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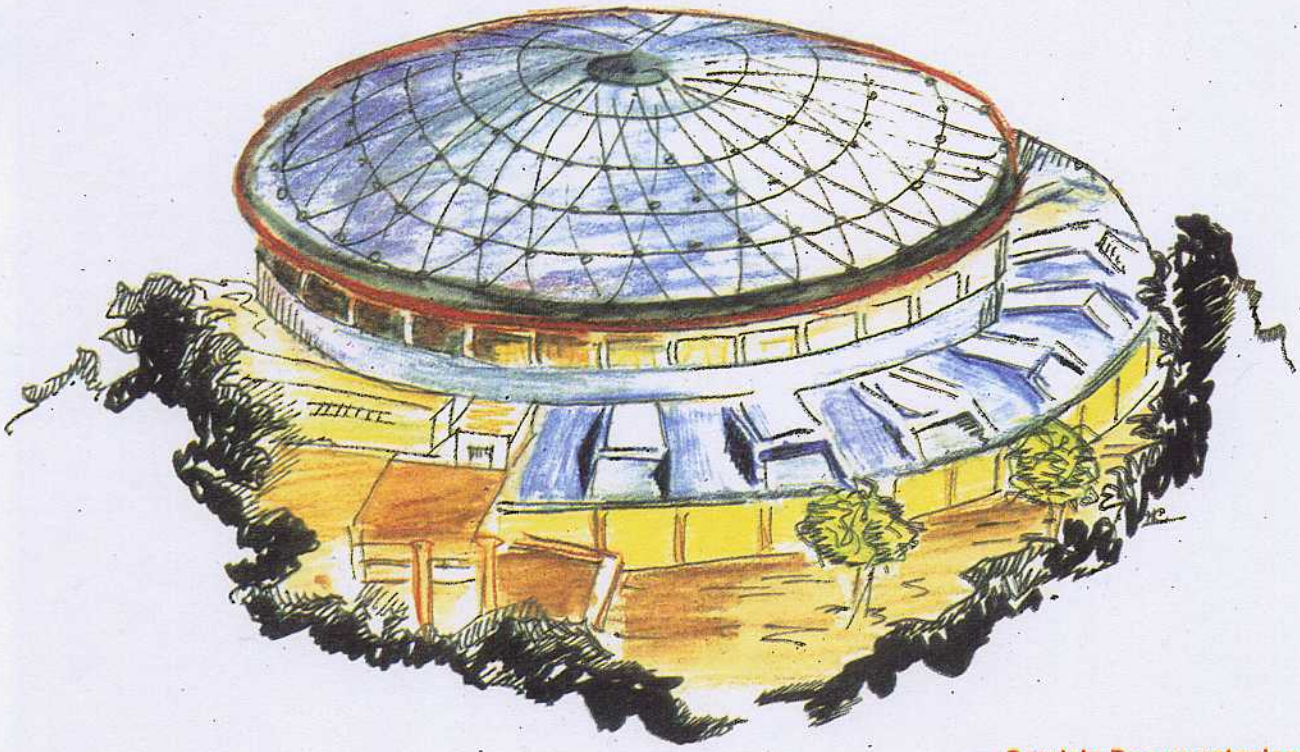
L. Maiani, N. Paver:

NON LEPTONIC K→ 3π DECAYS

Contribution to the DAΦNE Physics Handbook



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NON LEPTONIC $K \to 3\pi$ DECAYS

Luciano Maiani[1] and Nello Paver[2]

1. CP CONSERVING $K \to 3\pi$ DECAYS

Decay modes, branching ratios and expected fluxes

There are altogether five distinct CP conserving $K \to 3\pi$ modes: they are listed, with their branching ratios [1] and the corresponding number of events obtainable at a *phi*-factory with luminosity $\mathcal{L} = 5 \times 10^{32} cm^{-2} s^{-1}$, in Table I. Actually, the numbers there refer to "tagged" decays at a detector with 4π angular coverage such as the one planned for Daϕne (KLOE). In some channel "tagging" might not be needed, so that the corresponding number of events could be larger. In the last column of Table I we report as examples the typical statistics currently reached in $K \to 3\pi$ experiments for the different modes [1]. Indeed, the second figure in the $K_L \to 3\pi^0$ entry is the statistics reached for this channel by the $E731$ FNAL experiment [1], which appeared in the literature quite recently.

Concerning the decay $K_S \to \pi^+\pi^-\pi^0$, which has not been detected yet, the "present statistics" entry In Table I simply means the number of $K^0 \to \pi^+\pi^-\pi^0$ decays analysed in an experiment, and actually the experimental upper limit is obtained by combining such statistics of more experiments. Instead, for Daϕne we report the effective number of $K_S \to \pi^+\pi^-\pi^0$ events, as theoretically expected from the chiral Lagrangian approach which indicates $Br(K \to \pi^+\pi^-\pi^0) = (2-4) \times 10^{-7}$. These

[1] Dipartimento di Fisica, Università di Roma "La Sapienza" and INFN-Sezione di Roma, Italy 00185

[2] Dipartimento di Fisica Teorica, Università di Trieste and INFN-Sezione di Trieste, Italy 34100

predicted numbers indicate that the branching ratio of this transition should be measurable for the first time at the ϕ-factory.

The decay $K_S \to \pi^0\pi^0\pi^0$, not listed in Table I, is purely CP violating, with a predicted branching ratio of the order of 2×10^{-9} and an experimental limit of 3.7×10^{-5}.

Kinematics and Dalitz plot

For the transition

$$K(p) \to \pi_1(p_1)\pi_2(p_2)\pi_3(p_3) \tag{1.1}$$

one defines the following kinematical invariants:

$$s_i = (p - p_i)^2 = (m_K - m_\pi)^2 - 2m_K T_i, \quad (i = 1, 2, 3) \tag{1.2}$$

where T_i are the pion kinetic energies in the kaon rest frame: $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$; $E_1 + E_2 + E_3 = m_K$. Since $s_1 + s_2 + s_3 = 3s_0$ with $s_0 = \frac{1}{3}m_K^2 + m_\pi^2$, there are only two independent kinematical variables, which can be chosen as

$$Y = \frac{s_3 - s_0}{m_\pi^2}; \qquad X = \frac{s_2 - s_1}{m_\pi^2} \tag{1.3}$$

where "3" indicates for a given decay channel the "odd charge" pion. We are neglecting here pion (and kaon) mass differences except in the Q-values, that significantly depend on them:*

$$Q = T_1 + T_2 + T_3 = m_K - \sum_{i=1}^{3} m_i, \tag{1.4}$$

and, in an obvious notation referring to the charges of final pion states:

$$Q^{\pm\pm\mp} = 74.9\ MeV; \quad Q^{\pm 00} = 84.1\ MeV; \quad Q_{S,L}^{+-0} = 83.6\ MeV; \quad Q_{S,L}^{000} = 92.8\ MeV. \tag{1.5}$$

The Dalitz plot for $K \to 3\pi$ is the equilateral triangle with altitude Q shown in Figure 1, where the perpendiculars from internal points (representing the events) to the sides are the pions kinetic energies, obviously satisfying the energy conservation Eq.(1.4). Conventionally, the vertical perpendicular refers to the "odd charge" meson. The centre of the diagram, representing the origin of the three axes at 120° along which

* In the experimental analysis, in addition to using correct pion and kaon masses, one must account for isospin breaking and QED radiative corrections in order to extract the values of transition amplitudes from the data.

one plots T_1, T_2 and T_3, corresponds to the symmetric point $T_1 = T_2 = T_3 = \frac{1}{3}Q$. Cartesian coordinates of a point relative to this origin are easily seen to be proportional to the values of Y and of $X/\sqrt{3}$ respectively. All points inside the indicated boundary contour (resulting from three-momentum conservation) are kinematically allowed, and represent possible decay events. The diagram is divided into "sextants" labeled I to VI, which under permutations of indistinguishable pions are permuted into each other by reflections in the corresponding triangle median.

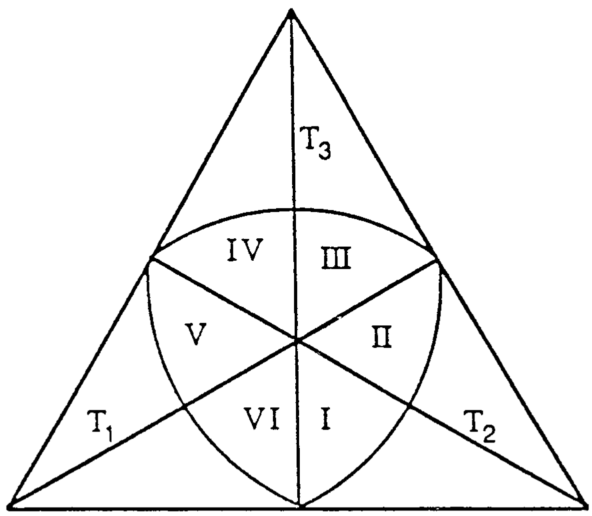


Figure 1: Dalitz plot for $K \to 3\pi$.

To evaluate phase space integrals it is often useful to define Dalitz variables by expressing kinetic energies in terms of polar coordinates r and ϕ, with the origin at the symmetric point [2]:

$$\begin{aligned} T_{1,2} &= \frac{Q}{3}\left(1 + r\cos\left(\frac{2}{3}\pi \mp \phi\right)\right) \\ T_3 &= \frac{Q}{3}\left(1 + r\cos\phi\right). \end{aligned} \tag{1.6}$$

In Eq.(1.6): $-\pi < \phi \leq \pi$ and $0 \leq r \leq r(\phi)$, with $r(\phi)$ the boundary curve in the Dalitz plot, implicitly defined by the equation

$$1 - (1 + \alpha)r^2 - \alpha r^3 \cos 3\phi = 0, \tag{1.7}$$

with $\alpha = \frac{2Q}{m_K}\left(2 - \frac{Q}{m_K}\right)^{-2}$. In the (non-relativistic) approximation of neglecting α, which actually is of the order of 0.1, the limiting curve in the plot would become a circle. Moreover, the variables X and Y defined in (1.3) are expressed in terms of polar coordinates as:

$$\begin{aligned} Y &= -\frac{2}{3}\frac{m_K}{m_\pi^2} Qr\cos\phi \\ X &= \frac{2}{\sqrt{3}}\frac{m_K}{m_\pi^2} Qr\sin\phi, \end{aligned} \tag{1.8}$$

so that the plot in terms of Y and $X/\sqrt{3}$ is quite similar to Figure 1 (except from the maximum radius of the contour of the allowed region, which in this case is obviously not equal to one).

