ISTITUTO NAZIONALE DI FISICA NUCLEARE - ISTITUTO NAZIONALE DI FISICA NUCLEARE - ISTITUTO NAZIONALE DI FISICA NUCLEARE - ISTITUTO NAZIONALE DI FISICA NUCLEARE - ISTITUTO NAZIONALE DI

Laboratori Nazionali di Frascati

Submitted to Physics Letters

LNF–93/040 (P)
28 Luglio 1993

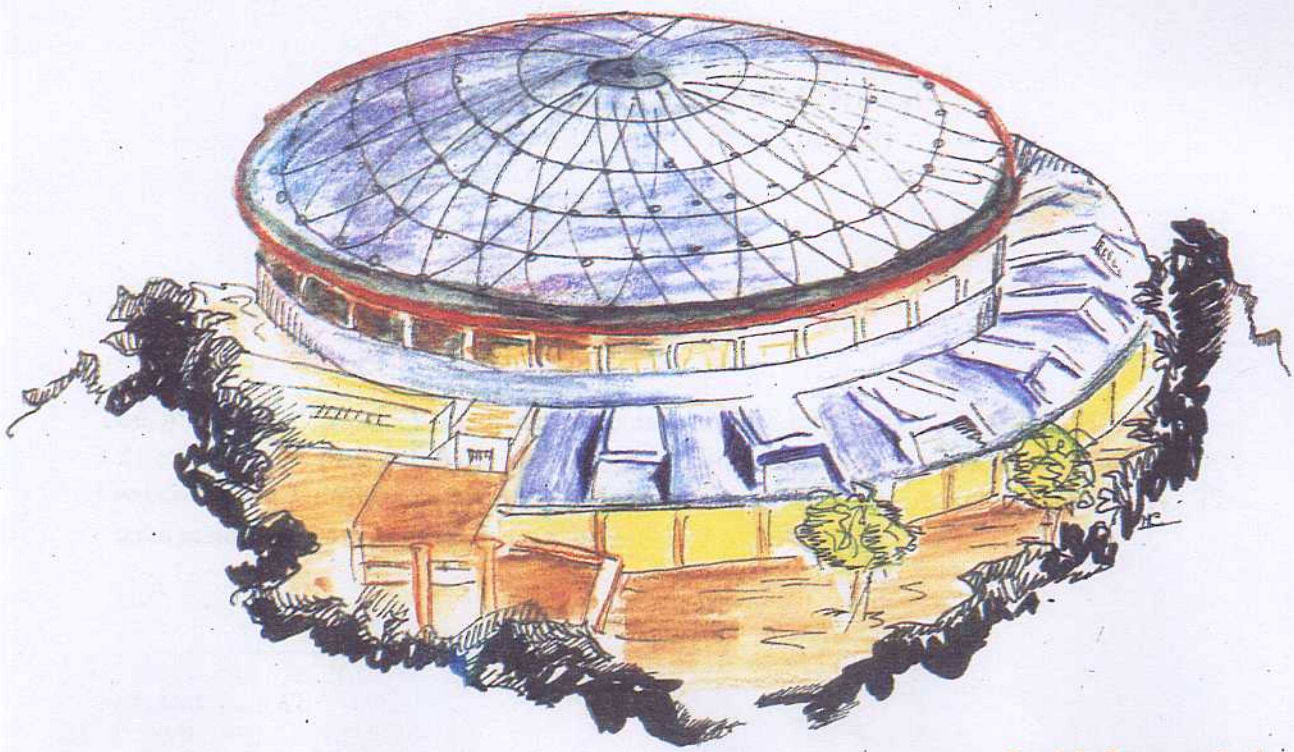
UAB-FT-309/93
UG-FT-29/93

A. Bramon, A. Grau, G. Pancheri:

THE $\rho^0 \to \pi^+\pi^-\pi^+\pi^-$ DECAY IN EFFECTIVE LAGRANGIANS INCORPORATING VECTOR-MESONS

PACS.: 13.20.JF

Contribution to the DAΦNE Theory Study Group



Servizio Documentazione
dei Laboratori Nazionali di Frascati
P.O. Box, 13 - 00044 Frascati (Italy)

