ISTITUTO NAZIONALE DI FISICA NUCLEARE – ISTITUTO NAZIONALE DI FISICA NUCLEARE – ISTITUTO NAZIONALE DI FISICA NUCLEARE – ISTITUTO NAZIONALE DI FISICA NUCLEARE – ISTITUTO NAZIONALE DI

Laboratori Nazionali di Frascati

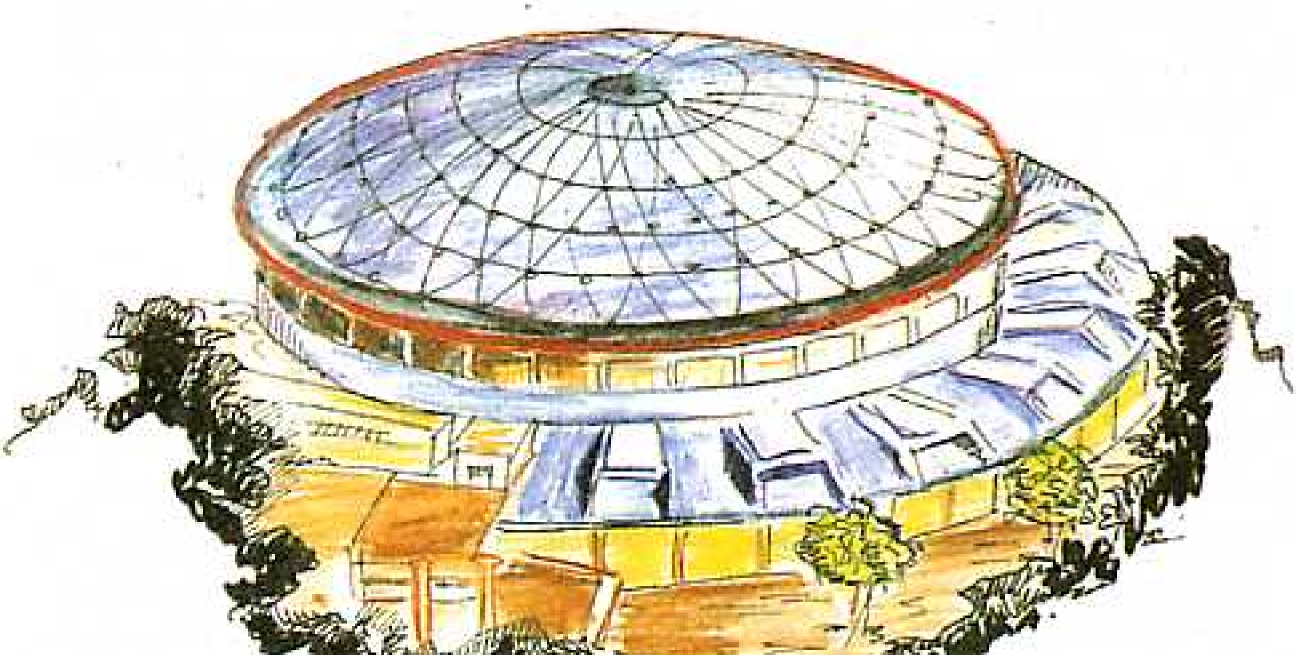
Submitted to Phys. Lett.

LNF-93/007 (P)
12 Febbraio 1993

Y.N. Srivastava, A. Widom:

INTERFERENCE BETWEEN PAST AND FUTURE EVENTS IN $\Phi \to K\bar{K}$ DECAYS

PACS.: 03.65.Bz; 11.90.+t; 13.20.Eb



Servizio Documentazione
dei Laboratori Nazionali di Frascati
P.O. Box, 13 - 00044 Frascati (Italy)

