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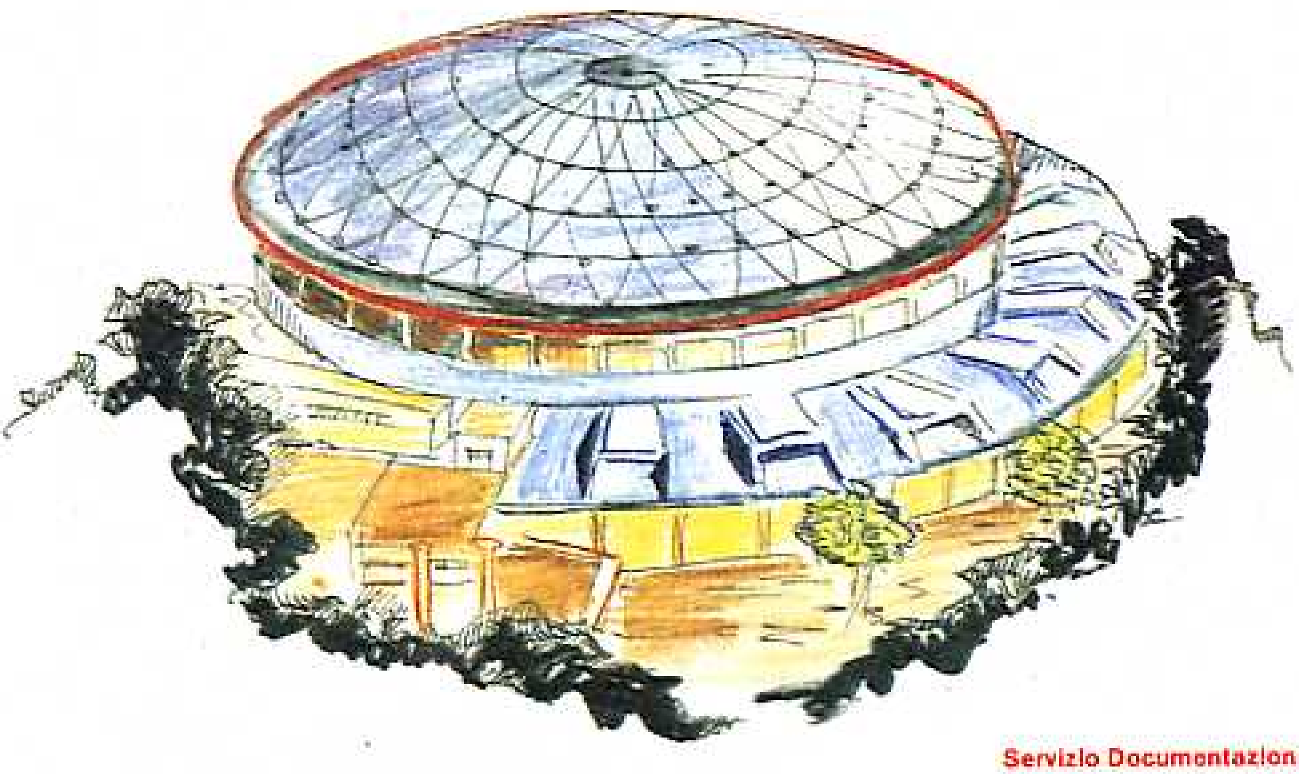

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THE $\gamma\gamma \to \pi^0\pi^0$ CONTRIBUTION TO THE PROCESS $e^+e^- \to \pi^0\pi^0$

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Abstract

We consider the $O(p^4)$ predictions of Chiral Perturbation Theory (χPT) in the process $\gamma\gamma \to \pi^0\pi^0$, together with the study of the existing experimental measurements and the planned new ones. At the Frascati Φ-factory DAΦNE the rate of the events $e^+e^- \to e^+e^-\pi^0\pi^0$ is large enough to allow for a measurement of the cross section with a good statistical sensitivity. However, a quantitative analysis of the commonly used approximation of considering only real photons for the detection of the $\gamma\gamma \to \pi^0\pi^0$ signal is still lacking. This motivates us to compute the cross section in χPT for the production of π^0 pairs due to off-shell photons. A measurement of the azymuthal correlations would test the higher order χPT corrections independently from the measurement of the cross section. We approximate these corrections by including the contribution of the low-lying vector resonances in the scattering amplitude.

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1 Introduction

Φ-factories are well suited for studying the chiral symmetry of the strong interactions. One very important test is given by the reaction $\gamma\gamma \to \pi^0\pi^0$, for which the data collected by the Crystal Ball collaboration [1] are not in good agreement with the prediction of Chiral Perturbation Theory (χPT) to lowest nonvanishing order, *i.e.* to $O(p^4)$ in the external momenta and quark masses [2], [3]. The considerable interest of this reaction stems from the fact that the $O(p^4)$ scattering amplitude is finite, hence it does not depend on the free parameters of the chiral lagrangian. In this case the $O(p^6)$ amplitude can give a numerically important contribution to the cross section in the threshold region. The calculation of the $O(p^6)$ corrections recently presented [4], reconcile the χPT theoretical predictions with the Crystal Ball measured cross section. Hopefully Φ-factories will improve the situation from the experimental point of view, producing much more precise data.

DAΦNE at Frascati provides a very good possibility to measure the production of (both charged and) neutral pion pairs [5]. The rate for the events $e^+e^- \to e^+e^-\pi^0\pi^0$, which we computed in χPT up to $O(p^4)$, is quite large and should yield a high statistical precision [6],[7]. The systematical errors can be reduced by tagging the scattered leptons. It has been pointed out that Φ-factories provide a unique possibility for the measurement of doubly-tagged events, thanks to the low energy of the lepton beams [8]. Equipping the KLOE detector [9] with a single or double tagging should allow to eliminate most of the background processes and obtain a clear signature of the $\gamma\gamma \to \pi^0\pi^0$ process [10].

In this paper we present the χPT predictions for the $\gamma\gamma \to \pi^0\pi^0$ one-loop cross section with both photons virtual, and this ingredient is necessary in order to give the complete one-loop cross section for the reaction $e^+e^- \to e^+e^-\pi^0\pi^0$. Our main motivation to do this is that we would like to discuss quantitatively all the approximations usually made in studying the process $e^+e^- \to e^+e^-\pi^0\pi^0$. The final assumption people use to make is that this process can be described only by the cross section for the process $\gamma\gamma \to \pi^0\pi^0$ with real photons. All the approximations needed to arrive to this result have been widely discussed in the literature, and the qualitative picture leading to them is clear and very well known. What is still missing (as far as we know) is a quantitative discussion of them. We felt urged to do this for the following reason. From the experimental point of view DAΦNE will offer the opportunity to have data more precise than ever, and one has then to check whether the theoretical predictions match this precision. Moreover, since χPT is not a model, rather a systematic field-theory approach to calculate amplitudes of processes involving pions, there appears no obstruction to considering non-real photons in this process.

In particular, it has been proposed that a measurement of the azymuthal correlations of the outgoing e^+e^- pair should be possible at DAΦNE and allow to test the predictions of various models for the $\gamma\gamma \to \pi^0\pi^0$ amplitude [11]. This topic is included in our description, within the general theoretical framework of χPT.

