left(\frac{1}{2}\sqrt{50\sigma_p^{+-}} - \sqrt{9\sigma_p^{00}}\right)^2 \tag{14}$$

Production cross-sections of charged c_2 meson components can be also expressed in terms of σ_p^{+-} and σ_p^{00}:

$$\sigma(c_2^-\pi^+) = \sigma[\bar{p}p \to (\pi^-\pi^0)\pi^+] = \frac{9}{2}\left(\sqrt{\sigma_p^{+-}} - \sqrt{2\sigma_p^{00}}\right)^2 \tag{15}$$

$$\sigma(c_2^-\pi^0) = \sigma[\bar{p}n \to (\pi^-\pi^0)\pi^0] = 9\left(\sqrt{\sigma_p^{+-}} - \sqrt{2\sigma_p^{00}}\right)^2 \tag{16}$$

$$\sigma(c_2^{--}\pi^+) = \sigma[\bar{p}n \to (\pi^-\pi^-)\pi^+] = 18\left(\sqrt{\sigma_p^{+-}} - \sqrt{2\sigma_p^{00}}\right)^2 \tag{17}$$

The large coefficient in eq.(17) makes it very sensitive to the admixture of $I = 2$ amplitude.

Existing experimental data show no peak in $\pi^-\pi^-$ invariant mass distribution in $\bar{p}n$ annihilation into $\pi^-\pi^-\pi^+$ [8]. This is the strong argument against the existence of isospin partners of $f/AX(1520)$.

If $f/AX(1520)$ is D-wave state of scalar diquark-antidiquark pair, there is no $I = 2$ partners of $f/AX(1520)$ and all cross-sections of eq.(15-17) are equal to zero.

In this case in addition to the normal isotopic relations

$$\sigma_p^{+-} = 2\sigma_p^{00}$$

$$\sigma_n^{+-} = 2\sigma_n^{00} \tag{18}$$

the model predicts the relation between three-pion annihilation cross-section in $\bar{p}p$ and $\bar{p}n$ channels in resonance peak:

$$\sigma_n^{00} = 2\sigma_p^{00} \tag{19}$$

Another sensitive test of the admixture of $I = 2$ amplitude in two-pion system is the double ratio

$$\frac{\sigma[(\pi^0\pi^0)\pi^0]}{\sigma[(\pi^0\pi^0)\eta]} \bigg/ \frac{\sigma[(\eta\eta)\pi^0]}{\sigma[(\eta\eta)\eta]} \qquad (20)$$

Phase space corrections are cancelled out of this ratio and the deviation of eq.(20) from unity is a clear signal of the presence of $I = 2$ amplitude. This ratio can be measured at different energies with Crystall Barrel detector in $\bar{p}p$ annihilations in flight at LEAR and in E-760 experiment at FNAL.

The energy dependence of four-quark meson production cross-section in the binary reaction $\bar{N}N \rightarrow M(4q)M(\bar{q}q)$ is due to excange in the t-channel by diquark and quark lines (see diagram shown in Fig.1). This is baryon exchange and at least at high energies the cross-section of the reaction $\bar{N}N \rightarrow M(4q)M(\bar{q}q)$ has the energy dependence

$$\sigma[\bar{N}N \rightarrow M(4q)M(\bar{q}q)] = \left(\frac{s}{s_0}\right)^{2\alpha_B(0)-2} \qquad (21)$$

here $\alpha_B(0) \approx -0.5$ is the intercept of baryon trajectory.

On the other hand the cross-section of production of two $\bar{q}q$ mesons in binary reaction $\bar{N}N \rightarrow M(\bar{q}q)M(\bar{q}q)$ is described by two diagrams shown in Fig. 2.
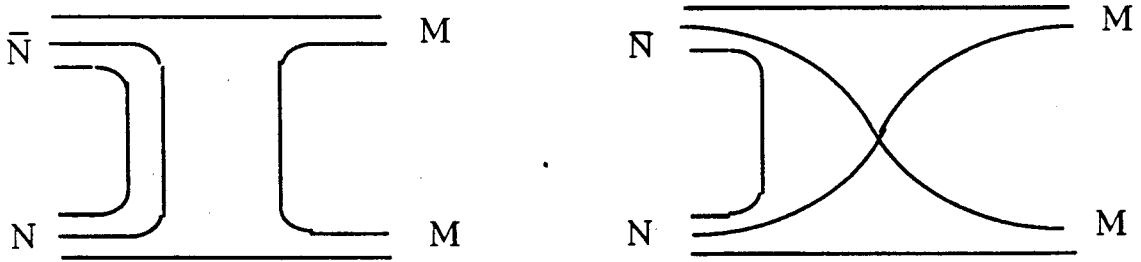

Fig. 2

There is also an exchange by three quarks in t-channel of these diargams and the energy behaviour of reaction $\bar{N}N \rightarrow M(\bar{q}q)M(\bar{q}q)$ at high energies should be the same:

$$\sigma[\bar{N}N \rightarrow M(\bar{q}q)M(\bar{q}q)] = \left(\frac{s}{s_0}\right)^{2\alpha_B(0)-2} \qquad (22)$$

This means that the ratio of these cross-sections at least at high enough energies does not depend on incoming antiproton energy. For example:

$$R_{f_2} = \frac{\sigma(\bar{p}p \rightarrow f_2(1270)\,\pi^0)}{\sigma(\bar{p}p \rightarrow f_2/AX\,\pi^0)} \approx const \tag{23}$$

Of course these arguments, strictly speaking, can not be applied to the annihilation at rest, but when the momentum of incoming antiproton becomes to be as large as some GeV's, the ratio eq.(23) should be constant with respect of energy.

4. Notations for exotic mesons.

In what follows we will discuss the other four-quark meson states with different quantum numbers. Now the standard notations of PDG exist only for non-exotic quantum numbers. In Table 1 we give the notations for all possible quantum numbers, which are the straightforward extension of the standard notations and coinside with them for non-strange quantum numbers. We used these notations partially when denoting as c_2 the isotensor exotic meson with $J^{PC} = 2^{++}$.

Table 1.

I^G/J^P	0^-	1^-	2^-	3^-	J^-	0^+	1^+	2^+	3^+	J^+
0^-	ω_0	ω	ω_2	ω_3	ω_J	h_0	$-h_1$	h_2	h_3	h_J
0^+	η	η_1	η_2	η_3	η_J	f_0	f_1	f_2	f_3	f_J
1^-	π	π_1	π_2	π_3	π_J	a_0	a_1	a_2	a_3	a_J
1^+	ρ_0	ρ	ρ_2	ρ_3	ρ_J	b_0	b_1	b_2	b_3	b_J
2^-	ϑ_0	ϑ	ϑ_2	ϑ_3	ϑ_J	c_0	c_1	c_2	c_3	c_J
2^+	λ	λ_1	λ_2	λ_3	λ_J	d_0	d_1	d_2	d_3	d_J

The unshadowed notations are used for already existing mesons taken from PDG Tables. Mesons with shadowed notations are the mesons with non-exotic quantum numbers which are not found experimentally up to now.