INFN - Laboratori Nazionali di Frascati
Servizio Documentazione

LNF–93/007 (P)
12 Febbraio 1993

Interference Between Past and Future Events in $\Phi \to K\bar{K}$ Decays

Y. Srivastava and A. Widom
Physics Department, Northeastern University, Boston, Mass., USA
&
Dipartimento di Fisica & INFN, Universita' di Perugia, Perugia, Italy

ABSTRACT

The Einstein-Tolman-Podolsky (ETP) effect showing the quantum interference of amplitudes for past and future (time-like separated) events is discussed for a proposed experiment on the $\Phi \to K\bar{K}$ decay processes. Within the proposed resolution of the experiment, and the known magnitudes of CP violating amplitudes, the ETP effect may be susceptible to laboratory tests.

1. Introduction

In the well known debates between Einstein and Bohr on the completeness of quantum mechanics as a description of nature, the Bohr view has clearly been maintained by most working physicists. The grounds are that quantum mechanical theory (although sometimes "counter intuitive") turns out to hold true in all known laboratory experiments. Ultimately, it is the experimental data that decide the validity of theoretical pictures.

In a remarkable paper by Einstein, Tolman and Podolsky [1] one such "counter intuitive" result of the quantum mechanical uncertainty principle was discussed, i.e. that uncertainties in "future" events can be reflected in the probability distributions of "present experiments". Below we refer to this feature of quantum mechanics as the ETP effect. The simple part of the argument is the formal derivation of the required uncertainty relations.

The measure of "uncertainty" in a physical quantity Q is defined as

$$\Delta Q = \sqrt{< Q^2 > - < Q >^2}. \tag{1}$$

For three Hermitian operators A, B and C obeying

$$[A, B] = i\hbar C, \tag{2}$$

the Weyl inequality reads

$$\Delta A \Delta B \geq (\hbar/2)| < C > |, \tag{3}$$

from which the standard uncertainty relations follow for equal times. For example, from

$$[p_x, x] = -i\hbar, \tag{4a}$$

follows

$$\Delta p_x \Delta x \geq (\hbar/2), \tag{4b}$$

while from

$$\dot{Q} = (i/\hbar)[H, Q], \tag{5a}$$

follows

$$\Delta Q \Delta E \geq (\hbar/2)| < \dot{Q} > |. \tag{5b}$$

A "clock" which reads "time" via the coordinate Q will be uncertain in its reading by an amount

$$\Delta t_Q = (\Delta Q/| < \dot{Q} > |), \tag{6}$$

so that Eq.(5b) reads

$$\Delta E \Delta t_Q \geq (\hbar/2), \tag{7}$$

independently of the coordinate Q which is used as a clock.

More interesting from the viewpoint of the work which follows are the uncertainties which follow for measurements made at different times. For dynamic uncertainties at times t_1 and t_2 we employ the time ordered (Feynman) Green's function for two physical quantities

$$G_{AB}(t_1, t_2) = (i/\hbar) < T\{A(t_1)B(t_2)\} > \tag{8}$$

which allows for amplitude propagation both forward and backward in time. Since the real part of the Feynman Green's function is determined by a commutator

$$\mathcal{R}eG_{AB}(t_1, t_2) = (i/2\hbar)sign(t_1 - t_2) < [A(t_1), B(t_2)] >, \tag{9}$$

we have from the Weyl inequality

$$\Delta A(t_1)\Delta B(t_2) \geq \hbar|\mathcal{R}eG_{AB}(t_1, t_2)|. \tag{10}$$

The central point of the ETP effect is that uncertainties in physical quantities at two different times must be viewed in a time symmetric manner. Past measured uncertainties will be reflected in future (to be measured) uncertainties which is all quite reasonable. That future measured uncertainties will be reflected in past measured uncertainties is perhaps less intuitive, but is also a consequence of quantum mechanics as described by Einstein, Tolman and Podolsky.