This paper is organized as follows: section 2 contains a description of the kinematics and the general formalism for a process of the kind $e^+e^- \to e^+e^-X$; in section 3 we carry out the calculation of the $\gamma\gamma \to \pi^0\pi^0$ order $O(p^4)$ cross section with both photons off-shell. In section 4 we present our numerical results and discuss the abovementioned approximations. Finally, in section 5 we discuss possible effects of higher orders in χPT estimating them by a vector resonance exchange in the scattering amplitude.

2 Kinematics and QED structure of the cross section

The generic process $e^+e^- \to e^+e^-X$ occurring via two photon fusion has been thoroughly studied in 1973 by Bonneau, Gourdin, Martin [12]. They developed a quite general formalism to calculate the cross section using the fact that the lepton photon vertex is very well known and that the $\gamma\gamma \to X$ reaction has to satisfy gauge invariance. So they wrote the most general gauge invariant $\gamma\gamma \to X$ amplitude in terms of 10 unknown form factors (they have to be determined case by case) and were then able to write down also the $e^+e^- \to e^+e^-X$ cross section in terms of these. We have used this formalism in the case in which the X final state is a pair of neutral pions. Before calculating the $\gamma\gamma \to \pi^0\pi^0$ process in the formalism of χPT, let us summarize the kinematics of the process and discuss the formalism of ref. [12].

The reaction under study is:

$$e^+(k_1)e^-(k_2) \to e^+(k_1')e^-(k_2')\pi^0(p_1)\pi^0(p_2). \tag{2.1}$$

The four-momentum transfers between the leptons are in this case equal to the momenta of the two photons:

$$q_i = k_i - k_i', \qquad i = 1,2. \tag{2.2}$$

So, later on we will consider the amplitude of the reaction:

$$\gamma(q_1)\gamma(q_2) \to \pi^0(p_1)\pi^0(p_2). \tag{2.3}$$

This, as a normal two body scattering process, is completely defined by the usual Mandelstam variables:

$$\begin{aligned} W^2 &= (p_1+p_2)^2 = (q_1+q_2)^2, \\ t &= (q_1-p_1)^2 = (q_2-p_2)^2, \\ u &= (q_2-p_1)^2 = (q_1-p_2)^2. \end{aligned} \tag{2.4}$$

Only two of these variables are independent if the masses of the particles are known. In our case we will consider q_1^2 and q_2^2 to be given, and use as variables of our process the following four ones:

$$W^2, \qquad t-u, \qquad q_1^2, \qquad q_2^2. \tag{2.5}$$

If we want to analyze only the distribution in W^2 of the $\pi^0\pi^0$ system, the kinematics of our total system is the same as that of a 3-body decay of a particle of mass $\sqrt{s}$,

$$s = (k_1+k_2)^2, \tag{2.6}$$

into an electron-positron pair and a particle of mass W. In order to determine completely the final state we need to measure, for example, energy and momentum of the two leptons:

$$E_1', \cos\theta_1, \phi_1, \qquad E_2', \cos\theta_2, \phi_2, \tag{2.7}$$

where

$$\cos\theta_i = \frac{\vec{k}_i \cdot \vec{k}_i'}{|\vec{k}_i||\vec{k}_i'|}, \tag{2.8}$$

and ϕ_i are two azimuthal angles. However one of these variables is trivial, corresponding to the total orientation of the system. So we need five independent variables to determine completely the final state. In the $\gamma\gamma \to \pi^0\pi^0$ amplitude we will be naturally led to choose as variables q_1^2, q_2^2 and W. For the remaining two, we will follow ref. [12] and use

$$u_1 = 2k_2 \cdot q_1, \qquad u_2 = 2k_1 \cdot q_2. \tag{2.9}$$

In the e^+e^- center of mass frame the variables we chose can be expressed in terms of the six lepton parameters in this way:

$$\begin{aligned} q_j^2 &= 2m_e^2 + 2|\vec{k}||\vec{k}_j'|\cos\theta_j - \sqrt{s}E_j', \qquad j = 1,2 \\ u_j &= s - 2m_e^2 - 2|\vec{k}||\vec{k}_j'|\cos\theta_j - \sqrt{s}E_j' \\ W^2 &= s - 2\sqrt{s}(E_1' + E_2') + 2E_1'E_2' + 2m_e^2 + \\ & \quad 2|k_1'||k_2'|[\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\cos(\phi_1 - \phi_2)]. \end{aligned} \tag{2.10}$$

The fully differential cross section in these variables can be written as follows:

$$\begin{aligned} \frac{d^5\sigma}{du_1du_2dq_1^2dq_2^2dW^2} &= \frac{\alpha^2}{2\pi^2}\frac{1}{s(s-4m_e^2)}\frac{qW}{q_1^2q_2^2}\frac{1}{\pi}\frac{1}{\sqrt{\mathcal{A} - 4m_e^2\mathcal{B}}}\{\sigma_{TT}K_{TT} + \sigma_{LT}K_{LT} + \\ & \sigma_{TL}K_{TL} + \sigma_{LL}K_{LL} + \tau_{TT}K_{TT}^{ex} + (\tau_0 + \tau_1)K_{LT}^{ex}\} \end{aligned} \tag{2.11}$$

where we used also the variable q, which is the photon momentum in the $\gamma\gamma$ center of mass system, that can be expressed as:

$$q = \frac{1}{2W}\sqrt{(W^2 - q_1^2 - q_2^2)^2 - 4q_1^2q_2^2} \tag{2.12}$$

and:

$$\begin{aligned} \mathcal{A} &= 4q_1^2q_2^2(s-u_1)(s-u_2) - (u_1u_2 + q_1^2q_2^2 - 2sq_1\cdot q_2) \\ \mathcal{B} &= 4s(q_1^2q_2^2 - (q_1\cdot q_2)^2) + q_1^2(u_2 - q_2^2)^2 + q_2^2(u_1 - q_1^2)^2 + 2q_1\cdot q_2(u_1 - q_1^2)(u_2 - q_2^2). \end{aligned} \tag{2.13}$$

Let us discuss a bit formula (2.11). The various terms inside the curly brackets have all the same structure, i.e. the product of a "cross section" $\sigma_{()}$ or a "correlation" $\tau_{()}$ times a "structure function". The subscripts refer to the helicities of the photons: T means transverse, L longitudinal. Note that σ_{TT} at $q_1^2 = q_2^2 = 0$ is exactly the cross section usually calculated for real photons. When both photons are real only σ_{TT} and τ_{TT} are different from zero. At low $\sqrt{-q_i^2}$ the Taylor expansion of the various cross sections and correlations is:

$$\begin{aligned} &\sigma_{TT} \sim const., && \tau_{TT} \sim const. \\ &\sigma_{LT} \sim q_1^2, && \sigma_{TL} \sim q_2^2 \\ &\sigma_{LL} \sim q_1^2 q_2^2, && \tau_0 \sim \tau_1 \sim \sqrt{q_1^2 q_2^2}. \end{aligned} \tag{2.14}$$