The mesons with double shadowed notations have exotic quantum numbers.

In these notations the exotic meson with $I^G J^{PC} = 1^- 1^{-+}$ which is denoted in PDG Tables as $\hat{\rho}(1405)$ should be denoted as $\pi_1(1405)$.

All $s\bar{s}$ isoscalar mesons should be denoted by corresponding notations with slash, for example: $f_2(1270)$ and $f_2'(1525)$. The only exception is for ω_J' which are traditionally denoted as ϕ_J.

5. Spectra of quark-antiquark mesons.

As it was discussed in Introduction both Regge-string type models and non-relativistic type quark models describe $\bar{q}q$ meson mass spectra rather well. To combine these approaches one has to assume that the members of multiplets with the same orbital quantum number L, but with different total angular momenta J lie on the daughter Regge trajectories. To obtain the correct Regge picture the mass spliting between different members of multiplet with given L should be small. Below we analize the $\bar{q}q$ meson spectra and show that this picture really takes place.

In potential models the splittings in S- P- and D-multiplets is given by the following expressions:

$$m\left(^{1}S_{0}\right) = m_{S} - 3/4\,a_{S}$$

$$m\left(^{3}S_{1}\right) = m_{S} + 1/4\,a_{S} \tag{24}$$

$$m\left(^{1}P_{1}\right) = m_{P} - 3/4\,a_{P}$$

$$m\left(^{3}P_{0}\right) = m_{P} + 1/4\,a_{P} - 2b_{P} + 4c_{P}$$

$$m\left(^{3}P_{1}\right) = m_{P} + 1/4\,a_{P} - b_{P} + 2c_{P} \tag{25}$$

$$m\left(^{3}P_{2}\right) = m_{P} + 1/4\,a_{P} + b_{P} + 2/5\,c_{P}$$

$$m\left(^{1}D_{2}\right) = m_{D} - 3/4\,a_{D}$$

$$m\left(^{3}D_{1}\right) = m_{D} + 1/4\,a_{D} - 3b_{D} - 2c_{D}$$

$$m\left(^{3}D_{2}\right) = m_{D} + 1/4\,a_{D} - b_{D} + 2c_{D} \tag{26}$$

$$m\left(^{3}D_{3}\right) = m_{D} + 1/4\,a_{D} + 2b_{D} + 4/7\,c_{D}$$

Here a_{S}, a_{P} and a_{D} are matrix elements of spin-spin potential for S-, P- and D- wave states correspondently and the coefficients before a_{L} are matrix elements of operator

$$s_{1} \cdot s_{2} = \frac{1}{2}\left(S^{2} - s_{1}^{2} - s_{2}^{2}\right) \tag{27}$$

in the triplet state ($S = 1$) and in the singlet state ($S = 0$).

b_{P} and b_{D} are matrix elements of spin-orbital potential for P- and D-wave states. The numerical coefficients are matrix elements of operator

$$L \cdot S = \frac{1}{2}\left(J^{2} - L^{2} - S^{2}\right) \tag{28}$$

in the states with fixed J, L and S.

c_P and c_D are matrix elements of tensor potential and coefficients are matrix elements of operator

$$S_{12} = 2\left[3\frac{(\mathbf{S}\cdot\mathbf{r})}{r^2} - \mathbf{S}^2\right] \tag{29}$$

in the states with fixed J, L and S.

In what follows we will conider as $\bar{q}q$ multiplets the meson states shown in Table 2.

Table 2

	$I = 1/2$	$I = 1$	$I = 0\ (\bar{q}q)$	$I = 0\ (\bar{s}s)$
$^1S_0$	K	π	$\eta(\bar{q}q)$	$\eta(\bar{s}s)$
$^3S_1$	$K^*(892)$	$\rho(770)$	$\omega(783)$	$\phi(1020)$
$^1P_1$	$K_1(1270)$	$b_1(1235)$	$h_1(1170)$	$h_1(1380)$
$^3P_0$	$K_0^*(1430)$	$a_0(1320)$	$f_0(1400)$	$f_0(1525)$
$^3P_1$	$K_1(1400)$	$a_1(1260)$	$f_1(1285)$	$f_1(1510)$
$^3P_2$	$K_2^*(1430)$	$a_2(1320)$	$f_2(1270)$	$f_2'(1525)$
$^1D_2$	$K_2(1580)$	$\pi_2(1670)$		
$^3D_1$	$K^*(1680)$	$\rho(1700)$	$\omega(1600)$	
$^3D_2$	$K_2(1580)$			
$^3D_3$	$K_3^*(1780)$	$\rho_3(1690)$	$\omega_3(1690)$	$\phi_3(1850)$

here $\eta(\bar{q}q)$ and $\eta(\bar{s}s)$ are $\bar{q}q$ and $\bar{s}s$ pseudoscalar states.

This assignment needs some comments.

1. The treatment of pseudoscalar meson as $^1S_0$ states of non-relativistic quark model is doubtful because they are Goldstone bosons of broken chiral symmetry.

2. The low-lying $f_0(975)$ and $a_0(980)$ mesons could be non-$\bar{q}q$ mesons and by this reason we considered scalar mesons $f_0(1400)$ and $a_0(1320)$ as scalar components of P-wave triplets.

3. We considered $\rho(1450)$ and $\omega(1390)$ as 2^3S_1 radial excitations.

4. We neglected with mixing between $K_1(1270)$ and $K_1(1400)$ and $K_2(1580)$ and $K_2(1580)$ states.

5. We neglected with hadron mass shifts which are unknow.

Now, using eq's (24-26) we come to the values of m_L, a_L, b_L and c_L matrix elements given in Table 3.

Table 3

	$I = 1/2$	$I = 1$	$I = 0\ (\bar{q}q)$	$I = 0\ (\bar{s}s)$
m_S (MeV)	792	612		
a_S (MeV)	398	630		
m_P (MeV)	1380	1282	1262	1485
a_P (MeV)	150	67	122	140
b_P (MeV)	5	14	- 23	4
c_P (MeV)	- 5	- 12	- 15	-3
m_D (MeV)	1713	1687		
a_D (MeV)	178	22		
b_D (MeV)	8	- 2		
c_D (MeV)	9	- 1		

When calculating the parameters of D-wave $I = 1$ multiplet we obtained the position of center of gravity using only three known D-states.

The uncertainities in values of parameters are not less than 10 MeV.