The notion that amplitudes can propagate from the future into the past is certainly not new in relativistic quantum mechanics. The Stückelberg-Feynman notion of an anti-particle as a particle traveling backward in time is a conventional part of the representation of amplitudes in terms of Feynman diagrams. From this view point, the uncertainty in the energy of an anti-particle in the past can arise as a result of its "preparation" at some future time. The notion that the above description may be only "formally true" and may not be an "intrinsic part" of quantum mechanics is quite incorrect. This will be discussed in detail for the case in which time reversal symmetry is broken by the (super) weak interactions. Amplitude interference between future and past events will be made manifest in the discussion of the probability of the ultimate decay products arising from $\Phi \to K\bar{K}$.

2. DAΦNE Configuration

The Feynman diagram (virtual internal propagator lines) for the decay of the Φ into $K\bar{K}$ in the proposed DAΦNE experiment is shown in Fig.1. The Φ is produced at space-time point $(\mathbf{r}_\Phi, t_\Phi)$ and (almost) immediately turns into a $K\bar{K}$ system. The $K\bar{K}$ system decays into two channels, c and d, respectively at $(\mathbf{r}_c, t_c)$ and $(\mathbf{r}_d, t_d)$. Without loss of generality we may take the coordinates (to within quantum uncertainties for a Φ "at rest") to be at the "origin"

$$(\mathbf{r}_\Phi, t_\Phi) = (\mathbf{0}, 0). \tag{11}$$

The "clock coordinates" which determine the times of the K decays into the c and d channels are merely the spatial distances to the Φ decay event, i.e. to within quantum uncertainties

$$t_c \simeq (r_c/v), \quad t_d \simeq (r_d/v), \tag{12}$$

where v is the speed of each of the K's produced by the Φ. (The two velocities are very near equal in magnitude and opposite in direction.)

The space time amplitude corresponding to the Feynman diagram of Fig.1 is then

$$\Psi_{cd}(x_c, x_d) = A_L^c A_S^d D_L(x_c) D_S(-x_d) - A_S^c A_L^d D_S(x_c) D_L(-x_d), \tag{13}$$

where the subscripts L and S represent the "long" and "short" K mesons and the A-coefficients are the amplitudes for decay into given channels. $D_{L,S}(x)$ are the Feynman propagators for the long and short K mesons and the minus sign between the exchange amplitudes is due to the fact that the Φ meson is odd under time reversal.

The Boson propagator for a particle of mass $M = (\hbar\kappa/c)$ and life-time $\tau = (1/c\eta)$ is given by

$$D(x) = \int \frac{d^4Q}{(2\pi)^4} \frac{e^{iQ\cdot x}}{(Q^2 + \kappa^2 - i\kappa\eta)}, \tag{14}$$

or equivalently

$$D(x) = \int_0^\infty (d\sigma/16\pi^2\sigma^2) exp\{-\kappa\eta\sigma - i[\kappa^2\sigma - (x^2/4\sigma)]\}. \tag{15}$$

For time-like regions

$$-x^2\kappa^2 = \kappa^2(c^2t^2 - r^2) >> 1, \tag{16a}$$

and for long life-times

$$\eta << \kappa, \tag{16b}$$

the propagator has an asymptotic form which can be derived from Eq.(15) using a stationary phase method for evaluating the integral; i.e.

$$D(x) \simeq exp[i(-\kappa + (i/2)\eta)\sqrt{-x^2} + ...], \tag{17}$$

with corrections "..." in the exponential factor of only logarithmic order $|ln(-\kappa^2 x^2)|$. Using the non-relativistic approximation

$$\sqrt{-x^2} = \sqrt{c^2t^2 - r^2} = ct\sqrt{1-(v/c)^2} \approx ct, \tag{18}$$

Eqs.(13),(17) and (18) yield the "two-time" amplitude

$$\Psi_{cd}(t_c, t_d) \approx \psi_L^c(t_c)\psi_S^d(t_d) - \psi_S^c(t_c)\psi_L^d(t_d), \tag{19}$$