The $K_{()}$'s do *not* depend on the dynamics of the $\gamma\gamma$ process and have been calculated and shown explicitly in [12], so we are not going to redisplay them here. They can be expressed as simple combinations of the variables we chose, and represent the probability for the two electrons to emit a pair of photons with the helicity indicated in the subscript. In this sense we called them "structure functions". The $\sigma_{()}$'s instead are nothing but cross sections of a two body scattering process in which the two initial particles are virtual photons with helicities given in the subscripts. They depend *only* on the $\gamma\gamma$ process and can be calculated only when one has an amplitude for the specific problem under consideration. The same can be said for the $\tau_{()}$'s with the only difference that they are not cross sections but interferences of amplitudes. It is worth mentioning at this point that the $K_{()}$'s multiplying them are proportional to $\cos 2\phi$ (K_{TT}^{ex}) and $\cos\phi$ (K_{LT}^{ex}) where the angle ϕ is the azymuthal angle between the outcoming leptons in the center of mass of the two photons (see for example [13]), so that they disappear when integrated over this angle. On the other hand they can be observed measuring the azymuthal angular distribution of the outcoming two leptons, and have been proposed as interesting, new tests of the various models built to describe the $\gamma\gamma \to hadrons$ process [11].

Note that while the $K_{()}$'s depend on all the five variables, the $\sigma_{()}$'s and $\tau_{()}$'s do not depend on u_1 and u_2. Hence one could make another step ahead in the calculation without loss of generality, namely the u_1, u_2 integration. This has been done also in ref. [12], and we are now going to display the formula for the triple differential cross section, since this will be our starting point for the numerical calculation of $d\sigma/dW^2$. The reason why we showed the fully differential cross sections is that it contains all the informations needed for a Montecarlo study of the tagging problem and of the measurement of azymuthal correlations. No integration in the lepton variables has been made, so every kind of detector-dependent cut can still be applied.

The triple differential cross section in q_1^2, q_2^2 and W^2 is:

$$\begin{aligned} \frac{d^3\sigma}{dq_1^2 dq_2^2 dW^2} = \frac{\alpha^2}{2\pi^2} \frac{1}{s(s-4m_e^2)} \frac{qW}{q_1^2 q_2^2} \{ &\sigma_{TT} J_{TT} + \sigma_{LT} J_{LT} + \\ &\sigma_{TL} J_{TL} + \sigma_{LL} J_{LL} + \tau_{TT} J_{TT}^{ex} + (\tau_0 + \tau_1) J_{LT}^{ex} \} \end{aligned} \tag{2.15}$$

where

$$J_{()}(q_1^2, q_2^2, W^2, s) = \frac{1}{\pi} \int \frac{du_1 du_2}{\sqrt{\mathcal{A} - 4m_e^2 \mathcal{B}}} K_{()}(u_1, u_2, q_1^2, q_2^2, W^2, s). \tag{2.16}$$

The $J_{()}$'s can be easily derived from a set of integrals $L_{\alpha\beta}(q_1^2, q_2^2, W^2, s), \alpha, \beta = 0, 1, 2$ calculated and given in [12], and we make reference to that paper for the complete expression. We have recalculated and checked explicitly all the expressions of [12] we used, including the integrals $L_{\alpha\beta}(q_1^2, q_2^2, W^2, s)$. We found a complete agreement with the formulae of ref. [12]. Notice that our convention for the metric tensor is different in sign from that used by Bonneau et al.

3 Off-shell $\gamma\gamma \to \pi^0\pi^0$ at one loop in χPT

Before making the calculation of the various $\sigma_{()}$'s in the framework of χPT, let us make some very general considerations about the $\gamma\gamma \to \pi^0\pi^0$ process. The amplitude of the process $\gamma\gamma \to \pi^0\pi^0$ can be written in terms of four different form factors:

$$M_{\mu\nu} = A(q_1^2, q_2^2, W, t-u)\left\{g_{\mu\nu} - \frac{q_{1\nu}q_{2\mu}}{q_1\cdot q_2}\right\} +$$

$$B(q_1^2, q_2^2, W, t-u)\frac{1}{(q_1\cdot q_2)^2}\{q_1^2 q_2^2 g_{\mu\nu} - q_1^2 q_{2\mu}q_{2\nu} - q_2^2 q_{1\mu}q_{1\nu} + q_1\cdot q_2 q_{1\mu}q_{2\nu}\}+$$

$$C(q_1^2, q_2^2, W, t-u)\frac{1}{(q_1\cdot q_2)^2}\{q_1^2 q_2\cdot\Delta g_{\mu\nu} - q_2\cdot\Delta q_{1\mu}q_{1\nu} - q_1^2 q_{2\mu}\Delta_\nu + q_1\cdot q_2 q_{1\mu}\Delta_\nu\}+$$

$$C(q_2^2, q_1^2, W, u-t)\frac{1}{(q_1\cdot q_2)^2}\{q_2^2 q_1\cdot\Delta g_{\mu\nu} - q_1\cdot\Delta q_{2\mu}q_{2\nu} - q_2^2 q_{1\nu}\Delta_\mu + q_1\cdot q_2 q_{2\nu}\Delta_\mu\}+$$

$$D(q_1^2, q_2^2, W, t-u)\frac{1}{(q_1\cdot q_2)^2}\{q_1\cdot\Delta q_2\cdot\Delta g_{\mu\nu} - q_2\cdot\Delta q_{1\nu}\Delta_\mu - q_1\cdot\Delta q_{2\mu}\Delta_\nu + q_1\cdot q_2\Delta_\mu\Delta_\nu\},$$

(3.1)

where $\Delta = p_1 - p_2$. The only constraints imposed here are Lorentz covariance, gauge invariance, Bose symmetry in the exchange of the two photons and Bose symmetry in the exchange of the two pions. The Bose symmetry properties are included in the form factors:

$$F(q_1^2, q_2^2, W, t-u) = F(q_2^2, q_1^2, W, u-t) \qquad F = A, B, D,$$

$$F(q_1^2, q_2^2, W, t-u) = F(q_1^2, q_2^2, W, u-t) \qquad F = A, B, D, \qquad (3.2)$$

$$C(q_1^2, q_2^2, W, t-u) = -C(q_1^2, q_2^2, W, u-t) \quad .$$

The form factor C has no definite property under the exchange of the two photons, but appears in the amplitude in such a way that the amplitude is symmetric in this exchange.