In next section we will apply these results to estimate the mass splittings in four-quark sector. While doing so we will use the dependence on quark (diquark) masses of spin-spin (V_{SS}), spin-orbital (V_{LS}) and tensor (V_T) potentials given by potential models:

$$V_{SS}, V_{LS}, V_T \propto (m_1 m_2)^{-1} \qquad (30)$$

6. Spectrum of non-strange four-quark mesons.

As it was mentioned already there are two kinds of non-strange color triplet diquarks $d_0 = (qq)_0$ with $S = I = 0$ and $d_1 = (qq)_1$ with $S = I = 1$ (S is the spin of diquark, I is the isospin). So it is possible to construct three different types of four-quark non-strange mesons:

$$
\begin{aligned}
A: &\quad d_0 - \overline{d_0} \\
B^{\pm}: &\quad \left(d_0 - \overline{d_1}\right) \pm \left(d_1 - \overline{d_0}\right) \\
C: &\quad d_1 - \overline{d_1}
\end{aligned}
\qquad (31)
$$

where the states A have isospin 0, the states $B^{\pm}$ have isospin 1 and the states C are degenerated with respect to isospin ($I = 0,1,2$).

The mass difference between non-strange vector and scalar diquarks can be estimated from the mass difference between octet and decuplet non-strange S-wave baryons:

$$\Delta m_d = m(d_1) - m(d_0) = \frac{2}{3}[m(\Delta) - m(N)] \approx 200 \text{ MeV} \qquad (32)$$

The masses and quantum numbers of A-type mesons are given by the following expressions:

$$m(^1S_0) = m_S^{00}; \qquad I^G; J^P = 0^+; 0^{++},$$
$$m(^1P_1) = m_P^{00}; \qquad I^G; J^P = 0^-; 1^{--}, \qquad (33)$$
$$m(^1D_2) = m_D^{00}; \qquad I^G; J^P = 0^+; 2^{++}.$$

where masses of orbital excitations are given by Regge formula:

$$\left(m_L^{00}\right)^2 = \left(m_S^{00}\right)^2 + \frac{1}{\alpha'}L \qquad (34)$$

For B-type mesons the spectrum is more complicated. Total spin momentum of two diquarks is equal to $S = 1$ and spin-orbital and tensor forces leads to splitting of level with the same L:

$$m(^3S_1) = m_S^{01}, \qquad\qquad I^G; J^P = 1^+; 1^{+-}, \quad 1^-; 1^{++},$$
$$m(^3P_0) = m_P^{01} - 2b_P^{01} - 4c_P^{01}, \quad I^G; J^P = 1^+; 0^{--}, \quad 1^-; 0^{-+},$$
$$m(^3P_1) = m_P^{01} - b_P^{01} + 2c_P^{01}, \quad I^G; J^P = 1^+; 1^{--}, \quad 1^-; 1^{-+},$$
$$m(^3P_2) = m_P^{01} + b_P^{01} - 2/5c_P^{01}, \quad I^G; J^P = 1^+; 2^{--}, \quad 1^-; 2^{-+}, \qquad (35)$$
$$m(^3D_1) = m_D^{01} - 3b_D^{01} - 2c_D^{01}, \quad I^G; J^P = 1^+; 1^{+-}, \quad 1^-; 1^{++},$$
$$m(^3D_2) = m_D^{01} - b_D^{01} + 2c_D^{01}, \quad I^G; J^P = 1^+; 2^{+-}, \quad 1^-; 2^{++},$$
$$m(^3D_3) = m_D^{01} + 2b_D^{01} - 4/7c_D^{01}, \quad I^G; J^P = 1^+; 3^{+-}, \quad 1^-; 3^{++}.$$

where $m_S^{01} = m_S^{00} + \Delta m_d$,

$$\left(m_L^{01}\right)^2 = \left(m_S^{01}\right)^2 + \frac{1}{\alpha'}L \qquad (36)$$

and b_P^{01}, c_P^{01} and b_D^{01}, c_D^{01} are the matrix elements of spin-orbital and tensor potentials for P- and D-orbital excitations.

The most complicated is the picture of splittings in C-type meson sector. There are three multiplets of C-type four-quark mesons which correspond to total diquark spin $S = 0$, 1 and 2.

For $S = 0$:

$$m(^1S_0) = m_S^{11} - 2a_S^{11}; \qquad I^G; J^P = 0^+, 1^-, 2^+; 0^{++},$$

$$m(^1P_1) = m_P^{11} - 2a_P^{11}; \qquad I^G; J^P = 0^-, 1^+, 2^-; 1^{--}, \qquad (37)$$

$$m(^1D_2) = m_D^{11} - 2a_D^{11}; \qquad I^G; J^P = 0^+, 1^-, 2^+; 2^{++}.$$

here $m_S^{11} = m_S^{01} + \Delta m_d = m_S^{00} + 2\Delta m_d$ and as before

$$\left(m_L^{11}\right)^2 = \left(m_S^{11}\right)^2 + \frac{1}{\alpha'} L \qquad (38)$$

a_L^{11} are matrix elements of spin-spin potential in states with orbital momentum of diquark-antidiquark pair equal to L.

For $S = 1$:

$$m(^3S_1) = m_S^{11} - a_S^{11}; \qquad I^G; J^P = 0^-, 1^+, 2^-; 1^{+-},$$

$$m(^3P_0) = m_P^{11} - a_P^{11} - 2b_P^{11} - 4c_P^{11}; \qquad I^G; J^P = 0^+, 1^-, 2^+; 0^{-+},$$

$$m(^3P_1) = m_P^{11} - a_P^{11} - b_P^{11} + 2c_P^{11}; \qquad I^G; J^P = 0^+, 1^-, 2^+; 1^{-+},$$

$$m(^3P_2) = m_P^{11} - a_P^{11} + b_P^{11} - 2/5c_P^{11}; \qquad I^G; J^P = 0^+, 1^-, 2^+; 2^{-+}, \qquad (39)$$

$$m(^3D_1) = m_D^{11} - a_D^{11} - 3b_D^{11} - 2c_D^{11}; \qquad I^G; J^P = 0^-, 1^+, 2^-; 1^{+-},$$

$$m(^3D_2) = m_D^{11} - a_D^{11} - b_D^{11} + 2c_D^{11}; \qquad I^G; J^P = 0^-, 1^+, 2^-; 2^{+-},$$

$$m(^3D_3) = m_D^{11} - a_D^{11} + 2b_D^{11} - 4/7c_D^{11}; \qquad I^G; J^P = 0^-, 1^+, 2^-; 3^{+-}.$$

