ISTITUTO NAZIONALE DI FISICA NUCLEARE - ISTITUTO NAZIONALE DI FISICA NUCLEARE - ISTITUTO NAZIONALE DI FISICA NUCLEARE - ISTITUTO NAZIONALE DI FISICA NUCLEARE - ISTITUTO NAZIONALE DI FISICA NUCLEARE - ISTITUTO NAZIONALE DI

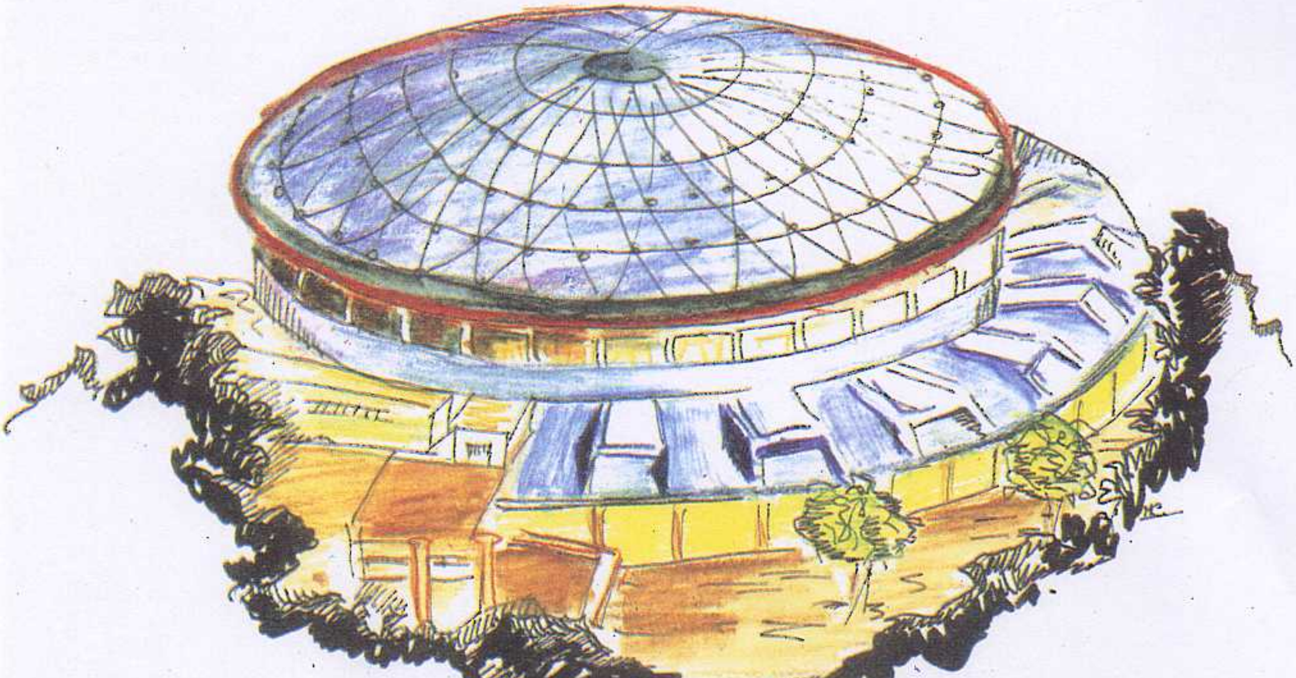
Laboratori Nazionali di Frascati

LNF–92/086 (P)
4 Novembre 1992

N. Brown, F.E. Close :

SCALAR MESONS AND KAONS IN Φ–RADIATIVE DECAY & THEIR IMPLICATIONS FOR STUDIES OF CP VIOLATION AT DAΦNE

Contribution to the DAΦNE Physics Handbook



Servizio Documentazione
dei Laboratori Nazionali di Frascati
P.O. Box, 13 - 00044 Frascati (Italy)

INFN - Laboratori Nazionali di Frascati
Servizio Documentazione

LNF-92/086 (P)
4 Novembre 1992

Scalar Mesons and Kaons in ϕ Radiative Decay & their implications for studies of CP violation at DAϕNE

N Brown† and F E Close
Rutherford Appleton Laboratory,
Chilton, Didcot, Oxon, OX11 OQX, England.

Prologue†

Tragically, Nick Brown died in an accident on 13 June 1991 just as his great potential as a physicist was beginning to flower. He was enthusiastically committed to science and this was his last and uncompleted research. He had written up rather complete notes which I include here as section 1 in his own words with only minor editing by me* and hope that he would have approved of it and the subsequent commentary.

We had become interested in the possibility that radiative decays of the ϕ meson might be so prominent that they would undermine the primary aim of the ϕ factory, namely the study of CP violation. As a result of Nick's work we can now be confident that the CP programme will not be significantly affected by this possibility. At the same time, the DAϕNE facility will create opportunities for studying ϕ radiative processes, in particular the production of the enigmatic scalar resonances, in their own right. Our joint understanding of this was primitive at the time of Nick's death and we did not perceive the subtleties and insights that this, still incompleted, work would bring. My personal sadness is that Nick was not able to witness the discoveries that

*Denoted by Enclosure within [...] in the closing paragraph of section 1 came from all this; he certainly would have enjoyed them and I would like to dedicate this paper to his memory.

Introduction

The existing literature makes predictions for the branching ratios $\phi \to K\bar{K}\gamma$ and $\phi \to S\gamma$ (where S denotes scalar mesons S^* or $f_0(975)$ and δ or $a_0(980)$) that vary by several orders of magnitude[1–5]. Clearly not all of these can be correct. In the spring of 1990 I began to look at these questions to try and isolate where the differences arise and decide which papers are most reliable: this problem has some practical urgency in view of the impending ϕ factory, DAϕNE, and the developing programme at VEPP4.

Nick Brown looked at the scalar resonance contributions to the $\phi \to K\bar{K}\gamma$ process, and noticed some inconsistencies in the literature: his conclusions are presented in section 1. I have studied the question of the intrinsic $\phi \to S\gamma$ rate to see if it can discriminate among models for the scalar mesons - e.g. are they $q\bar{q}, q^2\bar{q}^2$ or $K\bar{K}$ bound states? This latter work is summarised in sections 2 and 3 and will eventually be written up in detail with Isgur and Kumano[7]. It suggests that an accurate measurement of the absolute and relative branching ratios for $\phi \to \gamma f_0$ and γa_0 may indeed provide such discrimination and reinforces the conclusions of section 1 that, overall, $\phi \to \gamma K^0\bar{K}^0$ will pose **no** significant background to CP violation studies at DAϕNE.