INFN - Laboratori Nazionali di Frascati
Servizio Documentazione

LNF-93/040 (P)
28 Luglio 1993

UAB-FT-309/93
UG-FT-29/93

THE $\rho^0 \to \pi^+\pi^-\pi^+\pi^-$ DECAY IN EFFECTIVE LAGRANGIANS INCORPORATING VECTOR-MESONS

A. Bramon [1]
Grup de Física Teòrica, Universitat Autònoma de Barcelona,
08193 Bellaterra (Barcelona), Spain

A. Grau [2]
Departamento de Física Teórica y del Cosmos,
Universidad de Granada, 18071 Granada, Spain

and

G. Pancheri [3]
INFN, Laboratori Nazionali di Frascati,
P.O.Box 13, 00044 Frascati, Italy

Abstract

The $\rho^0 \to \pi^+\pi^-\pi^+\pi^-$ branching ratio is predicted to be $(0.5 \pm 0.1) \times 10^{-4}, (1.7 \pm 0.2) \times 10^{-4}$ and $(4.0 \pm 0.5) \times 10^{-4}$ in the "Hidden Symmetry" scheme, conventional Vector-Meson Dominance and the "Massive Yang-Mills" approach, respectively. The corresponding experimental upper bound is 2×10^{-4}.

[1] 16414::BRAMON
[2] 16487::IGRAU
[3] PANCHERI @ IRMLNF

The long-standing question of how to incorporate spin-1 mesons in effective, low-energy lagrangians has received considerable attention along the last years. On the experimental side, this revival seems related to the announced studies and construction of new, high-luminosity and low-energy e^+e^--machines [1]. From the theoretical point of view, on the other hand, the success of QCD-inspired effective theories, such as Chiral Perturbation Theory (ChPT) [2], has certainly contributed to the above mentioned revival. Indeed, spin-1 mesons and, more particularly, the low-lying vector mesons have been shown to play a crucial role when trying to understand and predict the values of most of the low-enegy constants or counterterms appearing in the ChPT lagrangian [2, 3, 4, 5]. This implicit presence of vector meson dynamics in the parameters of the theory clearly justifies the recent attempts aiming to incorporate explicit vector fields in effective chiral lagrangians.

Two main lines of thought have been developped in such attempts. One is the so-called "massive Yang-Mills approach" (YM) proposed, among others, by Schechter et al. [6] and by Meissner, and extensively reviewed by the latter in [7]. Spin-1 fields are incorporated in the ungauged chiral lagrangian through conventional covariant derivatives. This guarantees the simplicity and elegance of the approach but leaves the problem of giving a finite mass to the spin-1 gauge fields quite unsolved. The second approach – known as the "hidden symmetry scheme" (HS) – has been mainly worked out and recently reviewed by Bando and collaborators [8]. Vector mesons are the gauge fields of a "hidden" local symmetry in the chiral lagrangian. This allows to introduce the electro-weak gauge bosons through the usual covariant derivative procedure and, more important, generates automatically the appropriate mass terms for the vector mesons through a Higgs-like mechanism. A central issue remains open: the "dynamical" generation of the kinetic terms of these gauge fields is shown to be plausible [8], but far from being proved. And new questions immediately appear: are these two approaches, YM and HS, basically equivalent? If not, how could one experimentally discriminate between (thus favouring one of) the two?

The basic purpose of this note is to answer these questions. This doesn't seem to be a trivial task, as could be forseen from the extensive analysis by Meissner [7]. The basic features of each approach are usually buried in a large variety of secondary details aimed to enlarge their applicability, for instance, from two to three flavours, or from vectors to axial-vectors, or from normal, non-anomalous processes to others related to the axial anomaly. To avoid these complications we will restrict to the most basic version of both approaches involving only the photon field, A_μ, and the two SU(2)-triplet fields of pions, π, and ρ-mesons, ρ_μ,

$$\Pi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} & \pi^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} \end{pmatrix} \qquad \rho_\mu = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} & \rho^+ \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} \end{pmatrix} \tag{1}$$

Extensions to SU(3) or to axial-vector or anomalous sectors ($a_1 \to 3\pi$ or $\omega \to 3\pi, \pi^0 \to \gamma\gamma$) will not be discussed here, since the non-equivalence and possible experimental discrimination between the two approaches, can already be demonstrated at their most basic level.