And for $S = 2$:

$$m(^5S_1) = m_S^{11} + a_S^{11}; \qquad I^G; J^P = 0^+, 1^-, 2^+; 2^{++},$$

$$m(^5P_1) = m_P^{11} + a_P^{11} - 3b_P^{11} - 2c_P^{11}; \qquad I^G; J^P = 0^-, 1^+, 2^-; 1^{--},$$

$$m(^5P_2) = m_P^{11} + a_P^{11} - b_P^{11} + 2c_P^{11}; \qquad I^G; J^P = 0^-, 1^+, 2^-; 2^{--},$$

$$m(^5P_3) = m_P^{11} + a_P^{11} + b_P^{11} - 2/5c_P^{11}; \qquad I^G; J^P = 0^-, 1^+, 2^-; 3^{--},$$

$$m(^5D_0) = m_D^{11} + a_D^{11} - 6b_D^{11} - 12c_D^{11}; \qquad I^G; J^P = 0^+, 1^-, 2^+; 0^{++},$$

$$m(^5D_1) = m_D^{11} + a_D^{11} - 5b_D^{11} - 6c_D^{11}; \qquad I^G; J^P = 0^+, 1^-, 2^+; 1^{++}, \qquad (40)$$

$$m(^5D_2) = m_D^{11} + a_D^{11} - 3b_D^{11} + 18/7c_D^{11}; \qquad I^G; J^P = 0^+, 1^-, 2^+; 2^{++},$$

$$m(^5D_3) = m_D^{11} + a_D^{11} \qquad + 48/7c_D^{11}; \qquad I^G; J^P = 0^+, 1^-, 2^+; 3^{++},$$

$$m(^5D_4) = m_D^{11} + a_D^{11} + 4b_D^{11} - 24/7c_D^{11}; \qquad I^G; J^P = 0^+, 1^-, 2^+; 4^{++}.$$

Now we can construct the spectrum of non-starnge four-quark mesons. Two parameters: Regge slope $\alpha' = 1$ GeV^{-2} and vector and scalar diquark mass difference $\Delta m_d = 200$ MeV (eq.(32)) determine the splitting between different orbital excitations. The overall mass scale is fixed if we identify f/AX (1520) meson with D-wave A-type state. The values of masses are given in the Table 4.

Table 4.

	$IJ = 00$	$IJ = 01$	$IJ = 11$
m_S^{IJ} (GeV)	0.56	0.76	0.96
m_P^{IJ} (GeV)	1.14	1.26	1.39
m_D^{IJ} (GeV)	1.52	1.61	1.71

Parameters of spin-spin (a_L^{IJ}), spin-orbital (b_L^{IJ}) and tensor splitting (c_L^{IJ}) for four-quark mesons can be obtained with the help of eq.(30) if we assume that the wave function of orbitally excited diquark-antidiquark states is close to the same for quark-antiquark. As far ar uncertainities in the values of b_L and c_L parameters are about 100%, this assumption for the rough estimate of four-quark meson spectra looks reasonable.

The parameters, shown in Table 5 were obtained using a_L, b_L and c_L extracted from strange meson spectrum and given in Table 3.

Table 5.

	$IJ = 01$		$IJ = 11$		
	$L = P$	$L = D$	$L = S$	$L = P$	$L = D$
a_L^{IJ} (MeV)			240	90	107
b_L^{IJ} (MeV)	5	8		3	5
c_L^{IJ} (MeV)	−5	9		−3	5

If to use parameters obtained from $I = 1$ meson spectrum we will come to the results shown in the Table 6.

Table 6.

	$IJ = 01$		$IJ = 11$		
	$L = P$	$L = D$	$L = S$	$L = P$	$L = D$
a_L^{IJ} (MeV)			230	24	8
b_L^{IJ} (MeV)	8	−1.2		5	−0.7
c_L^{IJ} (MeV)	−7	−0.6		−0.7	−0.35

The resulting four-quark meson spectra for these two possibilities are shown in
Fig. 3
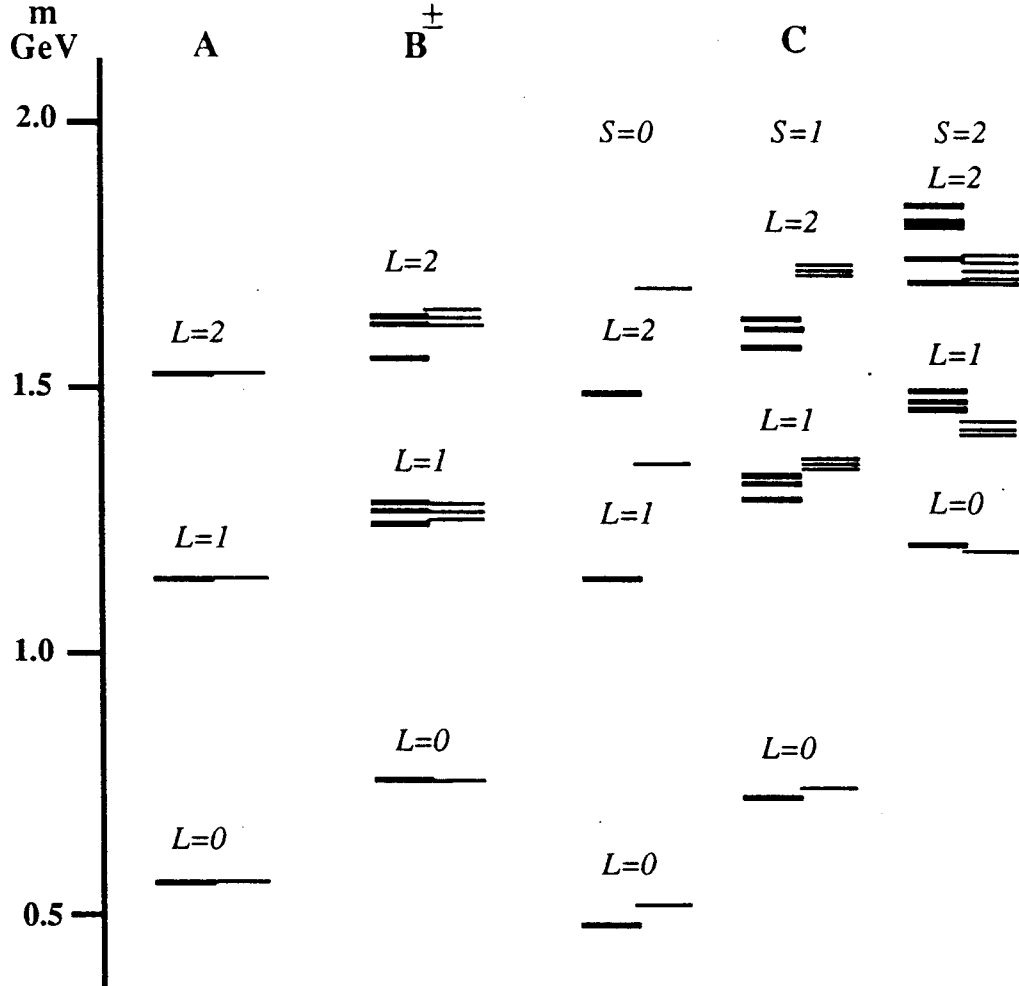
Fig. 3

Levels denoted by solid lines are masses of four-quark states obtained with
parameters from Table 5, while full-lines correspond to the massses of four-quark
states with parameters from Table 6. The two variants shown in Fig.3 illustrates the
accuracy of the model considered.