To calculate the $\sigma_{()}$'s and $\tau_{()}$'s we now have to contract the Lorentz indices with the polarization vectors of the photons in a definite helicity state. This definition is obviously gauge dependent, so we choose a Coulomb gauge: $\epsilon\cdot q = 0$. We have three independent polarization vectors (we are now considering the general case of virtual photons), two transverse and one longitudinal, defined in this way (we are in the center of mass of the two photons):

$$\begin{aligned} \epsilon_+(q_i) &= \left(0, -\frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0\right), & \epsilon_-(q_i) &= \left(0, \frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0\right), \\ \epsilon_0(q_1) &= \left(\frac{q}{\sqrt{-q_1^2}}, 0, 0, \frac{W}{2\sqrt{-q_1^2}}\right), & \epsilon_0(q_2) &= \left(-\frac{q}{\sqrt{-q_2^2}}, 0, 0, \frac{W}{2\sqrt{-q_2^2}}\right), \end{aligned} \tag{3.3}$$

having defined $q_1 \equiv (W/2, 0, 0, q)$. Then we introduce the helicity amplitudes as:

$$M_{\lambda_1\lambda_2} = \epsilon^{\mu}_{\lambda_1}(q_1)\epsilon^{\nu}_{\lambda_2}(q_2)M_{\mu\nu}. \tag{3.4}$$

To get the $\sigma_{()}$'s and $\tau_{()}$'s we simply have to multiply two helicity amplitudes and integrate over the $\pi^0\pi^0$ phase space. For example we have:

$$\sigma_{TT} \equiv \frac{1}{4}(\sigma_{++} + \sigma_{--} + \sigma_{+-} + \sigma_{-+}) \tag{3.5}$$

where

$$\begin{aligned} \sigma_{\alpha\beta} &\equiv \frac{e^4}{4qW}\int d\rho_{\pi^0\pi^0}|M_{\alpha\beta}|^2, \qquad \alpha, \beta = +, -, \\ d\rho_{\pi^0\pi^0} &= \frac{1}{2}\frac{d^3p_1}{(2\pi)^3 2p_1^0}\frac{d^3p_2}{(2\pi)^3 2p_2^0}(2\pi)^4\delta^4(q_1 + q_2 - p_1 - p_2). \end{aligned} \tag{3.6}$$

The detector acceptance may limit the phase space of the two pions, but, as observed in [14], this does not occur at a Φ-factory if one has a detector like KLOE that covers almost the full steradiant ($\cos\theta < 0.98$). From now on we will always integrate over the whole phase space.

Let us now abandon the general consideration and calculate instead the $\sigma_{()}$'s and $\tau_{()}$'s in the χPT framework to the $O(p^4)$ order. As noted in [3], dimensional considerations imply that at this order the amplitude contains only an S-wave. This means that only the A and B form factors can be different from zero. The authors of ref. [3] calculated A in the case of one off-shell photon; here we are going to give the result of a complete calculation of both form factors neglecting the contribution from kaon loops, i.e. in the $SU(2) \times SU(2)$ limit. They will be expressed as integrals in one Feynman parameter since the analytic solution of the integral is too cumbersome to be of any use:

$$\begin{aligned} A &= \frac{1}{8\pi^2 F_\pi^2}(W^2 - m_\pi^2)(1 + I_A), \\ B &= -\frac{1}{8\pi^2 F_\pi^2}(W^2 - m_\pi^2)(1 + I_B), \end{aligned} \tag{3.7}$$

where the value of the pion decay constant $F_\pi = (93.15 \pm 0.11) MeV$ is taken from ref. [15]. The integrals $I_{A,B}$ read as follows:

$$I_A = \int_0^1 dx[1-2x(1-x)\xi_1]f(x) \ + \ (\xi_1 \leftrightarrow \xi_2),$$
$$I_B = \int_0^1 dx\left[1+\frac{a}{4}-(2\xi_1+a)x+2\xi_1 x^2\right]f(x) \ + \ (\xi_1 \leftrightarrow \xi_2), \tag{3.8}$$

with

$$f(x) = \frac{1}{R}\log\left[\frac{2-[\xi_2+x(a+2\xi_1)+R](1-x)}{2-[\xi_2+x(a+2\xi_1)-R](1-x)}\right],$$
$$R = \{\xi_2(\xi_2-4)+2\xi_2(a+2\xi_1)x+(a^2-4\xi_1\xi_2)x^2\}^{1/2}. \tag{3.9}$$

We have introduced here the dimensionless variables:

$$\xi_i = \frac{q_i^2}{m_\pi^2},$$
$$a = \frac{q_1\cdot q_2}{m_\pi^2}. \tag{3.10}$$

Both A and B have an imaginary part due to a cut in the logarithm contained in $f(x)$. This cut goes from x_- to x_+, where:

$$x_\pm = \frac{1}{2}\left[1\pm\sqrt{1-\frac{4m_\pi^2}{W^2}}\right]. \tag{3.11}$$

With A and B being the only nonzero form factors in the amplitude, we may easily calculate the various $\sigma_{()}$'s and $\tau_{()}$'s. Note that since neither A nor B depend on $t-u$, the integral over the phase space factorizes. The only two $\sigma_{()}$'s that turn out to be nonzero are σ_{TT} and σ_{LL}. Their expression is:

$$\sigma_{TT} = \frac{\pi\alpha^2}{8}\frac{1}{qW}\sqrt{1-\frac{4m_\pi^2}{W^2}}\left|A+\frac{q_1^2q_2^2}{(q_1\cdot q_2)^2}B\right|^2, \tag{3.12}$$

$$\sigma_{LL} = \frac{\pi\alpha^2}{4}\frac{1}{qW}\sqrt{1-\frac{4m_\pi^2}{W^2}}\frac{q_1^2q_2^2}{(q_1\cdot q_2)^2}|A+B|^2. \tag{3.13}$$

In figs. 1-4 we plot σ_{TT} and σ_{LL} as functions of W and q_1^2 at different values of q_2^2. Notice that the solid line in fig. 3 agrees with previous calculations [2] [3]. Some comments are worth making at this point. The fact that the only nonzero $\sigma_{()}$'s are the two we found is in some sense a good feature, in order to measure just σ_{TT} at $q_1^2 = q_2^2 = 0$. In fact σ_{LL}, that starts with $q_1^2q_2^2$ near $q_1^2 = q_2^2 = 0$, can give a sizeable contribution only when both photons are off-shell (see fig. 4), but this event is strongly suppressed by the QED dynamics hidden in $J_{LL}/q_1^2q_2^2$. Were σ_{LT}, σ_{TL} nonzero, they would have given a sizeable contribution in the region where only one photon is off-shell, and this would be much less suppressed by QED. Their contribution to the total cross section would then be much bigger than that due to σ_{LL}, and this would make it more difficult to extract from the

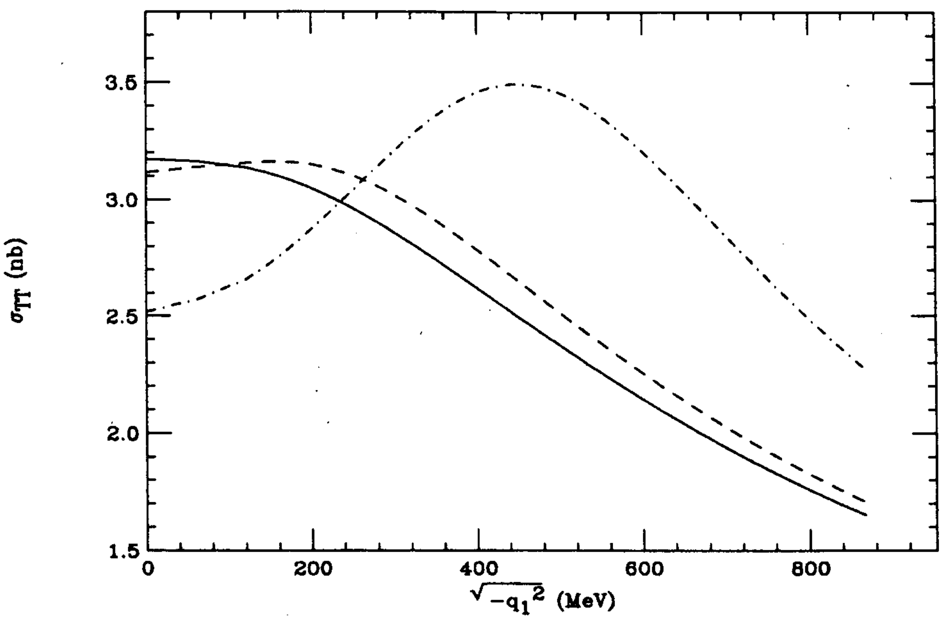

Figure 1: σ_{TT} as a function of $\sqrt{-q_1^2}$ for various values of q_2^2 and $W = 6m_\pi^2$. The solid, dashed and dotdashed lines correspond to $q_2^2 = -rm_\pi^2$, with $r = 0.1, 1.0, 10.0$ respectively.