where

$$\psi_k^a(t_a) = A_k^a exp[-i(M_k c^2 t_a/\hbar) - (t_a/2\tau_k)]. \tag{20}$$

The transition rate for K-decay channel c in time dt_c and K-decay channel d in time dt_d is computed as

$$d^2P_{cd}(t_c, t_d) = |\Psi_{cd}(t_c, t_d)|^2 dt_c dt_d. \tag{21}$$

Eqs.(19),(20) and (21) have been extensively studied[2-3], especially by workers preparing the DAΦNE experiment[4-5].Remarks on the meaning of these equations in the light of the ETP effect are worthy of note: (i) The amplitude is written in terms of "two times" t_c and t_d. (ii) The usual rules for taking the absolute value squared of an amplitude to obtain a quantum mechanical probability involve (say) position, spin,... at a given value of t. (iii) The "two time" procedure is nevertheless *perfectly correct* since "two times" are actually "two coordinate positions" in accordance with Eq.(12). Let us consider this last remark in more detail.

"Time" as a parameter t which appears in the Schrödinger equation has no intrinsic "uncertainty". The parameter time t is a c-number. On the other hand, the "time" read on a laboratory clock is in reality a coordinate (say Q), and is subject to probability distributions obtained by using quantum mechanical superposition of amplitudes (as are any other coordinates). For example, when "time" is read from an ordinary clock on the wall, it is the (angular) position of the second hand that is actually being observed. Thus, there is a conversion from a "coordinate" or "position" Q to the parameter t_Q that is introducing an energy-time uncertainty as in Eqs.(5) and (7). With this in mind, one may consider two different clocks with two different coordinates acting as "pointers" to the "time". There is no difficulty in using an amplitude (wave function) $\Psi(Q_1, Q_2, ...)$ for finding probability distributions for the two different clock coordinates, and thus the readings t_1 and t_2 on the two different clocks.

In this sense, Eq.(12) describes the conversion between the "positions" of the K-decays and the "times" of the K-decays, i.e. allows for the validity of Eq.(21). Furthermore, given superposition of amplitude interference for the wave function $\Psi(Q_1, Q_2, ...)$ with clock coordinates Q_1 and Q_2, one clearly has the possibility of amplitude interference at two laboratory clock times t_1 and t_2 approximating the c-number time parameter t in the Schrödinger equation. There is no difficulty in working out the energy-time uncertainty Eq.(7) for a "clock channel" in a K-decay event. The quantum uncertainties are

$$\delta t_a \sim (\delta r_a / v) \sim (\hbar / v \delta p_a) \sim (\hbar / \delta E_a). \tag{22a}$$

Thus,

$$\delta t_a [\delta E_a / (electron\ volts)] \sim 10^{-15} sec. \tag{22b}$$

which is much more than adequate for time resolution in (say) the nanosecond to picosecond range.

3. The ETP Effect

If we accept the view that a positron is an electron moving backward in time, then on an experimental level one might like to obtain information about events which will in future be in the positron's environment. However, the positron does not have the proper degrees of freedom to yield information about the future events in the environment, any more than does an electron have the proper degrees of freedom to inform an experimental apparatus about the history of past events in the environment. What we can be sure (beyond say the momentum) is spin and chirality for a stable particle, and that uses up all four of the Dirac spinor components, i.e. all of the internal two bits of information. There is no information about whether the electron originally came in the past from a piece of copper, or whether the positron came from the future (i.e. will eventually be annihilated) from a piece of iron.

However, due to CP symmetry breaking, or equivalently T symmetry breaking (under the assumption that $TCP = 1$), the uncharged K meson does carry some information about its *future* environment, and that information is reflected in the probability of *past* events. Consider the situation shown in Fig.2. The Φ decays at time zero into two uncharged K mesons, one of which decays $K \rightarrow \pi^o + \pi^o$ at time t and the other which decays at a later time T, induced to do so by a condensed matter absorbing bar. Will the probability of the *earlier* $K \rightarrow \pi^o + \pi^o$ event depend on where and when the other K meson hit *at a later time* T the absorbing bar? From the viewpoint of the Feynman diagrams in Fig.1, the interfering amplitudes do indeed exist from future to past events.