The decay rates are expressed in terms of polar variables as

$$\Gamma(K \to 3\pi) \equiv \frac{1}{(4\pi)^3 m_K}\frac{\sqrt{3}}{18}Q^2 \int\int r\ dr\ d\phi\ |A(r,\phi)|^2, \tag{1.9}$$

where the integration is over the full Dalitz plot. This integration, as well as integrations over the Dalitz plot with cuts, become particularly simple in the non-relativistic limit, which in most cases represents a good approximation. Explicit calculations of a set of relevant Dalitz plot integrals, also with cuts, can be found in the Appendix of Ref.[3].

Since the maximum allowed kinetic energies are rather small ($T_{i,max} \simeq 50\ MeV$), it is natural to expand Dalitz plot distributions in powers of Y and X as follows:

$$|A(K \to 3\pi)|^2 \propto 1 + gY + jX + hY^2 + kX^2, \tag{1.10}$$

where actually $j = 0$ if CP is conserved. The available experimental data on Γ, g, h and k for the different $K \to 3\pi$ modes do not require higher powers than included here, and are listed in Table II [1]. As one can see, present accuracies are at % level or better (depending on the different modes) for the widths Γ and the linear slopes g, but are somewhat worse (larger than 10%) for the quadratic slopes h and k.

Consistent with Eq.(1.10), also the transition amplitudes with definite isospin selection rules are expanded in X and Y, assuming the absence of nearby poles, up to quadratic terms. Limiting to $\Delta I = 1/2$ and $\Delta I = 3/2$ transitions, as also consistent with experimental data, there are three possible three pion final states with definite isospin: $|(3\pi)_{I=1}, symm. >$; $|(3\pi)_{I=1}, mixed\ symm. >$ antisymmetric in π_1 and π_2; and finally $|(3\pi)_{I=2} >$. Requiring Bose symmetry for the overall three-pion final state wavefunction, the expansions of the amplitudes for the different decay channels take the form [4,5,6]:

$$\begin{aligned}
A(K^+ \to \pi^+\pi^+\pi^-) =& (2\alpha_1 - \alpha_3)\, e^{i\delta_{1S}} + \left[\left(\beta_1 - \frac{1}{2}\beta_3\right) e^{i\delta_{1M}} + \sqrt{3}\gamma_3 e^{i\delta_2}\right] Y \\
& + 2(\zeta_1 + \zeta_3)\left(Y^2 + \frac{1}{3}X^2\right) - (\xi_1 + \xi_3 - \xi_3')\left(Y^2 - \frac{1}{3}X^2\right) \\
A(K^+ \to \pi^+\pi^0\pi^0) =& -\frac{1}{2}(2\alpha_1 - \alpha_3)\, e^{i\delta_{1S}} + \left[\left(\beta_1 - \frac{1}{2}\beta_3\right) e^{i\delta_{1M}} - \sqrt{3}\gamma_3 e^{i\delta_2}\right] Y \\
& - (\zeta_1 + \zeta_3)\left(Y^2 + \frac{1}{3}X^2\right) - (\xi_1 + \xi_3 + \xi_3')\left(Y^2 - \frac{1}{3}X^2\right) \\
A(K_L \to \pi^+\pi^-\pi^0) =& (\alpha_1 + \alpha_3)\, e^{i\delta_{1S}} - (\beta_1 + \beta_3)\, e^{i\delta_{1M}} Y \\
& + (\zeta_1 - 2\zeta_3)\left(Y^2 + \frac{1}{3}X^2\right) + (\xi_1 - 2\xi_3)\left(Y^2 - \frac{1}{3}X^2\right) \\
A(K_L \to \pi^0\pi^0\pi^0) =& -3(\alpha_1 + \alpha_3)\, e^{i\delta_{1S}} - 3(\zeta_1 - 2\zeta_3)\left(Y^2 + \frac{1}{3}X^2\right) \\
A(K_S \to \pi^+\pi^-\pi^0) =& \frac{2}{3}\sqrt{3}\gamma_3 X e^{i\delta_2} - \frac{4}{3}\xi_3' XY\,.
\end{aligned} \tag{1.11}$$

In Eq.(1.11) the subscripts 1 and 3 refer to $\Delta I = 1/2$ and $\Delta I = 3/2$ respectively, while α, β and γ represent transitions to the different isospin eigenstates introduced above (analogous meaning has the notation for the quadratic coefficients ξ and ζ). Furthermore, δ_{1S}, δ_{1M} and δ_2 are the phases due to final state strong interactions (as an approximation we omit the phases of quadratic terms). Actually, the representation (1.11) is a simplification, because it is correct only at the centre of the Dalitz plot. In general, different from $K \to 2\pi$, due to the possibility of mixing between the two $I = 1$ states *via* strong interactions, each of the amplitudes α and β for the different decay channels can have their corresponding phases [7]. Instead, for the amplitudes to the $I = 2$ state, which does not mix with the others, the simple representation (1.11) holds and the corresponding phase is unique for all processes.

The strong phases are expected to be quite small due to the smallness of the available phase space. Typical theoretical estimates, either in the non-relativistic approximation [8] and in chiral perturbation theory at one loop [9], numerically give for the relative phases at centre of the Dalitz plot:

$$\delta_{1S} - \delta_{1M} \cong \frac{\delta_{1S} - \delta_2}{2} \simeq 0.08. \tag{1.12}$$

If CP is conserved all coefficients in Eq.(1.11) can be chosen to be real numbers, so that the amplitudes for K^+ and the charge-conjugated K^- decays are the same. In the case of CP violation those coefficients will be complex, so that amplitudes for K^- decays are obtained from K^+ by complex-conjugating the coefficients α, β, etc. (but leaving the strong phase factors unchanged), and moreover the amplitude for $K_S \to \pi^+\pi^-\pi^0$ can acquire an additional (imaginary) constant term.

Experimental determinations

Since both kaons and pions are spinless, all observables for $K \to 3\pi$ (and therefore the decay dynamics) are embodied in the Dalitz plot distributions, i.e. in the decay rates Γ and in the linear and quadratic slopes for the various channels as in Eq.(1.10). From these, one has to reconstruct the coefficients of the amplitude expansions in Eq.(1.11). The relation is simply that, for any amplitude of the form

$$A = a + bY + cY^2 + dX^2, \tag{1.13}$$

as in (1.11), the corresponding coefficients in the Dalitz plot (1.10) are given by:

$$g = \frac{2b}{a}; \qquad h = \frac{g^2}{4} + \frac{2c}{a}; \qquad k = \frac{2d}{a}. \tag{1.14}$$

One can see that in the CP conserving case there are 15 experimentally measurable numbers from Dalitz plots (four for each of the modes $K^+ \to \pi^+\pi^+\pi^-$, $K^+ \to \pi^+\pi^0\pi^0$ and $K_L \to \pi^+\pi^-\pi^0$, two for the case $K_L \to 3\pi^0$ and one for $K_S \to \pi^+\pi^-\pi^0$). On the other side, in Eq.(1.11) there are 10 independent real isospin amplitudes, as displayed in Table III, plus the strong interaction relative phases $\delta_{1S} - \delta_{1M}$ and $\delta_{1S} - \delta_2$. An analogous counting of the number of observables and of independent isospin amplitudes can be done quite easily also for the CP violating case. It is interesting to notice that the measurement of $Br(K_S \to 3\pi)$ will allow the direct experimental determination of the $\Delta I = 3/2$ amplitude γ_3 (in modulus).

A recent fit of (1.11) and of the analogous decomposition of the theoretically related $K \to \pi\pi$ amplitudes to the experimental $K \to 3\pi$ and $K \to 2\pi$ data, updating the analysis of [5], has given for the different isospin amplitudes the results reported in Tables IV and V [10]. In Table IV are also reported, for a comparison, the theoretical predictions for $K \to 3\pi$ resulting from calculations based on chiral symmetry and chiral perturbation theory, which we briefly review in the next section. In the fit of Ref.[10] it was assumed that the (3π) final state phases are negligible. Also, the apparent discrepancy in Table V between $|\delta_1 - \delta_2|$ and the value obtained from the measured $\pi\pi$ phase shifts is probably connected to the neglect of isospin breaking effects in the $K \to \pi\pi$ amplitudes.

Actually, the authors of Ref.[10] could not include in their fit the new $E731$ determination of the quadratic slope in the $K_L \to 3\pi^0$ Dalitz plot, which was published later. Thus, in principle the overall fit should be repeated by taking into account also this data, which would lead to improved determinations of the quadratic amplitudes ζ_1 and ζ_3. Nevertheless, for the time being we still keep the results of [10] in the first column on Table IV, to give an impression of the phenomenological situation concerning the complex of $K \to 3\pi$ amplitudes. One can still read out from Table I the level of uncertainty affecting the amplitudes of Eq.(1.11) and notice, in particular, that the $\Delta I = 3/2$ linear coefficient β_3 is not as accurately determined as we would wish, and that knowledge of quadratic coefficients is somewhat poor. Also, as discussed later, being the $K_L \to 3\pi^0$ quadratic slope uncorrelated from either linear or constant amplitudes, it should still be possible to attempt the comparison of present experimental findings (including $E731$) with general results from chiral perturbation theory, which is one of the main points of interest in the following sections.

Theoretical predictions

In the Standard Model the non-leptonic weak Lagrangian for $\Delta S = 1$ transitions is originally written in terms of quark fields as [11,12,13,14]:

$$\mathcal{L}_W(\Delta S = 1)\Big|_{quark} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i(\mu) Q_i(\mu), \tag{1.15}$$

where $C_i(\mu)$ are numerical coefficients calculable in perturbative QCD and $Q_i(\mu)$ are local four-quark operators, with the selection rules $\Delta S = 1$ and $\Delta I = 1/2,\ 3/2$. Both C_i and Q_i separately depend on a renormalization scale μ, but their product must be μ-independent. At the scale $\mu < m_c$ the relevant independent four-quark operators can be chosen as [15]:

$$\begin{aligned}
Q_1 &= \bar{s}\gamma_\mu(1-\gamma_5)d\bar{u}\gamma^\mu(1-\gamma_5)u; & Q_2 &= \bar{s}\gamma_\mu(1-\gamma_5)u\bar{u}\gamma^\mu(1-\gamma_5)d \\
Q_3 &= \bar{s}\gamma_\mu(1-\gamma_5)d\sum_q \bar{q}\gamma^\mu(1-\gamma_5)q & & \\
Q_5 &= \bar{s}\gamma_\mu(1-\gamma_5)d\sum_q \bar{q}\gamma^\mu(1+\gamma_5)q; & Q_6 &= -2\sum_q \bar{s}(1+\gamma_5)q\bar{q}(1-\gamma_5)d \\
Q_7 &= \frac{3}{2}\bar{s}\gamma_\mu(1-\gamma_5)d\sum_q e_q\bar{q}\gamma^\mu(1+\gamma_5)q; & Q_8 &= -3\sum_q e_q\bar{s}(1+\gamma_5)q\bar{q}(1-\gamma_5)d,
\end{aligned} \tag{1.16}$$

where q runs over the light flavours u, d and s, and e_q is the corresponding quark charge. The operators Q_7 and Q_8 are induced by the electroweak (γ and Z°) penguin diagrams and, as being suppressed by a small coefficient of order α_{QED}, they are important only for the CP violating case. The non-leptonic Lagrangian thus involves both left-handed and right-handed quark fields. Under separate $SU(3)$ rotations of left-handed and of right-handed fields, i.e. chiral $SU(3)_L \times SU(3)_R$ symmetry transformations, the operators Q_1 and Q_2 behave as $(8_L, 1_R) + (27_L, 1_R)$ and thus contribute to both $\Delta I = 1/2$ and $\Delta I = 3/2$ transitions; Q_3, Q_5 and Q_6 transform as $(8_L, 1_R)$ leading to $\Delta I = 1/2$ only; finally Q_7 and Q_8 transform as $(8_L, 8_R)$, and contribute to both $\Delta I = 1/2$ and $\Delta I = 3/2$.