data a measurement of σ_{TT}. Moreover the dependence of σ_{TT} on $q_{1,2}^2$ is strong only for very large values of at least one of them (see fig. 1). Hence also for this cross section the QED dynamics makes very small any physical effect originated by its dependence on $q_{1,2}^2$.

Also two $\tau_{()}$'s are nonzero; their expression is:

$$\tau_{TT} = 2\sigma_{TT}, \tag{3.14}$$

$$\tau_1 = -\frac{\pi\alpha^2}{4}\frac{1}{qW}\sqrt{1-\frac{4m_\pi^2}{W^2}}\frac{\sqrt{q_1^2q_2^2}}{q_1\cdot q_2}\left[|A|^2 + \frac{q_1^2q_2^2}{(q_1\cdot q_2)^2}|B|^2 + Re(AB^*)\left(1+\frac{q_1^2q_2^2}{(q_1\cdot q_2)^2}\right)\right]. \tag{3.15}$$

It is very interesting that at this order τ_{TT} turns out to be equal (modulo a factor of two) to σ_{TT}. Thus a measurement of the azymuthal correlation would give an independent test of the χPT prediction for the $\gamma\gamma \to \pi^0\pi^0$ cross section, with different systematic errors from the experimental point of view, but also with different theoretical corrections coming from higher orders. This happens for the following reason: at this order τ_{TT} and σ_{TT} are equal because $M_{+-} = M_{-+} = 0$. This implies $\sigma_{TT} = \frac{1}{4}(\sigma_{++} + \sigma--)$. The definition of τ_{TT} in terms of helicity amplitudes is:

$$\tau_{TT} = \tau_{++--} \equiv \frac{e^4}{4qW}\int d\rho_{\pi^0\pi^0} M_{++}M_{--}^*. \tag{3.16}$$

But for parity reasons one has $M_{++} = M_{--}$. Hence it follows $\tau_{++--} = \sigma_{++} = \sigma_{--}$, and immediately formula (3.14). This result changes at higher orders, where σ_{+-} and σ_{-+}

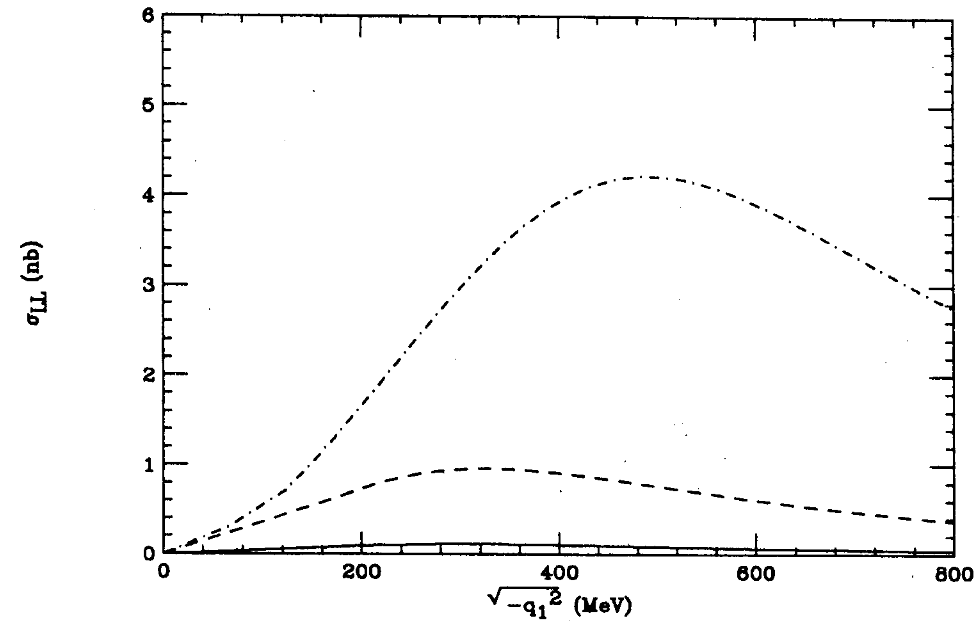

Figure 2: σ_{LL} as a function of $\sqrt{-q_1^2}$ for various values of q_2^2 and $W = 6m_\pi^2$. The solid, dashed and dotdashed lines correspond to $q_2^2 = -rm_\pi^2$, with $r = 0.1, 1.0, 10.0$ respectively.

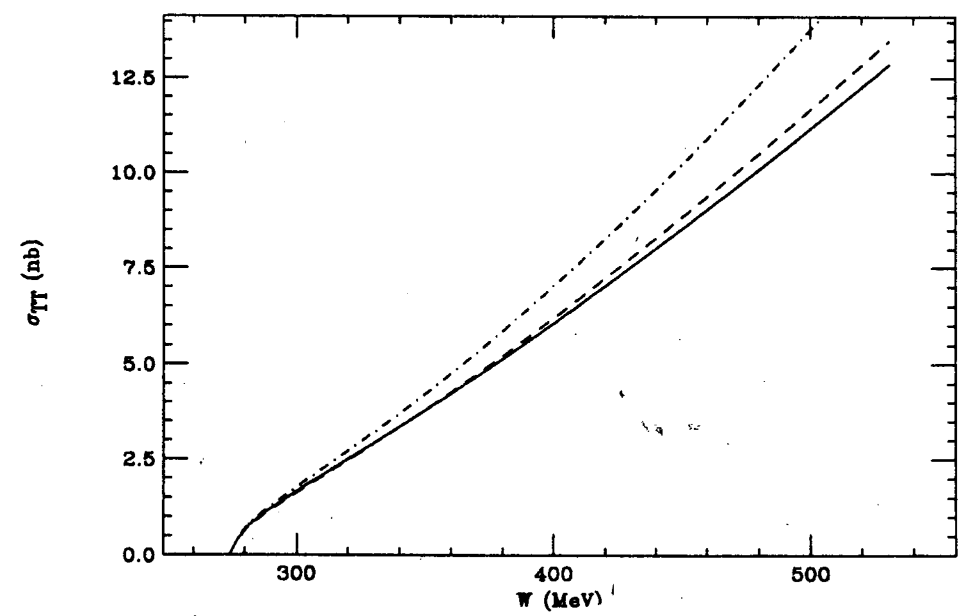

Figure 3: σ_{TT} as a function of W for various values of q_1^2, q_2^2. The solid, dashed and dotdashed lines correspond to $q_1^2 = q_2^2 = -rm_\pi^2$, with $r = 0.1, 1.0, 10.0$ respectively.

