ISTITUTO NAZIONALE DI FISICA NUCLEARE - ISTITUTO NAZIONALE DI FISICA NUCLEARE - ISTITUTO NAZIONALE DI FISICA NUCLEARE - ISTITUTO NAZIONALE DI FISICA NUCLEARE - ISTITUTO NAZIONALE DI FISICA NUCLEARE - ISTITUTO NAZIONALE DI

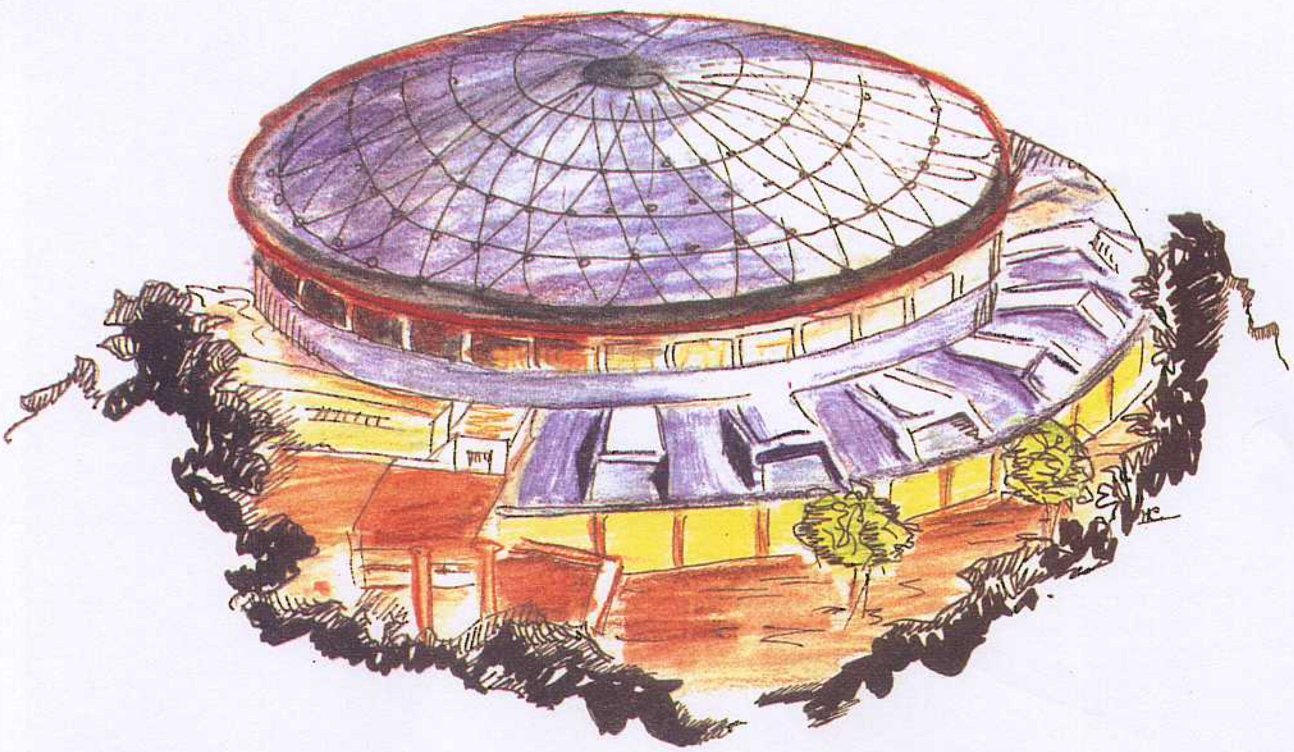
Laboratori Nazionali di Frascati

LNF-92/098 (P)
16 Novembre 1992

F. Palumbo:

NON-COMPACT GAUGE FIELDS ON THE LATTICE

Contribution to the
International Symposium on Lattice Field Theory
Amsterdam 15–19 September 1992



Servizio Documentazione
dei Laboratori Nazionali di Frascati
P.O. Box, 13 - 00044 Frascati (Italy)

