Laboratori Nazionali di Frascati

To be published in Z. für Physik C

LNF–93/023 (P)
12 Maggio 1993

INFNNA–IV–93/3
DSF–93–/3

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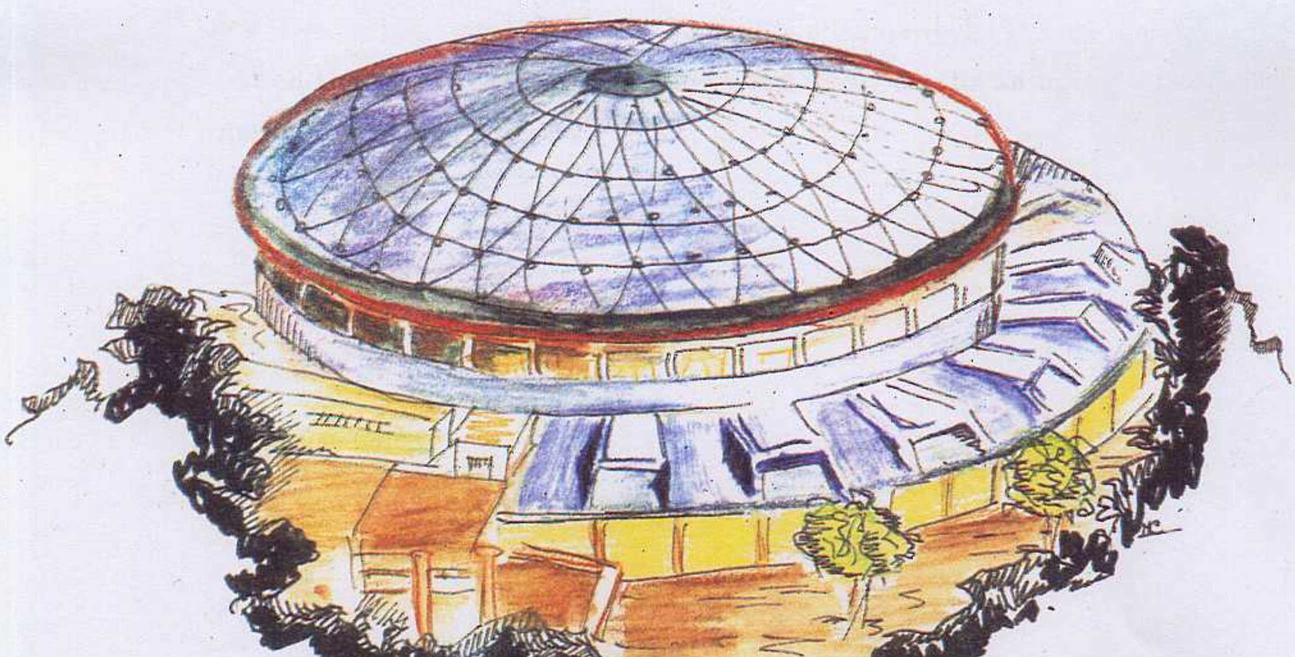
PACS N.: 13.20.Eb ; 13.25.+m

Contribution to the DAΦNE Theory Study Group



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$K_S \to \pi^+\pi^-\gamma$: A LABORATORY FOR MESON DYNAMICS

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Abstract

We discuss the decay $K_S \to \pi^+\pi^-\gamma$ in the framework of chiral perturbation theory. The direct emission in this decay can be an useful test of meson dynamics.

1 Introduction

The amplitudes for $K \to \pi\pi\gamma$ decays are generally defined as a superposition of two amplitudes: internal bremsstrahlung (A_{IB}) and direct emission (A_{DE}) [1, 2, 3, 4, 5, 6, 7, 8]. A_{IB} represents the contribution from just the bremsstrahlung of the external charged particles and it is predicted simply by QED [9]. A_{DE} is obtained by subtracting A_{IB} from the total amplitude. In this way one has disentangled the amplitude which depends from the direct $K \to \pi\pi\gamma$ coupling. This amplitude, differently from A_{IB}, is not predicted from $K \to \pi\pi$ amplitude and it furnishes a test for mesonic interaction models; historically they were studied to test the validity of the $\Delta I = \frac{1}{2}$ rule outside the area of purely hadronic weak processes. We will be mainly interested to the CP conserving decay $K_S \to \pi^+\pi^-\gamma$ in the framework of chiral perturbation theory; we will show that forthcoming experiments (E731, NA31, DAΦNE) should determine $A_{DE}(K_S \to \pi^+\pi^-\gamma)$, where only upper bounds are now existing [10]. Such on observation is interesting for meson dynamics and also for CP violation; indeed the operators contributing to $A_{DE}(K_S \to \pi^+\pi^-\gamma)$ could also generate a direct CP violating $A_{DE}(K_L \to \pi^+\pi^-\gamma)$ which would give rise to a CP violating interference between two $\Delta I = \frac{1}{2}$ transitions: A_{IB} and A_{DE}.