The mass of low-lying I^G; $J^{PC} = 1^-$; 1^{--} state in this model is about 1.2 GeV.
This value is not far from the mass of $\rho(1270)$ recently discovered by LASS group
meson[9]. Four-quark nature of this state explains the absence of $\rho(1270)$ meson in
e^+e^- annihilations on the present level of cross-sections.

There exist also experimental indications coming from e^+e^- data on the
resonance with $I = 2$ and $J^P = 2^+$ in $\rho\rho$ decay mode with the mass of the order of 1.6
GeV[10]. This exotic state in our model may be interpreted as D-wave state of C-type
meson.

The low-lying four-quark vector meson in this model is ω meson with mass about 1.2 GeV. The mass difference between strange and non-strange diquarks can be estimated from the masses of well-known mesons:

$$m\bigl(f_2'(1525)\bigr) - m\bigl(f_2(1270)\bigr) \approx m\bigl(\varphi(1020)\bigr) - m\bigl(\rho(770)\bigr) \approx$$
$$\approx 2\bigl[m\bigl(K^*(893)\bigr) - m\bigl(\rho(770)\bigr)\bigr] \approx 2\bigl[m\bigl(K_2^*(1430)\bigr) - m\bigl(f_2(1270)\bigr)\bigr] \approx 2\Delta m \approx 250 \text{ MeV} \quad (41)$$

where $\Delta m \approx 125$ MeV can be treated as the difference between masses of strange and non-strange quarks.

Thus the partner of $\omega(1200)$ meson with hidden strangeness should have the mass of 1.45 GeV. This is very close to the mass of C(1480) meson seen by LEPTON - F group[11] in $\varphi\pi$ decay channel. Our diquark string model predicts also the existence of isoscalar partner of C(1480) meson.

7. Decays of exotic mesons.

Tables A1 and A2 in Appendix 2 show all possible decays of mesons with different quantum numbers and meson spin less than 2 into S-wave and P-wave $\bar{q}q$ mesons.

There are some channels where the presence of peak clearly indicates the production of exotic resonance: P-wave in $\eta\pi$ system which is a signal of π_1 production and S-wave in $f_1\eta$ final state which is a signal of η_1 decay. P-waves in $b_1\pi$ ($h_1\eta$) and $b_1\eta$ ($h_1\pi$) systems could be the signals of h_0, h_2 and b_0. b_2 decays correspondently, but may arise also in decays of non-exotic h_1 and b_1 mesons.

May be the best way to search for ω_0 and ρ_0 exotic mesons is to look for D-wave in $f_2\omega$, $a_2\rho$ and $f_2\rho$, $a_2\omega$ systems correspondently. but in these cases there may be D-waves also from decays of non-exotic states: ω, ω_2 and ρ, ρ_2.

Our four-quark model predicts a large number of exotic $I = 2$ mesons which should decay into two $I = 1$ mesons: $\pi\pi$, $\pi\rho$, πa and πb final states. As is seen from Fig.3 the masses of these mesons are in the interval 1.2 - 1.8 GeV.

In the sector of $I = 0$ and $I = 1$ exotic mesons the model predicts the existence of two π_1 mesons: one with mass about 1250-1300 MeV and another, degenerated in mass with η_1 exotic resonance and having the mass in the interval 1300- 1350 MeV. The model predicts the mass of ρ_0 exotic meson degenerated with b_0 exotic meson in the same mass interval. There should be two b_2 exotic mesons: one with mass around 1600 MeV and another degenerated in mass with h_2 exotic state and with mass around 1650 MeV. The model does not predicts ω_0 and h_0 exotic mesons and this is a general feature of diquark-antidiquark models.

The search for 4-quark mesons in $\overline{N}N$ annihilations, especially those with exotic quantum numbers may be the very interesting problem for all groups, working with antiprotons.

Appendix 1. Cross-sections for three pseudoscalar meson production.

For $\bar{p}p$ annihilation we can write, neglecting with phase space corrections:

A. Reactions with f_2^s production.

$$\sigma[\bar{p}p \to (\pi^0\pi^0)_{I=0}\pi^0] = \sigma(f_2^s\pi^0)B(f_2^s \to \pi^0\pi^0) = 27\sigma_0 B_0^s$$

$$\sigma[\bar{p}p \to (\pi^+\pi^-)_{I=0}\pi^0] = \sigma(f_2^s\pi^0)B(f_2^s \to \pi^+\pi^-) = 27\sigma_0 2B_0^s$$

$$\sigma[\bar{p}p \to (\eta_8\eta_8)\pi^0] = \sigma(f_2^s\pi^0)B(f_2^s \to \eta_8\eta_8) = 27\sigma_0\frac{1}{9}B_0^s$$

$$\sigma[\bar{p}p \to (\eta_8\eta_0)\pi^0] = \sigma(f_2^s\pi^0)B(f_2^s \to \eta_8\eta_0) = 27\sigma_0\frac{2}{9}B_0^s$$

$$\sigma[\bar{p}p \to (\pi^0\pi^0)\eta_8] = \sigma(f_2^s\eta_8)B(f_2^s \to \pi^0\pi^0) = 9\sigma_0 B_0^s$$

$$\sigma[\bar{p}p \to (\pi^+\pi^-)\eta_8] = \sigma(f_2^s\eta_8)B(f_2^s \to \pi^+\pi^-) = 9\sigma_0 2B_0^s$$

$$\sigma[\bar{p}p \to (\eta_8\eta_8)\eta_8] = \sigma(f_2^s\eta_8)B(f_2^s \to \eta_8\eta_8) = 9\sigma_0\frac{1}{9}B_0^s \qquad \text{(A1)}$$

$$\sigma[\bar{p}p \to (\eta_8\eta_0)\eta_8] = \sigma(f_2^s\eta_8)B(f_2^s \to \eta_8\eta_0) = 9\sigma_0\frac{2}{9}B_0^s$$

$$\sigma[\bar{p}p \to (\pi^0\pi^0)\eta_0] = \sigma(f_2^s\eta_0)B(f_2^s \to \pi^0\pi^0) = 18\sigma_0 B_0^s$$

$$\sigma[\bar{p}p \to (\pi^+\pi^-)\eta_0] = \sigma(f_2^s\eta_0)B(f_2^s \to \pi^+\pi^-) = 18\sigma_0 2B_0^s$$

$$\sigma[\bar{p}p \to (\eta_8\eta_8)\eta_0] = \sigma(f_2^s\eta_0)B(f_2^s \to \eta_8\eta_8) = 18\sigma_0\frac{1}{9}B_0^s$$

$$\sigma[\bar{p}p \to (\eta_8\eta_0)\eta_0] = \sigma(f_2^s\eta_0)B(f_2^s \to \eta_8\eta_0) = 18\sigma_0\frac{2}{9}B_0^s$$

