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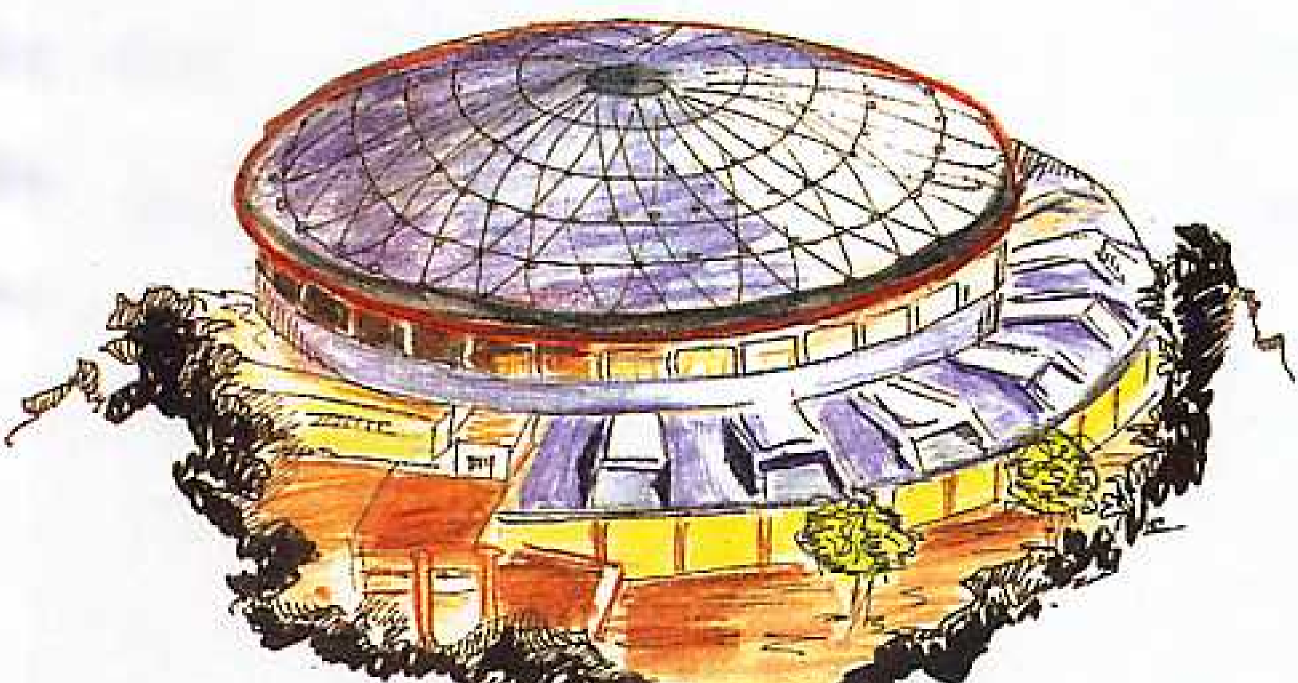

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LARGE N LATTICE QED

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ABSTRACT

We study the β, N critical behaviour of non compact QED with N species of light fermions, using a method we have proposed for unquenched simulations. We find that there exist two phase transition lines: one, second order, and the other, first order, that approaches asymptotically the $\beta = 0$ axis. These two lines have different physical origin, the second one being entirely due to fermions effects. We discuss the effect of the approximation used, in terms of an expansion of the effective action in powers of N, and conclude that the general features should not be affected by this approximation.

Strongly coupled lattice QED has been, in the last years, the object of much attention, both analytical and numerical. The main subject of these investigations has been the question if a strongly coupled abelian model can be constructed with non trivial interactions in the continuum limit.

During some time, the results about the nature of the fixed point of this model were controversial in the sense that different groups reported different values of critical exponents respectively consistent or inconsistent with a mean field description of this model. More recently, an extensive investigation of the quenched model in large lattices and at small fermion masses[1,2] gave evidence that a precise determination of the critical coupling β_c is needed in order to extract critical indices with good accuracy, since small variations in the value of β_c may induce strong changes in the values of the critical exponents towards their mean field values[3].