INFN - Laboratori Nazionali di Frascati
Servizio Documentazione

LNF-92/098 (P)
16 Novembre 1992

NON-COMPACT GAUGE FIELDS ON THE LATTICE

F. Palumbo
INFN-Laboratori Nazionali di Frascati, P.O.Box 13, 00044 Frascati, Italy

ABSTRACT

A noncompact regularization of gauge theories is presented where exact gauge invariance at finite lattice spacing is obtained by means of an auxiliary field which decouples in the continuum limit. The result of the evaluation of the renormalization scale parameter and of the string tension are reported, showing the same evidence for confinement as in Wilson's scheme. The new regularization is used to define composite gauge fields.

PACS.: 11.15.Ha

1. INTRODUCTION

Our original interest in a noncompact lattice regularization of gauge theories was related to the possibility that artifacts of the compact formulation present at strong coupling could persist at week coupling.This problem has attracted attention especially in connection with the search for an ultraviolet fixed point at finite β in abelian theories [1] and with the investigation of the confinement mechanism [2-3] in non abelian ones.Such a mechanism,in fact,which is quite simple at strong coupling where it is strictly related to the compactness of the gauge variables, in the scaling regime(which should approximate the continuum where gauge fields are non compact) has not yet been fully understood.To investigate this problem one would like to have a non compact formulation.Since such a formulation with exact gauge invariance at finite lattice spacing has long been thougth to be impossible [4],attempts have been made to define non compact gauge fields on the lattice by direct discretization [2] of the continuum action (explicitely breaking gauge invariance at finite lattice spacing) or by using gauge-invariant variables obtained by a nonrenormalizable gauge fixing [3].In all these cases a vanishing value of the string tension has been found.Such a negative result,although not very stringent in view of the mentioned features of the actions used, makes certainly desirable a lattice regularization with noncompact gauge fields,exactely gauge invariant at finite lattice spacing and renormalizable. Such a regularization moreover,being closer to the continuum, might simplify perturbative calculations making simpler Faddev-Popov terms and reducing the number of Feynmann graphs which in Wilson's regularization proliferate due to the expansion of the unitary link variables.

More recently we have found a new interest in a non compact regularization because it may allow us to define composite gauge fields in terms of anticommuting variables.

In this talk a lattice regularization with non compact gauge fields is presented [5] where exact gauge-invariance at finite lattice spacing is enforced by means of an auxiliary field,which is shown to decouple in the continuum limit.The result of the evaluation of the renormalization scale parameter [6] and of the string tension [7] are reported, showing an evidence for confinement analogous to the one obtained by Wilson's formulation.Finally an example of composite gauge fields in terms of anticommuting variables is given.

2. NON-COMPACT GAUGE FIELDS ON THE LATTICE

The problem is to define a parallel transporter D_μ such that the covariant derivative

$$\mathcal{D}_\mu(x)\psi(x) = D_\mu\psi(x+\mu) - \frac{1}{a}\psi(x), \tag{1}$$

transforms as the field ψ under gauge transformations

$$\psi(x) \to g(x)\psi(x). \tag{2}$$

This requires that under these transformations

$$D_\mu(x) \to g(x)D_\mu(x)g(x)^+. \tag{3}$$

*This work is carried out in the framework of the European Community Research Program"Gauge theories,applied supersymmetry and quantum gravity"with a financial contribution under contract SC1-CT92-0789 .

As it is well known the above equation has no solution with the gauge fields in the algebra of the group,which we assume to be SU(N). In Wilson's definition in fact D_μ belongs essentially to the group

$$D_\mu = \frac{1}{a} U_\mu, \qquad U_\mu \in SU(N). \tag{4}$$

We have instead considered the possibility that D_μ lives in the algebra of GL(N,c)

$$D_\mu = V_\mu + iA_\mu, \qquad A_\mu = A_{\mu a} T_a, \tag{5}$$

with V_μ ,$A_{\mu a}$ complex numbers and T_a the generators of SU(N).In the following we will restrict ourselves to the case of SU(2), where the generators can be normalized according to

$$\begin{aligned} [T_a, T_b] &= i\epsilon_{abc} T_c \\ \{T_a, T_b\} &= \frac{1}{2}\delta_{ab}, \end{aligned} \tag{6}$$

and V_μ, $A_{\mu a}$ can be taken real.It is then natural to identify A_μ with the gauge field ,so that V_μ should be an auxiliary field which should decouple in the continuum limit.