1 Some comments on calculations of $\phi \to K^0\bar{K}^0\gamma$

(N Brown notes as transcribed by F Close)

The decay $\phi \to K^0\bar{K}^0\gamma$ poses a possible background problem to tests of CP violation at future ϕ factories. The radiated photon allows the $K^0\bar{K}^0$ system to be in a CP even state, as opposed to the CP odd decay $\phi \to K^0\bar{K}^0$. This has been proposed as a good way to measure ε'/ε, but because this means looking for a small effect any appreciable rate for $\phi \to K^0\bar{K}^0\gamma$ (Br $(\phi \to K^0\bar{K}^0\gamma 10^{-6})$ will limit the precision of such an experiment.

Estimates of the non-resonant $\phi \to K^0\bar{K}^0\gamma$ branching fraction give, in the absence of any resonant contribution, a number $0(10^{-9})$, far too small to pose a problem. Estimates of the resonant decay chain $\phi \rightarrow S^*(\delta) + \gamma$, followed by the decays $S^*(\delta) \rightarrow K^0\bar{K}^0$, however, vary by three orders of magnitude, from $0(10^{-6})$ down to $0(10^{-9})$. Here we concentrate on this resonant process, noting, of course that if the contribution were of $0(10^{-9})$, interference effects between resonant and non-resonant amplitudes would be important. We will discuss differences between the calculations and show that there is, in fact, no large discrepancy. The smaller of the two estimates will turn out to be the more reliable, showing that $\phi \rightarrow K^0\bar{K}^0\gamma$ transitions are unlikely to be a problem in tests of CP violation.

The larger estimate for the $\phi \rightarrow K^0\bar{K}^0\gamma$ branching fraction comes from Lucio and Pestieau[1], based on work by Nussinov and Truong[2,3]. They calculate a differential decay width

$$\frac{d\Gamma_{LP}}{dQ^2} = \frac{|I(a,b)|^2 g_\varphi^2 g^2}{4m_{K^+}^4 \pi^4}\chi \tag{1}$$

Here Q^2 is the invariant mass squared of the $K^0\bar{K}^0$ system, and hence the resonance. The calculation presumes the transition goes through a virtual K^+K^- loop [fig 1] and g_φ is the coupling of ϕ to K^+K^- and g is the coupling of the resonance. $I(a,b)$ is a factor coming from the loop integral given by

$$I(a,b) = \frac{1}{2(a-b)} - \frac{2}{(a-b)^2}\{f(\frac{1}{b}) - f(\frac{1}{a})\} + \frac{a}{(a-b)^2}\{\{g(\frac{1}{b}) - g(\frac{1}{a})\} \tag{2}$$

where

$$\begin{aligned} f(x) &= \begin{cases} -(\arcsin(\frac{1}{2\sqrt{x}}))^2 & x > \frac{1}{4} \\ \frac{1}{4}[\ln(\frac{\eta+}{\eta-}) - i\pi]^2 & x < \frac{1}{4} \end{cases} \\ g(x) &= \begin{cases} -(4x-1)^{1/2}\arcsin(\frac{1}{2\sqrt{x}}) & x > \frac{1}{4} \\ \frac{1}{2}(1-4x)^{1/2}[\ln(\frac{\eta+}{\eta-}) - i\pi] & x < \frac{1}{4} \end{cases} \\ \eta\pm &= \frac{1}{2}(1 \pm (1-4x)^{1/2}) \quad a = \frac{m_\varphi^2}{m_{K+}^2}\ ,\ b = \frac{Q^2}{m_{K+}^2} \end{aligned} \tag{3}$$

The factor χ in eq (1.1) is given by

$$\chi = \frac{\alpha}{128\pi^2 m_\varphi^2}\frac{\frac{1}{3}(m_\varphi^2 - Q^2)^3(1 - \frac{4m_K^2}{Q^2})^{1/2}}{(Q^2 - m_{S^*}^2)^2 + m_{S^*}^2\Gamma_{S^*}^2} \tag{4}$$

For the moment we have assumed that only a single resonance $S^*(975)$ contributes. The differential width eq (1.1) must be integrated from $K^0\bar{K}^0$ threshold up to m_φ^2. We should note here that we are unable to get agreement with Lucio and Pestieau's final answer of $\Gamma(\phi \to K^0\bar{K}^0\gamma) \approx 6 \times 10^{-6} MeV$, unless we choose $\Gamma_{S^*} \approx 33 MeV$ (as quoted in the particle data tables) and integrating from the **K⁺K⁻ threshold**, as opposed to the **K⁰K̄⁰ threshold**.

Unfortunately this decay width is extremely sensitive to this threshold. This is partly because a lower threshold allows us to get closer to the resonance peak, as well as increasing the Q^2 range of integration, which enhances the rate considerably since we are then less suppressed by threshold factors $(1 - \frac{4mK_0^2}{Q^2})^{1/2}(m_\varphi^2 - Q^2)^3$.

Integrating correctly from the $K^0\bar{K}^0$ threshold gives us $\Gamma(\varphi \to K^0\bar{K}^0\gamma) = 7.7 \times 10^{-7} MeV$. However this is still not correct, since the "width" quoted in the particle data tables is a fixed number, whereas since we are considering transitions very close to the $K\bar{K}$ threshold, the width is strongly mass dependent. Indeed assuming, as Lucio and Pestieau do, that $g_{S^*K^0\bar{K}^0} = g_{S^*K^+K^-}$ we have

$$\begin{aligned} \Gamma(S^*(Q^2) \to K^0\bar{K}^0) &= \frac{g^2_{S^*K^0\bar{K}^0}}{16\pi Q^2}(Q^2 - 4m^2_{K_0})^{1/2} \\ \Gamma(S^*(Q^2) \to K^+K^-) &= \frac{g^2_{S^*K^+K^-}}{16\pi Q^2}(Q^2 - 4m^2_{K^+})^{1/2} \end{aligned} \tag{5}$$

Taking their value of $g^2_{S^*K^+K^-}/4\pi m_\phi^2 = 0.58$, these widths rise from zero at their respective thresholds to 32 MeV and 37 MeV respectively at $Q^2 = m_\varphi^2$. Adding these Q^2 dependent widths to a fixed $\pi\pi$ width of 33 MeV gives us a total decay width $\Gamma(\phi \to K^0\bar{K}^0\gamma) = 3.7 \times 10^{-7} MeV$, over an order of magnitude less than the result quoted in their paper. On the other hand, the calculation of Paver and Riazuddin[4] gives

$$\frac{d\Gamma_{PR}}{dQ^2} = \frac{\alpha}{128\pi^2 M_\varphi^3}\frac{1}{3}(M_\varphi^2 - Q^2)^3(1 - \frac{4m^2_{K^0}}{Q^2})^{1/2}|B|^2 \tag{6}$$

where

$$B = G_{S^*}(\frac{1}{m^2_{S^*}} - \frac{2}{m^2_{S^*} - m^2_{K^+}} + \frac{1}{m^2_{S^*} - Q^2 - im_{S^*}\Gamma_{S^*}}) \tag{7}$$