2 Lorentz and gauge invariants, Low theorem and kinematics

The total amplitude for the processes

$$K_{S,L}(P) \to \pi^+(p_+)\pi^-(p_-)\gamma(q,\epsilon) \tag{2.1}$$

[1, 2, 3, 4, 5, 6, 7, 8] is a linear combination of these three Lorentz and gauge invariants:

$$B = \frac{\varepsilon \cdot p_+}{q \cdot p_+} - \frac{\varepsilon \cdot p_-}{q \cdot p_-} \tag{2.2a}$$

$$\overline{B} = \varepsilon \cdot p_+ \; q \cdot p_- \; - \; \varepsilon \cdot p_- \; q \cdot p_+ \tag{2.2b}$$

$$B_{WZ} = \epsilon_{\alpha\beta\gamma\delta} p_+^\alpha p_-^\beta q^\gamma \varepsilon^\delta \tag{2.2c}$$

which are the possible invariants up to third order in momenta. The total invariant amplitude for the process must be then a superposition of these invariants multiplied some scalar functions. B correspond to the IB amplitude, $\overline{B}$ correspond to electric transitions, while B_{WZ} to magnetic transitions. B_{WZ}, in chiral lagrangian is generated by the Wess Zumino term [11, 12, 13]. Although

$$\overline{B} = (q \cdot p_+)(q \cdot p_-)B \tag{2.3}$$

one generally prefers to treat B and $\overline{B}$ separately due to the different behaviour with the photon energy going to zero; in this limit QED [9] establishes a correspondence among radiative and non-radiative decays (and cross-sections). In particular for $K_{S,L} \to \pi^+\pi^-\gamma$ it tells us that the amplitude can be written as:

$$A(K_{S,L} \to \pi^+\pi^-\gamma)_{q\to 0} \simeq eBA(K_{S,L} \to \pi^+\pi^-) \equiv A_{IB}(K_{S,L} \to \pi^+\pi^-\gamma) \tag{2.4}$$

where we have defined precisely the Internal Bremsstrahlung (IB) amplitude; $A(K_{S,L} \to \pi^+\pi^-)$ is the physical amplitude for $K_{S,L} \to \pi^+\pi^-$. If the polarization is not measured there is no interference among electric and magnetic transitions. Thus the total width can be written as the following sum

$$\Gamma_{Tot} = \Gamma_{IB} + \Gamma_{Int} + \Gamma_{WZ} + \Gamma_{|\overline{B}|^2} \tag{2.5}$$

Γ_{IB} is the internal bremsstrahlung contribution and is proportional to $|B|^2$ and Γ_{Int} comes from the interference between B and $\overline{B}$ and thus can be negative. $\overline{B}$ and B_{WZ} have CP eigenvalues $+1$ and -1 respectively. Limiting ourself to CP invariant and low angular momentum states [2, 5, 8] allow us to write:

$$A(K_S \to \pi^+\pi^-\gamma) = eBA(K_S \to \pi^+\pi^-) + \frac{e\overline{B}}{(4\pi f)^2 m_K^2} f_{DE}(E_\gamma^*, \cos\theta) \tag{2.6}$$

where $f_{DE}(E_\gamma^*, \cos\theta)$ is the structure dependent amplitude, E_γ^* is the photon energy in the K_S rest frame, θ is the angle between the photon and the π^+ in the dipion frame. One obtains for the double differential decay width for unpolarized photon from (2.4) and (2.6)

$$\frac{d^2\Gamma(K_S \to \pi^+\pi^-\gamma)}{dE_\gamma^* d\cos\theta} = \frac{2\alpha\beta^3}{\pi\beta_0}\left(1 - \frac{2E_\gamma^*}{m_K}\right)\sin^2(\theta)\Gamma(K_S \to \pi^+\pi^-)\times$$

$$\times\left[\frac{1}{E_\gamma^*(1-\beta^2\cos^2\theta)^2}+\frac{E_\gamma^*\mathrm{Re}[f_{DE}(E_\gamma^*,\cos\theta)A^*(K_S\to\pi^+\pi^-)]}{2(4\pi f)^2(1-\beta^2\cos^2\theta)|A(K_S\to\pi^+\pi^-)|^2}+\right.$$

$$\left.+\frac{|f_{DE}(E_\gamma^*,\cos\theta)|^2 {E_\gamma^*}^3}{16(4\pi f)^4|A(K_S\to\pi^+\pi^-)|^2}\right] \qquad (2.7)$$

$$E_{\gamma_{min}}^* \le E_\gamma^* \le \frac{m_K^2-4m_\pi^2}{2m_K} \quad \text{and} \quad -1\le\cos\theta\le 1$$

$$\beta_0=\sqrt{1-\frac{4{m_\pi}^2}{{m_K}^2}} \qquad \beta=\sqrt{1-\frac{4{m_\pi}^2}{{m_K}^2-2{E_\gamma}^*m_K}}$$