To make theoretical predictions, and thus to compare the constituent-level transition Lagrangian (1.15) with experimental data, one must estimate matrix elements of the four-quark operators between initial $|K>$ and final $<3\pi|$ hadronic states. The problem is that such matrix elements crucially depend on the non-perturbative structure of QCD, so that such a calculation can depend quite strongly on the particular hadronization models.

In this regard, great interest has been given recently to meson Lagrangian models, exploiting the notion of approximate $SU(3)_L \times SU(3)_R$ chiral symmetry of the strong interactions. In principle, this framework offers the possibility of evaluating hadronic matrix elements with a minimum amount of model dependence. In fact, *via* current algebra and PCAC, spontaneously broken chiral symmetry implies low energy theorems which are completely general, and rigorously valid for transition amplitudes involving $SU(3)$-octet pseudoscalar mesons (the would-be Goldstone bosons) at vanishing four-momenta. The low energy theorems allow to relate $K \to 3\pi$ to $K \to 2\pi$ amplitudes, making use only of the $SU(3)_L \times SU(3)_R$ transformation properties of the non-leptonic weak Lagrangian (1.15). For instance, neglecting in (1.16) the electroweak operators Q_7 and Q_8 (which are relevant only for CP violation), $\mathcal{L}_W(\Delta S = 1)$ behaves as a right-handed singlet leading to [16,6]

$$\lim_{p_3 \to 0} < \pi^+(p_1)\pi^-(p_2)\pi^0(p_3)|\mathcal{L}_W|K^0 > = \frac{-i}{2F_\pi} < \pi^+(p_1)\pi^-(p_2)|\mathcal{L}_W|K^0 >, \quad (1.17)$$

and to similar relations for the other pions becoming soft and for the other decay channels. F_π is here the pion decay constant ($F_\pi = 93.3\ MeV$).

Although the soft-pion points are somewhat away from the kinematically allowed region of the Dalitz plot, these low energy theorems turn out to be in a reasonable qualitative agreement with the data on $K \to 3\pi$. However, for a really quantitative calculation, these relations should be turned into predictions for physical (i.e. with finite mass and momentum) pions and kaons. By a procedure analogous to the derivation of the strong interaction meson-meson lagrangian [17], this is implemented by translating the weak four-quark Lagrangian (1.15) into an effective Lagrangian made of pion, kaon and eta fields, with the same transformation properties under chiral symmetry transformations. Such a weak non-leptonic Lagrangian thus accounts for the most general, and fundamental symmetry properties of non-perturbative QCD (in particular, it has the current algebra and PCAC soft pion theorems automatically built in), and is used as a computational tool to evaluate the relevant hadronic matrix elements of $\mathcal{L}_W(\Delta S = 1)$ by means of Feynman diagrams. The resulting predictions then depend on a limited number of phenomenological constants (counterterms), so that the scheme is expected to be both predictive and economical at the same time. These constants cannot be predicted by the Lagrangian itself, but should be taken from experiment. Actually, the comparison of the values so obtained with those resulting from Lattice QCD simulations, which should allow determinations of weak matrix elements from first principles, should represent a powerful consistency test of this theoretical description, and more in general of the effects of non-perturbative QCD.

The Goldstone-boson nature of pseudoscalar mesons naturally leads to the systematic expansion of transition amplitudes in powers of mesons momenta (chiral perturbation theory), which is quite suitable to the expansions of Eqs.(1.10) and (1.11). Another important aspect is represented by the inclusion in the chiral Lagrangian framework of vector mesons and vector dominance, which on the one side would enrich the theoretical description by accounting also for the role of resonances, and on the other side should lead to a determination from first principles of some of the phenomenological constants [18]. Thus, $K \to 3\pi$ can represent quite a significant (and detailed) test of the current ideas underlying the realization of the non-leptonic weak interaction Hamiltonian in terms of meson fields.

In the following we briefly sketch the basic features of chiral perturbation theory (χPT), also in the limit $N_c \to \infty$ with N_c the number of quark colors, and of chiral Lagrangian models including vector meson dominance. Predictions from Lattice QCD calculations, coupled to chiral perturbation theory expansions, are reported for CP violating $K \to 3\pi$ in Ref.[7].

a) *Chiral perturbation theory*

Neglecting the electroweak penguins Q_7 and Q_8, the non-leptonic $\mathcal{L}_W(\Delta S = 1)$ behaves under $SU(3)_L \times SU(3)_R$ as $(8_L, 1_R)$ or as $(27_L, 1_R)$. Operators in χPT reproducing these transformation properties involve, at the lowest order, two powers of meson momenta or meson masses, i.e. are $O(p^2)$. At this order there is only one $(8_L, 1_R)$ operator and one $(27_L, 1_R)$ operator, and the weak lagrangian takes the form [19]:

$$\mathcal{L}_W^{(2)}(\Delta S = 1) = c_2 \langle \lambda_6 L_\mu L^\mu \rangle + c_3 t^{jl}_{ik} \langle Q^i_j L_\mu \rangle \langle Q^k_l L^\mu \rangle, \tag{1.18}$$

where L_μ are the "left-handed" currents $L_\mu = iU^\dagger \partial_\mu U$, and $U = \exp(i\sqrt{2}\Phi/F_\pi)$ with Φ the familiar $SU(3)$ octet matrix:

$$\Phi = \begin{pmatrix} \dfrac{\pi^0}{\sqrt{2}} + \dfrac{\eta}{\sqrt{6}} & -\pi^+ & -K^+ \\ \pi^- & \dfrac{-\pi^0}{\sqrt{2}} + \dfrac{\eta}{\sqrt{6}} & -K^0 \\ K^- & -\bar{K}^0 & \dfrac{-2}{\sqrt{6}}\eta \end{pmatrix} \tag{1.19}$$

In (1.18) $\langle\ \rangle$ means *trace*, and λ_6 and $(Q^i_j)_{kl} = \delta_{il}\delta_{jk}$ are 3×3 matrices in flavour space projecting out, respectively, the octet and (with the help of the numerical coefficients

t_{ik}^{jl}) the 27-plet parts of the interaction. The explicit form of the matrices, as well as the values of t_{ik}^{jl}, can be found for example in Ref.[20]. c_2 and c_3 are the corresponding octet and 27-plet weak coupling constants.

By expanding the effective Lagrangian (1.18) to the right number of meson fields, one can easily derive the $O(p^2)$ contributions to the $K \to 2\pi$ and to the $K \to 3\pi$ amplitudes and linear slopes (the quadratic slopes being zero at this order, as they need operators with four derivatives of meson fields). In practice, the terms so obtained correspond to the "tree" diagrams depicted in Figure 2. As one can see, in the case of $K \to 3\pi$ there are "pole" diagrams, for which also the expression of the meson-meson strong interaction Lagrangian is required. At the same order p^2, this is fixed by chiral symmetry to be [21]:

$$\mathcal{L}_S^{(2)} = \frac{F_\pi^2}{4}\left(\langle \partial_\mu U \partial^\mu U^\dagger \rangle + \langle M(U^\dagger + U)\rangle\right), \qquad (1.20)$$

where $M = diag(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2)$ is the meson mass matrix, explicitly breaking chiral symmetry.

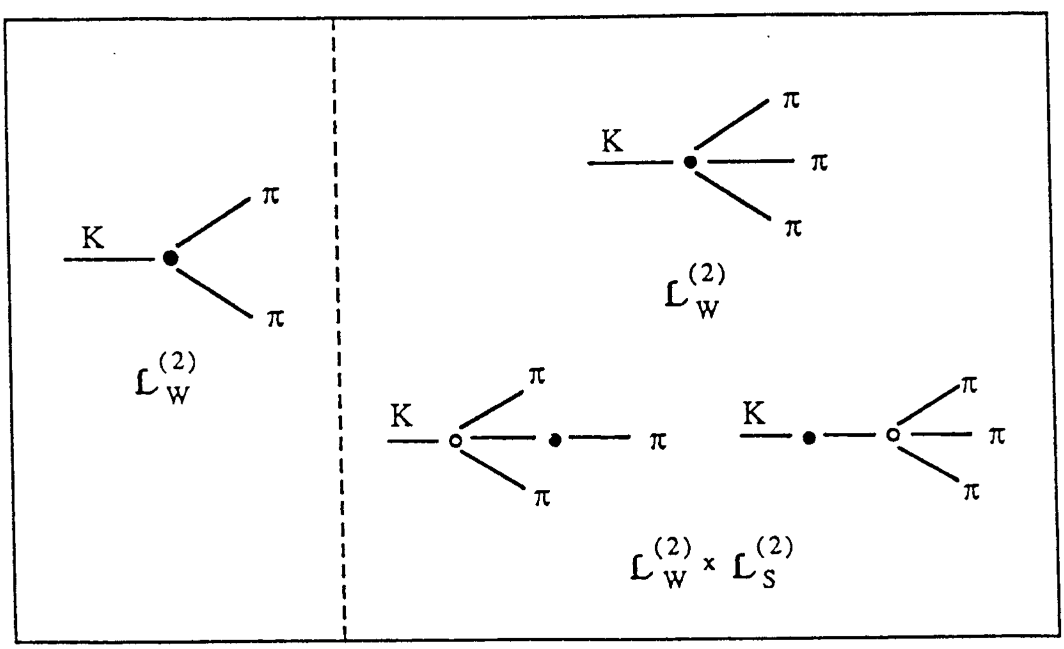


Figure 2: Lowest order diagrams ($O(p^2)$) for $K \to 2\pi$ and $K \to 3\pi$: the weak vertex is represented by •, the strong one by ∘.