The results of [1,2] reporting non mean field values for the critical exponents in the quenched model, have been improved for the unquenched case in [4,5] and also by us in [3,6] in a completely independent calculation which allowed the numerical determination of β_c from the results of the plaquette energy without the need of any kind of extrapolation at zero fermion mass. Additionally, an analysis, based on Renormalization Group approach, performed in [6] on the results for the fermionic effective action, pointed again to a non gaussian nature for the strongly coupled fixed point of this model. These results are in contrast with those in [7].

The agreement between the results of refs. [4,5] and those of refs. [3,6] for the two and four flavour cases was certainly encouraging and suggested the existence of a non trivial continuum limit for strongly coupled QED, at least if the number of dynamical flavours is less or equal than four.

Another intriguing result reported in [5] was that critical indices for the two and four flavours models, obtained from numerical simulations and the use of the equation of state, were found compatible between them and also with the critical exponents of the monopole percolation transition, thus suggesting that different flavour models are in the same universality class.

This result can be also understood in our approach, if the critical value of the plaquette energy is independent on the number of flavours, as suggested by the results of [3]. In such a case, the value (for instance) of the δ exponent which measures the response of the system to an external symmetry breaking field will be given by the dominant contribution to the expansion of the chiral condensate in powers of the number of flavours N at fixed pure gauge energy[3] E_c, and it should be independent of N.

The aim of this letter is to analyse in detail the dependence on the number of flavours N of the physical results and in particular to investigate the phase diagramm of this model in the N, β plane with special attention to the analysis of the physical origin of the different phase transitions observed.

The approach we use to simulate noncompact QED with dynamical fermions is that of refs.[3,6], originally tested in the compact model[8], and based on the introduction of an effective fermionic action $S^F_{eff}(E,N,m)$ which depends on the pure gauge energy E, fermion mass m and number of flavours N, and which is related to the gauge fields $A_\mu(x)$ by the relation[3]

$$e^{-S^F_{eff}(E,m)} \equiv < \left(\det\Delta\right)^{\frac{N}{4}} >_E =$$

$$\frac{\int[dA_\mu(x)](\det\Delta(m,A_\mu(x)))^{\frac{N}{4}}\delta(\frac{1}{2}\sum_{x,\mu<\nu}F^2_{\mu\nu}(x)-6VE)}{\int[dA_\mu(x)]\delta(\frac{1}{2}\sum_{x,\mu<\nu}F^2_{\mu\nu}(x)-6VE)} \tag{1}$$

where $\Delta(m, A_\mu(x))$ is the fermionic matrix (we use staggered fermions), E is the normalized pure gauge energy and the denominator in (1) is the density of states which can be analytically computed

$$N(E) = C_G E^{\frac{3}{2}(V-1)} \tag{2}$$

C_G in (2) is some irrelevant divergent constant and V is the lattice volume.

After the definition of the effective fermionic action, the partition function of this model can be written as a one-dimensional integral

$$\mathcal{Z} = \int dEN(E)e^{-6\beta VE - S^F_{eff}(E,N,m)} \tag{3}$$

from which we can define an effective full action per unit volume as

$$\bar{S}_{eff}(E,\beta,N,m) = -\frac{3}{2}\ln E + 6\beta E + \bar{S}^F_{eff}(E,N,m) \tag{4}$$

$\bar{S}^F_{eff}(E,N,m)$ in (4) is the effective fermionic action (1) normalized to the lattice volume.

The thermodynamics of this system can be studied now by means of the saddle point technique. The mean plaquette energy $< E_p >= E_0(m,\beta,N)$ will be given by the solution of the saddle point equation[3]

$$\frac{1}{4E} - \beta - \frac{1}{6}\frac{\partial}{\partial E}\bar{S}^F_{eff}(E,N,m) = 0 \tag{5}$$

satisfying the minimum condition

$$\frac{1}{4E^2} + \frac{1}{6}\frac{\partial^2}{\partial E^2}\bar{S}^F_{eff}(E,N,m) > 0 \tag{6}$$

By differentiating equation (5) respect to β we get for the specific heat

$$C_\beta = \frac{\partial}{\partial\beta} < E_p >= -\{\frac{1}{4E_0^2(m,\beta,N)} + \frac{1}{6}\frac{\partial^2}{\partial E^2}\bar{S}^F_{eff}(E,N,m)\Big|_{E_0(m,\beta),N}\}^{-1} \tag{7}$$

The effective fermionic action $\bar{S}^F_{eff}$ was computed in [3,6] as a power expansion on the flavour number N.