B. Reactions with f_2 production.

$$\sigma[\bar{p}p \to (\pi^0\pi^0)_{I=0}\pi^0] = \sigma(f_2\pi^0)B(f_2 \to \pi^0\pi^0) = \sigma_0 B_0$$

$$\sigma[\bar{p}p \to (\pi^+\pi^-)_{I=0}\pi^0] = \sigma(f_2\pi^0)B(f_2 \to \pi^+\pi^-) = \sigma_0 2B_0$$

$$\sigma[\bar{p}p \to (\eta_8\eta_8)\pi^0] = \sigma(f_2\pi^0)B(f_2 \to \eta_8\eta_8) = \sigma_0\frac{1}{9}B_0$$

$$\sigma[\bar{p}p \to (\eta_8\eta_0)\pi^0] = \sigma(f_2\pi^0)B(f_2 \to \eta_8\eta_0) = \sigma_0\frac{2}{9}B_0$$

$$\sigma[\bar{p}p \to (\pi^0\pi^0)\eta_8] = \sigma(f_2\eta_8)B(f_2 \to \pi^0\pi^0) = 3\sigma_0 B_0$$

$$\sigma[\bar{p}p \to (\pi^+\pi^-)\eta_8] = \sigma(f_2\eta_8)B(f_2 \to \pi^+\pi^-) = 3\sigma_0 2B_0$$

$$\sigma[\bar{p}p \to (\eta_8\eta_8)\eta_8] = \sigma(f_2\eta_8)B(f_2 \to \eta_8\eta_8) = 3\sigma_0\frac{1}{9}B_0 \qquad \text{(A2)}$$

$$\sigma[\bar{p}p \to (\eta_8\eta_0)\eta_8] = \sigma(f_2\eta_8)B(f_2 \to \eta_8\eta_0) = 3\sigma_0\frac{2}{9}B_0$$

$$\sigma[\bar{p}p \to (\pi^0\pi^0)\eta_0] = \sigma(f_2\eta_0)B(f_2 \to \pi^0\pi^0) = 6\sigma_0 B_0$$

$$\sigma[\bar{p}p \to (\pi^+\pi^-)\eta_0] = \sigma(f_2\eta_0)B(f_2 \to \pi^+\pi^-) = 6\sigma_0 2B_0$$

$$\sigma[\bar{p}p \to (\eta_8\eta_8)\eta_0] = \sigma(f_2\eta_0)B(f_2 \to \eta_8\eta_8) = 6\sigma_0\frac{1}{9}B_0$$

$$\sigma[\bar{p}p \to (\eta_8\eta_0)\eta_0] = \sigma(f_2\eta_0)B(f_2 \to \eta_8\eta_0) = 6\sigma_0\frac{2}{9}B_0$$

C. Reactions with a_2 production.

$$\sigma[\bar{p}p \to (\pi^0\eta_8)\pi^0] = \sigma(a_2^0\pi^0)B(a_2^0 \to \pi^0\eta_8) = 6\sigma_0 B_1$$

$$\sigma[\bar{p}p \to (\pi^0\eta_0)\pi^0] = \sigma(a_2^0\pi^0)B(a_2^0 \to \pi^0\eta_0) = 6\sigma_0 2B_1$$

$$\sigma[\bar{p}p \to (\pi^-\eta_8)\pi^+] = \sigma(a_2^-\pi^+)B(a_2^- \to \pi^-\eta_8) = 6\sigma_0 B_1$$

$$\sigma[\bar{p}p \to (\pi^-\eta_0)\pi^+] = \sigma(a_2^-\pi^+)B(a_2^- \to \pi^-\eta_0) = 6\sigma_0 2B_1 \qquad (A3)$$

$$\sigma[\bar{p}p \to (\pi^0\eta_8)\eta_8] = \sigma(a_2^0\eta_8)B(a_2^0 \to \pi^0\eta_8) = 2\sigma_0 B_1$$

$$\sigma[\bar{p}p \to (\pi^0\eta_0)\eta_8] = \sigma(a_2^0\eta_8)B(a_2^0 \to \pi^0\eta_0) = 2\sigma_0 2B_1$$

$$\sigma[\bar{p}p \to (\pi^0\eta_8)\eta_0] = \sigma(a_2^0\eta_0)B(a_2^0 \to \pi^0\eta_8) = 4\sigma_0 B_1$$

$$\sigma[\bar{p}p \to (\pi^0\eta_0)\eta_0] = \sigma(a_2^0\eta_0)B(a_2^0 \to \pi^0\eta_0) = 4\sigma_0 2B_1$$

D. Reactions with c_2 production.

$$\sigma[\bar{p}p \to (\pi^0\pi^0)_{I=2}\pi^0] = \sigma(c_2^0\pi^0)B(c_2^0 \to \pi^0\pi^0) = 8\sigma_0 B_2$$

$$\sigma[\bar{p}p \to (\pi^+\pi^-)_{I=2}\pi^0] = \sigma(c_2^0\pi^0)B(c_2^0 \to \pi^+\pi^-) = 8\sigma_0\frac{1}{2}B_2 \qquad (A4)$$

$$\sigma[\bar{p}p \to (\pi^0\pi^-)_{I=2}\pi^+] = \sigma(c_2^-\pi^+)B(c_2^- \to \pi^0\pi^-) = 6\sigma_0\frac{3}{2}B_2$$

Corresponding relations for $\bar{p}n$ annihilations have the form:

A. Reactions with f_2^s production.

$$\sigma[\bar{p}n \to (\pi^0\pi^0)_{I=0}\pi^-] = \sigma(f_2^s\pi^-)B(f_2^s \to \pi^0\pi^0) = 54\sigma_0 B_0^s$$

$$\sigma[\bar{p}n \to (\pi^+\pi^-)_{I=0}\pi^-] = \sigma(f_2^s\pi^-)B(f_2^s \to \pi^+\pi^-) = 54\sigma_0 2B_0^s$$

$$\sigma[\bar{p}n \to (\eta_8\eta_8)\pi^-] = \sigma(f_2^s\pi^-)B(f_2^s \to \eta_8\eta_8) = 54\sigma_0\frac{1}{9}B_0^s \qquad (A5)$$

$$\sigma[\bar{p}n \to (\eta_8\eta_0)\pi^-] = \sigma(f_2^s\pi^-)B(f_2^s \to \eta_8\eta_0) = 54\sigma_0\frac{2}{9}B_0^s$$

B. Reactions with f_2 production.

$$\sigma[\bar{p}n \to (\pi^0\pi^0)_{I=0}\pi^-] = \sigma(f_2\pi^-)B(f_2 \to \pi^0\pi^0) = 2\sigma_0 B_0$$

$$\sigma[\bar{p}n \to (\pi^+\pi^-)_{I=0}\pi^-] = \sigma(f_2\pi^-)B(f_2 \to \pi^+\pi^-) = 2\sigma_0 2B_0$$

$$\sigma[\bar{p}n \rightarrow (\eta_8\eta_8)\pi^-] = \sigma(f_2\pi^-)B(f_2 \rightarrow \eta_8\eta_8) = 2\sigma_0\frac{1}{9}B_0 \qquad \text{(A6)}$$

$$\sigma[\bar{p}n \rightarrow (\eta_8\eta_0)\pi^-] = \sigma(f_2\pi^-)B(f_2 \rightarrow \eta_8\eta_0) = 2\sigma_0\frac{2}{9}B_0$$