Defining the $K \to 2\pi$ amplitudes $A_0 = ia_{1/2}e^{i\delta_0}$ and $A_2 = -ia_{3/2}e^{i\delta_2}$, with δ_0 and δ_2 the $\pi\pi$ phaseshifts in $I = 0,\ 2$ at $s = m_K^2$, such that:

$$\begin{aligned} A(K_S \to \pi^0\pi^0) &= \sqrt{\frac{2}{3}}A_0 + \frac{2}{\sqrt{3}}A_2 \\ A(K_S \to \pi^+\pi^-) &= -\sqrt{\frac{2}{3}}A_0 + \frac{1}{\sqrt{3}}A_2 \\ A(K^+ \to \pi^+\pi^0) &= \frac{\sqrt{3}}{2}A_2, \end{aligned} \tag{1.21}$$

one easily obtains for the $\Delta I = 1/2$ amplitudes:

$$\begin{aligned} a_{1/2} &= \frac{1}{F_\pi^2 F_K}\sqrt{6}\left(m_K^2 - m_\pi^2\right)\left(c_2 - \frac{2}{3}c_3\right) \\ \alpha_1 &= \frac{1}{F_\pi^3 F_K}\frac{m_K^2}{3}\left(c_2 - \frac{2}{3}c_3\right) \\ \beta_1 &= \frac{1}{F_\pi^3 F_K}\left(-m_\pi^2\right)\left(c_2 - \frac{2}{3}c_3\right), \end{aligned} \tag{1.22}$$

and, for the $\Delta I = 3/2$ amplitudes:

$$\begin{aligned} a_{3/2} &= \frac{1}{F_\pi^2 F_K}\left(m_K^2 - m_\pi^2\right)\left(-\frac{20}{\sqrt{3}}c_3\right) \\ \alpha_3 &= \frac{1}{F_\pi^3 F_K}m_K^2\left(\frac{20}{9}c_3\right) \\ \beta_3 &= \frac{1}{F_\pi^3 F_K}\frac{m_\pi^2}{m_K^2 - m_\pi^2}\left(5m_K^2 - 14m_\pi^2\right)\left(\frac{5}{3}c_3\right) \\ \gamma_3 &= \frac{1}{F_\pi^3 F_K}\frac{m_\pi^2}{m_K^2 - m_\pi^2}\left(3m_K^2 - 2m_\pi^2\right)\left(\frac{-15}{2\sqrt{3}}c_3\right). \end{aligned} \tag{1.23}$$

These simple equations manifestly contain the current-algebra and PCAC soft pion theorems relating $K \to 3\pi$ amplitudes for vanishing pion four-momentum to $K \to 2\pi$, plus finite pion mass corrections extrapolating those relations back into the physical Dalitz plot.

The values of the coupling constants c_2 and c_3 are free parameters, not fixed by chiral symmetry. Their values are obtained phenomenologically by fitting (1.22) and (1.23) to the experimental values of $K \to 2\pi$ amplitudes, and then used to predict $K \to 3\pi$. The results for c_2 and c_3, clearly displaying the $\Delta I = 1/2$ enhancement in $K \to 2\pi$, are reported in Table V. The corresponding $O(p^2)$ predictions for $K \to 3\pi$ are listed in Table IV. Although, in principle, at this level $F_\pi = F_K = F_0$, $SU(3)$ breaking has been phenomenologically included by using in (1.22) and (1.23) $F_K \cong 1.22\ F_\pi$.

One can see that there is agreement (or disagreement) at the 20-40% level, which is the kind of uncertainty one naively expects at this order in χPT. If, conversely, one would attempt to fit c_2 and c_3 from $K \to 3\pi$ data, the determination of c_2 would remain consistent with that in Table V, whereas that of c_3 would change considerably.

One can notice that the amplitude α_1 is so well measured that it really represents a challenge to the theory. Also noticeable is the discrepancy with the central value of the $\Delta I = 3/2$ slope β_3. For this reason it would be of great importance to increase the experimental accuracy on this parameter. Furthermore, we remind that final state strong interaction phases vanish at this order in χPT.

Recently, the general form of the $O(p^4)$ contribution to the $\Delta S = 1$ non-leptonic Lagrangian has been worked out in Ref.[20]. It turns out that the number of new independent weak operators contributing in general to $\mathcal{L}_W^{(4)}(\Delta S = 1)$, and whose coupling constants are not determined by chiral symmetry alone, is unfortunately unmanageably large. As an example, to give an idea of the form of these operators, we show below a set of independent $O(p^4)$ $(8_L, 1_R)$ operators with four derivatives in the pseudoscalar meson fields [22]:

$$\begin{aligned} O_1 &= \langle \lambda_6 L_\mu L^\mu L_\nu L^\nu \rangle & \qquad O_2 &= \langle \lambda_6 L_\mu L_\nu L^\mu L^\nu \rangle \\ O_3 &= \langle \lambda_6 L_\mu L_\nu L^\nu L^\mu \rangle & \qquad O_4 &= \langle \lambda_6 L_\mu L^\mu \rangle \langle L_\nu L^\nu \rangle . \end{aligned} \tag{1.24}$$

Notice that operators of this form vanish for $K \to 3\pi$ at the soft pion points, hence they do not contribute to $K \to 2\pi$ and therefore change at $O(p^4)$ the relation between $K \to 3\pi$ and $K \to 2\pi$ from the form given at $O(p^2)$ by Eqs.(1.22) and (1.23). They also generate terms of order Y^2 and X^2 and thus determine the quadratic amplitudes in (1.11). In addition to operators of the form (1.24) there are a multitude of operators containing higher derivatives of meson fields, which would contribute to both $K \to 3\pi$ and $K \to 2\pi$.

However, in the specific case of $K \to 2\pi$ and $K \to 3\pi$ decays, the situation simplifies considerably. To just emphasize the main aspects, symbolically denoting any of the $K \to 2\pi$ and $K \to 3\pi$ amplitudes by A_i, we have up to order p^4:

$$A_i = A_i^{(2)} + A_i^{(4)}, \tag{1.25}$$

where, as pictorially represented by the diagrams in Figure 3, the $O(p^4)$ correction term can be further separated into

$$A_i^{(4)} = A_i^{CT}(\mu) + A_i^{loop}(\mu) + A_i^{poles}. \tag{1.26}$$

A_i^{CT} represents the tree diagram $O(p^4)$ weak counterterms: it is found that in practice only seven linear combinations of $O(p^4)$ weak counterterms, to be determined phenomenologically, are effective for $K \to 2\pi$ and $K \to 3\pi$. In addition, there are a number of strong meson-meson interaction $O(p^4)$ counterterms, which determine $\mathcal{L}_S^{(4)}$ appearing in the pole diagrams, but their values are already available from previous analyses [21] and therefore they do not add anything unknown.

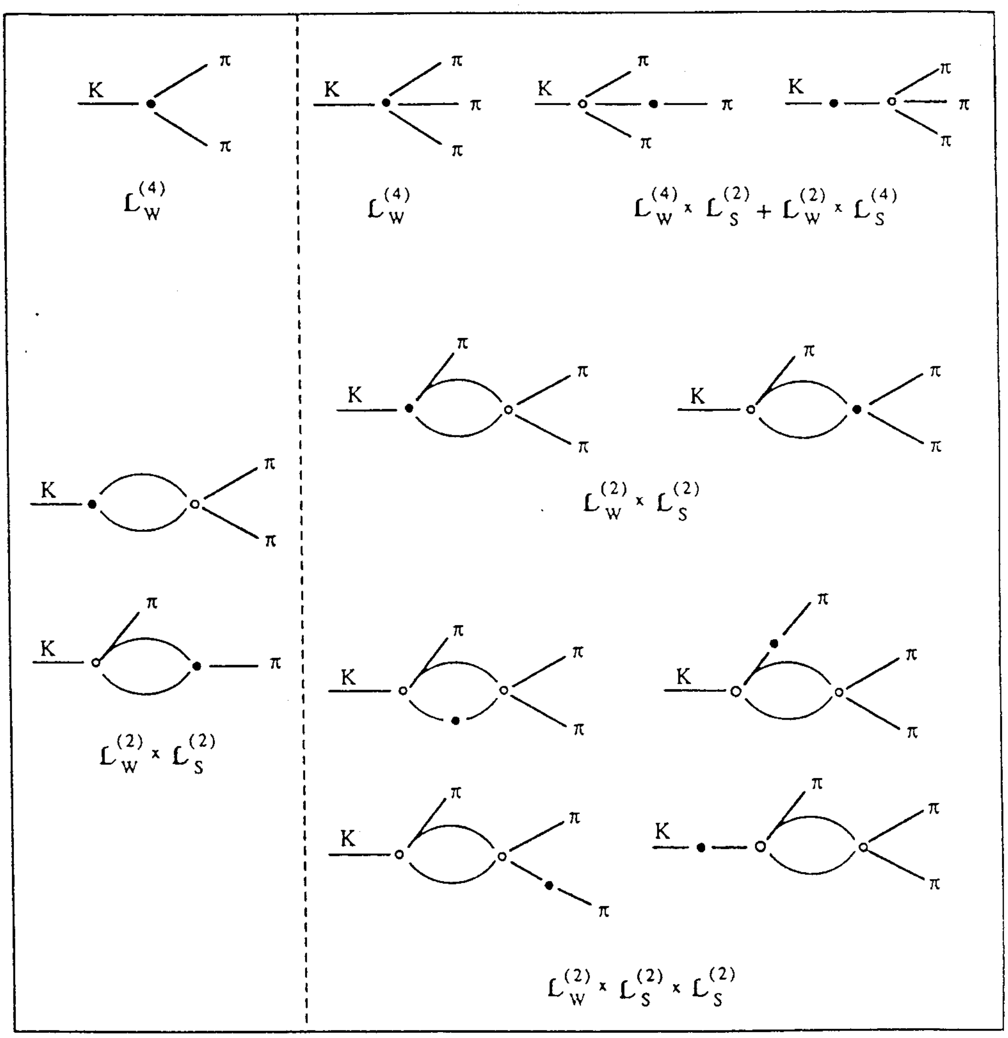


Figure 3: $O(p^4)$ contributions to $K \to 2\pi$ and $K \to 3\pi$: • and ∘ as in Figure 2.

A_i^{loop} are the contributions of chiral loops which, among other things, account for rescattering corrections, and are essentially governed by $\mathcal{L}_S^{(2)} \times \mathcal{L}_W^{(2)}$. Both counterterms and loops separately depend (logarithmically) on a renormalization scale μ, but their sum is μ-independent.