The first term is the Internal Bremsstrahlung, the second the interference term between B and $\overline{B}$; the third is the pure Direct Emission rate. Indeed according to our calculation f_{DE} is independent of θ; thus we can integrate (2.7) obtaining:

$$\frac{d\Gamma}{dE_\gamma^*}(K_S\to\pi^+\pi^-\gamma)=\frac{2\alpha}{\pi}\frac{\Gamma(K_S\to\pi^+\pi^-)\beta^3}{\beta_0}\left(1-\frac{2E_\gamma^*}{m_K}\right)\times$$

$$\times\left\{\frac{1}{E_\gamma^*}\left[\frac{1+\beta^2}{2\beta^3}ln\frac{1+\beta}{1-\beta}-\frac{1}{\beta^2}\right]+\right.$$

$$+\frac{E_\gamma^*\mathrm{Re}[f_{DE}(E_\gamma^*)A^*(K_S\to\pi^+\pi^-)]}{2(4\pi f)^2|A(K_S\to\pi^+\pi^-)|^2}\left[\frac{2}{\beta^2}-\frac{1-\beta^2}{\beta^3}ln\frac{1+\beta}{1-\beta}\right]+$$

$$\left.+\frac{|f_{DE}(E_\gamma^*)|^2{E_\gamma^*}^3}{12(4\pi f)^4|A(K_S\to\pi^+\pi^-)|^2}\right\} \qquad (2.8)$$

We remark that the behaviour $\frac{1}{E_\gamma^*}$ for $E_\gamma^*\to 0$ in (2.7) and (2.8), peculiar of bremsstrahlung processes will tend strongly to enhance the IB rates compared to DE ones, which depend on meson dynamics and have no poles for $E_\gamma^*\to 0$.

3 CHPT: $o(p^2)$, $o(p^4)$ counterterms

CHPT is an effective quantum field theory [14, 15] of hadron interactions based on symmetry arguments. Indeed the CHPT lagrangian is required to be invariant under the symmetry $SU(3)_L \times SU(3)_R$ which is spontaneously broken to $SU(3)_V$ and the Goldstone bosons of the broken symmetry are the pseudoscalar mesons. One considers the lagrangian as a perturbative expansion of the external momenta and masses. Already at tree level all the low energy theorems, PCAC and soft pion properties are recovered. Furthermore, though the theory is not renormalizable, we do require unitarity, which is obtained perturbatively by considering pseudoscalar meson loops. Divergences in the loops are absorbed by corresponding counterterms, which then depend on the renormalization scale μ of the loops. Due to the non-renormalizability, new counterterms, whose coefficients can be determined from experiments, have to be added order by order to the theory. Actually, the $o(p^4)$ counterterm corrections to the lowest order strong lagrangian can be predicted reasonably well by vector meson exchange [17], which also gives an $o(p^4)$ contribution to the $A(K \to 3\pi)$ improving the lowest order weak lagrangian result [18]. The lagrangian is a sum of a strong lagrangian ($\Delta S = 0$) and a weak lagrangian ($\Delta S = 1$), including electromagnetic interactions. At order p^2 one has

$$L_{\Delta S=0} = \frac{1}{4} f^2 Tr D_\mu U D^\mu U^\dagger + \frac{f^2}{2} Tr U^\dagger \mu M + \frac{f^2}{2} Tr U \mu M \tag{3.1}$$

where

$$U = e^{\frac{2i}{f}\pi_a T_a}, \quad D_\mu U = \partial_\mu U + ieA_\mu[Q, U] \tag{3.2}$$

$$M = diag(m_u, m_d, m_s) \quad Q = diag(2/3, -1/3, -1/3)$$

$$Tr T_a T_b = \frac{1}{2}\delta_{ab}, \quad f \simeq F_\pi = 93.3\text{MeV} \quad T_a = \lambda_a/2 \tag{3.3}$$

λ_a are the Gell-Mann matrices, μ is the right factor to reproduce the observed meson masses and A_μ is the electromagnetic field*. Neglecting the part of the non-leptonic $\Delta S = 1$ weak lagrangian transforming as 27, we are left with the octet contribution

$$L^{(8)}_{\Delta S=1} = \frac{1}{4} f^2 h_8 Tr \lambda_6 D_\mu U D^\mu U^\dagger \tag{3.4}$$

* The Condon-Shortley-De Swart phase convention is not satisfied.

From $K \to \pi\pi$ decays we have at order p^2

$$h_8 = 3.2 \cdot 10^{-7} \tag{3.5}$$

At order p^2 since there are not enough powers of momenta, only an IB amplitude will appear for the $K \to \pi\pi\gamma$ decays. At this order for the CP conserving decay $K_S \to \pi^+\pi^-\gamma$ the diagrams in Fig.1 will contribute; diagram 1.a is needed to make the amplitude gauge invariant. In agreement with Low theorem, the total contribution at this order is

$$A(K_S \to \pi^+\pi^-\gamma)^{o(p^2)} = eA(K_S \to \pi^+\pi^-)^{o(p^2)} B \tag{3.6}$$

$$A(K_S \to \pi^+\pi^-)^{o(p^2)} = \frac{-h_8}{2f}(m_K^2 - m_\pi^2)$$

$A(K_S \to \pi^+\pi^-)^{o(p^2)}$ is the $o(p^2)$ CHPT amplitude for $K_S \to \pi^+\pi^-$. Thus diagram 1.a, contrary to appearance, contributes to the IB amplitude; its existence is due to the derivative couplings in CHPT. Actually we take (2.4) as a definition of the IB amplitudes meaning that these relations hold order by order in CHPT. At order p^4, where the loop diagrams in Fig. 2 and counterterms will appear, the amplitudes IB and DE in (2.6), will be present. After the explicit calculation of the loop diagrams in Fig. 2 one would find the two gauge invariant loop contributions to IB and DE in (2.6).

All the possible $o(p^4)$ local operators, for the lagrangian $\Delta S = 0$ [15] and $\Delta S = 1$ [16] have been classified; the interaction with external fields, like for instance the electromagnetic field A_μ, appears both through the covariant derivative and a direct gauge invariant coupling. Accordingly, their coefficients can be determined experimentally from amplitudes with no external fields or they have to involve the external fields. Vector meson dominance (VMD) has successfully predicted these $\Delta S = 0$ coefficients [17] (for a recent update see also [12]).

In the $K_S \to \pi^+\pi^-\gamma$ decay, only the $\Delta S = 1$ p^4 counterterms contribute. The operators where the external field appear in the covariant derivative, will give contributions to the IB amplitude; these summed to the IB loop contribution will give the (2.4), where $A(K_S \to \pi^+\pi^-)$ is evaluated at $o(p^4)$. Indeed we have verified that the loop contribution is consistent with the result for $K_S \to \pi^+\pi^-$ of ref.[19]. As we shall discuss in the next section for the $o(p^4)$ IB contribution, there are two possible attitudes: either we take $A(K_S \to \pi^+\pi^-)$ from the experiments or we assume the $o(p^4)$ CHPT result for this amplitude taking for the coefficients of the corresponding counterterms the result of the fit of ref.[19].

At order p^4 however new gauge invariant counterterms involving the external field $F_{\mu\nu}$ [21, 16, 23] will contribute to processes with photons (real and virtual).

The $\Delta S = 1$ quark lagrangian is invariant under CPS symmetry even if CP is violated [20, 21]: this consists of a CP transformation plus the interchange of d and s quarks. Strong and electromagnetic lagrangian is CPS invariant too. Thus imposing this symmetry to the effective lagrangian only four chiral and gauge invariant p^4 independent counterterms, will contribute to electric transitions of radiative decays with at least one photon in the final state. Defining* C_i as [21]

$$
\begin{aligned}
C_{1L} &= F^{\mu\nu} < Q\lambda_{6-i7} L_\mu L_\nu > \\
C_{2L} &= F^{\mu\nu} < Q L_\mu \lambda_{6-i7} L_\nu > \\
C_{1R} &= F^{\mu\nu} < Q U^\dagger \lambda_{6-i7} U R_\mu R_\nu > \\
C_{1\hat{R}} &= F^{\mu\nu} < Q R_\mu R_\nu U^\dagger \lambda_{6-i7} U > \\
C_{2R} &= F^{\mu\nu} < Q R_\mu U^\dagger \lambda_{6-i7} U R_\nu >
\end{aligned}
\tag{3.7}
$$

where

$$
L_\mu = i f^2 U D_\mu U^\dagger \qquad R_\mu = i f^2 U^\dagger D_\mu U \qquad < A > := tr A
$$

due to CPS symmetry only the combination $C_{1R} + C_{1\hat{R}}$ appears and the lagrangian is written as

$$
L_{CT} = \frac{ieh_8}{8f^4}\left[\omega_{1L} C_{1L} + \omega_{2L} C_{2L} + \frac{\omega_{1R} + \omega_{1\hat{R}}}{2}(C_{1R} + C_{1\hat{R}}) + \omega_{2R} C_{2R}\right] + h.c. \tag{3.8}
$$

ω_i are dimensionless coupling constants which could be determined from experiments: in the decays $K \to \pi e^+ e^-$ [21] only two of these four operators are independent:

$$
\omega_1 = \omega_{1L} + \omega_{1R} + \omega_{1\hat{R}} \qquad , \omega_2 = \omega_{2L} + \omega_{2R}. \tag{3.9}
$$

The other combinations [23]

* Note that we have a different definition of L and R compared to [21].

$$\omega_1' = \omega_{1L} - (\omega_{1R} + \omega_{1\hat{R}}) \qquad \omega_2' = \omega_{2L} - \omega_{2R}. \tag{3.10}$$

can be determined in $K^+ \to \pi^+\gamma\gamma$ [22] and/or $K \to 2\pi\gamma$ decays*. It was noted that the combination of counterterms appearing in the electric transitions of the decay $K^+ \to \pi^+\pi^0\gamma$:

$$A_{DE}(K^+(P) \to \pi^+(p_+)\pi^0(p_0)\gamma(q))_{CT} = -e\frac{h_8}{8f^3}\overline{B}_c(\omega_1 + 2\omega_2 - \omega_1' + 2\omega_2') \tag{3.11}$$

was scale independent [16, 24] ($\overline{B}_c = \epsilon \cdot p_+ q \cdot P - \epsilon \cdot Pq \cdot p_+$). Very interestingly this is the same combination which appears in the $K_S \to \pi^+\pi^-\gamma$ counterterm contribution

$$A_{DE}(K_S \to \pi^+\pi^-\gamma)_{CT} = -e\frac{h_8}{4f^3}\overline{B}(\omega_1 + 2\omega_2 - \omega_1' + 2\omega_2') \qquad (3.12).$$

Indeed as we will see in the next paragraph the loop contribution is finite confirming the scale independence of the combinations (3.11) and (3.12). Thus measuring DE emission in $K_S \to \pi^+\pi^-\gamma$ will give us information on electric transitions in $K^+ \to \pi^+\pi^0\gamma$ decays. We emphasize that if CP violation is considered, due to the imaginary part of the combination $\omega = \omega_1 + 2\omega_2 - \omega_1' + 2\omega_2'$, (3.11) and (3.12) could interfere with the respective CP conserving A_{IB}'s generating a CP violating interference. Thus the experimental determination of the CP conserving part of (3.11) and (3.12) might bring light to the underlying dynamics which could generate CP violation.

4 Loop contributions and discussions

We have neglected the kaon and kaon-pion loop contribution since their absorptive part is vanishing in the physical region; this should imply also that the respective dispersive integrals, receiving a non-vanishing contribution only when the absorptive part is far

* The overall sign of the lagrangian in (3.8) has been taken such to be consistent with the observable counterterm coefficients chosen in [21, 23] and with our definition for the electric charge in (2.4), (3.2) and (3.6).

from the physical region, should be suppressed. The total contribution to f_{DE} in (2.6) is obtained by adding the corresponding f_{DE}^{CT} from (3.12) to the contributions from the pion loop in Fig.2:

$$f_{DE}^{loop}(E_\gamma^*) = \frac{h_8}{2E_\gamma^{*2} f} \left\{ (m_K^2 - m_\pi^2)(m_K^2 - 2E_\gamma^* m_K) \left[\beta ln \left(\frac{1+\beta}{\beta - 1} \right) - \beta_0 ln \left(\frac{1+\beta_0}{\beta_0 - 1} \right) \right] \right.$$

$$\left. - E_\gamma^* m_K (3m_\pi^2 - 2m_K^2) + m_\pi^2 (m_K^2 - m_\pi^2) \left[ln^2 \left(\frac{1+\beta_0}{\beta_0 - 1} \right) - ln^2 \left(\frac{1+\beta}{\beta - 1} \right) \right] \right\} \qquad (4.1)$$

β and β_0 are defined in (2.7). We notice that it is finite, confirming the scale independence [16, 24] of the counterterm combination (3.11) and (3.12). It is θ independent since there are not enough powers of momenta at this order. Due to the E_γ^* dependence the loop contribution could be disentangled from the counterterm contribution, which is constant. Furthermore we remark that this is going like $\frac{1}{E_\gamma^*}$ for E_γ^* going to zero. Thus the interference term in (2.8) goes as a constant in this limit.