The calculation proceeds as follows: (i) Let $\dot{P}_c^+(t)$ denote the transition rate for a channel c event at time t (with no accompanying decay previous to t);

$$\dot{P}_c^+(t) = \int_t^\infty dt_d \sum_d (d^2 P_{cd}(t, t_d)/dt dt_d). \tag{23}$$

(ii) The probability of a channel c event in the interval $t_i < t < t_f$ with no other decay having taken place is then

$$P_c(t_f, t_i) = \int_{t_i}^{t_f} dt \dot{P}_c^+(t). \tag{24}$$

(iii) The probabilities are computed by breaking up Eqs.(19),(20),and (21) into a direct term and an interference term

$$(d^2 P_{cd}(t_c, t_d)/dt_c dt_d) = (d^2 P_{cd}(t_c, t_d)/dt_c dt_d)_{direct} + (d^2 P_{cd}(t_c, t_d)/dt_c dt_d)_{interference}, \tag{25a}$$

$$(d^2 P_{cd}(t_c, t_d)/dt_c dt_d)_{direct} = |\psi_L^c(t_c)\psi_S^d(t_d)|^2 + |\psi_S^c(t_c)\psi_L^d(t_d)|^2, \tag{25b}$$

$$(d^2 P_{cd}(t_c, t_d)/dt_c dt_d)_{interference} = -2\mathcal{R}e[\psi_L^c(t_c)\psi_S^d(t_d)\psi_S^c(t_c)^*\psi_L^d(t_d)^*]. \tag{25c}$$

Eq.(25c) is crucial for the interference between *time-like* separated events, i.e. the ETP effect.

For purposes of illustration and to avoid tedious expressions, (which are not difficult to obtain) let us omit the relative phases between the decay channels for K_L and K_S and assume an ideal absorber at T. Calling ΔN the number of events from the interference term and N the number of events from the direct term, we have for the $\pi^o\pi^o$ decay channel

$$\Delta N = (-2N_o\sqrt{B_S^o B_L^o})[X_1^{int} + X_2^{int} + X_3^{int}] \tag{26}$$

where (setting $\hbar = c = 1$)

$$X_1^{int} = (b/2)\frac{\Gamma_L\Gamma_S}{(\Gamma^2 + \Delta m^2)}(\exp(-2\Gamma t_i) - \exp(-2\Gamma t_f)), \tag{27a}$$

$$X_2^{int} = (-b)\frac{\Gamma_L\Gamma_S}{(\Gamma^2 + \Delta m^2)}[\exp(-\Gamma(T+t_i)cos\Delta m(T-t_i) - \exp(-\Gamma(T+t_f)cos\Delta m(T-t_f)], \tag{27b}$$

$$X_3^{int} = \frac{\sqrt{\Gamma_L\Gamma_S}\Gamma}{(\Gamma^2 + \Delta m^2)}[\exp(-\Gamma(T+t_i)\{cos\Delta m(T-t_i) + \frac{\Delta m}{\Gamma}sin\Delta m(T-t_i)\} - (t_i \rightarrow t_f)]. \tag{27c}$$

and

$$N = N_o[B_S^o(\Gamma_S/2\Gamma) + B_L^o(\Gamma_L/2\Gamma)](\exp(-2\Gamma t_i) - \exp(-2\Gamma t_f)) \tag{28}$$

In the above formulae, $\Gamma = (1/2)(\Gamma_S + \Gamma_L) \equiv (1/\tau)$, $\Delta m = m_L - m_S$ and $b = \Sigma_d\sqrt{(B_S^d B_L^d)}$, where $B_{S,L}^d$ denote the branching ratios of $K_{S,L}$ into the channel d.