C. Reactions with a_2 production.

$$\sigma[\bar{p}n \rightarrow (\pi^-\eta_8)\eta_8] = \sigma(a_2^-\eta_8)B(a_2^- \rightarrow \pi^-\eta_8) = 4\sigma_0 B_1$$

$$\sigma[\bar{p}n \rightarrow (\pi^-\eta_0)\eta_8] = \sigma(a_2^-\eta_8)B(a_2^- \rightarrow \pi^-\eta_o) = 4\sigma_0 2B_1$$

$$\sigma[\bar{p}n \rightarrow (\pi^-\eta_8)\eta_0] = \sigma(a_2^-\eta_8)B(a_2^- \rightarrow \pi^-\eta_8) = 8\sigma_0 B_1 \qquad \text{(A7)}$$

$$\sigma[\bar{p}n \rightarrow (\pi^-\eta_0)\eta_0] = \sigma(a_2^-\eta_8)B(a_2^- \rightarrow \pi^-\eta_o) = 8\sigma_0 2B_1$$

D. Reactions with c_2 production.

$$\sigma[\bar{p}n \rightarrow (\pi^0\pi^0)_{I=2}\pi^-] = \sigma(c_2^0\pi^-)B(c_2^0 \rightarrow \pi^0\pi^0) = 4\sigma_0 B_2$$

$$\sigma[\bar{p}n \rightarrow (\pi^+\pi^-)_{I=2}\pi^-] = \sigma(c_2^0\pi^-)B(c_2^0 \rightarrow \pi^+\pi^-) = 4\sigma_0\frac{1}{2}B_2$$

$$\sigma[\bar{p}n \rightarrow (\pi^0\pi^-)\pi^0] = \sigma(c_2^-\pi^0)B(c_2^- \rightarrow \pi^0\pi^-) = 12\sigma_0\frac{3}{2}B_2 \qquad \text{(A8)}$$

$$\sigma[\bar{p}n \rightarrow (\pi^-\pi^-)\pi^+] = \sigma(c_2^{--}\pi^+)B(c_2^{--} \rightarrow \pi^-\pi^-) = 24\sigma_0\frac{3}{2}B_2$$

here σ_0 is the cross-section of the reaction $\bar{p}p \rightarrow f_2\pi^0$, B_0 is the branching ratio of decay $f_2 \rightarrow \pi^0\pi^0$, B_0^s is the branching ratio of decay $f_2^s \rightarrow \pi^0\pi^0$ (in general, B_0^s may be different than B_0), B_1 is the branching ratio of decay $a_2^0 \rightarrow \pi^0\eta_8$ and B_2 is the branching ratio of decay $c_2^0 \rightarrow \pi^0\pi^0$.

When deriving eq's (A1-A8) we used amplitudes given by eq's (8,9) for four-quark meson production cross-sections and Clebsh-Gordan coefficients and eq.(7) to obtain the branching ratios of four-quark meson decays into pions, η_8's and η_0's.

Appendix 2. Decays of meson resonances into S- and P-wave mesons.

Table A1.

Decays of meson resonances with all possible quantum numbers with spin less than 2 into two S-wave $\bar{q}q$ mesons. The exotic mesons are shadowed.

m $J^{PC} I^G$	$\rho\rho$	$\rho\omega$	$\rho\pi$	$\rho\eta$	$\omega\omega$	$\omega\pi$	$\omega\eta$	$\pi\pi$	$\pi\eta$	$\eta\eta$
ω_0 $0^{--}0^-$	G	I	1	G	G	G	1	G	I	G
ω $1^{--}0^-$	G	I	1	G	G	G	1	G	I	G
ω_2 $2^{--}0^-$	G	I	1	G	G	G	1	G	I	G
η $0^{-+}0^+$	1	G	G	I	1	I	G	P-I	G	P
η_1 $1^{-+}0^+$	1	G	G	I	1	I	G	P-I	G	P
η_2 $2^{-+}0^+$	1	G	G	I	1 .	I	G	P-I	G	P
π $0^{-+}1^-$	G	1	1	G	G	G	I	G	P	G
π_1 $1^{-+}1^-$	G	1	1	G	G	G	I	G	1	G
π_2 $2^{-+}1^-$	G	1	1	G	G	G	I	G	P	G
ρ_0 $0^{--}1^+$	P-I	G	G	1	I	1	G	P-I	G	I
ρ $1^{--}1^+$	1	G	G	1	I	1	G	1	G	I
ρ_2 $2^{--}1^+$	1	G	G	1	I	1	G	P-I	G	I
h_0 $0^{+-}0^-$	G	I	P	G	G	G	P	G	I	G
h_1 $1^{+-}0^-$	G	I	0	G	G	G	0	G	I	G
h_2 $2^{+-}0^-$	G	I	2	G	G	G	2	G	I	G
f_0 $0^{++}0^+$	0	G	G	I	0	I	G	0	G	0
f_1 $1^{++}0^+$	2	G	G	I	P	I	G	P-I	G	P
f_2 $2^{++}0^+$	0	G	G	I	0	I	G	2	G	2
a_0 $0^{++}1^-$	G	0	P	G	G	G	I	G	0	G
a_1 $1^{++}1^-$	G	0	0	G	G	G	I	G	P	G
a_2 $2^{++}1^-$	G	0	2	G	G	G	I	G	2	G
b_0 $0^{+-}1^+$	P-I	G	G	P	I	P	G	P-I	G	I
b_1 $1^{+-}1^+$	0	G	G	0	I	0	G	P-I	G	I
b_2 $2^{+-}1^+$	2	G	G	2	I	2	G	P-I	G	I

Here numbers are the lowest possible orbital momenta in corresponding two-particle decay, G stands for G-parity conservation forbidden decays, P stands for parity conservation forbidden decay and P-I denotes symmetry forbidden decays for identical particles in the final state.