Theoretically one would have expected that the dispersive contribution of (4.1) was suppressed compared to the absorptive one since for this channel the loop contribution is finite (separately pion and kaon loop contribution are finite). This statement is true, for instance, for the corresponding imaginary and real part of the decays $K_S \to \gamma\gamma$ [25], $K_L \to \pi^0\gamma\gamma$ [26] and $K^+ \to \pi^+\gamma\gamma$ [22]. Surprisingly and interestingly the dispersive contribution in this case turns out to be larger than the corresponding absorptive one. We remark also that while the absorptive part is vanishing in SU(3) limit as result of the Cabibbo Gell-Mann theorem [27], which it tells us that $K_S \to \pi^+\pi^-$ is proportional to $m_K^2 - m_\pi^2$, the dispersive part, where pions in the loop are off-shell, is not constrained by the theorem.

The interference term in (2.8) depends on $A(K_S \to \pi^+\pi^-)$; for this amplitude one can either use the CHPT result [19] or the experimental one. Since the real target of this paper is $A_{DE}(K_S \to \pi^+\pi^-\gamma)$ we feel that the second possibility is probably wiser. $A(K_S \to \pi^+\pi^-)$ is generally written (for instance [27]) in the isospin decomposition

$$A(K_S \to \pi^+\pi^-) = \sqrt{\frac{2}{3}} A_0 e^{i\delta_0} + \frac{1}{\sqrt{3}} A_2 e^{i\delta_2}. \qquad (4.2)$$

A_0 and A_2 are determined experimentally [19] from $K \to \pi\pi$ decays*. The energy dependent experimental $\pi\pi$ phase shifts at different energies have been fitted to curves [29],

* The sign for A_0 is chosen to be negative consistently with (3.6).

called Roy's curves, based on very general assumptions like analyticity, unitarity and crossing symmetries. From a general analysis using experimental data and Roy's equations the following phases at the kaon mass scale are quoted [30]:

$$\delta_0 = (39 \pm 5)^0 \qquad \delta_2 = (-8 \pm 2)^0. \tag{4.3}$$

Since CHPT expansion for the separate phase shifts is not rapidly converging [31] at the scale m_K (4.3), looks the best one can do now. In Fig.3 we show how this uncertainty affects our interference by choosing $\omega_i = 0$ and plotting the interference differential branching ratio as a function of the photon energy for different values of the pair (δ_0, δ_2): $(34^0, -6^0)$, $(39^0, -8^0)$ and $(44^0, -10^0)$. The relative integrated interference branching ratios with a photon energy cut of $E_\gamma^* > 20\ MeV$ are:

(δ_0, δ_2)	$Br(K_S \to \pi^+\pi^-\gamma : E_\gamma^* > 20 MeV)_{Interf.}$
$(34^0, -6^0)$	-6.7×10^{-6}
$(39^0, -8^0)$	-5.5×10^{-6}
$(44^0, -10^0)$	-4.2×10^{-6}

(4.4)

In Fig.4 we keep fixed the phase shifts to the central value and we allow the counterterm combination ω to assume the following values:

$$\omega \equiv \omega_1 + 2\omega_2 - \omega_1' + 2\omega_2' = 0, \pm 2L_9, \pm 5L_9. \tag{4.5}$$

L_9 is the coefficient of a p^4 $\Delta S = 0$ counterterm and its experimental value determined to be [15] $(6.9 \pm 0.7) \cdot 10^{-3}$ can be considered as a typical size of p^4 counterterms. In (4.6) we show the integrated interference branching ratio for a cut in the photon energy $E_\gamma^* > 20 MeV$, assuming for the phases the central value in (4.3) and varying the counterterm combination ω

ω	$Br(K_S \to \pi^+\pi^-\gamma : E_\gamma^* > 20 MeV)_{Interf.}$
$5L_9$	5.5×10^{-6}
$2L_9$	-1.1×10^{-6}
L_9	-3.3×10^{-6}
0	-5.5×10^{-6}
$-L_9$	-7.7×10^{-6}
$-2L_9$	-9.8×10^{-6}
$-5L_9$	-1.6×10^{-5}

(4.6)

The previous experiment [10] gave a bound larger than the values in (4.6): $|Br(K_S \to \pi^+\pi^-\gamma : E_\gamma^* > 50 MeV)_{Interf.}| < 9 \cdot 10^{-5}$; this was found using a counterterm-like amplitude. With the upcoming facilities, one should be able to sharpen the values in (4.3);

then by studying the photon spectrum one can check the CHPT prediction in (4.1) and fix the value of the counterterm combination ω, constraining theoretical models [32]. For instance DAΦNE [33], should have about 2×10^9 $K_S/year$, which could be enough to do this research.

Acknowledgements

We would like to thank G. Ecker, J. Gasser and T. Pich for important remarks, F. Buccella for continuous encouragement and suggestions, L. Cappiello and G. Isidori for helpful discussions.

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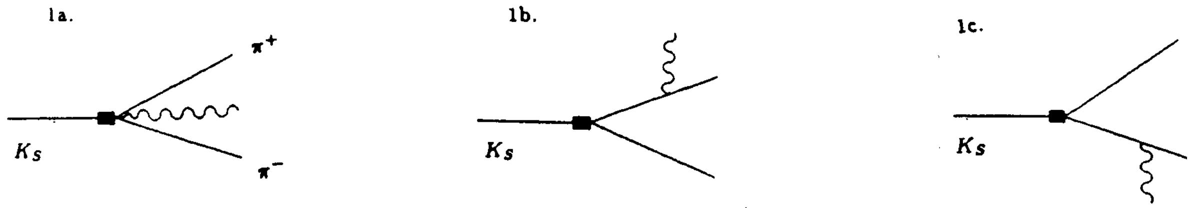

Fig.1: $o(p^2)$ CHPT Feynman diagrams for $K_S \to \pi^+\pi^-\gamma$.

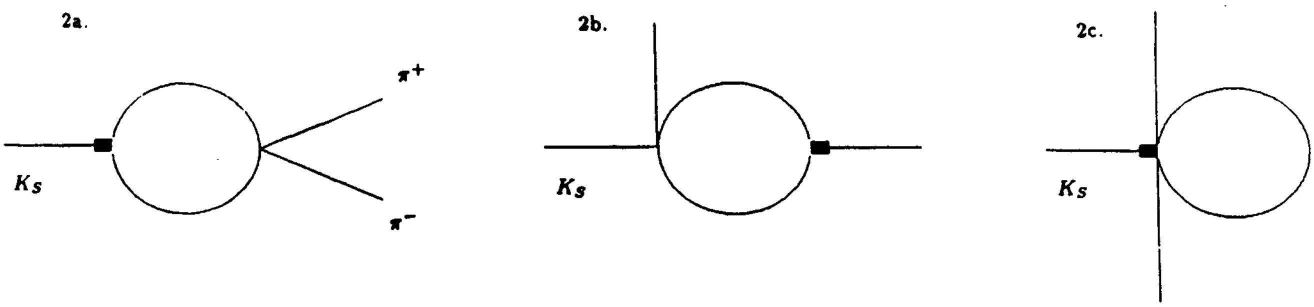

Fig.2: $o(p^4)$ CHPT loops for $K_S \to \pi^+\pi^-\gamma$. The photon has to be attached to the charged lines or to the strong or to the weak vertex.