The transformations of A_μ and V_μ which follow from Eq.(3) are

$$\begin{aligned} \delta A_\mu &= i[A_\mu, \theta] + a(V_\mu \triangle_\mu \theta + \frac{i}{2}[A_\mu, \triangle_\mu \theta]) \\ \delta V_\mu &= -\frac{1}{8} a Tr A_\mu \triangle_\mu \theta, \end{aligned} \tag{7}$$

where θ_a are the parameters of the transformation. We see that for $a \to 0$, they do not reproduce the gauge transformations. These can be recovered if the auxiliary field V_μ acquires a nonvanishing expectation value

$$< V_\mu >= \frac{1}{a}, \tag{8}$$

so that defining the shifted field

$$W_\mu = \frac{1}{a} - V_\mu, \tag{9}$$

we have

$$\delta A_\mu = \triangle_\mu \theta + i[A_\mu, \theta] - a(W_\mu \triangle_\mu \theta - \frac{i}{2}[A_\mu, \triangle_\mu \theta])$$

$$\delta W_\mu = \frac{1}{8} a Tr A_\mu \triangle_\mu \theta. \tag{10}$$

So a non compact regularization can be constructed if a)spontaneous breaking of GL to iGL can be ensured by a suitable potential and b)the auxiliary field decouples in the continuum limit.

We will now construct the action in such a way that the above conditions be satisfied. The strength and the Yang-Mills Lagrangian density can be written in analogy to the continuum

$$F_{\mu\nu}(x) = \frac{1}{i}[D_\mu(x)D_\nu(x+\mu) - D_\nu(x)D_\mu(x+\nu)] \tag{11}$$

$$L_{YM} = \frac{1}{8}\beta \sum_{\mu,\nu} F^+_{\mu\nu}(x)F_{\mu\nu}(x). \tag{12}$$

The possibility of enforcing conditions a) and b) is related to the existence of the other invariant

$$t_\mu = \frac{1}{2}Tr[D^+_\mu D_\mu - \frac{1}{a^2}] = A^2_\mu + W^2_\mu - \frac{2}{a}W_\mu, \tag{13}$$

so that the total action will contain an arbitrary function of this invariant. We determine this function by requiring i)minimal couplings of dimension not greater than 4 to have renormalizability ii) parity invariance and Euclidean invariance in the continuum limit iii)lower boundedness for the potential (lagrangian density at constant fields) iv)a divergent mass$\sim 1/a$ for W_μ to ensure its decoupling v)cancellation of the kinetic terms of W_μ in order to have a simple propagator .The resulting total lagrangian is

$$L_G = L_{YM} - \frac{1}{16}\beta a^2 \sum_{\mu\nu}(\triangle_\mu t_\nu - \triangle_\nu t_\mu)^2 + \frac{1}{2}\gamma^2 \sum_\mu t^2_\mu. \tag{14}$$

The minimum of the potential occurs at W_μ =0,-2/a.Since the second root can be shown to add only unessential complications (which will not be discussed here) the conditions a) and b) are satisfied.It is perhaps worth while noticing how the requirements iv) and v) are implemented in eq. (14).The second term does not vanish for $a \to 0$ because $t_\mu \sim 1/a$,and its finite contribution exactely cancels the kinetic terms of W_μ which remains with only a mass term $\sim 1/a$ arising from the third term.It is also worth while noticing that in the