Table A2
Decays of meson resonances into one S-wave and one P-wave $\bar{q}q$ mesons

m	$J^{PC}\,I^G$	$f_0\rho$	$f_0\omega$	$f_0\pi$	$f_0\eta$	$a_0\rho$	$a_0\omega$	$a_0\pi$	$a_0\eta$	$f_1\rho$	$f_1\omega$	$f_1\pi$	$f_1\eta$
ω_0	$0^{--}0^-$	G	P	I	G	P	G	G	I	G	0	I	G
ω	$1^{--}0^-$	G	0	I	G	0	G	G	I	G	0	I	G
ω_2	$2^{--}0^-$	G	2	I	G	2	G	G	I	G	0	I	G
η	$0^{-+}0^+$	I	G	G	0	G	I	0	G	I	G	G	P
η_1	$1^{-+}0^+$	I	G	G	P	G	I	P	G	I	G	G	0
η_2	$2^{-+}0^+$	I	G	G	2	G	I	2	G	I	G	G	2
π	$0^{-+}1^-$	G	I	0	G	P	G	G	0	G	I	P	G
π_1	$1^{-+}1^-$	G	I	P	G	0	G	G	P	G	I	0	G
π_2	$2^{-+}1^-$	G	I	2	G	2	G	G	2	G	I	2	G
ρ_0	$0^{--}1^+$	P	G	G	I	G	P	0	G	0	G	G	I
ρ	$1^{--}1^+$	0	G	G	I	G	0	P	G	0	G	G	I
ρ_2	$2^{--}1^+$	2	G	G	I	G	2	2	G	0	G	G	I
h_0	$0^{+-}0^-$	G	1	I	G	1	G	G	I	G	1	I	G
h_1	$1^{+-}0^-$	G	1	I	G	1	G	G	I	G	1	I	G
h_2	$2^{+-}0^-$	G	1	I	G	1	G	G	I	G	1	I	G
f_0	$0^{++}0^+$	I	G	G	P	G	I	P	G	I	G	G	1
f_1	$1^{++}0^+$	I	G	G	1	G	I	1	G	I	G	G	1
f_2	$2^{++}0^+$	I	G	G	P	G	I	P	G	I	G	G	1
a_0	$0^{++}1^-$	G	I	P	G	1	G	G	P	G	I	1	G
a_1	$1^{++}1^-$	G	I	1	G	1	G	G	1	G	I	1	G
a_2	$2^{++}1^-$	G	I	P	G	1	G	G	P	G	I	1	G
b_0	$0^{+-}1^+$	1	G	G	I	G	1	P	G	1	G	G	I
b_1	$1^{+-}1^+$	1	G	G	I	G	1	1	G	1	G	G	I
b_2	$2^{+-}1^+$	1	G	G	I	G	1	P	G	1	G	G	I

Table A2 (continued)

m	$J^{PC} I^G$	$a_1\rho$	$a_1\omega$	$a_1\pi$	$a_1\eta$	$f_2\rho$	$f_2\omega$	$f_2\pi$	$f_2\eta$	$a_2\rho$	$a_2\omega$	$a_2\pi$	$a_2\eta$
ω_0	$0^{-+}0^-$	0	G	G	I	G	2	I	G	2	G	G	I
ω	$1^{--}0^-$	0	G	G	I	G	0	I	G	0	G	G	I
ω_2	$2^{--}0^-$	0	G	G	I	G	0	I	G	0	G	G	I
η	$0^{-+}0^+$	G	I	P	G	I	G	G	2	G	I	2	G
η_1	$1^{-+}0^+$	G	I	0	G	I	G	G	2	G	I	2	G
η_2	$2^{-+}0^+$	G	I	2	G	I	G	G	0	G	I	0	G
π	$0^{-+}1^-$	0	G	G	P	G	I	2	G	2	G	G	2
π_1	$1^{-+}1^-$	0	G	G	0	G	I	2	G	0	G	G	2
π_2	$2^{-+}1^-$	0	G	G	2	G	I	0	G	0	G	G	0
ρ_0	$0^{--}1^+$	G	0	P	G	2	G	G	I	G	2	2	G
ρ	$1^{--}1^+$	G	0	0	G	0	G	G	I	G	0	2	G
ρ_2	$2^{--}1^+$	G	0	2	G	0	G	G	I	G	0	0	G
h_0	$0^{+-}0^-$	1	G	G	I	G	1	I	G	1	G	G	I
h_1	$1^{+-}0^-$	1	G	G	I	G	1	I	G	1	G	G	I
h_2	$2^{+-}0^-$	1	G	G	I	G	1	I	G	1	G	G	I
f_0	$0^{++}0^+$	G	I	1	G	I	G	G	P	G	I	P	G
f_1	$1^{++}0^+$	G	I	1	G	I	G	G	1	G	I	1	G
f_2	$2^{++}0^+$	G	I	1	G	I	G	G	1	G	I	1	G
a_0	$0^{++}1^-$	1	G	G	1	G	I	P	G	1	G	G	P
a_1	$1^{++}1^-$	1	G	G	1	G	I	1	G	1	G	G	1
a_2	$2^{++}1^-$	1	G	G	1	G	I	1	G	1	G	G	1
b_0	$0^{+-}1^+$	G	1	1	G	1	G	G	I	G	1	P	G
b_1	$1^{+-}1^+$	G	1	1	G	1	G	G	I	G	1	1	G
b_2	$2^{+-}1^+$	G	1	1	G	1	G	G	I	G	1	1	G

Table A2 (continued)

m	$J^{PC}\,I^G$	$h_1\rho$	$h_1\omega$	$h_1\pi$	$h_1\eta$	$b_1\rho$	$b_1\omega$	$b_1\pi$	$b_1\eta$
ω_0	$0^{--}0^-$	I	G	G	P	G	I	P	G
ω	$1^{--}0^-$	I	G	G	0	G	I	0	G
ω_2	$2^{--}0^-$	I	G	G	2	G	I	2	G
η	$0^{-+}0^+$	G	0	I	G	0	G	G	I
η_1	$1^{-+}0^+$	G	0	I	G	0	G	G	I
η_2	$2^{-+}0^+$	G	0	I	G	0	G	G	I
π	$0^{-+}1^-$	0	G	G	I	G	0	P	G
π_1	$1^{-+}1^-$	0	G	G	I	G	0	0	G
π_2	$2^{-+}1^-$	0	G	G	I	G	0	2	G
ρ_0	$0^{--}1^+$	G	I	P	G	0	G	G	P
ρ	$1^{--}1^+$	G	I	0	G	0	G	G	0
ρ_2	$2^{--}1^+$	G	I	2	G	0	G	G	2
h_0	$0^{+-}0^-$	I	G	G	1	G	I	1	G
h_1	$1^{+-}0^-$	I	G	G	1	G	I	1	G
h_2	$2^{+-}0^-$	I	G	G	1	G	I	1	G
f_0	$0^{++}0^+$	G	1	I	G	1	G	G	I
f_1	$1^{++}0^+$	G	1	I	G	1	G	G	I
f_2	$2^{++}0^+$	G	1	I	G	1	G	G	I
a_0	$0^{++}1^-$	1	G	G	I	G	1	1	G
a_1	$1^{++}1^-$	1	G	G	I	G	1	1	G
a_2	$2^{++}1^-$	1	G	G	I	G	1	1	G
b_0	$0^{+-}1^+$	G	I	1	G	1	G	G	1
b_1	$1^{+-}1^+$	G	I	1	G	1	G	G	1
b_2	$2^{+-}1^+$	G	I	1	G	1	G	G	1

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