The structure here is different to that from a simple scalar pole expression. The other terms arise from demanding that the complete amplitude satisfy certain current algebra conditions. For the moment, and for the purposes of comparison, we restrict ourselves to the scalar pole term, in which case we obtain

$$\frac{d\Gamma_{PR}}{dQ^2} = G^2_{S^*}\chi \qquad (8)$$

where $G_{S^*} = G_{S^*K^0\bar{K}^0}.G_{\phi S^*\gamma}$, and χ is given in eq (4). Since the function $|I(a,b)|^2$ varies smoothly and slowly over the Q^2 range of interest (it falls by a factor of 2 from $Q^2 = 4m^2_{K^0}$ to $Q^2 = m^2_\phi$ see table 1), replacing it by a constant should be a good approximation. Any difference, then, in the two approaches lies in the values chosen for the various coupling constants and the width, Γ_{S^*}.

We have already discussed how the width Γ_{S^*} should correctly incorporate the Q^2 dependence due to the opening up of the $K\bar{K}$ channels.

Q^2	$\chi I(Q^2)$
0.99	0.0134
1.01	0.008
1.025	0.0075
1.037	0.0064

Table 1: The Q^2 dependence of $\chi I(Q^2) \equiv \frac{|I(a,b)|^2}{4\pi^4}\frac{m^4_\phi}{m^4_{K+}}$.

We parameterise this decay by g_{S^*KK} and $R = g^2_{S^*\pi\pi}/g^2_{S^*KK}$, then the S^* width becomes

$$\Gamma_{S^*}(Q^2) = \frac{g^2_{S^*KK}}{16\pi Q^2}\{(Q^2 - 4m^2_{K_0})^{1/2} + (Q^2 - 4m^2_{K+})^{1/2} + 2R(Q^2 - 4m^2_\pi)^{1/2}\}$$

where the factor $2R$ is for charged and neutral pion decays. Physically we expect $R << 1 (R \approx 1/4?)$ and we consider the range $0.3\frac{g^2_{S^*KK}}{4\pi m^2_Q} \leq 3$ to cover a range of possibilities (Note Ref 5 quotes couplings in the range $0.3g^2/4\pi 2.3$).

Although some of these coupling values would give a $\Gamma(S^* \to \pi\pi) >> 33$ MeV, it is well-known that the effect of the nearby $K\bar{K}$ threshold distorts the $S^* \to \pi\pi$ shape, and if $K\bar{K}$ is more strongly coupled than $\pi\pi$ one can get a 'cusp' effect[6].

$R\backslash g^2_{s^*KK}/4\pi$	0.3	0.58	1.0	1.5	2.5	3.0
0.125	18	18.8	43.2	47	49	49.5
0.25	12.5	20	21	21.8	23	22.4
0.5	6.37	7.7	8.065	8.18	8.25	8.24
1.0	2.36	2.53	2.57	2.58	2.58	26

Table 2: $\Gamma(\phi \to K^0\bar{K}^0\gamma)$ in units of 10^{-8} MeV as a function of $g^2(SKK)$ and $R = g^2(S\pi\pi)/g^2(SKK)$.

[There are also possible interferences between δ and S^* and if

$$\begin{aligned} g_{S^*K^+K^-} &= g_{\delta K^+K^-} \\ g_{S^*K^0\bar{K}^0} &= -g_{\delta K^0\bar{K}^0} \end{aligned}$$

then we can compute $\Gamma(\phi \to \gamma_+(S^*, \delta) \to \gamma K^0\bar{K}^0)$ as a function of $R_{S^*}, R_\delta, g^2_{S^*}/4\pi, g^2_\delta/4\pi$ where

$$\begin{aligned} R_{S^*} &= g^2_{S^*\pi\pi}/g^2_{S^*KK} \\ R_\delta &= g^2_{\delta\pi\eta}/g^2_{\delta KK} \end{aligned}$$

A representative sample is given in table 3.]

$R_{S^*}, R_\delta\backslash g^2_{S^*}/4\pi, g^2_\delta/4\pi$	(0.3,0.3)	(0.3,1)	(1,0.3)	(1,1)	(2.5,1)	(1, 2.5)
(1,1)	3.5	5	4	4.6	4.6	4.7
(1/4,1)	20	40	20	40	40	44
(1,1/4)	0.8	4.4	0.9	0.9	1	1.4
(1/4, 1/4)	4.5	24	9.4	11	12	14

Table 3: The width ($\Gamma(\phi \to K^0\bar{K}^0\gamma)$ including possible interference effects in units of 10^{-8} MeV.

Note that for many of the combinations the effects of the interference are small (the δ actually gives a larger contribution than the S^* for many cases, which is why we have

$$|A_{s^*}|^2 \leq |A_{s^*} + A_\delta|^2 \leq |A_\delta|^2$$

where A = amplitude).

[The conclusions to be drawn from these calculations are that the branching ratio is sensitive to detailed modelling, resonance coupling strengths and interference effects. Predictions of branching ratios in excess of 10^{-7} are in error (a conclusion that is reinforced by the subsequent work on $\phi \to \gamma S^{(7)}$ and described elsewhere in these proceedings). The process $\phi \to \gamma K^0 \bar{K}^0$ will therefore pose no significant background to CP violation studies at DAϕNE.]

2 Probing the nature of the scalar mesons below 1 GeV

Some of the $\phi \to \gamma K\bar{K}$ will arise from the transition $\phi \to \gamma S$ where S denotes the scalar mesons $f_0(975)$, a_0 (980) each of which couples strongly to $K\bar{K}$. There is considerable uncertainty in the literature as to the expected branching ratio for $\phi \to \gamma S$, estimates ranging from $0(10^{-3})$ [ref 2] to $0(10^{-6})$ [10]. These variations are in part due to errors and in part due to modelling. We have studied the literature, identified which calculations are mathematically reliable and determined whether the relative and absolute magnitudes of the branching ratios $\phi \to \gamma f_0$ and $\phi \to \gamma a_0$ can help probe whether the scalars are for example $q\bar{q}, qq\bar{q}\bar{q}$ [9] or $K\bar{K}$ [8] systems.