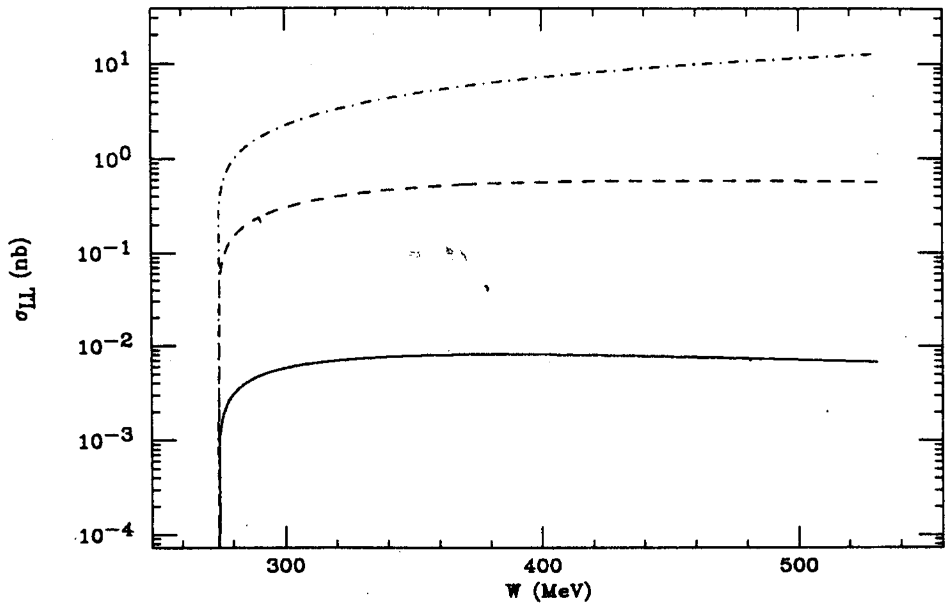

Figure 4: σ_{LL} as a function of W for various values of q_1^2, q_2^2. The solid, dashed and dotdashed lines correspond to $q_1^2 = q_2^2 = -rm_\pi^2$, with $r = 0.1, 1.0, 10.0$ respectively.

are no longer zero. This can be seen in the last section, where we approximate the order $O(p^6)$ with vector resonance exchanges, but also from the direct calculation of this order in χPT [4]. We thus believe that a measurement of the azymuthal correlation would be highly valuable both from the theoretical and experimental point of view (see also ref. [16] for a detailed analysis of the interest and feasibility of such a measurement).

4 Numerical results

As we said in the introduction, the final assumption often made in treating this kind of processes is that they can be described only in terms of a $\gamma\gamma$ cross section with both photons real. To arrive to this result one needs two different approximations:

i) that all the cross sections, but σ_{TT} give a negligible contribution;

ii) that the q_1^2 and q_2^2 dependence of σ_{TT} can be neglected. We are now in a position to discuss them both from a quantitative point of view.

Our starting point for the numerical calculation is formula (2.15), in which we use the $J_{()}$'s given in [12] and σ_{TT} and σ_{LL} calculated in the preceding section. Our first result is the prediction for the total cross section at fixed W for the process $e^+e^- \to e^+e^-\pi^0\pi^0$. We calculate and compare the contributions coming from σ_{TT} and σ_{LL}:

$$\frac{d\sigma_{TT,LL}}{dW^2} = \frac{\alpha^2}{2\pi^2}\frac{1}{s(s-4m_e^2)}\int\int\frac{dq_1^2}{q_1^2}\frac{dq_2^2}{q_2^2}qW\sigma_{TT,LL}J_{TT,LL}. \tag{4.1}$$

We integrated numerically over the q_1^2, q_2^2 phase space, whose boundary is given by the equation:

$$\frac{2s\left[1-\frac{2m_e^2}{s}+\sqrt{1-\frac{4m_e^2}{q_2^2}}\right]}{(q_1q_2+qW)\left[1+\sqrt{1-\frac{4m_e^2}{q_1^2}}\right]\left[1+\sqrt{1-\frac{4m_e^2}{q_2^2}}\right]}=1. \tag{4.2}$$

We did not attempt to consider any realistic possibility to detect the leptons and made the integration over the whole phase space, corresponding to the experimental situation in which the leptons are not tagged.

In table I we give the results for $d\sigma_{TT}/dW^2$ and $d\sigma_{LL}/dW^2$ at $\sqrt{s}=1.019\ GeV$ for W varying between threshold and 500 MeV. As one can easily see, σ_{LL} gives tipically a 1% correction to the σ_{TT} cross section, an effect too small to be measured, and even to disturb the program of extracting σ_{TT} from the $e^+e^-\to e^+e^-\pi^0\pi^0$ data. So the assumption i) seems to be a very good one, but let us stress once more that this is not a result predictible before making the actual calculation of the amplitude. Were $\sigma_{LT,TL}$ nonzero at this order, they would have produced a much bigger contribution to the total cross section.

Now let us consider the approximation ii). It has appeared many times in the literature, but usually people make reference to the paper of Brodsky, Kinoshita and Terazawa (BKT) [17] (it is cited also by the PDG booklets), in which they defined an equivalent photon luminosity function that has to be multiplied by the $\gamma\gamma$ cross section in order to obtain the $e^+e^-\to e^+e^-X$ cross section.

The connection with the formalism used here is the following: BKT neglected all the contributions to the cross section but σ_{TT} (the only one different from zero at $q_1^2\sim q_2^2\sim 0$), considering it to be a constant in q_1^2, q_2^2 and using $\sigma_{TT}(q_1^2,q_2^2,W)\sim\sigma_{TT}(0,0,W)$. The motivation for this is that in the integral σ_{TT} is multiplied by $\frac{qW}{q_1^2q_2^2}J_{TT}$, a function strongly peaked at $q_1^2=q_2^2=0$. Put it another way, this simply corresponds to the fact that electrons prefer to emit quasi-real photons, so it does not seem to be a bad approximation to consider the photons after the emission as being real. Then they calculated the leading term in the integral and called this the $\gamma\gamma$ luminosity function $\mathcal{L}_{\gamma\gamma}$. The expression they obtained is [1]:

$$\mathcal{L}_{\gamma\gamma}=\frac{1}{4}ln^2\left(\frac{s}{4m_e^2}\right)\left[\left(2+\frac{W^2}{s}\right)ln\left(\frac{s}{W^2}\right)-2\left(3+\frac{W^2}{s}\right)\left(1-\frac{W^2}{s}\right)\right]. \tag{4.3}$$

Bonneau et al. [12] noted that the BKT approximation can be very bad in the high energy domain (which is already reached at DAΦNE, as we will see), and were able to give a complete analytic expression for the integral of J_{TT}, that they called F_{TT}:

$$F_{TT}\equiv\frac{W^4}{4s^2}\int\int\frac{dq_1^2}{q_1^2}\frac{dq_2^2}{q_2^2}J_{TT}. \tag{4.4}$$

As one may easily check comparing (4.1) with (4.4), the expression for $d\sigma/dW^2$ is obtained in each case using

$$\frac{d\sigma}{dW^2}=\left(\frac{\alpha}{\pi}\right)^2\frac{1}{W^2}\sigma_{TT}(0,0,W)(\mathcal{L}_{\gamma\gamma},F_{TT}). \tag{4.5}$$

[1] The expression normally used for $\mathcal{L}_{\gamma\gamma}$ contains a $(\alpha/\pi)^2 1/W^2$ factor in front. We omitted it here for later convenience (see eq. (4.5) below).