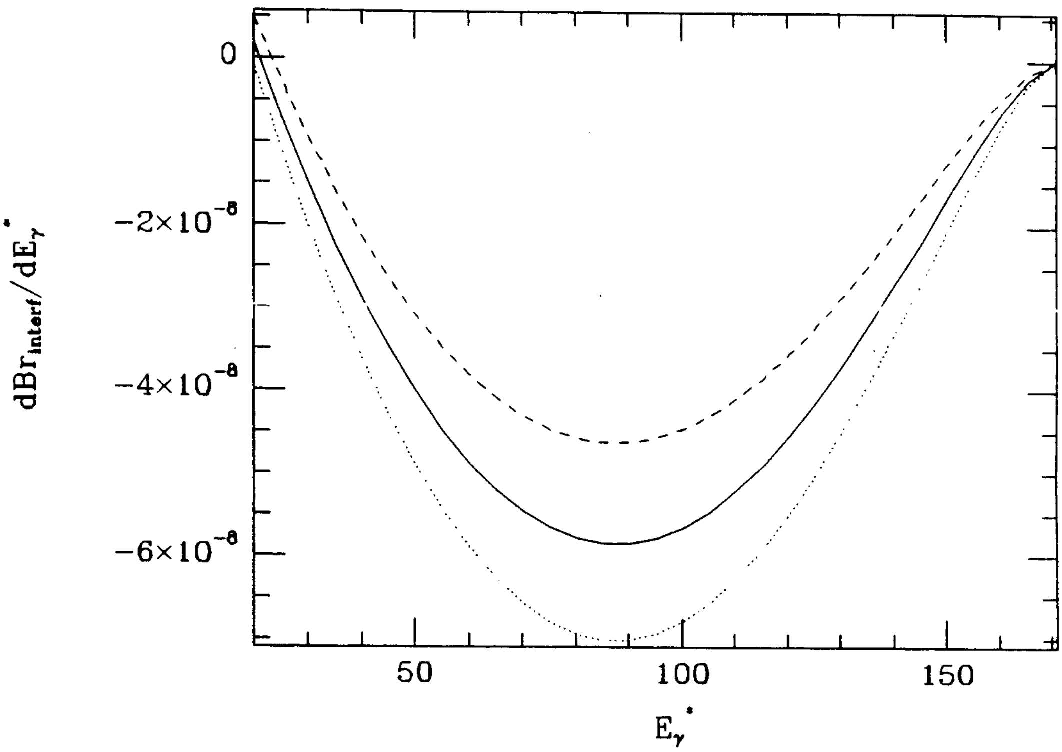

Fig.3: Differential branching ratio for the interference between $A_{DE}(K_S \to \pi^+\pi^-\gamma)$ and $A_{IB}(K_S \to \pi^+\pi^-\gamma)$ as function of the photon energy (E_γ^*) in the kaon rest frame. The scale independent counterterm is set to zero. The dashed curve correspond to a value for the phase shift pair (δ_0, δ_2) of $(44^0, -10^0)$, the full curve to $(39^0, -8^0)$ and dotted curve to $(34^0, -6^0)$.

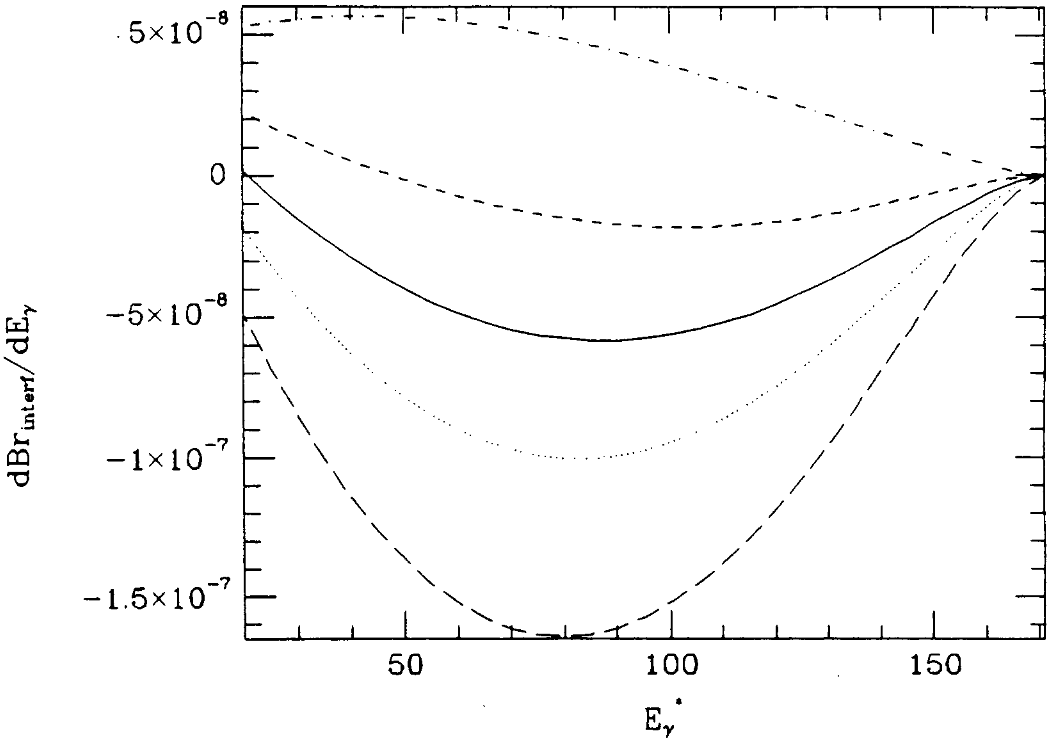

Fig.4: Differential branching ratio for the interference between $A_{DE}(K_S \to \pi^+\pi^-\gamma)$ and $A_{IB}(K_S \to \pi^+\pi^-\gamma)$ as function of the photon energy (E_γ^*) in the kaon rest frame. The phase shift pair (δ_0, δ_2) is chosen to $(39^0, -8^0)$. The scale independent counterterm is set to -5 L_9 (long dash curve),-2 L_9 (dotted curve), 0 (full line), $+2$ L_9 (short dash line), 5 L_9 (dot-short dash curve) .