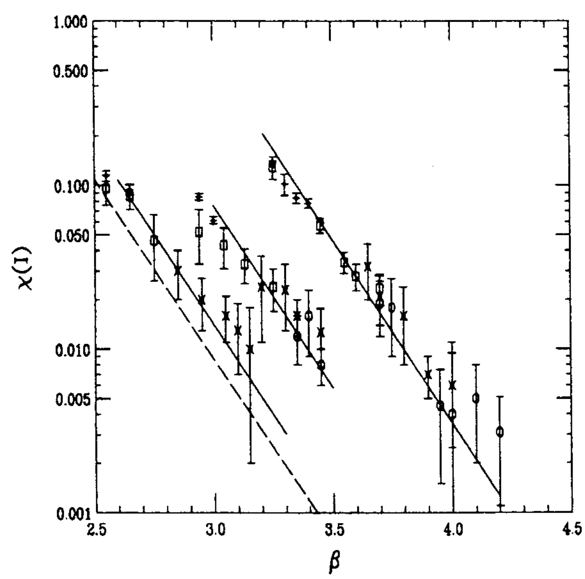


Figure 1. The value χ_I as a function of β for decreasing values of γ_1 from left to right (γ_1 = 5; 2,5; 1). The fit to Wilson's values is given by the slashed line. Full circles, squares, crosses and empthy circles refer to I = 3, 4, 5, 6 respectively.

limit $\gamma \to \infty$ we recover Wilson's definition. In such a limit in fact the partition function contains a δ-function of t_μ . Integrating over W_μ the jacobian from the δ -function gives the Haar measure over SU(2),while

$$W_\mu = \frac{1}{a}[1 \pm (1 - a^2 A_\mu^2)^{1/2}] \tag{15}$$

is real only if A_μ is compact.Inserting this solution in Eq.(5) we can write D_μ in the form of Wilson.

To conclude this section let us mention that nonunitary link variables already appeared in the color dielectric theory [8].In such a context an action very similar to ours has been constructed as an effective action (rather than a true regularized action) to be used in the phase of unbroken GL. Here the potential has its minimum at $t_\mu = -1/a^2$ and as a consequence there is no particle interpretation,a feature which in such a theory is assumed to characterize the QCD vacuum autside the hadrons.

3. RENORMALIZATION SCALE PARAMETER AND STRING TENSION

Retaining the auxiliary field W_μ the renormalization scale parameter Λ_{NC} of the present non compact regularization has been evaluated to one loop following the procedure of

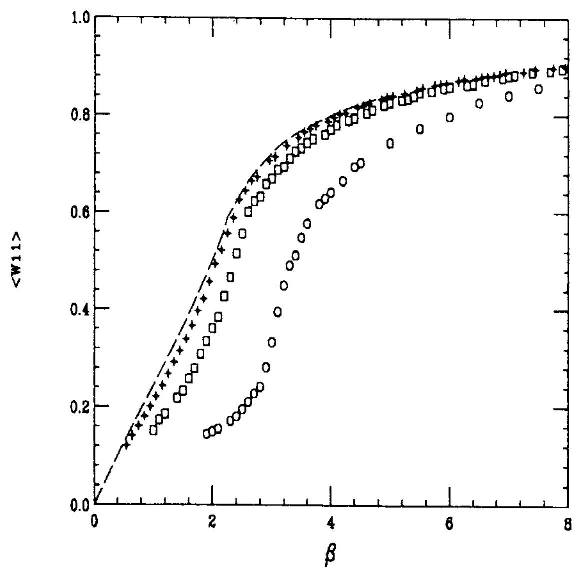


Figure 2. The expectaton value of $< \frac{1}{2}\, T_r\, U_P >$ as a function of β for $\gamma_1 = 5$ (crosses); 2.5 (squares); 1 (circles). The shaded line corresponds to Wilson's model.

Dashen and Gross [9]with the result

$$\Lambda_{NC} = \Lambda_W exp(-\frac{12\pi^2}{11}E), \tag{16}$$

where

$$E = 0.2208\frac{1}{\gamma^2}\beta \tag{17}$$

The same calculation shows that in the scaling limit

$$\gamma = \gamma_1\beta + \gamma_2 \tag{18}$$

where γ_1 is an arbitrary parameter while γ_2 is determined but it has not been evaluated.The ultraviolet fixed point is therefore the same in the two regularizations.

The plaquette energy and the Creutz ratio have been evaluated by a Monte Carlo simulation on a 12^4 lattice with the results reported in Figs.1,2. The same evidence for confinement is found as in Wilson's regularization,but the agreement with the scaling of Eq.(16) is rather poor.

4. COMPOSITE GAUGE FIELDS

Composite gauge fields have been considered often related to their dynamical generations in different perspectives.Here an example is given in terms of a commuting or anticommuting field λ in the fundamental representation of SU(2)

$$D_\mu = \frac{1}{2a}[\bar{\lambda}^* \otimes \lambda(x+\mu) + \epsilon\lambda \otimes \lambda^*(x+\mu)]. \tag{19}$$

In the above equation

$$\bar{\lambda} = \sigma_2\lambda, \tag{20}$$

and $\epsilon = \pm 1$ for the commuting or anticommuting case.Such a parametrization is the most general one which is invariant under the global transformation

$$\lambda \rightarrow e^{i\alpha}\lambda, \tag{21}$$

and which gives hermitean fields V_μ and A_μ

$$\begin{aligned} A_\mu &= i(\lambda^*\sigma_a \triangle_\mu \lambda - \triangle_\mu\lambda^*\sigma_a\lambda) \\ V_\mu &= \frac{1}{a}\lambda^*\lambda + \frac{1}{2}(\lambda^* \triangle_\mu \lambda + \triangle_\mu\lambda^*\lambda) \end{aligned} \tag{22}$$

The bosonic case has been included to illustrate two points.The first one is that while Wilson's regularization can be obtained in the bosonic case by requiring $\lambda\lambda^* = 1$, compositness with anticommuting variables requires the gauge fields to be noncompact.The second point concerns the particle content of such a simple example.In the bosonic parametrization it is trivial, because the D_μ of Eq.(19) corresponds to a pure gauge field, which is not true for the anticommuting variables,whose dynamics ,however,has not yet been investigated.

REFERENCES

1 S. Hands, J.B.Kogut and J.Sloan, Nucl.Phys. B344 (1990) 255; Erratum B352 (1991) 528

2 A.Patrascioiu, E.Seiler and I.O. Stamatescu, Phys. Lett. B107 (1981) 364; E.Seiler, I.O.Stamatescu and D.Zwanziger, Nucl. Phys. B239 (1984) 177; Y.Yotsuyanagi, Phys. Lett. B135 (1984) 141; K.Cahill, S.Prasad, R.Reeder and B.Richter, Phys. Lett. B181 (1986) 333; K.Cahill, Phys. Lett. B231 (1989) 294

3 F.Palumbo, M.I.Polikarpov and A.I.Veselov, Phys. Lett. B258 (1991) 189

4 A.M.Polyakov, Gauge fields and strings, Harwood Academic Publishers (1987), p.63

5 F.Palumbo, Phys.Lett. B244 (1990) 55 ; B250 (1990) 212(E) ; C.M.Becchi and F.Palumbo, Phys.Rev.D44 (1991) R946

6 C.M.Becchi and F.Palumbo, Nucl.Phys.B,in press

7 F.Palumbo, M.I.Polikarpov and A.I.Veselov, Phys.Lett.B, in press

8 H.B.Nielsen and A.Patkos, Nucl.Phys.B195 (1982) 137; J.M.Drouffe, Nucl.Phys.B218 (1983) 89; G.Mack, Nucl.Phys. B235[FS11] (1984) 197; H.J.Pirner, J.Wroldsen and M.Ilgenfritz, Nucl.Phys. B294 (1987) 905; B.Grossman, H.J.Pirner, A.I.Signal, R.Baier and J.Wroldsen, Int.Journal of Mod.Phys.A6, No.15 (1991) 2649

9 R.Dashen and D.J.Gross, Phys. Rev. D23 (1981) 2340