The strategy followed by the authors of Ref.[10], to empirically determine the seven $O(p^4)$ weak coupling constants, was to fit the full chiral structures found for the $K \to 2\pi$ and $K \to 3\pi$ isospin amplitudes to the experimental values in the first column of Table IV and in Table V. Basically, still neglecting the $K \to 3\pi$ strong phases, there are twelve experimental isospin amplitudes, plus the $K \to 2\pi$ relative phase, to fit in terms of the seven constants plus the octet and 27-plet couplings c_2 and c_3 (of course the latter have to be redetermined at this p^4 order). Although not including, in the sector of quadratic amplitudes, the latest $E731$ determination of $K_L \to 3\pi^0$, we nevertheless report in the third column of Table IV the results so obtained in [10] (corresponding to the choice for the renormalization scale $\mu = m_\eta$ to minimize the logarithmic dependence and neglecting the strong phases of $K \to 3\pi$).

It is interesting to notice from these numbers that, although this order in chiral perturbation theory is not quite sufficient to account for the $K \to 2\pi$ phase $\delta_0 - \delta_2$, the constant and the linear amplitudes for $K \to 3\pi$ are now well reproduced and that, in general, phenomenology requires the $O(p^4)$ corrections to be somewhat larger for the $\Delta I = 3/2$ transitions. Also, it appears that the situation for the quadratic amplitudes is not as well-defined as it is for the linear ones and for the widths, due to large uncertainties.

In fact in this regard, since the number of fitted parameters is anyway smaller than the number of isospin amplitudes, one can go beyond this global fitting procedure and derive parameter-free consistency conditions among amplitudes [23]. These constraints would represent specific predictions of chiral perturbation theory at $O(p^4)$. Basically, two of the seven weak counterterms are reabsorbed in the definition of the coupling constants c_2 and c_3 at this order. The remaining five can be isolated by defining, for each of the ten $K \to 3\pi$ amplitudes A_i:

$$\tilde{A}_i = A_i^{exp} - A_i^{(2)} - A_i^{loop} - A_i^{pole}, \tag{1.27}$$

which are known completely. The chiral relations derived in [10] then imply the five consistency relations (to order m_π^2/m_K^2) [23]:

$$\tilde{\zeta}_1 = -\frac{9}{4}\frac{m_\pi^4}{m_K^4}\tilde{\alpha}_1; \qquad \tilde{\xi}_1 = \frac{3}{2}\frac{m_\pi^2}{m_K^2}\tilde{\beta}_1 \tag{1.28}$$

for the $\Delta I = 1/2$ amplitudes, and

$$\tilde{\zeta}_3 = \frac{9}{8}\frac{m_\pi^4}{m_K^4}\tilde{\alpha}_3; \qquad \tilde{\xi}_3 = -\frac{3}{4}\frac{m_\pi^2}{m_K^2}\tilde{\beta}_3; \qquad \tilde{\xi}'_3 = -\frac{3\sqrt{3}}{2}\frac{m_\pi^2}{m_K^2}\tilde{\gamma}_3 \tag{1.29}$$

for the amplitudes with $\Delta I = 3/2$.

Thus, once the $\Delta I = 1/2$ and $\Delta I = 3/2$ weak counterterms are extracted from the experimental determinations of the constant and linear amplitudes α_1, β_1 and α_3, β_3 and γ_3 respectively, the quadratic amplitudes are predicted, so that (not surprisingly) the test of the chiral method at order p^4 is determined in practice by the amplitudes ζ_1, ζ_3 and ξ_1, ξ_3 and ξ'_3. The predictions are reported in the fourth column of Table IV, and clearly indicate that the situation concerning the "chiral test" is much more favorable for the $\Delta I = 1/2$ relations than for the $\Delta I = 3/2$ ones, the theoretical uncertainty being quite large in this case.

Specifically, for the $K_L \to 3\pi^0$ quadratic amplitude (referring to Eq.(1.11)) the chiral symmetry prediction is [10,23]:

$$\frac{\zeta_1 - 2\zeta_3}{\alpha_1 + \alpha_3} = -(6 \pm 2) \times 10^{-3}, \tag{1.30}$$

and the corresponding Dalitz plot quadratic slope $h = -(1.2 \pm 0.4) \times 10^{-3}$. Although of the same sign, this prediction appears to be in contradiction with the recent $E731$ measurement $h = -(3.3 \pm 1.1 \pm 0.7) \times 10^{-3}$, reported in Table II. However, due to the large uncertainty mentioned above, the significance of such a disagreement between the two values might be not fully significant at the present stage.

Identifying the source of the disagreement should certainly be an interesting problem. Indeed, combining their result with charged kaon experiments, the $E731$ collaboration suggests a sizable violation of the $\Delta I = 1/2$ rule in the quadratic term, connected to an unexpectedly large value of the ratio ζ_3/ζ_1. Thus, as anticipated, the $\Delta I = 3/2$ has a crucial role for χPT.

All this calls for improved experimental determinations of $K \to 3\pi$ amplitudes, in particular of the $\Delta I = 3/2$ ones and of the quadratic slopes, hopefully leading more accurate (and more stringent) tests of the theoretical framework. In this regard, also more accurate phenomenological determinations of the strong interaction counterterms, involved in the pole term contributions of Eq.(1.26), would be useful in order to reduce the theoretical uncertainties on $K \to 3\pi$ amplitudes to the lowest possible level.

In closing this section, we finally remark that an experimental determination of the strong relative phases $\delta_{1S} - \delta_{1M}$ and $\delta_{1S} - \delta_2$ should also be of relevance to the chiral test.

b) *Calculations in the $N_c \to \infty$ expansion*

A definite prediction of $O(p^4)$ terms can be obtained using the framework of chiral perturbation theory in the leading $1/N_c$ expansion ($N_c \to \infty$), N_c being the number of quark colors [24,25].

Basically, in this approach a "bosonization" prescription is applied to the quark left-handed "currents" $J_\mu = \bar{q}\gamma_\mu(1-\gamma_5)q$ in Eq.(1.16), which leads to a chiral expansion of the form:

$$J_\mu = iF_\pi^2 \left[L_\mu + \frac{N_c}{6\Lambda_\chi^2} \left(terms\ cubic\ in\ L_\mu \right) + \cdots \right], \tag{1.31}$$

where L_μ has been defined previously, and Λ_χ is a scale characterizing chiral symmetry breaking that is expected to be of the order of 1 GeV. An analogous "bosonization" is written for the right-handed current operators. Assuming for $\mathcal{L}_W(\Delta S = 1)$ the form of a *current*$\times$*current* product analogous to (1.18)

$$\mathcal{L}_W(\Delta S = 1) = c_2' \langle \lambda_6 J_\mu J^\mu \rangle + c_3' t_{ik}^{jl} \langle Q_j^i J_\mu \rangle \langle Q_l^k J^\mu \rangle \tag{1.32}$$

and using (1.31), one reobtains the leading Lagrangian $\mathcal{L}_W^{(2)}(\Delta S = 1)$ of Eq.(1.18), plus the correction $\mathcal{L}_W^{(4)}(\Delta S = 1)$. The latter is expressed by a sequence of four-derivative meson field operators of the kind represented in (1.24), with coupling constants numerically determined. Instead, the couplings c_2' and c_3' are left as undetermined degrees of freedom. These are fit, as in the preceding section, from the experimental $K \to 2\pi$, and in this way the amplitudes of $K \to 3\pi$ can be predicted to $O(p^4)$ without extra parameters.*

The numerical results obtained in the $1/N_c$ approach are reported Table IV. The comparison of theoretical predictions with experimental data looks encouraging. Concerning the needed significant accuracies to make a test of the $\Delta I = 3/2$ sector, the same comments hold, as made previously with regard to the results of [10]. Certainly, a nice feature of this theoretical framework, which is inspired by the principle of factorization and the vacuum saturation approximation of hadronic matrix elements,

* Actually in [25] the possibility is considered of different chiral symmetry breaking suppression scales for the $O(p^4)$ operators, leading to two such parameters to be extracted phenomenologically.

is the fact that the $O(p^4)$ operator coupling constants are calculable (once c_2' and c_3' are extracted phenomenologically from the data), so that there is a substantial reduction in the number of free parameters to order p^4. However, there is the difficulty to conceptually justify the use of the experimental values of c_2' and c_3' (otherwise the $\Delta I = 1/2$ enhancement would not be accounted for), instead of the actual $1/N_c$ values of the coefficients in Eq.(1.16). Also, it should be interesting to quantitatively assess the next-to-leading $1/N_c$ corrections for finite $N_c = 3$, in order to test the convergence of such an expansion. One can notice that, since there are no chiral loops at the leading order in $1/N_c$, strong phases vanish so that eventually they must be included by hand. In the absence of the mentioned meson loops, the operators coupling constants become mass-scale independent, and consequently at his order it is ambiguous to compare with the results obtained in the full theory in Ref.[10].

c) *Vector meson dominance models*

A suggestive way to overcome the difficulties with the order p^4 discussed above, and to dynamically estimate the counterterm coupling constants from first principles, should be the inclusion of vector mesons in the chiral Lagrangian framework. Using the strong chiral Lagrangian for vector and pseudoscalar mesons, it has been shown in Ref.[26] that the couplings of the strong interaction p^4 counterterms calculated from vector meson exchanges are in good agreement with the values determined phenomenologically from experimental data in [21], if the scale μ is chosen between m_η and 1 GeV.

In Ref.[18] it has been assumed that the same kind of vector dominance holds also for the non-leptonic weak Lagrangian. Working out all lowest order $(8_L, 1_R)$ weak couplings between vector and pseudoscalar mesons, which are not known, it turns out that the $O(p^4)$ corrections induced by vector meson exchanges on the $K \to 3\pi$ amplitudes in the $\Delta I = 1/2$ sector depend on only one unknown constant. These corrections affect only β_1 and ξ_1 because vector mesons can contribute only to final states antisymmetric under the exchange of two pions. Therefore, in this framework corrections to the symmetric amplitudes are determined just by the loop contributions. In fact, for α_1 these turn out not to go in the right direction [10].

Nevertheless, the antisymmetric amplitudes are the ones for which the discrepancies between experimental results and $O(p^2)$ predictions are the largest, so that it is still sensible to investigate the corrections from vector exchanges.

The unknown constant mentioned above can be determined by the *current* $\times$ *current* hypothesis of Eq.(1.32), using for J_μ an expression in terms of vector fields. The consequence of this hypothesis, which follows directly from the vacuum insertion approximation, is that the weak contributions of vector mesons to the $K \to 3\pi$ amplitudes with $\Delta I = 1/2$ vanish, leaving only the strong contribution of the pole terms in Figure 4 [18].