In table II we compare $\mathcal{L}_{\gamma\gamma}$ and F_{TT} at various energies. It is clearly seen that at DAΦNE the BKT approximation gives a result that is about 20% too high. Let us immediately say that this observation does not affect the Crystal Ball data [1], since they used the analytic expression of F_{TT} given in [12] to analyze the data.

Now, forgetting about the BKT approximation, in table III we compare the complete calculation with the factorized one at both DAΦNE and PETRA energies. The error is always between 5% and 10%, something smaller than the statistical error at Crystal Ball, but hopefully an effect that should be taken care of at DAΦNE.

This calculation needs three numerical integrations of functions that are not very regular over the domain of integration. This requires good numerical routines, a good computer and also a certain amount of time. In order to save time in this and future applications of the formulae given here, and also as a check of our numerical routines, we tried to approximate A and B with analytical expressions. As noted in [3], in the case when only one photon is virtual, B is equal to zero and for A a simple analytical expression can be given (just integrate I_A putting ξ_1 or ξ_2 equal to zero). The expression obtained in this way for σ_{TT} would then be wrong only in the region of phase space where both photons are off-shell. But this region is very much suppressed by QED, as we already mentioned. So the error made with this approximation should be reasonably small. This is in fact the case, as we show in table IV, where the error is typically 1% (note that this is of the same order as the correction given by σ_{LL}, which starts with $q_1^2 q_2^2$, i.e. a term we left out of σ_{TT} in this approximation).

At this point a new question naturally arises: since it is established both experimentally and theoretically that higher orders in χPT are important, we would like to see how they could affect the present analysis. The first thing to say is that they would probably switch on the other form factors (C and/or D), and consequently other $\sigma_{()}$'s and $\tau_{()}$'s. The effect of this could be to hide better σ_{TT} inside the $e^+e^- \to e^+e^-\pi^0\pi^0$ cross section and make it more difficult to extract σ_{TT} from the data. As an estimate of these effects we will use vector resonance exchanges, and discuss them in the next section.

5 Estimating higher orders by vector resonance exchange

The contribution of the low-lying vector resonances to the process $\gamma\gamma \to \pi^0\pi^0$ has been computed in χPT for real photons [18], [19], [5]. The correction to the scattering amplitude due to the exchange of a vector resonance in the t, u-channel to order $O(p^6)$ yields nonvanishing contributions to three form factors in eq. (3.1)

$$
\begin{aligned}
A^{(V)} &= C\tfrac{1}{2}(3W^2 - q_1^2 - q_2^2 - 8m_\pi{}^2)(W^2 - q_1^2 - q_2^2), \\
D^{(V)} &= -B^{(V)} = C\tfrac{1}{2}(W^2 - q_1^2 - q_2^2)^2, \qquad (5.1) \\
C^{(V)} &= 0.
\end{aligned}
$$

Here we denote, as in ref. [5],

$$
C = \frac{320}{9}\pi\alpha\left(\frac{h_V}{M_V F_\pi}\right)^2 , \qquad (5.2)
$$

where $M_V(\approx M_\rho) = 770MeV$ is the common mass of the octet and singlet vector mesons.

The effective coupling relevant for the $V \to \gamma P$ decay reads, following ref. [20],

$$L_3 = h_V \epsilon_{\mu\nu\rho\sigma} Tr V^\mu \{u^\nu, f_+{}^{\rho\sigma}\} \quad , \tag{5.3}$$

where

$$u_\mu = i u^\dagger D_\mu U u^\dagger \quad , f_\pm{}^{\mu\nu} = u F_L{}^{\mu\nu} u^\dagger \pm u^\dagger F_R{}^{\mu\nu} u \quad , \tag{5.4}$$

with the covariant derivative $D_\mu U$ and the corresponding nonabelian field strengths defined as in ref. [21]. The absolute value of h_V has been fixed in refs. [20], [5] using the decay rates with the smallest experimental errors, i.e. $\rho^+ \to \pi^+\gamma$, $\omega^0 \to \pi^0\gamma$, and making the nonet assumption for the ω. This yields the coupling constant values

$$h_V = \pm 0.037 \quad . \tag{5.5}$$

Notice the agreement of the scattering amplitude obtained from the form factors (5.1) in the limit of two real photons, with the corresponding result in ref. [5]. The various cross-section contributions can be straightforwardly calculated, including the 1-loop amplitude and the correction due to the vector-resonance exchange. Here we only display the contributions that are numerically more important, within the framework of a power counting in the constant C whose value is obtained from Eqs. (5.2), (5.5). Both σ_{TT} and σ_{LL} receive contributions to $O(C)$. We recall from the previous section that σ_{LL} is negligible with respect to σ_{TT}, to the 1-loop order. On the other hand, it is clear from earlier work [18], [19], [5] that the correction due to the low-lying vector resonances to σ_{TT} is relatively small (i.e. less than 35% of the 1-loop value of σ_{TT}) over the whole energy range where a χPT calculation of the $\gamma\gamma \to \pi^0\pi^0$ cross section can be considered as a sound approximation (i.e. $W \leq 0.6\ GeV$). Hence we neglect here the correction to eq. (3.13) and only write the $O(C)$ correction to σ_{TT} in eq. (3.12)

$$\begin{aligned} \sigma_{TT}{}^{(1)} = C \frac{\pi\alpha^2}{8} \frac{1}{qW} \sqrt{1 - \frac{4m_\pi^2}{W^2}} Re \left[A + \frac{q_1^2 q_2^2}{(q_1 \cdot q_2)^2} B \right] \times \\ \left[2(q_1 \cdot q_2)(2W^2 - q_1^2 - q_2^2 - 4m_\pi^2) - 4q_1^2 q_2^2 + \tfrac{1}{3}(W^2 - 4m_\pi^2)\left(q_1^2 + q_2^2 - \tfrac{(q_1^2 - q_2^2)^2}{W^2} \right) \right] . \end{aligned} \tag{5.6}$$

Notice that, since $\sigma_{+-}{}^{(1)} = \sigma_{-+}{}^{(1)} = 0$, we have $\tau_{TT}{}^{(1)} = 2\sigma_{TT}{}^{(1)}$, just as for the pure 1-loop contributions.