The change in the observed number of events can be seen in Fig. 3, where we show $(\Delta N/N)$, the ratio of the interference versus the direct one, as a function of T for $t_i = 0$ and $t_f = 3\tau$. The main contribution to b comes from the charged and neutral pion-pair branching ratios. All the parameters (mass differences, decay widths and branching ratios) are from the particle data group tables [6]. Even with a limited angular range covered by the absorber, for a total of 10^9 neutral kaon decays contemplated at DAΦNE, the effect should be visible.

4. Conclusions

The example considered above illustrates the central point raised by ETP, i.e., the interference between future and past events through the time-reversal violating decays of neutral kaons may indeed modify the number of decays at an earlier time due to changes made in its future. It is conceivable that regeneration type experiments might offer alternative experimental set ups for further probing of this important effect at yet larger times.

Acknowledgements

It is a pleasure to thank R. Baldini Ferroli and G. Pancheri for helpful discussions. This work was supported by INFN in Italy and by DOE in the US.

References

1. A. Einstein, R. C. Tolman, and B. Podolsky: Phys. Rev. **37** (1931) 780.

2. T. Kamae, T. Kifume and K. Tsumemoto: Prog. Theo. Phys. **41** (1969)1267.

3. I. Dunietz, J. Hauser and J. L. Rosner: Phys. Rev. **D35** (1987) 2166.

4. See contributions by P. Franzini and by F. J. Botella, J. Rolda'n and J. Barnabe'u, in Workshop on Physics and Detectors for DAΦNE, April 1991, Frascati, Italy, Edited by G. Pancheri.

5. V. Patera and A. Pugliese in The DAΦNE Physics Handbook,(1992) Volume 1, Edited by L. Maiani, G. Pancheri and N. Paver.

6. Review of Particle Properties, Particle Data Group: Physics Letters, **B239** (1990)1.

Figure Captions

Fig. 1 The two Feynman diagrams are shown for the propagation of K_L and K_S at two times through the decay of a Φ in e^+e^- annihilation.

Fig. 2 An early decay of one kaon into a pion pair while the other kaon gets absorbed later at T.

Fig. 3 Fractional shift in the number of neutral pion-pair decay events as a function of T for $t_i = 0$ and $t_f = 3\tau$. T is in the future with respect to t_f.