As previously mentioned, the "massive Yang-Mills approach" incorporates the ρ-meson fields in the derivative term

$$\mathcal{L} = \frac{f^2}{8}\, tr\left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right) \tag{2}$$

of the lowest-order, ungauged chiral lagrangian through the conventional substitution

$$\partial_\mu \Sigma \rightarrow D_\mu \Sigma \equiv \partial_\mu \Sigma + ig\left[\rho_\mu, \Sigma\right], \tag{3}$$

where $\Sigma \equiv exp(2i\Pi/f)$ contains the triplet (1) of Goldstone pions and their decay constant [9] f = 132 MeV. One is then lead to the lowest-order lagrangian

$$\begin{aligned} \mathcal{L}^{YM} &= \frac{f^2}{8}\, tr\left(D_\mu \Sigma D^\mu \Sigma^\dagger\right) \\ &= ig\sqrt{2}\rho^0_\mu\left(\pi^+\partial^\mu\pi^- - \pi^-\partial^\mu\pi^+\right)\left[1 - \frac{1}{3f^2}\left(2\pi^0\pi^0 + 4\pi^+\pi^-\right)\right] \\ &\quad + 2g^2\rho^0_\mu\rho^{0\mu}\pi^+\pi^- + \ldots \end{aligned} \tag{4}$$

In (4) the dots refer to terms which are not relevant to our present purpose. In particular, they include pure pionic interactions, which are already contained in the ungauged lagrangian (2), and $\rho - \pi$ interactions other than those explicitly shown in (4) concerning neutral ρ mesons. The $\rho - \pi$ interactions above proceed, as usual, through the gauge coupling-constant g, whose value $g \simeq 4.2$ could be fixed by [9] $\Gamma(\rho^0 \rightarrow \pi^+\pi^-) = g^2 p_\pi^3/3\pi M_\rho^2 = 152$ MeV. Photonic interactions are introduced by invoking Vector Meson Dominance (VMD), i.e., forcing the $\pi^+\pi^-\gamma$ vertex to proceed exclusively through the $\pi^+\pi^-\rho^0$ one, complemented with the $\rho - \gamma$ conversion lagrangian

$$\mathcal{L}_{\rho\gamma} = -\sqrt{2}ef^2 g A^\mu \rho^0_\mu = -\frac{eM_\rho^2}{\sqrt{2}g} A^\mu \rho^0_\mu \tag{5}$$

This lagrangian implies $M_\rho^2 = 2g^2 f^2 = (0.75 \pm 0.04\ GeV)^2$ and $\Gamma(\rho \rightarrow e^+e^-) = 2\alpha^2\pi M_\rho/3g^2 = 5.4 \pm 0.6$ keV, also quite in line with the experimental data [9] $M_\rho =$ 770 MeV and $\Gamma(\rho \rightarrow e^+e^-) = 6.77 \pm 0.32$ keV, if we finally adopt $g = 4.0 \pm 0.2$.

All these results are also contained in the basic lagrangian of the "hidden symmetry scheme",

$$\begin{aligned} \mathcal{L}^{HS} &= \frac{a}{2} f^2\, tr\left(g\,\rho_\mu - e\tau_3 A_\mu + \frac{i}{2}\left(\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi\right) + \ldots\right)^2 \\ &= \frac{1}{2} M_\rho^2\, tr \rho_\mu\rho^\mu - 2egf^2\rho^0_\mu A^\mu + \sqrt{2}ig\rho^0_\mu\left(\pi^+\partial^\mu\pi^- - \pi^-\partial^\mu\pi^+\right) + \ldots \end{aligned} \tag{6}$$

where again the dots refer to irrelevant terms and $\xi \equiv \sqrt{\Sigma} = exp(i\Pi/f)$. Notice that the free parameter a in (6) has been fixed to $a = 2$ in its last line, thus reobtaining the correct relations among M_ρ and ρ-couplings as before. Similarly, all the (non ρ-mediated) π-interactions generated through the Σ and ξ matrices turn out to be the same. A main difference however appears: there are no quadratic terms in the ρ-field in the lagrangian (6) other than the first, mass-term one. This is in sharp contrast with the conventional massive YM lagrangian (4), where vertices containing two ρ-mesons coupled to an even number of pions are obviously present. In a sense, the ability of the HS scheme in generating M_ρ has to be paid by the absence of the above couplings. In principle, the latter could be added by hand but this possibility could be against the spirit of the model and will certainly destroy the attractiveness shown in its recent and successful extension at the one-loop-level [10].