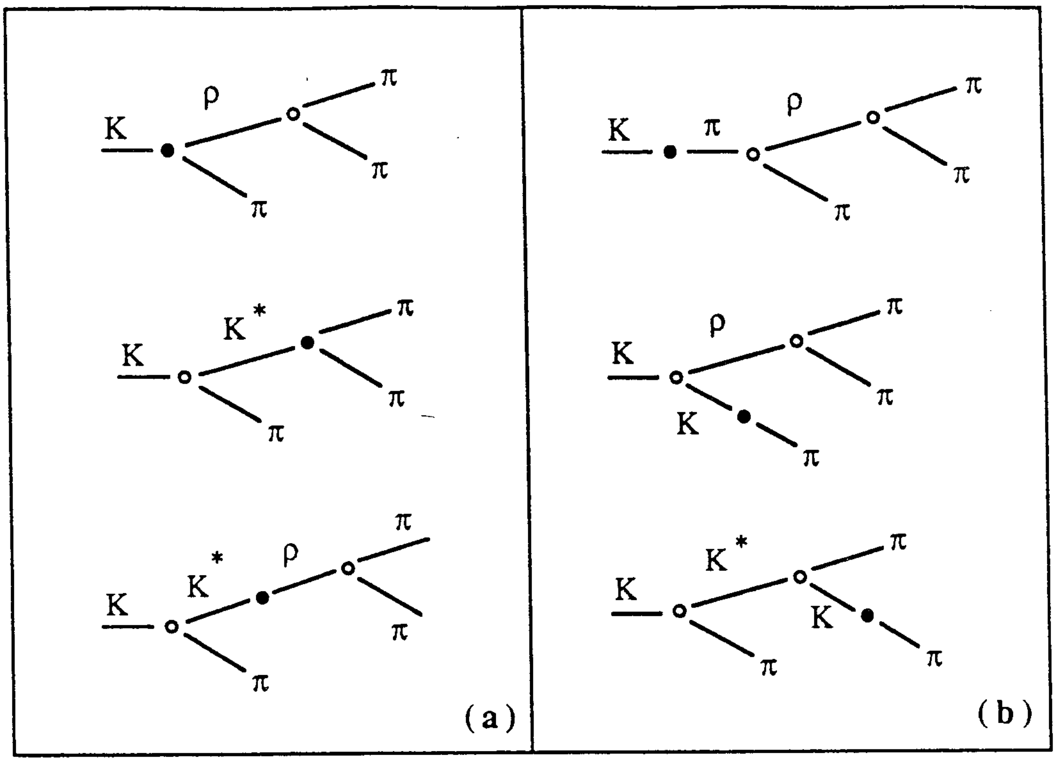


Figure 4: Vector meson exchanges for $K \to 3\pi$: diagrams with weak vector meson couplings (a) and pole diagrams with vector meson exchange (b).

In the last column of Table IV we report the results for the $\Delta I = 1/2$ amplitudes to order p^4, calculated with the vector meson counterterms plus the pseudoscalar meson loops (the latter are taken from [10]), for the scale parameter μ between m_η and 1 GeV. One can see that the chiral corrections calculated in this framework significantly improve the $O(p^2)$ predictions for the amplitudes β_1 and ξ_1. In conclusion, vector meson dominance seems a good model to determine the p^4 counterterms for the antisymmetric $K \to 3\pi$ amplitudes, but cannot cannot solve the problem for the symmetric ones. The latter would be determined by e.g. scalar and pseudoscalar resonances, whose weak couplings, however, are not known so that in practice there would be no reduction in the number of parameters. Therefore, attempts to evaluate those couplings would be of great interest in this regard.

$K \to 3\pi$ studies at a ϕ factory

According to the considerations made in the previous sections, accurate measurements of $K \to 3\pi$ Dalitz plots would allow a stringent test of the chiral lagrangian description of $\Delta S = 1$ non-leptonic weak interactions to order p^4. A desirable accuracy should be the knowledge of linear slopes to (much) better than 1 % and the quadratic ones to about a few %.* It would also be quite interesting to measure the strong phases, related to the chiral structure of the pion rescattering interaction.

Present experimental $K \to 3\pi$ Dalitz plot analyses rely on samples of about $10^4 - 10^6$ events, depending on the particular decay modes (see Table I), while at the ϕ-factory one expects in principle a substantial improvement in the statistical accuracy, assuming high detection efficiencies. However, the experimental accuracy is mostly controlled by the systematic error, which must be decreased accordingly. Moreover, due to correlations, reducing the error on one parameter of the Dalitz plot (1.10) automatically requires a corresponding reduction in the uncertainty on the other parameters. Thus, a fit procedure is necessary in any case.

To give an example of these correlations, one can see how the request for a good determination of the linear slope g implies having a better precision also on the quadratic slopes h and k, and that conversely linear slopes affect quadratic ones. This is made clear by considering that the linear slope can be defined in principle by weighting the distribution (1.10) with the variable Y, i.e. through the relation:

$$\Sigma_1 \equiv \frac{\int Y|A|^2 d\Gamma}{\int |A|^2 d\Gamma} = gR\left[1 + \left(k + \frac{h}{3}\right) R\right]^{-1}, \qquad (1.33)$$

where $d\Gamma = r\ dr\ d\phi$ is the phase space element, and R is the ratio of integrals:

$$R = \frac{\int Y^2 d\Gamma}{\int d\Gamma} = \frac{1}{3}\frac{\int X^2 d\Gamma}{\int d\Gamma}. \qquad (1.34)$$

From (33), by making an expansion, one obtains for the relative uncertainties:

$$\frac{\delta g}{g} \simeq \frac{\delta \Sigma_1}{\Sigma_1} + \left(k + \frac{h}{3}\right) R \frac{\delta\left(k + \frac{h}{3}\right)}{\left(k + \frac{h}{3}\right)}, \qquad (1.35)$$

so that errors on linear and quadratic slopes are correlated. Inserting numerical values in (1.35), the indication is that in order to reach an accuracy of (say) 10^{-3} on g the

* It should be noticed, especially concerning quadratic slopes, that some channels might possibly be affected by the presence of significant QED radiative corrections [27].

error on the quadratic slopes must be of the order of 5×10^{-2}. Clearly, in the special case of $K_L \to 3\pi^0$ the correlation above does not exist, because the quadratic term in the Dalitz plot is not contaminated by the linear amplitude which is zero for this mode (see Eq.(1.11)).* However, the determination of isospin amplitudes requires in general the combination of data for both $K^\pm$ and K^0 decays. Thus, although possibly not unsurmountable, these difficulties, and in particular the systematic errors, should be carefully taken into account in studies of $K \to 3\pi$ Dalitz plot distributions at the ϕ-factory.

On the other hand, the measurement of the rate of $K_S \to \pi^+\pi^-\pi^0$ certainly represents a very significant achievement, as it could be done for the first time at Daϕne. As indicated in Eq.(1.11), this decay can occur via a CP conserving interaction, however it is suppressed by the $\Delta I = 3/2$ rule (the three pions are in $I = 2$), and by an angular momentum barrier (recall that CP($\pi^+\pi^-\pi^0$)=$(-1)^I = (-1)^{(l+1)}$, where l is the orbital angular momentum of π^+ and π^-). The predicted branching ratio, $Br(K_S \to \pi^+\pi^-\pi^0) = (2-4) \times 10^{-7}$ makes it well accessible at this facility, with many hundreds of events as shown in Table I, and would allow, as previously pointed out, the direct test of the $\Delta I = 3/2$ part of the transition Hamiltonian.

As an alternative to the direct width measurement, one could try the measurement of the CP conserving $K_S \to \pi^+\pi^-\pi^0$ amplitude *via* the observation of the time dependent interference of this decay channel with $K_L \to \pi^+\pi^-\pi^0$ at the ϕ-factory, in analogy to the time asymmetries discussed for CP (and CPT) violation involving $K \to 2\pi$ and $K \to \pi l \nu_l$ [28,29,30]. An appealing aspect of this method is that such interference depends linearly on the (expected small) final state interaction $K \to 3\pi$ phases as $\sim \sin(\Delta m \cdot t + \delta)$, while the width depends quadratically as $\cos\delta \sim 1 - \frac{\delta^2}{2}$. Thus one might expect to have access to these phases also, and be able to make a direct determination or at least to set an upper limit on their values. This would be particularly desirable in view of the previous discussion and of the fact that these phases determine the size of direct CP violation asymmetries in $K \to 3\pi$.

The basic idea uses the fact that the initial $K\bar{K}$ state, immediately following ϕ decay, is represented by the superposition of K_L and K_S (assuming CPT conservation):

$$|i> \equiv |K^0\bar{K}^0(C = odd)> = \frac{1+|\epsilon|^2}{1-\epsilon^2}\,\frac{|K_L(\hat{z})K_S(-\hat{z})> - |K_S(\hat{z})K_L(-\hat{z})>}{\sqrt{2}}, \quad (1.36)$$

* Also, electromagnetic corrections are not needed for this channel.

so that the subsequent K_L and K_S decays are correlated, and their quantum interferences show up in relative time distributions and time asymmetries. In (1.36) $\hat{z}$ is the direction of the momenta of the kaons in the c.m. system, and ϵ is the CP violating K^0-$\bar{K}^0$ mass mixing parameter,

$$|K_S>=\frac{1}{\sqrt{1+|\epsilon|^2}}(|K_1>+\epsilon|K_2>), \qquad |K_L>=\frac{1}{\sqrt{1+|\epsilon|^2}}(|K_2>+\epsilon|K_1>), \tag{1.37}$$

with $|K_{1,2}>$ the CP even and odd eigenstates respectively.

Specifically, we consider the transition amplitude for the initial state to decay into the final states f_1 at times t_1 and f_2 at times t_2 respectively and, in order to maximize the interference effect we are looking for, we choose $f_1 = \pi^{\pm}l^{\mp}\nu$ and $f_2 = \pi^+\pi^-\pi^0$ (simply denoted as $f_2 = 3\pi$ in the sequel). Defining the "intensity" of time correlated events $I(\Delta t)$ as:

$$I(\Delta t) \equiv \frac{1}{2}\int_{|\Delta t|}^{\infty} dt\ |<f_1(t_1,\hat{z}), f_2(t_2,-\hat{z})|T|i>|^2 \tag{1.38}$$

where $t = t_1 + t_2$ and $\Delta t = t_2 - t_1$, and making use of the exponential time dependences of the mass eigenstates K_S and K_L, one easily finds in the CP conserving case $\epsilon = 0$:

$$I(\pi^{\pm}l^{\mp}\nu, 3\pi; \Delta t<0) = \frac{|A(K^0\to\pi l\nu)|^2}{8\gamma}\cdot\{|A(K_L\to 3\pi)|^2 e^{-\gamma_S|\Delta t|}$$
$$+|A(K_S\to 3\pi)|^2 e^{-\gamma_L|\Delta t|} \pm 2e^{-\gamma|\Delta t|}[Re(A(K_L\to 3\pi)^* A(K_S\to 3\pi))\cos(\Delta m|\Delta t|)$$
$$+Im(A(K_L\to 3\pi)^* A(K_S\to 3\pi))\sin(\Delta m|\Delta t|)]\} \tag{1.39}$$

and

$$I(\pi^{\pm}l^{\mp}\nu, 3\pi; \Delta t>0) = \frac{|A(K^0\to\pi l\nu)|^2}{8\gamma}\cdot\{|A(K_L\to 3\pi)|^2 e^{-\gamma_L\Delta t}$$
$$+|A(K_S\to 3\pi)|^2 e^{-\gamma_S\Delta t} \pm 2e^{-\gamma\Delta t}[Re(A(K_L\to 3\pi)^* A(K_S\to 3\pi))\cos(\Delta m\Delta t)$$
$$-Im(A(K_L\to 3\pi)^* A(K_S\to 3\pi))\sin(\Delta m\Delta t)]\}. \tag{1.40}$$

Here $\gamma = \frac{(\gamma_L+\gamma_S)}{2}$; $\Delta m = m_L - m_S$; and Eq.(1.11) must be used for the isospin decomposition and the pion momentum dependences of $K\to 3\pi$ amplitudes.