In summary, we find that for the f_0 (975) the B.R. will typically be $0(10^{-4})$ if $q^2\bar{q}^2$, $0(10^{-5})$ if $s\bar{s}$ and below $0(10^{-5})$ for a spatially diffuse $K\bar{K}$ system. If the a_0 (980) is $q\bar{q}$, it will be Zweig decoupled from the ϕ. The production rate via the $K\bar{K}$ loop, viz $\phi \to \gamma K\bar{K} \to \gamma a_0$, may be calculated but has some interesting points of principle which shed light on the role of finite hadron size in such loop calculations. The relative size of $\phi \to \gamma a_0/\gamma f_0 \simeq 1$ if they are $K\bar{K}$ systems; for $q^2\bar{q}^2$ the ratio is sensitively dependent on the internal structure of the states. If the state's internal charge is distributed about the centre of mass thus $(q\bar{s})(\bar{q}s)$ - where q denotes u or d - the ratio $\phi \to \gamma f_0/\gamma a_0$ will be unity and only the absolute branching ratio will distinguish $q^2\bar{q}^2$ from $K\bar{K}$. If it is $(q\bar{q})(s\bar{s})$ the process will be dominated by the $K\bar{K}$ loop and the absolute rate will depend on the Zweig rule dynamics. However if the structure is $(qs)(\bar{q}\bar{s})$ the ratio of widths $\phi \to \gamma f_0/\gamma a_0 = 1/9$. The absolute rate depends on an unknown overlap for $\phi \to K^+K^- \to D\bar{D}$ (where D denotes diquark), nonetheless the dominance of a_0 over f_0 would be rather distinctive.

We now survey the literature and evaluate the claims. This section is based, in part, on work done in collaboration with Nathan Isgur and Shunzo Kumano, which will appear in ref. 7.

Several papers[1-3,5] have computed the amplitude $M(\phi \to S^*\gamma)$ by assuming the decay to proceed through the charged K loop (fig 1), $\phi \to K^+K^- \to S(k) + \gamma \to K^0\bar{K}^0\gamma$ where the $K^\pm$ are real or virtual and S is the scalar meson with four momentum k. The amplitude describing the decay can be written

$$M(\phi(p,\eta)) \to S(k) + \gamma(q,\epsilon) = \frac{eg_\phi g}{2\pi^2 m_K^2} I(a,b)[(p.q)(\epsilon.\eta) - (p.\epsilon)(q.\eta)] \quad (9)$$

where $\epsilon(q), \eta(p)$ denote γ and ϕ polarisations (momenta), the g_ϕ, g couplings for ϕK^+K^- and SK^+K^- are related to the widths by

$$\Gamma(\phi \to K^+K^-) = \frac{g_\phi^2}{48\pi m_\phi^2}(m_\phi^2 - 4m_{K^+}^2)^{3/2} \quad (10)$$

and

$$\Gamma(S \to K^+K^-) = \frac{g^2}{16\pi m_S^2}(m_S^2 - 4m_{K^+}^2)^{1/2} \quad (11)$$

The quantities a, b are defined as $a = \frac{m_\phi^2}{m_K^2}, b = \frac{m_S^2}{m_K^2}$ so that $a - b = \frac{2p.q}{m_K^2}$ is proportional to the photon energy. The loop integral $I(a,b)$ is given in eq (1-3); note that m_S^2 is in general virtual though we shall here concentrate on the real resonance production where $m_S \simeq 975$ or 980 MeV.

First we summarise limitations and problems in the existing literature concerning attempts to calculate the above. Refs (2,3) have made technical errors; ref (1) obtains the $I(a,b)$ correctly as above but has a numerical error in computing the ensuing width; ref (5) uses a different approach (see later) and obtains an expression which agrees with that of ref 1. We confirm this result and its numerical value too - however we disagree with the physical interpretation.

2.1 Calculation of the integral $I(a,b)$

Upon making the ϕ and K interactions gauge invariant, one finds for charged kaons

$$H_{int} = (eA_\mu + g_\phi\phi_\mu)j^\mu - 2eg_\phi A^\mu \phi_\mu K^+K \quad (12)$$

where A^μ, ϕ_μ and K are the photon, phi and charged kaon fields, $j^\mu = iK^+(\vec{\partial}^\mu - \overleftarrow{\partial}^\mu)K$. Upon recombining the two kaons to form a **pointlike** scalar field, gauge invariance generates no extra diagram and the resulting diagrams are in figs (1). Immediately one notes a problem: the contact diagram fig 1a diverges. The trick has been to calculate the finite fig (1b) and then, by appealing to gauge invariance, to abstract a finite answer. This is done either by

a) [Refs 1-3], Fig 1a contributes, to $A^\mu \phi^\nu g_{\mu\nu}$ whereas Fig 1b contributes both to this and to $p_\mu q_\nu A^\mu \phi^\nu$. Therefore one need calculate only fig 1b, abstract the finite coefficient of the $p_\mu q_\nu$ term and, by gauge invariance, one is assured that this must be the finite result.

b) [Ref 5] Compute the imaginary part of the amplitude (which arises only from fig 1b) and write a **subtracted** dispersion relation, with the subtraction constrained by gauge invariance.

We[7] have considered the case where the scalar meson is an extended object, in particular a $K\bar{K}$ bound state. The $SK\bar{K}$ vertex therefore involves a momentum dependent form factor $f(k)$, where k is the kaon, or loop, momentum which will be scaled in $f(k)$ by k_0, the mean momentum in the bound state wavefunction or, in effect, the inverse size of the system. In the limit where $R \to 0$ (or $k_0 \to \infty$) we recover the formal results of approaches (a, b) above, as we must, but our approach offers some possible new insights into the physical processes at work. In particular there is a further diagram (fig 2c) proportional to $f'(k)$ since the minimal substitution yields

$$f(k - eA) - f(k) = -eA.\hat{k}\frac{\partial f}{\partial k} \tag{13}$$

As we shall see, this exactly cancels the contribution from the seagull diagram fig 2a in the limit where $q_\gamma \to 0$, and gives an expression for the finite amplitude which is explicitly in the form of a difference $M(q) - M(q = 0)$; this makes contact with the subtracted dispersion relation approach of ref. 5.

First let us briefly summarise the Feynman diagram in the standard pointlike field theory as it has caused some problems in refs (2,3). If we denote $M_{\mu\nu} = [p_\nu q_\mu - (p.q)g_{\mu\nu}]H(m_\phi, m_S, q)$ then the tensor for fig (3) may be written (compare refs 2,3 eqs 8 & 6)

$$M_{\mu\nu} = egg_\phi \int \frac{d^4k}{(2\pi)^4} \frac{(2k-p)_\mu (2k-q)_\nu}{(k^2-m^2)[(k-q)^2-m^2][(k-p)^2-m^2]} \tag{14}$$

We will read off the coefficient of $p_\nu q_\mu$ after combining the denominators by the standard Feynman trick so that

$$M_{\mu\nu} = \frac{egg_\phi}{(2\pi)^4} 8 \int_0^1 dz \int_0^{1-z} dy \int_{-\infty}^{\infty} \frac{d^4k k_\mu k_\nu}{(k^2 - c + i\epsilon)^3} \tag{15}$$

where $c \equiv m^2 - z(1-z)M_\phi^2 - zy(M_S^2 - M_\phi^2)$, and the $p_\nu q_\mu$ term appears when we make the shift $k \to k + qy + pz$. One obtains

$$H = \frac{egg_\phi}{4\pi^2 i} \int_0^1 dz \int_0^{1-z} dy\; yz[m^2 - z(1-z)M_\phi^2 - zy(M_S^2 - M_\phi^2)]^{-1} \tag{16}$$

Note that $M_S^2 < M_\phi^2$ and so one has to take care when performing the $\int dy$. One obtains (recall $a = m_\phi^2/m^2, b = m_S^2/m^2$)

$$H = \frac{egg_\phi}{4\pi_i^2 m^2} \frac{1}{(a-b)} \{ \int_0^1 \frac{dz}{z} [z(1-z) - \frac{(1-z(1-z)a)}{(a-b)} \ln(\frac{1-z(1-z)b}{1-z(1-z)a})] \\ \cdot \frac{-i\pi}{(a-b)} \int_{1/\eta_+}^{1/\eta_-} (1-z(1-z)a)dz \} \tag{17}$$

where $\eta_\pm \equiv \frac{a}{2}(1 \pm \rho)$ with $\rho \equiv \sqrt{1-4/a}$.

I have exhibited these manipulations in order to assess the existing literature. In ref (2), the (unnumbered) eq $8\frac{1}{2}$ has omitted the imaginary part - the final integral above. In ref (3) at eq 7 one can see how this has arisen: the coefficient of their term $\alpha\beta(\mu^2 - \mu'^2)$ corresponds to $zy(M_\phi^2 - M_S^2)$ in our eq (2.8) i.e. it has the opposite sign. However, as ref (3) then proceeds to neglect this term one might think that the discrepancy would not matter - but there is no need to ignore this term as the integrals can be performed analytically and one sees that the said term is not negligible.

In performing the integrals, take care to note that $a > 4$ whereas $b < 4$ (which causes $\rho_a^2 > 0, \rho_b^2 < 0$). Hence one confirms the $I(a,b)$ as given in ref (1) [our calculation above has referred only to the diagram where the K^+ emits the γ; the contribution for the K^- gives the same and so the total amplitude is double that of eq (2.9), hence in quantitative agreement with eqs (3 and 4) of ref (1)]. Straightforward algebra confirms that this equals the eqs (9-11) of ref (5).

Numerical evaluation, using $m(f_0) = 975MeV$, $g^2/4\pi = 0.6GeV^2$ leads to

$$\Gamma(\phi \to f_0\gamma) = 6 \times 10^{-4}MeV \tag{18}$$

in contrast to the value of $8.5 \times 10^{-4}MeV$ (J Pestieau, private communication, confirms our value at eq 2.10). In ref (5) the rate for $\phi \to f_0\gamma$ is not directly quoted, though as their expression agrees with ours, one supposes that the value at eq (2.10) should obtain. Instead, ref (5) gives values for $\phi \to \gamma f_0 \to \gamma\pi\pi$ (for example) and claims that this depends upon the $q\bar{q}$ or $q^2\bar{q}^2$ structure of the f_0. However, the differences in rate (which vary by an order of magnitude between $q\bar{q}$ and $q^2\bar{q}^2$ models) arise because different magnitudes for the fKK couplings have entered. In the $q^2\bar{q}^2$ model a value for $g^2(fKK)$ was used identical to ours and, if one assumes 100% branching ratio for $f_0 \to \pi\pi$, the rate is consistent with our eq (2.10) [ref 5 has integrated over the resonance]: Ref 5 notes that in the $q^2\bar{q}^2$ model the relation between $g^2(a_0KK)$ and $g^2(a_0\pi\eta)$ imply $\Gamma(a_0 \to \pi\eta) \simeq 275$ MeV. In the $q\bar{q}$ model ref 5 uses as **input** the experimental value of $\Gamma(a_0 \to \pi\eta) \simeq 55$ MeV which implies a reduced value for $g^2(a_0\pi\eta)$ and, therefore, for $g^2(a_0\bar{K}K)$: the predicted rate for $\phi \to \gamma a_0 \to \gamma\pi\eta$ is correspondingly reduced.

Thus we believe that the apparent discrimination among models in ref 5 is rather indirect and logically suspect in the case of $\phi \to \gamma S$. The calculation has assumed a pointlike scalar field which couples to pointlike kaons with a strength that can be extracted from experiment. The computation of a rate for $\phi \to KK \to \gamma S$ will depend upon this strength and cannot of itself discriminate among models of internal structure for the S.

We[7] have considered the production of an extended scalar meson which is treated as a KK system (following the work of ref 8). Our results numerically agree with the pointlike field theory as $R_S \to 0$, and the branching ratio falls for $R_S \neq 0$. A genuine KK molecule will have $R2fm$ in order that two colour singlet $q\bar{q}$ states are meaningful - the resulting width $10^{-5}MeV$. At the other extreme, where the $q^2\bar{q}^2$ are all contained within a single 1 fm confinement domain one recovers the result of eq (2.10) and ref 5. Thus we suspect that that large branching ratio corresponds to a P-matrix state (ref 9).

2.2 The production of an extended scalar meson via a KK loop

Suppose that K^+ and K^- with three momenta $\pm\vec{k}$ produce an extended scalar meson in its rest frame. The interaction Hamiltonian $H = g\phi(|\vec{k}|)$ is in general a function of momentum. Now make the replacement $\vec{k} \to \vec{k} - e\vec{A}$, expand $\phi(k - eA)$ to leading order in e and one finds a new electromagnetic contribution

$$H_{K^+K^-f_0\gamma} = -eg\phi'(k)\hat{k}.\vec{A} \tag{19}$$

The effect of this form factor is readily seen in time ordered perturbation theory. There are four contributions: ($H_{1,4}$ are figs 2a, c, while $H_{2,3}$ are fig b where the γ is emitted from the K^+ or K^- leg). We write these (for momentum routing see fig 3)

$$H_{2,3} = 2egg_\phi \int d^3k \frac{\phi(k)2\vec{\epsilon}_\gamma.\vec{k}(\vec{k}.\vec{\epsilon}_\phi \pm \frac{1}{2}\vec{q}.\vec{\epsilon}_\phi)}{D(E)D_1D(q\pm)} \tag{20}$$

$$H_1 = 2egg_\phi \int d^3k \frac{\phi(k)\vec{\epsilon}_\gamma.\vec{\epsilon}_\phi}{D_1} \tag{21}$$

$$H_4 = 2egg_\phi \int d^3k \frac{\phi'(k)\vec{\epsilon}_\gamma.\hat{k}\vec{\epsilon}_\phi.\vec{k}}{D(0)} \tag{22}$$

where

$$D_1 \equiv m_\phi - q - D(E)$$

$$D(q^\pm) \equiv m_\phi - 2E(k \pm q/2) \tag{23}$$

$$D(0) \equiv m_\phi - 2E(k) \tag{24}$$

$$D(E) \equiv E(k + q/2) + E(k - q/2) \tag{25}$$

where $E(P)$ is the energy of a kaon with momentum P. Note that H_1 is the (form factor modified) contact diagram and H_4 is the new contribution arising from the extended $SK\bar{K}$ vertex.

After some manipulations their sum can be written

$$H = 2egg_\phi\vec{\epsilon}_\gamma.\vec{\epsilon}_\phi \int d^3k[\frac{\phi(k)}{D_1}\{1 + \frac{\vec{k}^2 - (\vec{k}.\hat{q})^2}{D(E)}(\frac{1}{D(q^+)} + \frac{1}{D(q^-)})\} + \frac{\phi'(k)|\vec{k}|}{3D(0)}] \tag{26}$$

If $\lim_{k^2\to\infty}(k^2\phi(k)) \to 0$ we may integrate the final term in eq (2.18) by parts and obtain for it

$$H_4 = 2egg_\phi \vec{\epsilon}_\gamma.\vec{\epsilon}_\phi \int d^3k \frac{\phi(k)}{D(0)}\{-1 - \frac{\vec{k}^2 - (\vec{k}.\hat{q})^2}{E(k)D(0)}\} \tag{27}$$

This is identical to the $\vec{q} \to 0$ limit of $H_1 + H_2 + H_3$, and hence we see explicitly that the $g_{\mu\nu}$ term (i.e. the $\vec{\epsilon}_\gamma.\vec{\epsilon}_\phi$ as calculated above) is effectively subtracted at $q = 0$ due to the partial integration of the $\phi'(k)$ contribution, H_4.

If one now has a model for $\phi(k)$ one can perform the integrals in eq (2.18) numerically.

For the KK molecule the wavefunction

$$\psi(r) = \frac{1}{\sqrt{4\pi}}\frac{u(r)}{r} \tag{28}$$

is a solution of the Schrodinger equation

$$\{-\frac{1}{m_k}\frac{d^2}{dr^2} + v(r)\}u(r) = Eu(r) \tag{29}$$

where (ref 13) one approximates

$$v(r) = -440(MeV)\exp(-\frac{1}{2}(\frac{r}{r_0})^2) \tag{30}$$

with $r_0 = 0.57fm$ and hence $E = -10MeV$. This equation may be solved numerically and, for analytic purposes, we find that the $\psi(r)$ is well approximated by[14] (fig 4)

$$\psi(r) = (\frac{\mu^3}{\pi})^{1/2}\exp(-\mu r); \quad \mu \equiv \frac{\sqrt{3}}{2R_{rms}} \tag{31}$$

where $R_{rms} \simeq 1.2fm$ (thus $\psi(0) = 3 \times 10^{-2}Gev^{3/2}$, see also ref 13). The momentum space wave function that is used in our computation is thus

$$\phi(k) = \mu^4/(k^2 + \mu^2)^2 \tag{32}$$

The rate for $\Gamma(\phi \to S\gamma)$ is shown as a function of $R_{K\bar{K}}$ in fig (5). The non relativistic approximation eqs (2.12-2.19) is valid for $R0.3fm$ which is applicable to the KK molecule: for $R \to 0$ the fully relativistic formalism is required and has been included in the curve displayed in fig 5. As $R \to 0$ and $\phi(k) \to 1$ we recover the numerical result of the pointlike field theory whereas for the specific $K\bar{K}$ molecule wavefunction above one predicts a branching ratio of some 4×10^{-5} (width $\simeq 10^{-4} MeV$). This is only $\frac{1}{5}$ of the pointlike field theory result but is larger than that expected for the production rate of a $\phi \to \gamma S(s\bar{s})$ scalar meson.

Note that at $1.2 fm$ the system is in some sense a mixture between genuinely separate $K\bar{K}$ (which require $R2fm$) and a compact $q^2\bar{q}^2$ system. These results and their interpretation are still preliminary. Some of the questions that they bear on are discussed in the next section.

3 The Zweig rule

If the $a_0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$ its production in $\phi \to a_0\gamma$ will vanish modulo corrections from $K\bar{K}$ loop contributions - called Zweig violating processes. In the pointlike field theory calculations one would expect a branching ratio of order 10^{-4} via this loop process. For the $f_0 = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$ a similar expectation obtains. However, if $f_0 = s\bar{s}$ it can be produced directly from the $\phi(s\bar{s})$ and the rate calculated rather reliably. One may scale[10] the results of ref (11) for $b\bar{b}$ and $c\bar{c}$ to $s\bar{s}$, or compare with calculations[11] for light quark states: independent of how one chooses to do this, the resulting branching ratio is $0(10^{-5})$ (between 0.5 to 2×10^{-5}).

Thus one has an interesting conundrum. Here is a process that can be computed via the, supposedly dominant, $s\bar{s} \to s\bar{s} + \gamma$ transition but which is overwhelmed by an order of magnitude by a $K\bar{K}$ loop contribution. The latter are known to be suppressed in the $\phi \leftrightarrow \omega$ mixing and an ad hoc rule - the Zweig or OZI rule[12] - has been invented to "legitimise" this. However, in the present example of $\phi \to S\gamma$ one can calculate the loop diagram explicitly and one finds that it can be dominant: what then for the OZI rule?

First we see an important message from our computation. If the $K\bar{K}$ system is diffuse, $R2fm$, then the loop calculation gives a branching ratio $< 10^{-5}$ and the empirical OZI rule is good. Physically, the rate is suppressed due to the poor spatial overlap between the $K\bar{K}$ system and the ϕ - intuitively there is small probability to find the extended $K\bar{K}$ system "in" the small ϕ wavefunction. The pointlike field theory does not allow for this - superficially the loops have large magnitude and the empirical result that the loops must be small causes the OZI "rule" to be shouted. But shout as loud as you like - such a "rule" needs justification and the present calculation may be giving the essential clue that the confinement scale of $1fm$ is important.

If the size of the $K\bar{K}$ system is $\Lambda_{QCD}^{-1} \simeq 1fm$, then it is really a $q^2\bar{q}^2$ overall colour $\underset{\sim}{1}$ and separate identifiable kaons are not present. The $\phi \rightarrow \gamma S$ branching ratio is then elevated above the 10^{-5} barrier: there is non-negligible amplitude for a **compact** $q^2\bar{q}^2$ intermediate state to cause $\phi \rightarrow \gamma a_0$ for example.

That the $\phi \leftrightarrow \omega$ states are ideal (pure $s\bar{s}$, pure $n\bar{n}$) is because there are no compact $s\bar{s}n\bar{n}$ states with $J^P = 1^-$ quantum number. The $K\bar{K}$ intermediate state that superficially has $s\bar{s}n\bar{n}$ constituents must extend over $2fm$ in order that meaningful kaons develop: this reduces the spatial overlap and suppresses the $K\bar{K}$ loop. Thus, at low energies at least, the empirical OZI rule is a statement that colour singlet states saturate at $0(1fm)$: a collection of ≥ 2 colour singlet hadrons requires spatial extent $\geq 2fm$ and hence has reduced overlap with the $0(1fm)$ state.

These remarks may have significant application for $\gamma\gamma \rightarrow \pi\pi, K\bar{K} \rightarrow S$ and for the radiative interactions of (composite) Higgs scalars.

There is still much thought needed on the correct modelling of the KK or $q^2\bar{q}^2$ scalar meson and the resulting rate for $\phi \rightarrow S\gamma$. There are interesting interference effects possible between $I = 1$ and $I = 0$ states which have not been examined in detail. At this state I would merely assert that the $\phi \rightarrow S\gamma$ provides a great opportunity for probing the nature of the scalar mesons below 1 GeV which will complement that of $\gamma\gamma \rightarrow S$ (e.g. refs 13,14). The branching ratio will be between 10^{-4} and 10^{-5} ; the precise predictions are still awaited but will (hopefully) exist **before the data are taken.**

4 Acknowledgements

I am indebted to the Institute for Nuclear Theory in Seattle for their hospitality and my collaborators there, N Isgur and S Kumano. I would also like the organisers of the DAϕNE workshops, and also M Pennington and G Preparata for their interest in the Zweig rule and some technical aspects of this work.

References

[1] J. Lucio and J. Pestieau, Phys. Rev. **D42**, 3253 (1990).

[2] S. Nussinov and T.N. Truong, Phys. Rev. Lett. **63**, 1349 (1989).

[3] S. Nussinov and T.N. Truong, Phys. Rev. Lett. **63**, 2003 (1989).

[4] N. Paver and Riazuddin, Phys. Lett. **B246**, 240 (1990).

[5] N.N. Achasov and V. Ivanchenko, Nucl. Phys. **B315**, 465 (1989).

[6] S. Flatte, Phys. Lett. **63B**, 234 (1976).

[7] F.E. Close, N. Isgur and S. Kumano (in preparation).

[8] J. Weinstein and N. Isgur, Phys. Rev. **D41**, 2236 (1990).

[9] R.L. Jaffe, Phys. Rev. **D15**, 281 (1977);
R.L. Jaffe and F.E. Low, Phys. Rev. **D19**, 2105 (1979).

[10] F.E. Close and N. Isgur, "Probing the Nature of the Scalar Mesons below 1 GeV", Oxford 1987 (unpublished).

[11] S. Godfrey and N. Isgur, Phys. Rev. **D32**, 189 (1985).

[12] H. Lipkin, Phys. Lett. **B225**, 287 (1989); Nucl. Phys. **B291**, 720 (1987).

[13] T. Barnes, Phys. Lett. **165B**, 434 (1985)

[14] S. Kumano (private communications).

[15] N.N. Achasov and G.N. Shestakov, Z. Phys. **C41**, 309 (1988).

[16] E.P. Shabalin, Sov. J. Nucl. Phys. **46**, 485 (1987).

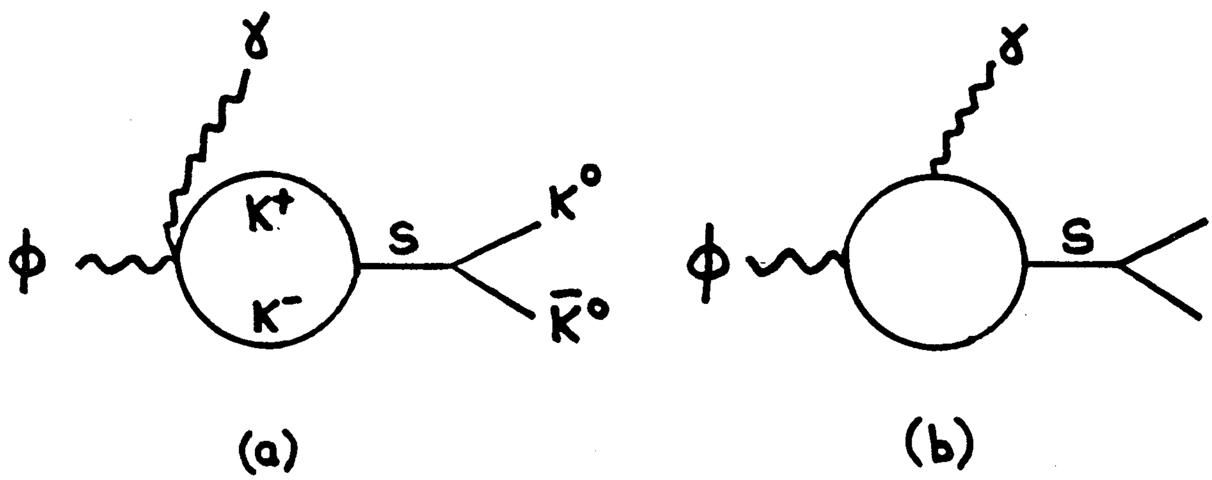


Figure 1. The contact (a) and loop radiation (b) contributions.

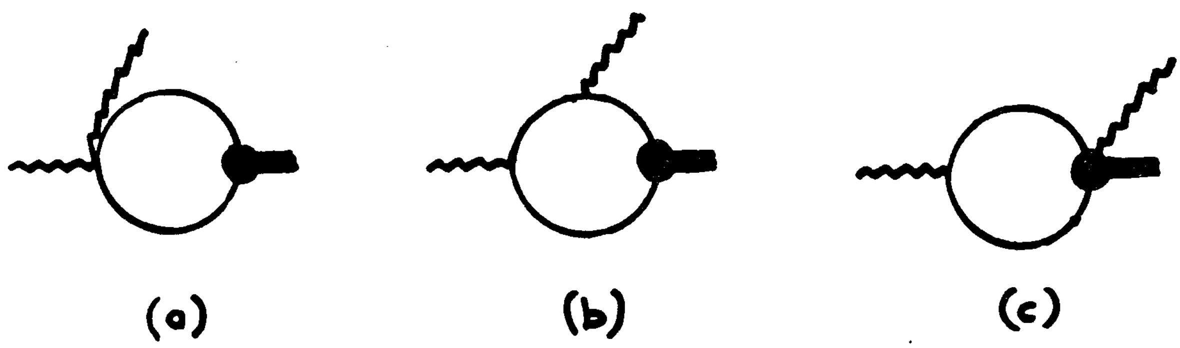


Figure 2. As fig 1 but with an extended scalar meson. Note the new diagram (c).