For completeness — and also to be fair with Sakurai and other pioneers of the central issues of the above two schemes — we will also discuss conventional VMD [11, 12]. In its most simple and restrictive formulation, vector-mesons are assumed to dominate both the photonic interactions of the pseudoscalar mesons and the strong interactions among themselves. This is most simply achieved by introducing the covariant derivative $D_\mu \Pi = \partial_\mu \Pi + ig[\rho_\mu, \Pi]$ in the kinetic term of the free pion lagrangian $\mathcal{L} = \frac{1}{2} f^2 \; tr(\partial_\mu \Pi \partial^\mu \Pi)$. One obtains the interaction lagrangian given by

$$\mathcal{L}_{int}^{VMD} = ig \; tr \left(\rho_\mu (\Pi \partial^\mu \Pi - \partial^\mu \Pi \; \Pi) \right) - \frac{g^2}{2} \; tr \left([\rho_\mu, \Pi] \right)^2 \tag{7}$$

The photon field A_μ is introduced via eq.(5) thus leading to the same $\gamma\rho$ and $\rho\pi\pi$ couplings as in the two previous and more sophisticated schemes. The latter, however, contain identical (non ρ-dominated) multipion interactions, which in conventional VMD are required to be frozen out.

All the three schemes so far presented are fully equivalent when applied to two of the limited number of processes involving only photons, pions and ρ-mesons. One concerns the well known coupling of charged pions to off-shell photons, $k^2 \neq 0$, which is found to be given by the same ρ^0-dominated form factor, $F(k^2) = 2f^2g^2/(M_\rho^2 - k^2)$, where the correct normalization, i.e. $F(0) = 1$, is ensured in all three schemes by the satisfaction of the relation $2f^2g^2 = M_\rho^2$. The second, less well known, process is the radiative $\rho^0 \to \pi^+\pi^-\gamma$ decay (as well as its equivalent isospin rotated versions $\rho^\pm \to \pi^\pm\pi^0\gamma$, that will not be considered because no experimental data are available). The corresponding common amplitude is easily found to be [13]

$$\begin{aligned} A(\rho^0 \to \pi^+\pi^-\gamma) \;\; = \;\; & 2\sqrt{2}eg \Big[\frac{1}{qp_+} \epsilon \left(p_+ + \frac{1}{2} q \right) \; \rho \left(p_- - \frac{Q}{2} \right) + \\ & \frac{1}{qp_+} \epsilon \left(p_- + \frac{1}{2} q \right) \; \rho \left(p_+ - \frac{Q}{2} \right) + \rho\epsilon \Big] \end{aligned} \tag{8}$$

where Q and ρ stand for the four-momentum and polarization of the ρ^0- meson, q and ϵ for those of the photon and $p_+ + p_- = Q - q$ is the sum of the two pion four-momenta. Notice the gauge-invariance of the amplitude (8), which goes to zero both under the exchange $\epsilon_\mu \to q_\mu$ as well as $\rho_\mu \to Q_\mu$. As discussed in [13], the amplitude (8) leads to a decay width $\Gamma(\rho^0 \to \pi^+\pi^-\gamma) = 1.6$ MeV for photon energies larger than 50 MeV, in good agreement with the corresponding experimental rate of 1.5 ± 0.2 MeV [14, 9].