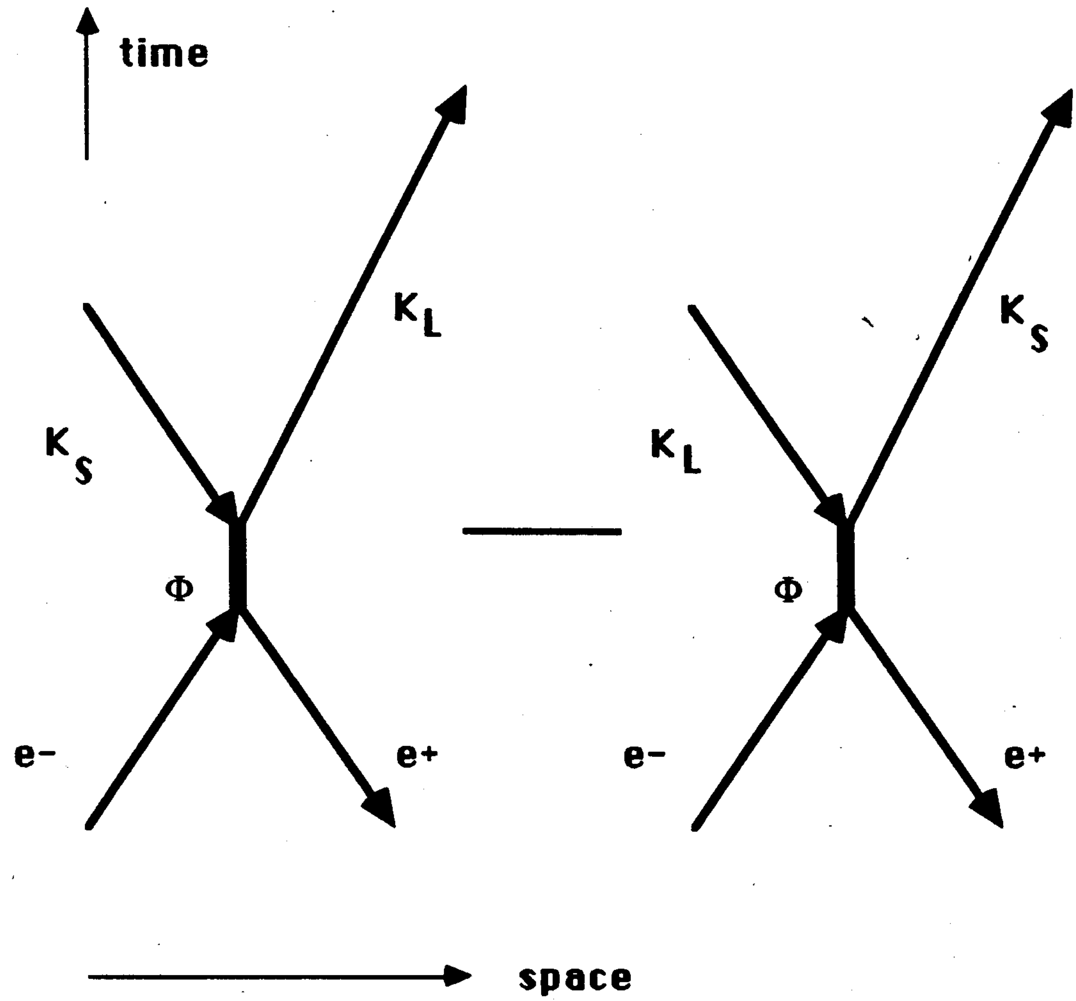


FIG. 1

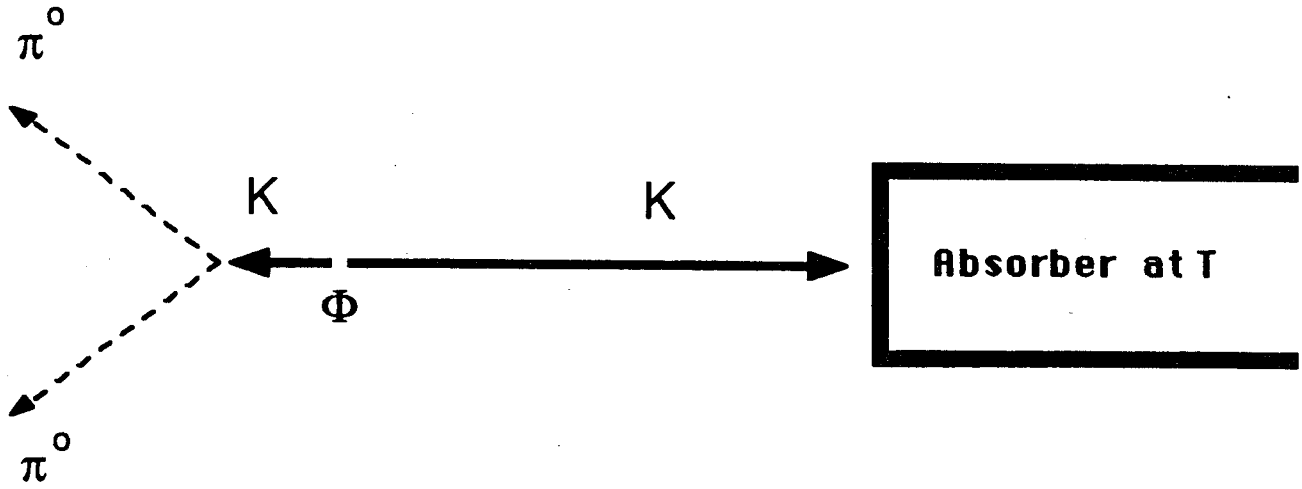

π°
π°
K
K
Φ
Absorber at T

FIG. 2

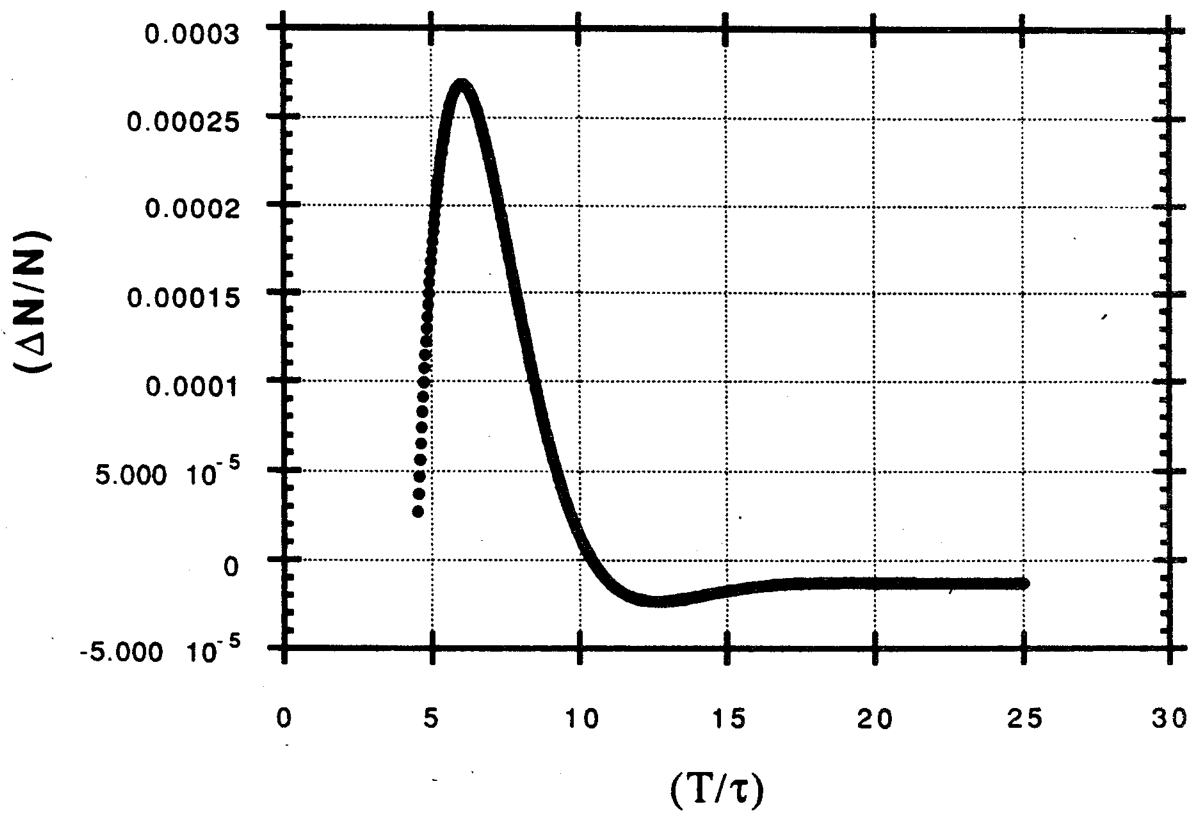

0.0003
0.00025
0.0002
0.00015
0.0001
5.000 10⁻⁵
0
-5.000 10⁻⁵
(ΔN/N)
0
5
10
15
20
25
30
(T/τ)

FIG. 3