The observation of two different regimes in the behaviour of $\bar{S}^F_{eff}$ as a function of E, linear in the small energy region and nonlinear at large energies, and a carefull analysis of the numerical results allowed us the determination of the critical value of the energy (which turned out to be independent on the number of flavours N) and of the critical coupling β_c. Furthermore it was established in [3] that the results for $\bar{S}^F_{eff}$ reported in Fig.1 can be very well fitted by two polynomials with a gap in the second energy derivative of $\bar{S}^F_{eff}$ at $E = E_c$ which manifests itself in the specific heat C_β as a second order phase transition.

The origin of this non analyticity in the effective fermionic action is not clear at present. However, our numerical results suggest that this singular behaviour could have the same origin as the monopole percolation transition of the quenched model extensively analysed in [9]. In fact if we take our value of the critical energy $E_c = 1.016(10)$ (independent on the number of flavours) and consider the zero flavour limit, we get $\beta_c = 0.246(2)$ in very good agreement with the critical β_c obtained in [9] from the results for the monopole susceptibility in the quenched model.

Going back again to expression (7) it should be noticed that a non analyticity of the effective fermionic action is not the only way to get a discontinuity in the specific heat. A discontinuity in C_β can in fact be produced by a zero in the denominator of (7) through exact cancellation of the two terms in this expression[3]. To this end, a negative value of the second energy derivative of the effective fermionic action is necessary. But this is just what happens in the large energy region as it can be deduced from the results for the effective fermionic action reported in Fig. 1.

The above arguments can be made quantitative and at the same time the critical number of flavours N_c can be simply estimated. We start from the cumulant expansion of the effective fermionic action[3]

$$-S^F_{eff}(E, N, m) = \frac{N}{4} < \ln \det \Delta(m, A_\mu(x)) >_E$$

$$+\frac{N^2}{32}\{< (\ln \det \Delta)^2 >_E - < \ln \det \Delta >_E^2\} + ... \tag{8}$$

where $< O >_E$ means the mean value of the operator $O(A_\mu(x))$ computed with the probability distribution $[dA_\mu(x)]\delta(\frac{1}{2}\sum_{x,\mu<\nu} F^2_{\mu\nu}(x) - 6VE)$. The numerical results for the succesive terms in the expansion (8) in a 8^4 lattice and massless fermions have been reported in [3], and are here extended to larger energies. As a first approximation, we will consider only the first contribution to (8), which is reported in Fig. 1 for $N = 4$.