From Eqs.(1.39) and (1.40) one can notice that in the intensity of events for time differences $\Delta t < 0$ the $|A(K_S\to 3\pi)|^2$ term is enhanced by $\gamma_L \ll \gamma_S$ with respect to $|A(K_L\to 3\pi)|^2$, a situation which is complementary to that of $K\to 2\pi$ [31]. The total number of events is obtained by integrating (1.39) in $|\Delta t|$ and over the full $K \to 3\pi$ Dalitz plot (in this case the interference term drops). Taking to a good approximation the non-relativistic limit, so that the allowed domain in the Dalitz plot reduces to the unit circle, the integration in the polar coordinates Eq.(1.9) becomes trivial. For example, taking (1.11) into account:

$$\Gamma(K_L \to \pi^+\pi^-\pi^0) = \frac{1}{(4\pi)^3 m_K} \frac{\sqrt{3}}{18} Q^2_{+-0} \pi |\alpha|^2, \tag{1.41}$$

with $|\alpha| \equiv |\alpha_1 + \alpha_3| = 8.5 \times 10^{-7}$ from Table IV. The integration of (1.39) gives:

$$\begin{aligned} N(\pi^\pm l^\mp \nu, 3\pi; \Delta t < 0) &= N_{\phi \to K^0 \bar{K^0}} \frac{1}{2} BR(K_L \to \pi l \nu) \\ &\times \left[\left(\frac{\gamma_L}{\gamma_S}\right)^2 BR(K_L \to 3\pi) + BR(K_S \to 3\pi) \right] \times \Omega, \end{aligned} \tag{1.42}$$

where, with the assumed luminosity, $N_{\phi \to K^0 \bar{K^0}} \simeq 7.5 \times 10^9/year$. In (1.42) $\Omega < 1$ is a factor which accounts for the experimental acceptance. For the predicted values of the $K_S \to 3\pi$ width, the two terms in (1.42) have indeed the same size and we expect about $1600 \times \Omega$ events/year. The $K_S \to \pi^+\pi^-\pi^0$ amplitude γ_3 and the strong relative phases $\delta_2 - \delta_{1S}$ and $\delta_2 - \delta_{1M}$ (in the approximation $\sin\delta \simeq \delta$) appear linearly in the interference terms of (1.39) and (1.40), and are there multiplied by typical time dependent coefficients. In particular, the $\sin(\Delta m \Delta t)$ time dependent term is the one containing to the $K \to 3\pi$ strong phases. To extract the interference terms one can integrate the intensities (1.39) and (1.40) over the $K \to 3\pi$ Dalitz plot, with suitable cuts.

Specifically [32], one can can make a cut in the Dalitz variable X, by defining:

$$\Sigma_X(\Delta t) = \frac{1}{(4\pi)^3 m_K} \frac{\sqrt{3}}{18} Q^2_{+-0} \int\int r \; dr \; d\phi \; \text{sign}(X) \; I(\pi^\pm l^\mp \nu, 3\pi; \Delta t), \tag{1.43}$$

or a cut in XY by defining:

$$\Sigma_{XY}(\Delta t) = \frac{1}{(4\pi)^3 m_K} \frac{\sqrt{3}}{18} Q^2_{+-0} \int\int r \; dr \; d\phi \; \text{sign}(XY) \; I(\pi^\pm l^\mp \nu, 3\pi; \Delta t). \tag{1.44}$$

Explicitly using (1.11), with α defined as for (1.41) and $\beta \equiv \beta_1 + \beta_3$ (we fix the strong phases at the centre of the Dalitz plot):

$$\begin{aligned} \Sigma_X(\Delta t < 0) = &\pm \frac{\gamma_L^2}{4\gamma} BR(K_L \to \pi l \nu) BR(K_L \to \pi^+\pi^-\pi^0) \left(\frac{2 m_K Q_{+-0}}{\sqrt{3} m_\pi^2}\right) \times \\ &\frac{16\gamma_3}{3\sqrt{3}|\alpha|\pi} e^{-\gamma|\Delta t|} [\cos(\Delta m |\Delta t|) + (\delta_2 - \delta_{1S}) \sin(\Delta m |\Delta t|)], \end{aligned} \tag{1.45}$$

$$\Sigma_X(\Delta t > 0) = \pm \frac{\gamma_L^2}{4\gamma} BR(K_L \to \pi l \nu) BR(K_L \to \pi^+\pi^-\pi^0)(\frac{2m_K Q_{+-0}}{\sqrt{3}m_\pi^2})\times$$
$$\frac{16\gamma_3}{3\sqrt{3}|\alpha|\pi} e^{-\gamma\Delta t}[\cos(\Delta m \Delta t) - (\delta_2 - \delta_{1S})\sin(\Delta m \Delta t)], \qquad (1.46)$$

and

$$\Sigma_{XY}(\Delta t < 0) = \pm \frac{\gamma_L^2}{4\gamma} BR(K_L \to \pi l \nu) BR(K_L \to \pi^+\pi^-\pi^0)(-\frac{4m_K^2 Q_{+-0}^2}{3\sqrt{3}m_\pi^4})\times$$
$$\frac{2\beta\gamma_3}{\sqrt{3}|\alpha|^2\pi} e^{-\gamma|\Delta t|}[\cos(\Delta m|\Delta t|) + (\delta_2 - \delta_{1M})\sin(\Delta m|\Delta t|)], \quad (1.47)$$

$$\Sigma_{XY}(\Delta t > 0) = \pm \frac{\gamma_L^2}{4\gamma} BR(K_L \to \pi l \nu) BR(K_L \to \pi^+\pi^-\pi^0)(-\frac{4m_K^2 Q_{+-0}^2}{3\sqrt{3}m_\pi^4})\times$$
$$\frac{2\beta\gamma_3}{\sqrt{3}|\alpha|^2\pi} e^{-\gamma\Delta t}[\cos(\Delta m \Delta t) - (\delta_2 - \delta_{1M})\sin(\Delta m \Delta t)]. \qquad (1.48)$$

For the expected number of events at the ϕ-factory, obtainable by integrating the above relations in $|\Delta t|$, one can use input values for γ_3, α and β from Table IV, and the value (1.12) for the final state interaction phases at the centre of the Dalitz plot. By taking appropriate sums and differences of number of events we would find from (1.45) and (1.46) about $245 \times \Omega$ and $35 \times \Omega$ events/year available to the determinations of γ_3 and $\delta_2 - \delta_{1S}$ respectively, where Ω is the acceptance factor introduced in (1.42), and analogously from (1.47) and (1.48) about $4 \times \Omega$ events/year for the determination of $\delta_2 - \delta_{1M}$. Thus, for Ω not much smaller than unity, it might require a couple of years integrated luminosity (or alternatively the full 10^{33} luminosity) to measure $\delta_2 - \delta_{1S}$ and to bound $\delta_2 - \delta_{1M}$ to the level of Eq.(1.12). In any case, also experimental bounds on those phases would represent a progress over the present situation, specially having in mind the applications to CP violating asymmetries in $K \to 3\pi$.

An additional nice feature is that Eqs.(1.45)-(1.48) are essentially free from the background decay $\phi \to K^0\bar{K^0}\gamma$, leading to the $C = even$ state:

$$|K^0\bar{K^0}(C = even) >= \frac{|K_S(\hat{z})K_S(-\hat{z}) > + |K_L(\hat{z})K_L(-\hat{z}) >}{\sqrt{2}}. \qquad (1.49)$$

In fact, besides being suppressed by the small branching ratio of the originating process $\phi \to K^0\bar{K^0}\gamma$ [33], the contribution of the $|K_SK_S>$ state is still further suppressed by the small K_L branching ratios, while that of the $|K_LK_L>$ state is cut out.

In conclusion, it should be useful to study the mentioned interferences described above as an alternative means to measure the CP conserving $K^0 \to 3\pi$ amplitude. Clearly, this possibility directly relates to the explicit Dalitz plot distributions which enable to make appropriate cuts.

2. $K \to 3\pi$: CP VIOLATING EFFECTS

The observation of CP violation in $K \to 3\pi$ would greatly enrich our understanding of this, still mysterious, phenomenon.

CP-violation in $K_S \to 3\pi$ decays, even at the level where mass-mixing effects only are observed, is very interesting, for any deviation of ϵ_S from ϵ is a direct signal of CPT violation. On the other side, CP-odd charge asymmetries in $K^\pm$ decays would give a univocal sign of the milliweak nature of CP violation. If current Standard Theory estimates are correct, the direct CP-violation parameter for $K \to 2\pi$, ϵ'/ϵ, tends to be small because of a cancellation of the dominant penguin diagram contribution with the one arising from the electroweak penguin diagrams, made relevant by the large value of the t quark mass. The same two classes of contributions do not cancel in the asymmetry of $K^\pm$ decays, which then could provide the crucial test of the scheme.

Unfortunately, it turns out that Daϕne is still a long way from the crucial region, with presently foreseen luminosities. As we shall see, the predicted Dalitz-plot slope asymmetries are in the order of few units per million (subject to a theoretical error which could be perhaps of a factor 10), while, on purely statistical grounds, Daϕne experiments can reach the level of a few units in 10^{-4}. On neutral kaons, Daϕne experiments can observe a few $K_S \to 3\pi^0$ decays [34], enough to establish the effect, but to a precision unlikely to reveal any CPT anomaly in ϵ_S.

Nonetheless, Daϕne will improve a great deal on presently existing data (reported in Table VI limiting to $K^\pm$), and it is appropriate to discuss in this Report the present status-of-the-art of this field.

In the following, we review in detail the Standard Theory estimates of CP violation in charged K decays. Neutral K decays will be considered briefly in the end.