The $O(C^2)$ corrections spoil the validity of the relation (3.14) holding, both to order $O(p^4)$ and including the $O(C)$ corrections, between τ_{TT} and σ_{TT}. This is so because $\sigma_{+-}{}^{(2)} = \sigma_{-+}{}^{(2)}$ are not vanishing

$$\sigma_{+-}{}^{(2)} = C^2 \frac{\pi\alpha^2}{15} \frac{2}{qW} \sqrt{1 - \frac{4m_\pi^2}{W^2}} (q_1 \cdot q_2)^2 (W^2 - 4m_\pi^2)^2 . \tag{5.7}$$

As a consequence, we can now determine the $O(C^2)$ correction to σ_{TT}, in terms of the analogous contribution to τ_{TT} and eq. (5.7), as follows:

$$\sigma_{TT}{}^{(2)} = \frac{1}{2}(\tau_{TT}{}^{(2)} + \sigma_{+-}{}^{(2)}), \tag{5.8}$$

where

$$\tau_{TT}{}^{(2)} = C^2 \frac{\pi\alpha^2}{4} \frac{1}{qW} \sqrt{1 - \frac{4m_\pi^2}{W^2}} \Big[\left((q_1 \cdot q_2)(2W^2 - q_1^2 - q_2^2 - 4m_\pi^2) - 2q_1^2 q_2^2 \right)^2$$

$$+ \tfrac{1}{20}(W^2 - 4m_\pi^2)^2 \left(q_1^2 + q_2^2 - \tfrac{(q_1^2 - q_2^2)^2}{W^2} \right)^2 \tag{5.9}$$

$$+ \tfrac{1}{3}(W^2 - 4m_\pi^2) \left((q_1 \cdot q_2)(2W^2 - q_1^2 - q_2^2 - 4m_\pi^2) - 2q_1^2 q_2^2 \right) \left(q_1^2 + q_2^2 - \tfrac{(q_1^2 - q_2^2)^2}{W^2} \right) \Big].$$

To order $O(C^2)$ there are also nonvanishing contributions to σ_{LT} and σ_{TL}

$$\sigma_{LT} = C^2 \frac{\pi\alpha^2}{60} \frac{1}{qW} \sqrt{1 - \frac{4m_\pi^2}{W^2}} \frac{(-q_1^2)}{W^2} (W^2 - 4m_\pi^2)^2 (W^2 - q_1^2 + q_2^2)^2, \tag{5.10}$$

$$\sigma_{TL} = C^2 \frac{\pi\alpha^2}{60} \frac{1}{qW} \sqrt{1 - \frac{4m_\pi^2}{W^2}} \frac{(-q_2^2)}{W^2} (W^2 - 4m_\pi^2)^2 (W^2 - q_2^2 + q_1^2)^2. \tag{5.11}$$

Although they are less suppressed by QED than σ_{LL}, these contributions represent a very small correction with respect to the size of σ_{TT} and τ_{TT}. In a similar fashion, one can neglect the contribution of the vector resonances in the calculation of the azymuthal correlation $\tau_0 + \tau_1$ whose value is dominated by the expression (3.15). Notice that τ_0 becomes nonzero to order $O(C^2)$

$$\tau_0 = C^2 \frac{\pi\alpha^2}{60} \frac{1}{qW} \sqrt{1 - \frac{4m_\pi^2}{W^2}} \frac{\sqrt{q_1^2 q_2^2}}{W^2} (W^2 - 4m_\pi^2)^2 (W^2 - q_1^2 + q_2^2)^2. \tag{5.12}$$

In concluding this section, two remarks are in order. In the first place, it is clear that, from the numerical standpoint, the higher order corrections we calculated in this section do not change sizeably the lowest order prediction. Hence we do not go into the numerical details of these corrections. Secondly, including the contribution of the low-lying vector resonances in the scattering amplitude bares as an important consequence the unbalancing of the relative size of the $\gamma\gamma \to \pi^0\pi^0$ cross section σ_{TT} and the azymuthal correlation τ_{TT}, whose lowest-order values are related by eq. (3.14). This means that an independent measurement (with enough precision) of these quantitites would provide two distinct handles to test the χPT prediction of the higher order corrections. We should add finally the remark that the corrections due to the inclusion of a vector meson dominance within a χPT calculation of the photon scattering amplitude yields also an important contribution to the π^0 polarizability, accounting for about $\frac{2}{3}$ of the π^0 forward-angle dispersion sum rule [22].

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TABLE I - Values of $\frac{d\sigma_{TT}}{d(W^2)}$ and $\frac{d\sigma_{LL}}{d(W^2)}$ from Eqs. (4.1), (4.2) at $\sqrt{s} = 1.019$ GeV.

W (GeV)	$\frac{d\sigma_{TT}}{d(W^2)}$	$\frac{d\sigma_{LL}}{d(W^2)} + \frac{d\sigma_{TT}}{d(W^2)}$
0.28	1.035	1.044
0.30	2.026	2.043
0.32	2.467	2.492
0.34	2.680	2.706
0.36	2.762	2.792
0.38	2.756	2.785
0.40	2.703	2.731
0.42	2.603	2.632
0.44	2.479	2.505
0.46	2.338	2.362
0.48	2.196	2.218
0.50	2.055	2.079

TABLE II - Values of the $\gamma\gamma$ luminosity function $L_{\gamma\gamma}$ and F_{TT} from Eqs. (4.3), (4.4) at various energies.

($\sqrt{s}$,W)	$L_{\gamma\gamma}$	F_{TT}
1.019, 0.3	238.875	188.047
1.019, 0.4	159.329	121.33
30.0, 0.3	3264.259	2838.29
30.0, 0.4	3020.998	2642.8991

TABLE III - Comparison between the factorized calculation of Eqs. (4.5), (4.4) and the full calculation according to Eq. (4.1), at both DAΦNE and PETRA energies.

	DAΦNE		PETRA	
W (GeV)	$\sigma_{TT} F_{TT}$	$\int \int \sigma_{TT} J_{TT}$	$\sigma_{TT} F_{TT}$	$\int \int \sigma_{TT} J_{TT}$
0.28	0.98	1.03	13.8	14.4
0.30	1.88	2.03	28.4	29.8
0.32	2.28	2.47	37.1	39.0
0.34	2.48	2.68	43.3	45.8
0.36	2.55	2.76	47.9	50.9
0.38	2.53	2.76	51.3	54.8
0.40	2.48	2.70	54.0	58.0
0.42	2.38	2.60	55.9	60.4
0.44	2.26	2.48	57.4	62.3
0.46	2.14	2.34	58.4	63.8
0.48	2.00	2.20	59.2	64.9
0.50	1.86	2.05	59.6	65.8

TABLE IV - Comparison of the result of the full calculation according to Eq. (4.1) and the approximation of one virtual and one real photon discussed in Ref. [15].

W (GeV)	$\int \sigma_{TT} J_{TT}$	$\int \sigma_{TT}^{DHL} J_{TT}$
0.28	1.035	1.033
0.30	2.026	1.993
0.32	2.467	2.435
0.34	2.680	2.654
0.36	2.762	2.740
0.38	2.756	2.740
0.40	2.703	2.690
0.42	2.603	2.600
0.44	2.479	2.480
0.46	2.338	2.346
0.48	2.196	2.208
0.50	2.055	2.071