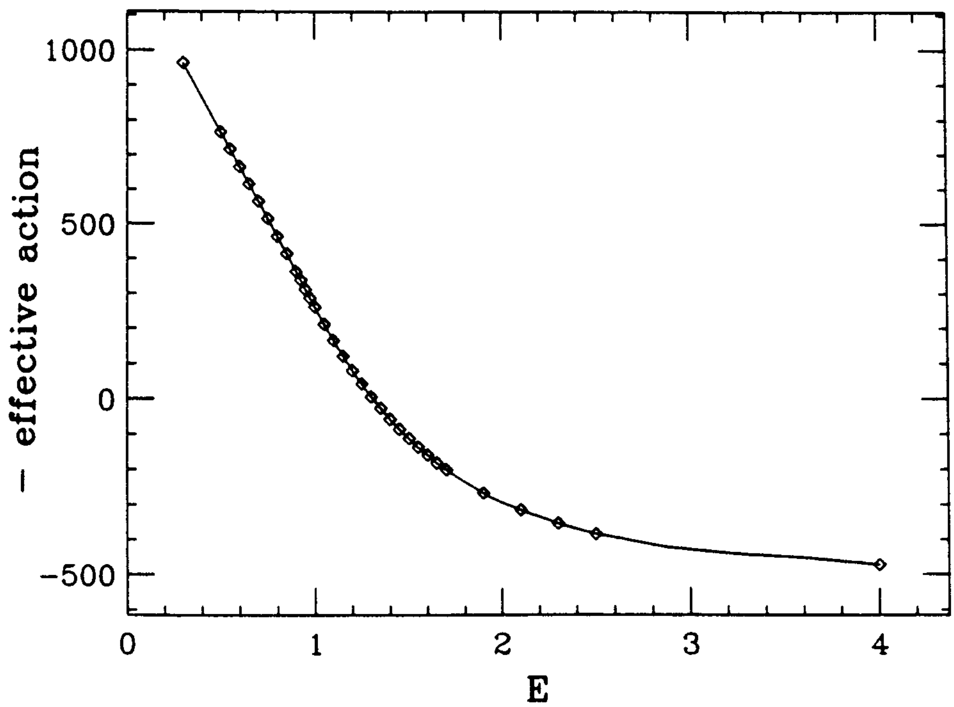

FIG. 1 - First contribution to the effective fermionic action (Equation 8) in a 8^4 *lattice,* $m = 0.0$ *and* $N = 4$. *Errors are smaller than symbols.*

Fitting the results for the mean logarithm of the fermionic determinant at $m = 0$ by two polynomials: first order for $E < E_c = 1.016$ and fifth for $E > E_c$ we get for the second energy derivative of the effective fermionic action

$$\frac{\partial^2}{\partial E^2}\bar{S}^F_{eff} = 0 \qquad (E \leq E_c)$$

$$\frac{\partial^2}{\partial E^2}\bar{S}^F_{eff} = \frac{N}{4}[-1.669 + 3.877E - 2.494E^2 + 0.494E^3] \quad (E_c < E < 2.5) \qquad (10)$$

A simple analysis of these results tell us that in order to compensate the pure gauge contribution to the denominator of the specific heat in (7) we need $N = 13.1$. If N is large but less than 13.1, the height of the peak increases with N and just at this critical value, the specific heat diverges. Now, if we increase again the value of N, there will be an energy interval where the denominator of the specific heat in (7) will be negative and therefore no solutions of the saddle point equations (5), (6) will exist in this energy interval. This means that these energies will not be accessible to the system and hence, for $N > N_c$ a first order transition will appear.

The phase diagram of massless noncompact QED in the N, β plane which emerges from our results is plotted in Fig. 2. The continuous (broken) lines represent first (second) order phase transitions respectively. The end point of the first order phase transition line is a second order phase transition point with a divergent specific heat. The first order line ends at some finite β since $\frac{\partial^2}{\partial E^2}\bar{S}^F_{eff} = 0$ for $E < E_c$. On the other hand, the second order line merges, for large N, into the first order one since E_c falls into the energy interval not accessible to the system, which widens as N increases.

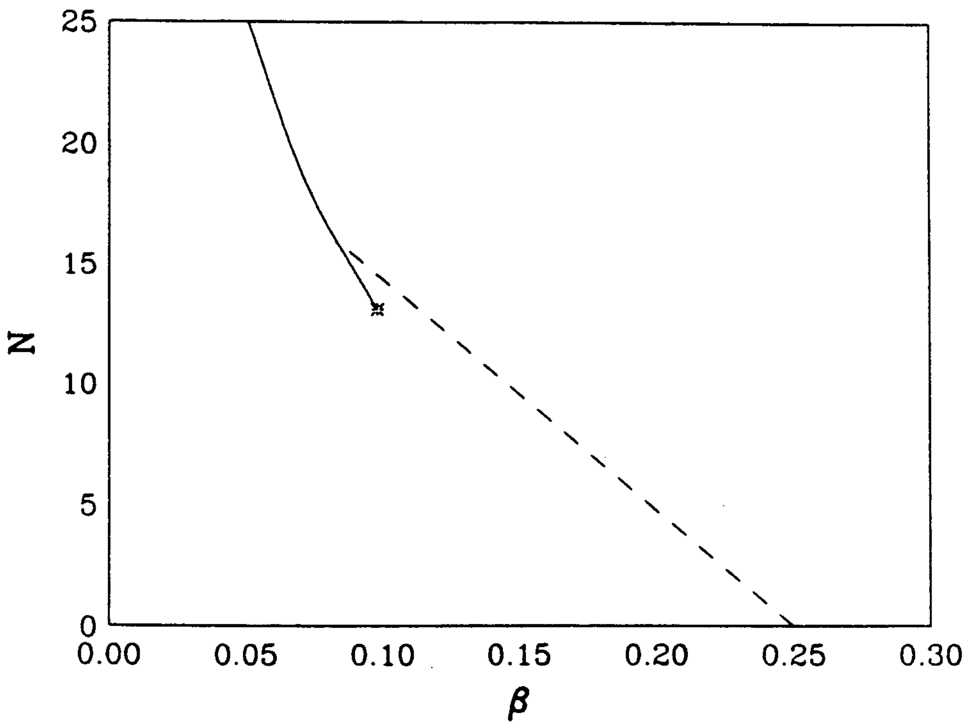

FIG. 2 - Phase diagram in the (β, N) plane at $m = 0.0$.

Of course the quantitative results of this simple analysis could change if we take into account higher order contributions to the effective fermionic action (8), but the physics behind these phase transitions can be understood at this simple level. What is independent on approximations is the fact that there is no first order phase transition at $\beta = 0$ for any finite value of N.

This result can be rigorously proved since the effective fermionic action $\bar{S}^F_{eff}(E, m, N)$ is bounded for any finite N. Therefore, when β goes to zero, the effective action (4) has only one minimum at $E = \infty$, whereas for any finite value of E the effective action is finite. Thus, at $\beta = 0$ there is no first order transition for any finite N.

A stronger result can be proved, under the very natural assumption (corroborated by the experimental data) that the effective fermionic action is monotonically increasing with E, namely that at $N \to \infty$ $< E >= 0$ for all β. This implies that, in this limit only one phase (Coulomb) does exist.

In fact, since the value of the fermionic determinant is bounded, it follows that

$$e^{N<\log\det\Delta>} \le e^{-S^F_{eff}(E,m)} \le e^{N\log\Delta_{max}} \tag{11}$$

which implies that the fermionic effective action diverges linearly with N.

As a consequence, for $N \to \infty$

$$\bar{S}_{eff}\Big|_{E=\frac{1}{N}} \approx \frac{3}{2}\ln N + NK(E,m)\Big|_{E=0} \tag{12}$$

and

$$\bar{S}_{eff}\Big|_{E=E_0} \approx NK(E_0,m) \tag{13}$$

The monotonicity of $\bar{S}^F_{eff}(E)$ then implies that

$$\lim_{N\to\infty}\left(\bar{S}_{eff}\Big|_{E=E_0} - \bar{S}_{eff}\Big|_{E=\frac{1}{N}}\right) = \infty \qquad \forall E_0 \neq 0 \tag{14}$$

implying that $E = 0$ is the absolute minimum of $\bar{S}_{eff}$ for any $\beta \neq 0$. This, together with the previous discussion, implies that the first order line reaches asymptotically the $\beta = 0$ axis.

To compare with published results on this subject, Kondo, Kikukava and Mino[10] found, within the Schwinger-Dyson approach, a continuous phase transition line which approches the $\beta = 0$ axis asymptotically. The general discussion of the above paragraph excludes such a behaviour for large N.

On the other hand Dagotto, Kocic and Kogut[11], in the framework of a numerical simulation, found evidence for a second order transition line wich becomes first order at large N, crossing the $\beta = 0$ axis ($N_c \sim 30$ at $\beta = 0$). The probable origin of the disagreement between this result and ours is that the critical β at $N = 30$ is so small that it is difficult to distinguish it from zero in a numerical simulation where metastability signal can be observed also at $\beta = 0$[11].

From a physical point of view, these two phase transition lines have a different origin. The second order line, as pointed before, could have the same origin as the monopole percolation transition of the pure gauge model[5,9] . Indeed our results for the critical energy and their independence on the flavour number N favour this interpretation. The first order line is on the other hand produced by pure fermionic effects. When the number of dynamical fermions increases, the fermionic contribution to the denominator of the specific heat becomes more and more important and has the correct sign to cancel the pure gauge contribution.

To finish let us say that these results have been also confirmed by our numerical simulations of this model. In these numerical simulations we have taken into account the first contribution to the effective fermionic action (8). In our opinion, the contribution of higher powers in N is not likely to change the qualitative behaviour depicted above.

In fact, since the effective fermionic action diverges linearly with N, is bounded as $E \to \infty$ and is monotonically increasing, then, barring very peculiar behaviours, it is convex for large E (corresponding to $\beta \to 0$). In such a case, for N large enough the two terms in (7) can always compensate, so a first order transition line will be in general present, at large N and small β. In conclusion, although the numerical structure of the phase diagram might vary, it is very likely that the qualitative features remain unchanged.

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