All the three schemes under consideration lead, however, to different predictions when dealing with the experimentally investigated $\rho^0 \to \pi^+\pi^-\pi^+\pi^-$ decay [15, 9]. According to the naive versions [7] of "massive YM approach" all the diagrams shown in Fig.1 contribute to the corresponding amplitude,

$$A(\rho^0 \to \pi^+\pi^-\pi^+\pi^-)_{YM} = A_a + A_b + A_c + A_d, \tag{9}$$

in contrast with what happens in the other two schemes. Indeed, the spontaneous absence of quadratic vertices in the ρ-field in HS [8] (and, exceptionally, in the specific version [6] of the YM approach) and of multipion interactions in VMD [11], leads to

$$\begin{aligned} A(\rho^0 \to \pi^+\pi^-\pi^+\pi^-)_{HS} &= A_b + A_c + A_d \\ A(\rho^0 \to \pi^+\pi^-\pi^+\pi^-)_{VMD} &= A_a + A_b \end{aligned} \tag{10}$$

respectively. From the various lagrangians one immediately obtains

$$\begin{aligned} A_a &= 2\sqrt{2}g^3\Bigg[(Q-2p_1^+)\epsilon\left(\frac{1}{(p_1^+ + p_2^-)^2 - M_\rho^2} + \frac{1}{(p_1^+ + p_1^-)^2 - M_\rho^2}\right) \\ &\quad +(1 \leftrightarrow 2)\Bigg] + CR \\ A_b &= 2\sqrt{2}g^3\Bigg[\frac{(Q-2p_1^+)\epsilon}{(Q-p_1^+)^2 - m_\pi^2}\left(\frac{2p_1^-(p_2^+ - p_2^-)}{(p_2^+ + p_2^-)^2 - M_\rho^2} + \frac{2p_2^-(p_2^+ - p_1^-)}{(p_2^+ + p_1^-)^2 - M_\rho^2}\right) \\ &\quad +(1 \leftrightarrow 2)\Bigg] + CR \\ A_c &= \frac{8\sqrt{2}g}{3f^2}\,\epsilon[p_1^+ + p_2^+ - p_1^- - p_2^-] \\ A_d &= \frac{4\sqrt{2}g}{3f^2}\left[\frac{(Q-2p_1^+)\epsilon}{(Q-p_1^+)^2 - m_\pi^2}[2p_1^-p_2^- - p_2^+(Q-p_1^+)] + (1 \leftrightarrow 2)\right] + CR \end{aligned} \tag{11}$$

where $Q = p_1^+ + p_2^+ + p_1^- + p_2^-$ is the four-momentum of the ρ^0-meson decaying into four charged pions of four-momenta $p_{1,2}^\pm$, and CR stays to indicate that, because of the odd charge conjugation of ρ^0, one has to *substract* similar crossed terms with p^+ and p^- interchanged.

Once these permutations have been performed, the scarce phase space available in $\rho^0 \to 4\pi$ makes it most convenient to expand the denominators in (11) in terms of the ratios $|\vec{p}_{1,2}^{\,\pm}|^2/M_\rho^2$. Using $2g^2f^2 = M_\rho^2$, and neglecting higher order terms in

$|\vec{p}_{1,2}^{\pm}|^2/M_\rho^2$, one then finds $A_b \approx A_d \approx 0$, and one can approximately write

$$\begin{aligned} 2\ A(\rho^0 \to \pi^+\pi^-\pi^+\pi^-)_{YM} &= 6\ A(\rho^0 \to \pi^+\pi^-\pi^+\pi^-)_{HS} \\ &= 3\ A(\rho^0 \to \pi^+\pi^-\pi^+\pi^-)_{VMD} \\ &= 3\ A_a = 6\ A_c \\ &= 16\sqrt{2}\frac{g}{f^2}\rho^0_\mu(p_1^+ + p_2^+ - p_1^- - p_2^-)^\mu \end{aligned} \qquad (12)$$

Using $g = 4.0$ and $f = 132$ MeV these approximate amplitudes lead to the following estimates for the decay widths

$$\begin{aligned} &\Gamma(\rho^0 \to \pi^+\pi^-\pi^+\pi^-)_{YM} \simeq 57\ keV \\ &\Gamma(\rho^0 \to \pi^+\pi^-\pi^+\pi^-)_{HS} \simeq 6\ keV \\ &\Gamma(\rho^0 \to \pi^+\pi^-\pi^+\pi^-)_{VMD} \simeq 25\ keV, \end{aligned} \qquad (13)$$

thus showing that discrimination is indeed possible and justifying a more accurate evaluation in terms of the complete amplitudes (9),(10) and (11).

After a straightforward but time-consuming calculation we find our final results

$$\begin{aligned} &\Gamma(\rho^0 \to \pi^+\pi^-\pi^+\pi^-)_{YM} = 60 \pm 7\ keV \\ &\Gamma(\rho^0 \to \pi^+\pi^-\pi^+\pi^-)_{HS} = 7.5 \pm 0.8\ keV \\ &\Gamma(\rho^0 \to \pi^+\pi^-\pi^+\pi^-)_{VMD} = 25 \pm 3\ keV \end{aligned} \qquad (14)$$

where the common relative error comes only from $g = 4.0 \pm 0.2$ since we have fixed $2g^2f^2$ to the physical M_ρ^2 in A_a and A_b. These results have to be compared with the recent experimental analysis [15, 9] leading to

$$\begin{aligned} &\Gamma(\rho^0 \to \pi^+\pi^-\pi^+\pi^-)_{EXP} \leq 30\ keV \\ &BR(\rho^0 \to \pi^+\pi^-\pi^+\pi^-)_{EXP} < 2 \times 10^{-4} \end{aligned} \qquad (15)$$

and improving an older upper bound [16] by more than one order of magnitude. All the three models we have discussed predict essentially the same pionic spectra (see Fig.2) but only conventional VMD and the HS scheme respect the experimental upper bound (15) for the branching ratio.

The isospin rotated processes $\rho^0 \to \pi^+\pi^-\pi^0\pi^0$ and $\rho^\pm \to \pi^\pm\pi^+\pi^-\pi^0$ are not discussed because they are considerably more complicated than $\rho^0 \to \pi^+\pi^-\pi^+\pi^-$. They should involve the triple ρ vertices of any typical YM theory and, more importantly, they will receive sizable contributions from the $\rho \to \omega\pi$ followed by $\omega \to 3\pi$ decay chain. These amplitudes belong to the anomalous sector, for which it is conceivable that additional free parameters could be introduced and adjusted to save any model.

In summary, we have discussed the $\rho^0 \to \pi^+\pi^-\pi^+\pi^-$ decay in the context of several models incorporating vector-mesons in effective chiral lagrangians. All the models turn out to predict similar spectra and angular distributions but a clearly distinct branching ratio. Conventional VMD and, more interesting, the present day "Hidden Symmetry" scheme of Bando et al. [8] are seen to be compatible with the experimentally available upper bound. By contrast, "massive Yang Mills" approaches (other than sophisticated versions such as in ref. [6]) seem to be excluded. Further data from planned low-energy e^+e^- machines, such as the Frascati Φ-factory, should definitely clarify the issue.

Acknowledgements

This work has been supported in part by the Human Capital and Mobility Programme, EEC Contract # CHRX-CT920026.

References

[1] Proposal for a Phi factory, Laboratori Nazionali di Frascati-INFN report LNF-90/031(R) (April 1990).

[2] J. Gasser and H. Leutwyler, Ann. Phys. 158 (1984) 142.
J. Gasser and H. Leutwyler, Nucl. Phys. *B*250 (1985) 465, 517, 539.

[3] G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. *B*321 (1989) 311.

[4] J.F. Donoghue, C. Ramirez and G. Valencia, Phys. Rev. *D*39 (1989) 1947.

[5] For a recent review, see J. Bijnens, Int. J. Mod. Phys. *A*8 (1993) 3045.

[6] O. Kaymakcalan and J. Schechter, Phys. Rev. *D*31 (1985) 1109.

[7] U.-G. Meissner, Phys. Rep. 161 (1988) 213.

[8] M. Bando et al., Phys. Rep. 164 (1988) 217.

[9] *Particle Data Group*, Phys. Rev. *D*45 (1992) 1.

[10] M. Harada and K. Yamawaki, Phys. Lett. *B*297 (1992) 151.

[11] J.J. Sakurai, *Currents and Mesons*, University of Chicago, Chicago 1969.

[12] T.H. Bauer et al., Rev. Mod. Phys. 50 (1978) 261.

[13] A. Bramon, G. Colangelo and M. Greco, Phys. Lett. *B*287 (1992) 263.