$K^\pm$ decays

One can study CP-odd charge asymmetries, in the total rates and in the linear slope of the Dalitz plot:

$$\frac{\Delta\Gamma}{2\Gamma} = \frac{\Gamma(K^+ \to 3\pi) - \Gamma(K^- \to 3\pi)}{\Gamma(K^+ \to 3\pi) + \Gamma(K^- \to 3\pi)}, \tag{2.1}$$

and

$$\frac{\Delta g}{2g} = \frac{g(K^+ \to 3\pi) - g(K^- \to 3\pi)}{g(K^+ \to 3\pi) + g(K^- \to 3\pi)}. \tag{2.2}$$

for both τ (i.e. $\pi^\pm\pi^\pm\pi^\mp$) and τ' ($\pi^\pm\pi^0\pi^0$) decay modes.

In the presence of direct CP violation, the coefficients in Eq.(1.11) are complex numbers, and the asymmetry at the center of the Dalitz plot is easily seen to be:

$$\left(\frac{\Delta g}{2g}\right)_\tau = \frac{Im[(2\alpha_1-\alpha_3)^*(\beta_1-\frac{1}{2}\beta_3)\sin(\delta_{1S}-\delta_{1M})+(2\alpha_1-\alpha_3)^*\sqrt{3}\gamma_3\sin(\delta_{1S}-\delta_2)]}{(2\alpha_1-\alpha_3)(\beta_1-\frac{1}{2}\beta_3+\sqrt{3}\gamma_3)}, \quad (2.3)$$

$$\left(\frac{\Delta g}{2g}\right)_{\tau'} = -\frac{1}{2}\times(\mathit{same\ expression\ with}\ \gamma_3\to-\gamma_3).$$

Quantities in the denominator are taken in the CP-exact limit and therefore are understood to be real.

To make contact with the notation of Ref.[7], we set:

$$(2\alpha_1-\alpha_3) = -a_C \quad (2.4)$$

$$\left(\beta_1-\frac{1}{2}\beta_3\right)+\sqrt{3}\gamma_3 = -\frac{m_\pi^2}{m_K^2}b_C \quad (2.5)$$

$$\left(\beta_1-\frac{1}{2}\beta_3\right)-\sqrt{3}\gamma_3 = \frac{m_\pi^2}{m_K^2}b_N. \quad (2.6)$$

When phases are neglected, a_C and b_C ($a_N = a_C/2$ and b_N) are the constant and the linear terms in the $K^+\to\pi^+\pi^+\pi^-$ ($K^+\to\pi^+\pi^0\pi^0$) amplitude. The sign convention adopted in Ref.[7] and in the following corresponds to replace: $A(K^+\to\pi^+\pi^+\pi^-)\to -A(K^+\to\pi^+\pi^+\pi^-)$ in Eq.(1.11). With this convention, the isospin relations are:

$$b_C+b_N=0; \qquad (\Delta I = 1/2) \quad (2.7)$$

and

$$a_N-\frac{1}{2}a_C=0. \qquad (\Delta I\leq 3/2) \quad (2.8)$$

Furthermore, lowest order in chiral perturbation theory and the $\Delta I = 1/2$ rule give:

$$a_C+\frac{2}{3}b_C=0; \qquad (\Delta I = 1/2\ \mathit{and}\ O(p^2)\ \chi\mathrm{PT}) \quad (2.9)$$

Specializing to the τ slope asymmetry, we obtain:

$$\left(\frac{\Delta g}{2g}\right)_\tau = \frac{\sin(\delta_{1S}-\delta_{1M})}{a_C}\left[\frac{a_C}{b_C}\frac{Im(b_C-b_N)}{2}-Ima_C\frac{(b_C-b_N)}{2b_C}\right] + \frac{\sin(\delta_{1S}-\delta_2)}{a_C}\left[\frac{a_C}{b_C}\frac{Im(b_C+b_N)}{2}-Ima_C\frac{(b_C+b_N)}{2b_C}\right]. \quad (2.10)$$

The r.h.s. of Eq.(2.10) and the analogous asymmetry for τ' decay vanish explicitly in the limit in which relations (2.7) to (2.9) are obeyed, since then there is only one amplitude whose phase can be transformed away [6,35].

The effective Hamiltonian at the constituent quark level consists, in fact, of various components, transforming according to the $(8_L, 1_R)$, $(27_L, 1_R)$ and $(8_L, 8_R)$ representations of chiral SU(3)xSU(3) (see Eqs.(1.15) and (1.16)).

For small values of the t quark mass the $(8_L, 8_R)$ component, which arises from the electroweak penguin diagrams, can be safely neglected. In lowest order χPT, a non-vanishing result for the asymmetry arises from the interference of the $(8_L, 1_R)$ and $(27_L, 1_R)$ amplitudes, so that, in this limit, the asymmetry is necessarily suppressed by the small ratio (see Table V):

$$\omega = \frac{a_{3/2}}{a_{1/2}} = 0.045, \tag{2.11}$$

similarly to ϵ'/ϵ. When m_t increases above 150 GeV or so, the effect of the electroweak operators becomes non-negligible. The rise of the $(8_L, 8_R)$ contribution is in fact responsible for the decrease of the Standard Theory prediction of ϵ'/ϵ, which tends to vanish for $m_t \approx 200\ GeV$. As for the asymmetry, it turns out that the contribution of the $(8_L, 8_R)$ operator increases the prediction of χPT for the Dalitz plot slope, and decreases the prediction for the asymmetry of the widths. Another source of corrections to the lowest order χPT prediction is the isospin breaking u-d quark mass difference, which feeds contributions proportional to the (large) coefficient of the $(8_L, 1_R)$ into the $\Delta I = 3/2$ channel.

We describe now with some detail the lowest order χPT calculation, accounting for the electroweak penguin diagram contribution and for isospin breaking corrections [7,36,37,38]. The first ingredient is the evaluation of strong interaction rescattering phases. At the centre of the Dalitz plot one finds in χPT [7,9]:

$$\delta_{1S} = \frac{q_0}{32\pi F_\pi^2}(2s_0 + m_\pi^2) \simeq 0.13 \tag{2.12}$$

$$\delta_{1M} = -\delta_2 = \frac{q_0}{32\pi F_\pi^2}(s_0 - m_\pi^2) \simeq 0.048 \tag{2.13}$$

where s_0 is defined in (1.2) and

$$q_0 = \sqrt{1 - \frac{4m_\pi^2}{s_0}} \tag{2.14}$$

The $(8_L, 8_R)$ component of the effective Hamiltonian is accounted for by adding in Eq.(1.18)

$$\mathcal{L}_W^{(2)}|_{(8,8)} = 2c_8 F_\pi^2 \langle \lambda_6 U^\dagger Q U \rangle \qquad (2.15)$$

with $Q = diag(2, -1, -1)$.

In terms of the coefficients c_2, c_3 and c_8 defined in Eqs.(1.18) and (2.15), one has:*

$$a_C = \frac{1}{F_\pi^3 F_K} m_K^2 \left(-\frac{2}{3} c_2 + \frac{24}{9} c_3 \right) - \frac{12}{F_\pi F_K} c_8, \qquad (2.16)$$

$$b_C = \frac{1}{F_\pi^3 F_K} m_K^2 \left(c_2 + 26 c_3 \right), \qquad (2.17)$$

$$b_N = \frac{1}{F_\pi^3 F_K} m_K^2 \left(-c_2 + 19 c_3 - \frac{9}{2\sqrt{2}} \omega \Omega_{IB} c_2 \right) - \frac{9}{F_\pi F_K} c_8, \qquad (2.18)$$

where $\Omega_{IB} \simeq 0.25$ represents the effect of isospin breaking.

Introducing the dimensionless quantities:

$$Im L_2 = \frac{m_K^2}{F_\pi^3 F_K} Im c_2 \qquad (2.19)$$

$$Im L_3 = \frac{m_K^2}{F_\pi^3 F_K} \frac{10}{3} Im c_3 \qquad (2.20)$$

$$Im L_8 = \frac{2}{F_\pi F_K} Im c_8 \qquad (2.21)$$

one can express the asymmetry as a superposition of the $Im L_i$ with coefficients determined by the experimental (real) amplitudes. In turn, the $Im L_i$ are derived from a theoretical determination of the matrix elements of the various components of the effective Hamiltonian ($1/N_c$ expansion [37], or Lattice QCD [39,40,41,42]).

The final expression is then:

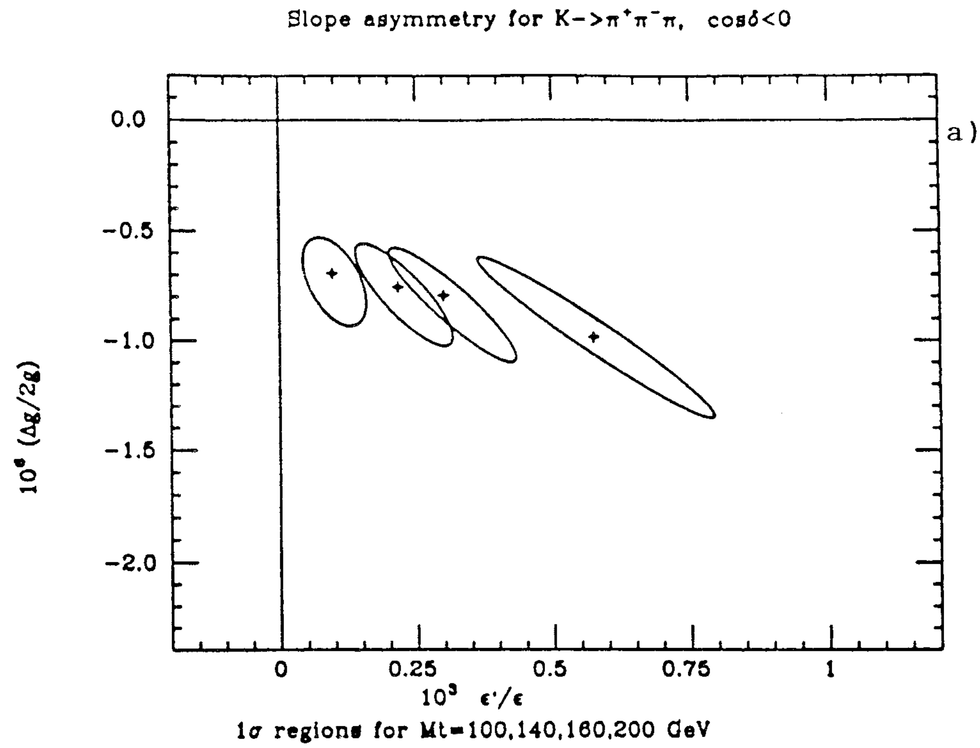
$$\frac{\Delta g}{2g} = \sin(\delta_{1S} - \delta_{1M}) \frac{1}{a_C} \sum_{2,3,8} