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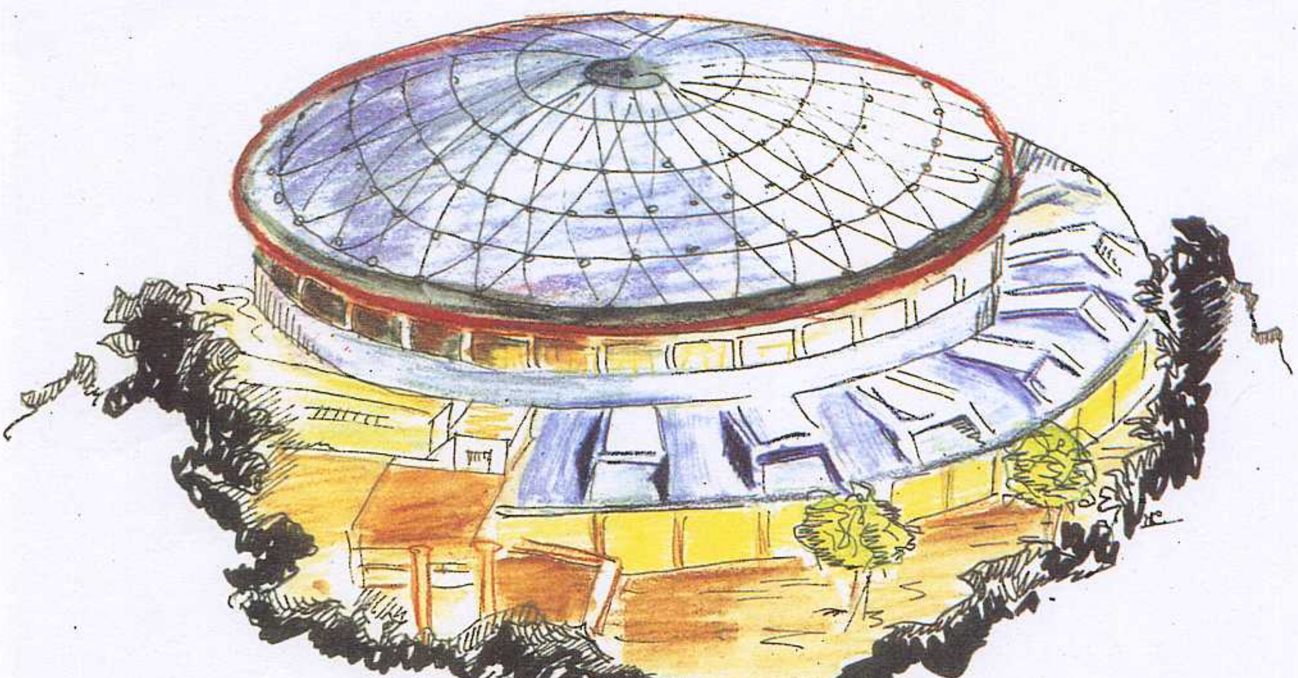

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Can One Test Quantum Mechanics at the Φ-Factory?

GianCarlo Ghirardi
Dipartimento di Fisica Teorica, Universita' di Trieste, Italy
and
International Centre for Theoretical Physics, Trieste, Italy

Renata Grassi
Dipartimento di Fisica Teorica, Universita' di Trieste, Italy

and

Renzo Ragazzon
Dipartimento di Fisica Teorica, Universita' di Trieste, Italy

ABSTRACT

The problem of the possibility of testing Bell's type inequalities in $\Phi \rightarrow K - \overline{K}$ is discussed. A comparison with EPR-Bohm type experiments for the singlet state and with Aspect's experiments with optical photons is made. We show that, due to the specific features of the two mesons system, no violation of Bell's type inequalities is implied by quantum mechanics. Finally, the deviations from quantum predictions which are implied by the assumption of "spontaneous factorization" are analyzed.

In 1964 Bell[1] derived a famous inequality which allows to test quantum mechanics versus any local realistic theory for the case of the Einstein-Podolsky-Rosen-Bohm[2] *Gedankenexperiment*. Bell's type inequalities apply to any correlated measurement on two correlated systems. In this paper, following the general discussion of ref. (3), we will analyze whether in the case of the $K-\overline{K}$ mesons system produced in the decay of the vector meson Φ, $J^{PC} = 1^{--}$, such inequalities can be used to test the predictions of quantum mechanics versus those of an arbitrary local realistic theory.

1. Bell's inequalities for the singlet-spin case.

Let us consider a system of two spin 1/2 particles in the singlet state, which propagate in opposite L (left) and R (right) directions. At left an observer can measure the spin component of the particle in arbitrarily chosen directions ("at free will") a_1 or a_2 respectively. Similarly, at right, a measurement of the spin component in the directions b_1 or b_2 can be performed. Let A(a) and B(b) be the observables which take the value +1 if the spin is found "up" in the measures in the chosen directions at left and at right respectively and -1 if the spin is found "down". Let E(a,b) be the correlation observable which assumes the value + 1 if both spins are found "up" or both are found "down" and -1 otherwise. In any local realistic theory the correlation function E(a,b) must satisfy the Bell's type inequality[4]:

$$|E(a_1,b_1) - E(a_1,b_2)| + |E(a_2,b_1) + E(a_2,b_2)| \leq 2. \quad (1.1)$$

We remark that according to quantum mechanics, in the case of the singlet state, the probability of finding, in a simultaneous spin measurement of the two particles, spins "up" ($\uparrow$) or "down" ($\downarrow$) along two chosen directions a and b is given by

$$\begin{aligned} P(a,\uparrow;b,\uparrow) &= P(a,\downarrow;b,\downarrow) = 1/4(1-\cos\theta_{ab}) \\ P(a,\uparrow;b,\downarrow) &= P(a,\downarrow;b,\uparrow) = 1/4(1+\cos\theta_{ab}), \end{aligned} \quad (1.2)$$

where θ_{ab} is the angle between the two directions a and b. From equations (1.2) it follows

$$E(a,b) = -\cos\theta_{ab}. \quad (1.3)$$

The predictions (1.3) of quantum mechanics for the correlations E(a,b) lead to a violation of the Bell inequality (1.1). In Fig. 1 we plot the values taken by the expression at the left hand side of eq.(1.1) for $\theta_{a1b1}=\theta_{a2b1}=\theta_{a2b2}=\Delta$, $\theta_{a1b2}=3\Delta$, with Δ varying in the interval $(0°,90°)$. For $\Delta = 45°$ one gets the greatest violation $(2\sqrt{2})$ of the considered inequality.

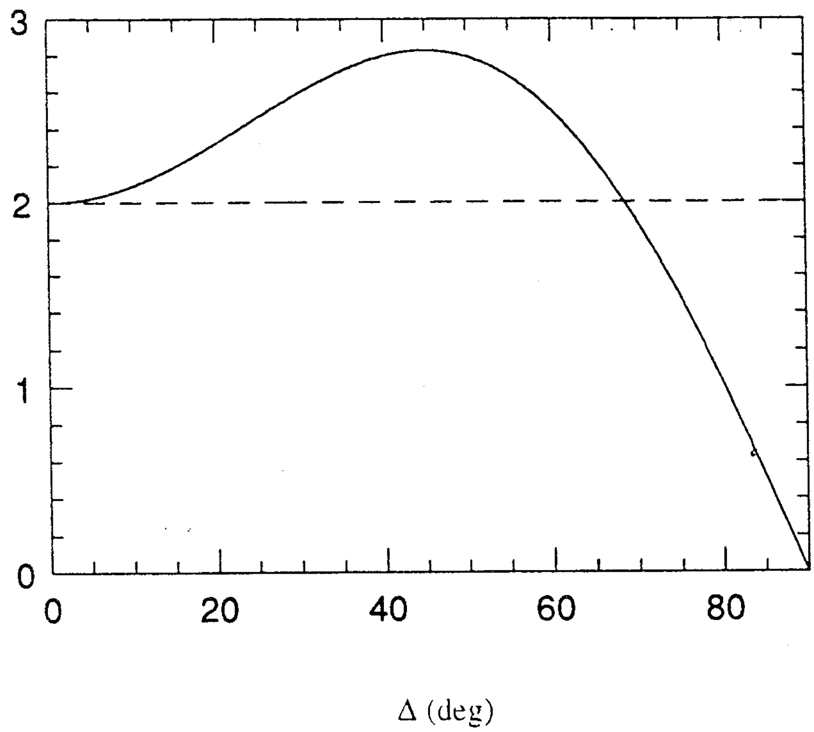

Fig. 1. Violation of Bell's inequality for the singlet state of two 1/2-spin particles. The continuous line represents the l.h.s. of eq. (1.1) as a function of Δ (see the text).

2. Aspect's experiments.

The most important experiment testing quantum mechanics against local realistic theories looking for a violation of a Bell inequality is that performed by Aspect and coworkers[5] in 1982. In the experiment they measure correlations of linear polarizations of pairs of optical photons (produced in an atomic radiative cascade $(J=0)\rightarrow(J=1)\rightarrow(J=0)$) with time-varying analyzers. In each leg of the experimental apparatus there is an acousto-optical switch followed by two linear polarizers. Each analyzer amounts to a polarizer which jumps between two orientations in a time short compared with the photon transmit time. This experiment is more decisive than the preceding ones[6] with static orientations of the polarizers because it satisfies to a higher degree of accuracy the condition that the choice of the directions in which the measurements are performed is made at "free will". The photon propagating in one direction cannot be "informed" by a "luminal" or "subluminal" message about the orientation of the polarizer in the opposite direction. For experiments with optical photons, to take into account the low photoelectric efficiencies, one must use the Clauser, Horne, Shimony and Holt[4] Bell's type inequality

$$S = P(a_1,b_1) - P(a_1,b_2) + P(a_2,b_1) + P(a_2,b_2) - P(a_1) - P(b_2) \leq 0 \qquad (2.1)$$

to test quantum mechanics against local realistic theories. In eq. (2.1) P(a,b) is the probability of detecting both photons with the polarizers in the directions a and b, respectively, and P(a) is the probability of detecting both photons when one polarizer is removed and the other has orientation a. Let E(a,b) be the correlation function which assumes the value + 1 when both photons are detected or when neither of them is detected, and -1 otherwise:

$$E(a,b) = 1 - 2(P(\neg a,b) + P(a,\neg b)) \qquad (2.2)$$

where, e.g., $P(a,\neg b)$ represents the probability of detecting the photon after the polarizer with orientation a and of not detecting the photon after the polarizer with orientation b. As in the case of the spin, in any local realistic theory the correlation functions E(a,b) must satisfy inequality (1.1). Inequality (2.1) follows then from eq. (1.1) when one replaces $P(a,\neg b)$ with P(a) - P(a,b) and $P(\neg a,b)$ with P(b) - P(a,b) in the expression (2.2) for E(a,b).

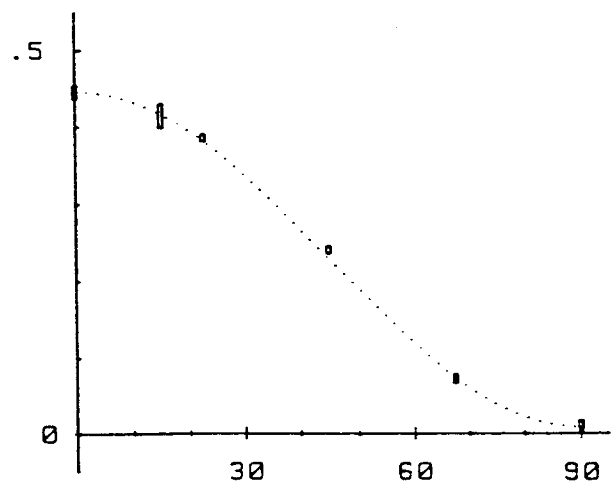

Fig. 2. Experimental values for P(a,b) found by Aspect and his coworkers[5]. The dashed line represents the prediction of quantum mechanics.

Aspect and his collaborators, in their experiment, have chosen a set of orientations leading to the greatest predicted conflict between quantum mechanics and Bell's inequalities ($(a_1,b_1) = (a_2,b_1) = (a_2,b_2) = 22{,}5^o$; $(a_1,b_2) = 67{,}5^o$), obtaining for the expression S at the left hand side of (2.1)

$$S_{exp} = 0.101 \pm 0.020 \tag{2.3}$$

violating the inequality $S \leq 0$ by 5 standard deviations. This result is also in excellent agreement with the value S_{QM} predicted by quantum mechanics when one takes into account the geometrical features of the experiment and the real efficiencies of the polarizers. Furthermore, the agreement between the probabilities $P(a,b)_{exp}$ found for various angles θ_{ab} with those predicted by quantum mechanics for the actual experiment is very good, as shown in Fig. 2.

3. Testing quantum mechanics versus local realistic theories at the Φ-factory.

In connection with the problem of Bell's type experiments at the Φ-factory, we follow the detailed discussion of ref. (3) and we consider the strong decay of the Φ(1020), $J^{PC} = 1^-$, vector meson into a pair of neutral pseudoscalar mesons K^0, $\overline{K}^0$. Let us denote by

$$\lambda_S = m_S - (i/2)\gamma_S, \ \lambda_L = m_L - (i/2)\gamma_L \tag{3.1}$$

the eigenvalues of the non-hermitian hamiltonian H_W which describes the $K - \overline{K}$ strangeness oscillations and also the loss of probability in the decay channels.

We consider, as in ref. (3), the correlation functions $E(t_l,t_r)$ defined in the following way. $E(t_l,t_r)$ takes the value +1 when either two $\overline{K}^0$ mesons or no $\overline{K}^0$ mesons are found at the measurements at right and at left, and -1 otherwise. In a local realistic theory these correlation functions(3) must satisfy a Bell type inequality:

$$|E(t_1,t_3) - E(t_1,t_4)| + |E(t_2,t_3) + E(t_2,t_4)| \leq 2 \tag{3.2}$$

when the instants t_l, t_r ($l = 1,2$; $r = 3,4$) correspond to the left and right mesons being at space-like separated points. It is important to remark that, would a violation of the inequality (3.2) be found, this would not necessarily imply a conflict with the locality assumption. In fact, if one identifies t_l and t_r with the instants in which the mesons decay, the times in which measurements apt to detect $\overline{K}^0$ mesons are performed would not be chosen arbitrarily (at "free-will") by the experimenter. Then, e.g., the meson at left could "carry", without entailing non local effects, "information" about the instant of the measure at right.

To make evident the similarities with the case of the singlet state of two spin-1/2 particles, we consider only the strangeness oscillations, disregarding the meson decays. One has:

$$E(t_l,t_r) = -\cos\theta_{lr} \tag{3.3}$$

where

$$\theta_{lr} = (m_L - m_S)(t_l - t_r). \tag{3.4}$$

For $\theta_{13} = \theta_{23} = \theta_{24} = \pi/4$ and $\theta_{14} = 3\pi/4$ one would have the greatest violation $(2\sqrt{2})$ of the inequality (3.2), as in the case of the singlet state of two spin-1/2 particles. If one takes into account the decay of the mesons, the correlation function (3.3) becomes

$$E(t_l,t_r) = \frac{1}{2}(1-e^{-\gamma_S t_l})(1-e^{-\gamma_L t_r}) + \frac{1}{2}(1-e^{-\gamma_S t_r})(1-e^{-\gamma_L t_l}) \\ -e^{-(\gamma_S+\gamma_L)(t_l+t_r)/2}\cos\Delta m(t_l - t_r) \tag{3.5}$$

where $\Delta m = m_L - m_S$. From eq. (3.5) one sees that, if $\gamma_S t_k$ $(k = l,r)$ is appreciably larger than 1, the terms describing the strangeness oscillations disappear. As a consequence, the times which have to be considered must be such that $\gamma_S t_k$ is smaller or of the order of 1. Since $\gamma_S \approx 582\gamma_L$, one can then put in expression (3.5) $\gamma_L \approx 0$. With these approximations, substitution of (3.5) in (3.2) gives

$$|e^{-\gamma_S(t_1+t_3)/2}\cos(\Delta m(t_1-t_3)) - e^{-\gamma_S(t_1+t_4)/2}\cos(\Delta m(t_1-t_4))| \\ +|e^{-\gamma_S(t_2+t_3)/2}\cos(\Delta m(t_2-t_3)) + e^{-\gamma_S(t_2+t_4)/2}\cos(\Delta m(t_2-t_4))| \leq 2. \tag{3.6}$$

Due to the specific values of the parameters γ_S and Δm, there is no possibility of choosing the four times appearing in eq.(3.6) in such a way that the inequality be violated. In analogy with what has been done in Fig. 1, in Fig. 3, putting

$$\Delta m t_1 = \theta_1 = 0,\ \ \Delta m t_2 = \theta_2 = 2\Delta,\ \ \Delta m t_3 = \theta_3 = \Delta,\ \ \Delta m t_4 = \theta_4 = 3\Delta, \tag{3.7}$$

and taking into account that

$$\gamma_S t_k = \frac{\gamma_S}{\Delta m}\theta_k \approx 2.11\theta_k. \tag{3.8}$$

we plot the values taken by the left hand side of eq. (3.6) when Δ varies in the interval $(0, \pi/2)$.

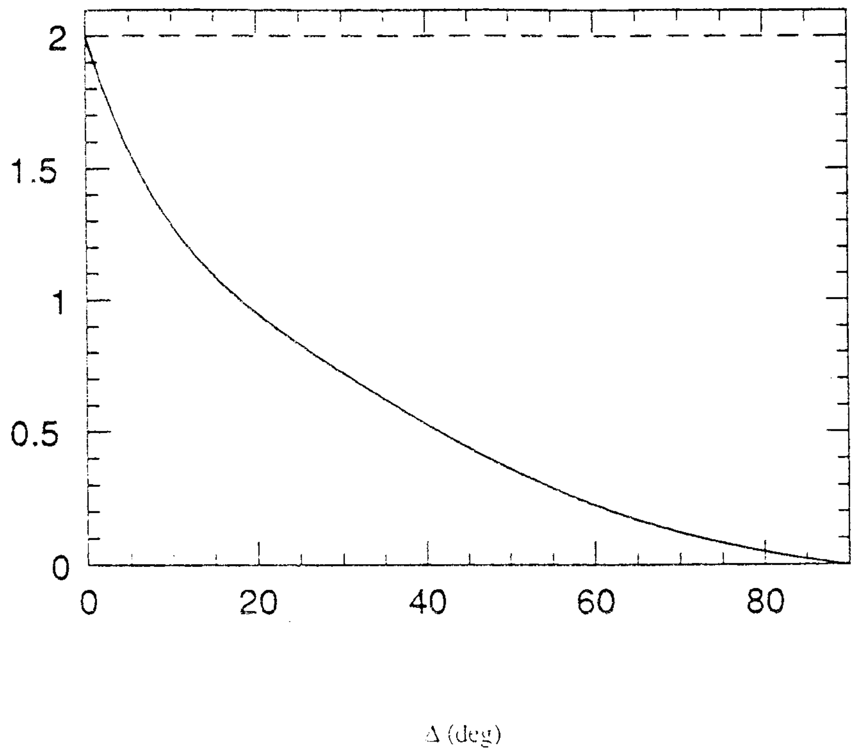

Fig. 3. Plot of the l.h.s. of eq. (3.6) for various Δ (see the text). Due to the specific values of γ_S, γ_L and Δm, no violation of inequality (3.6) can be observed.

For those values of the times (i.e. for $\Delta = \pi/4$) which would entail the maximum possible violation of the inequality, if all the exponential factors in eq. (3.6) would equal 1, one gets the value 0.438, which is much smaller than 2.

For very small values of t_k the expression at the left hand side of eq. (3.2) tends to 2. Thus, this region too deserves attention. It is, however, easy to prove that the value is attained from below. In fact, for very small t_l and t_r, the correlation function $E(t_l,t_r)$ takes, at first order, the form:

$$E(t_l,t_r) \approx -1 + \frac{1}{2}\gamma_S t_l + \frac{1}{2}\gamma_S t_r + \frac{1}{2}\gamma_L t_l + \frac{1}{2}\gamma_L t_r. \qquad (3.9)$$

Substitution of (3.9) in eq. (3.2) gives, for the left hand side, the expression

$$\left|\frac{1}{2}\gamma_S t_3 + \frac{1}{2}\gamma_L t_3 - \frac{1}{2}\gamma_S t_4 - \frac{1}{2}\gamma_L t_4\right| + 2$$
$$-\gamma_S t_2 - \gamma_L t_2 - \frac{1}{2}\gamma_S t_3 - \frac{1}{2}\gamma_L t_3 - \frac{1}{2}\gamma_S t_4 - \frac{1}{2}\gamma_L t_4 \qquad (3.10)$$

which is obviously smaller than 2.

To stress that the fact that the quantum correlations $E(t_l,t_r)$ do not give rise to a violation of the Bell's type inequality (3.2) is a specific feature of the system under investigation, i.e. it originates from the actual values taken by γ_S, γ_L and Δm, we plot in Fig. 4 the left hand side of eq. (3.2) (for the same choices of the angles made in Fig. 1 and in Fig. 3) assuming $\Delta m = 10\gamma$ ($\gamma=\gamma_S=\gamma_L$). It is seen that, would such a relation between the decay widths and the mass difference hold, one would have a neat violation of inequality (3.2), even though not as relevant as for $\gamma_S=\gamma_L=0$.

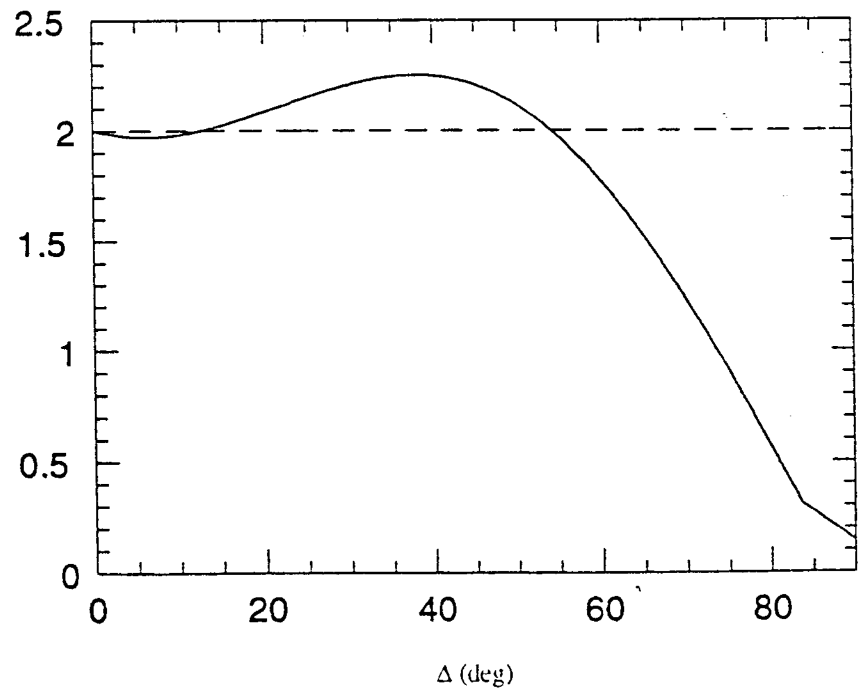

Fig. 4. Plot of the l.h.s. of eq. (3.2) for a hypothetical meson-antimeson system with the decay widths and mass difference satisfying the relation $\gamma_S = \gamma_L = \Delta m/10$. In this case one could actually observe a violation of the Bell's type inequality (3.2).

4. The "spontaneous factorization" hypothesis.

As pointed out in ref. (3) (see also ref. (7)), the "spontaneous factorization" or Furry's[8] hypothesis meets serious conceptual difficulties. However, it has been suggested[9] that the state of the $K-\overline{K}$ mesons system

$$1/\sqrt{2}(|K_S>_l|K_L>_r - |K_L>_l|K_S>_r) \tag{4.1}$$

could collapse, shortly after the decay of the Φ meson, in the two factorized states

$$|K_S>_l|K_L>_r, \ |K_L>_l|K_S>_r \tag{4.2}$$

with equal probabilities (1/2). The predictions of quantum mechanics for double observations of K^0, $\overline{K}^0$ mesons at left and at right differ from those which one gets under Furry's hypothesis, so that one could test its validity.

The $K^0$and $\overline{K}^0$ mesons can be identified by their characteristic semileptonic mode of decays: $K^0 \to l^+ \nu \pi^-$, $\overline{K}^0 \to l^- \overline{\nu} \pi^+$,where l denote the lepton. We consider the parameter R(T) defined as follows:

$$R(T) = \frac{N_{++}(T) + N_{--}(T)}{N_{+-}(T) + N_{-+}(T)} \tag{4.3}$$

where $N_{++}(T)$ represents the number of double $\overline{K}^0$ decays in the interval (0,T); $N_{--}(T)$ the number of double K^0 decays, $N_{+-}(T)$ the number of $\overline{K}^0$ decays on the left associated with K^0 decays on the right and $N_{-+}(T)$ the number of K^0 decays on the left associated with $\overline{K}^0$ decays on the right. (in the same time interval). The quantities N_{ij} (i,j=±1) are given by