[14] S.I. Dolinsky et al., Phys. Rep. 202 (1991) 99.

[15] L.M. Kurdadze et al., JETP Lett. 47 (1988) 512.

[16] R. Erbe et al., Phys. Rev. 188 (1969) 2060.

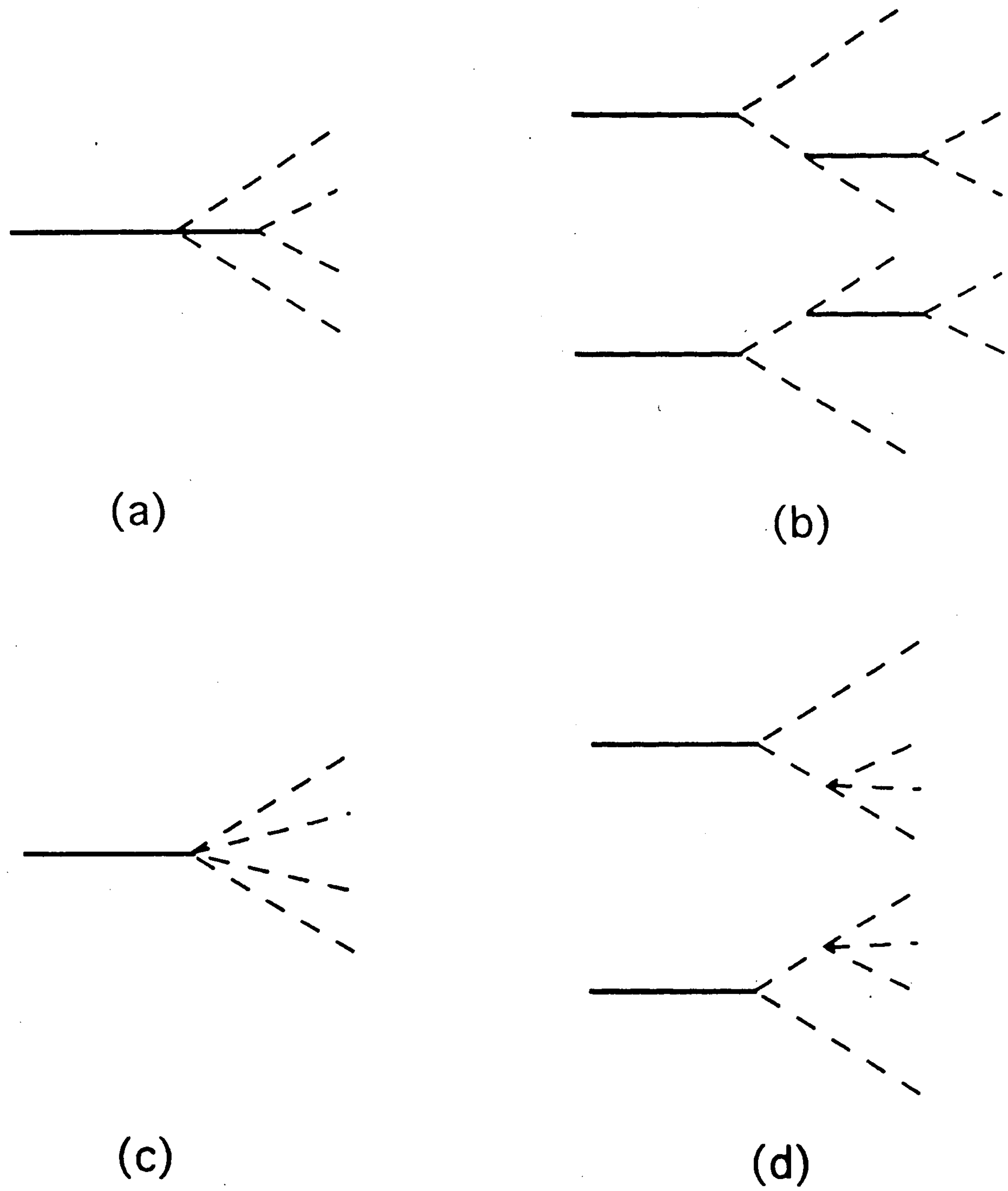


Fig. 1 Set of diagrams contributing to $\rho^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ according to the various models discussed in the text. Solid lines stand for neutral ρ-mesons and dashed lines for charged pions. Crossed diagrams for a), b) and d) are not shown.

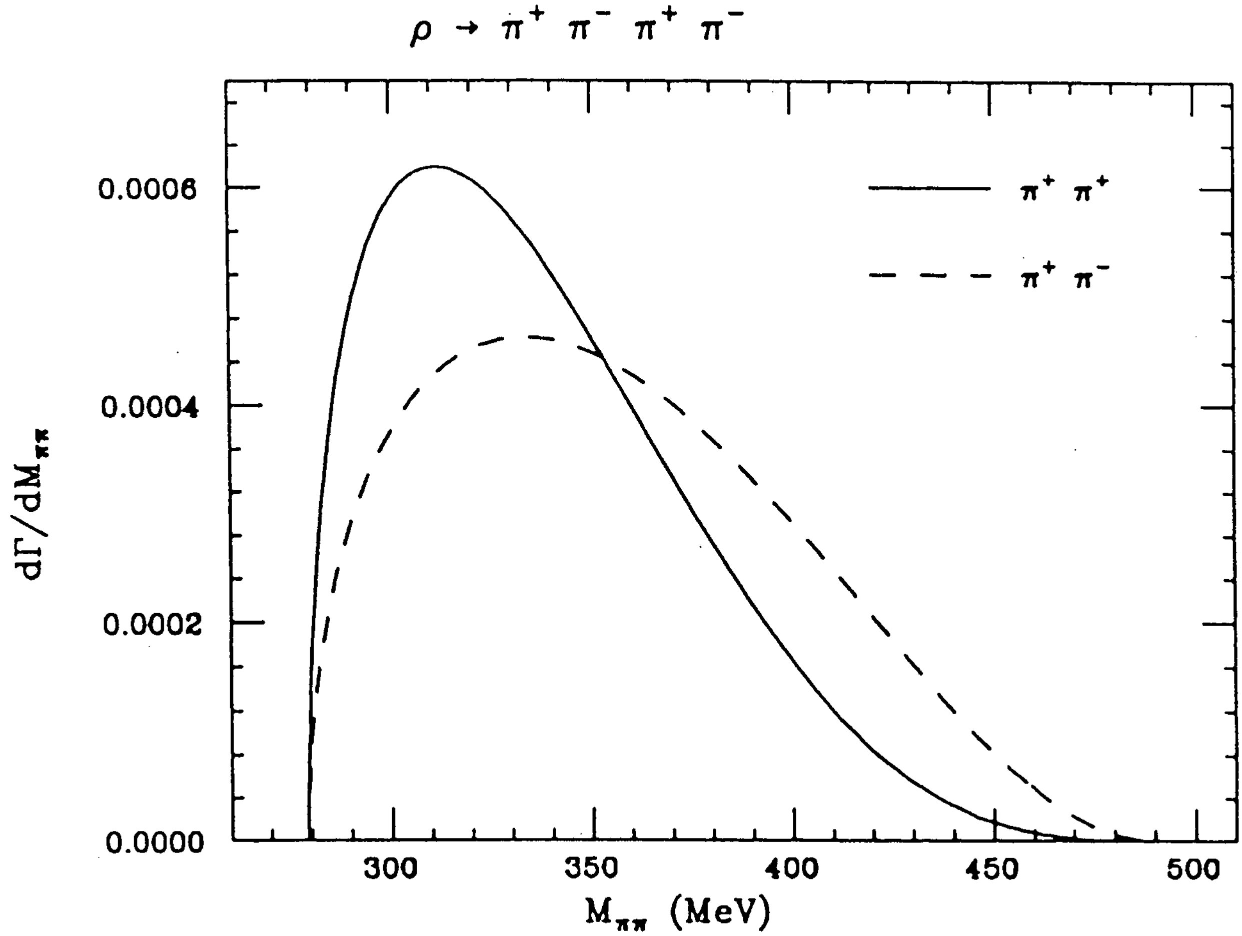


Fig.2 Common di-pion mass spectra in $\rho^0 \to \pi^+\pi^-\pi^+\pi^-$ as predicted by the various models discussed in the text.