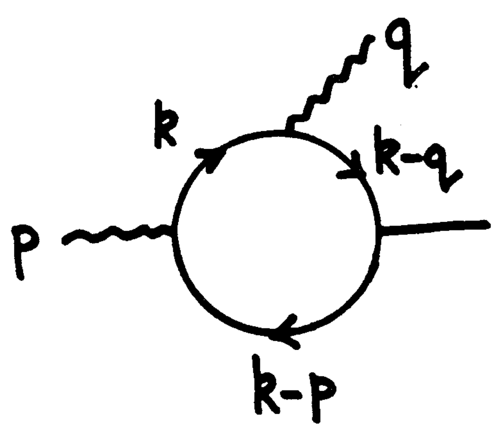


Figure 3. Momentum routing.

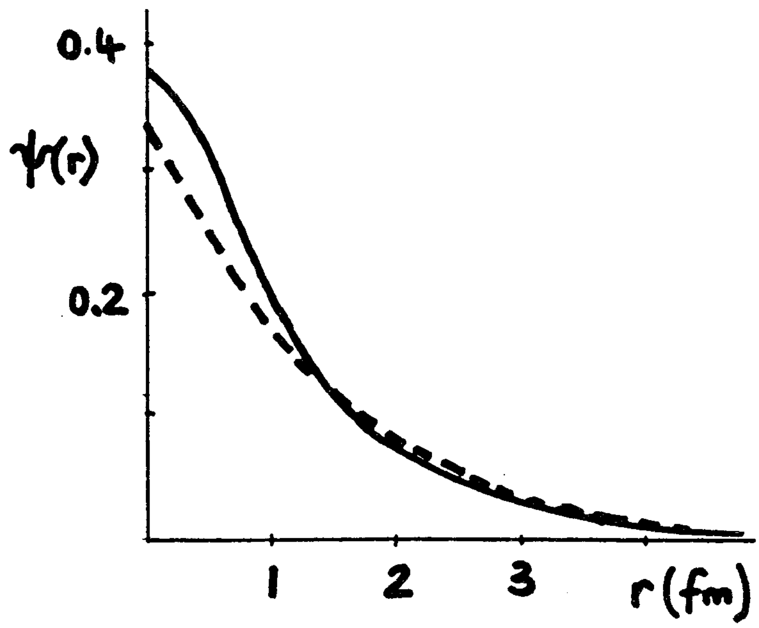


Figure 4. Comparison between exact $\psi(r)$ (solid) and the approximation of eq (2.23). **r** is the KK molecule radius in fm, $\psi(r)$ is in $(fm)^{-3/2}$.

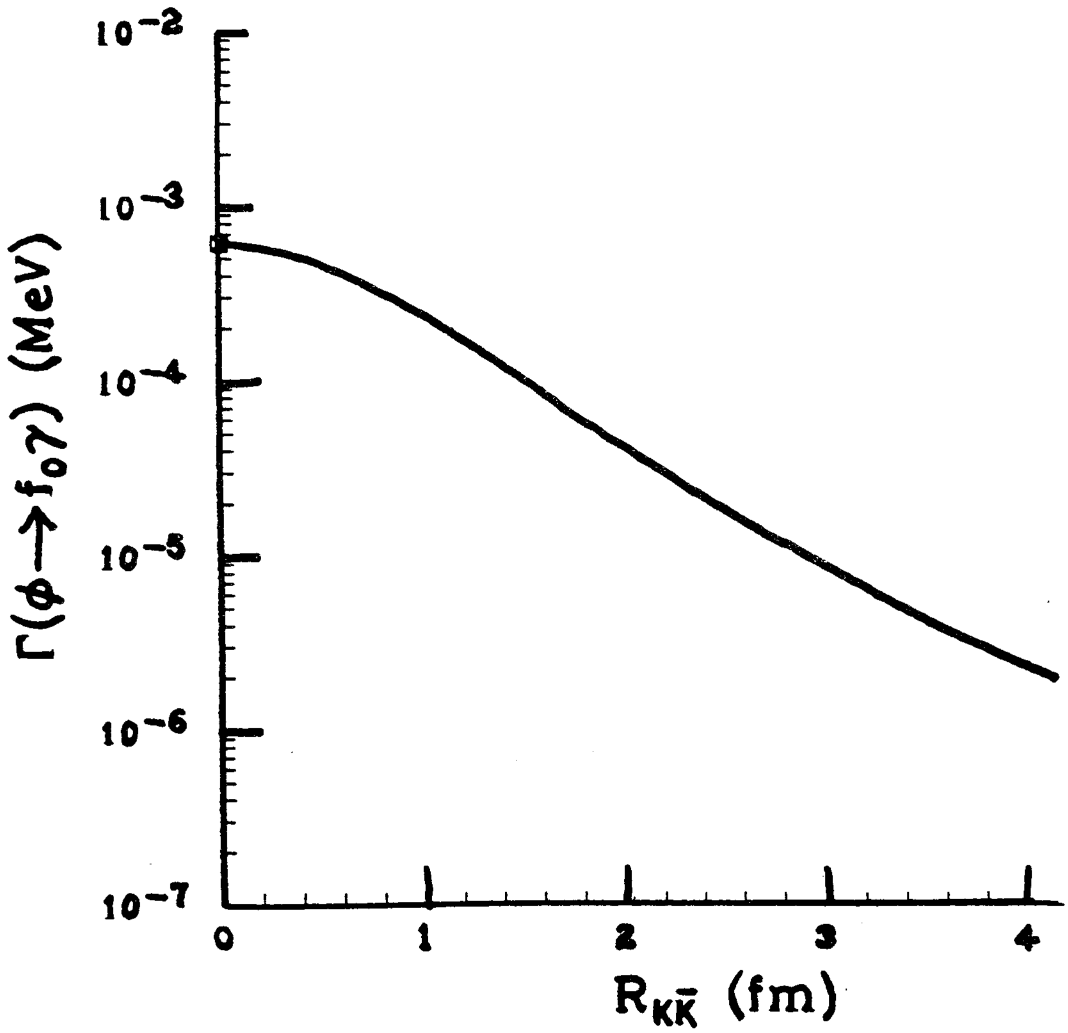


Figure 5. $\Gamma(\phi \to S\gamma)$ in MeV versus $R(K\bar{K})$ in fm.