$$N_{ij.} = N_0 \lambda^2 \int_0^T dt_1 \int_0^T dt_2 P_{ij}(t_1, t_2) \tag{4.4}$$

where, e.g., $P_{++}(t_1,t_2)$ is the joint probability of finding $\overline{K}^0$ mesons on the left at t_1 and on the right at t_2, respectively; N_0 is the number of $K - \overline{K}$ systems produced in the Φ decays and λ is the semileptonic width of K^0 decaying into an l^+ (equal to the semileptonic width of $\overline{K}^0$ decaying into a l^-). In the Furry's hypothesis one has:

$$R_F(T)=1, \ \forall T, \tag{4.5}$$

which has to be confronted with the prediction of quantum mechanics:

$$R_{QM}(T) = \frac{(1-e^{-\gamma_S T})(1-e^{-\gamma_L T}) - (\gamma_S \gamma_L / \alpha^2)(1+e^{-2\gamma T} - 2e^{-\gamma T}\cos \Delta m T)}{(1-e^{-\gamma_S T})(1-e^{-\gamma_L T}) + (\gamma_S \gamma_L / \alpha^2)(1+e^{-2\gamma T} - 2e^{-\gamma T}\cos \Delta m T)} \tag{4.6}$$

where $\alpha^2 = \Delta m^2 + \gamma^2$ with $\gamma = (\gamma_L + \gamma_S)/2$. For $T \to \infty$ $R_{QM}(T)$ tends to

$$R_{QM}(\infty) = \frac{1 - \gamma_S \gamma_L / \alpha^2}{1 + \gamma_S \gamma_L / \alpha^2} \approx 0.993, \tag{4.7}$$

which is very close to 1 (i.e. to R_F). In Fig. 5 we plot the values of $R_{QM}(T)$ for T varying in the interval (0, $20/\gamma_S$).

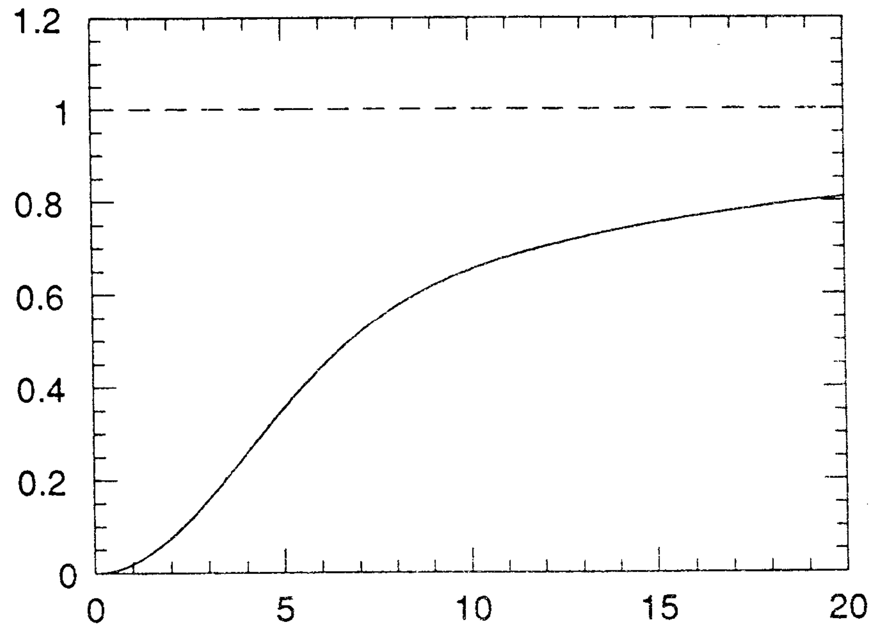

Fig. 5. Plot of R_{QM} (continuous line) as a function of T/τ_S. The dashed line represents the Furry's prediction $R_F = 1$.

As one can see, for $T \lesssim 5/\gamma_S$, $R_{QM}(T)$ is appreciably different from R_F. Therefore, as noted in ref.(9), if an appropriate number of semileptonic decays are collected in such a range, one could test the Furry's hypothesis against quantum mechanics. At any rate we think that Furry's mechanism does not deserve too much attention since it meets serious difficulties in its theoretical description (3) and, moreover, appears to have already been experimentally disproved[10] for analogous processes, typically B-meson decays.

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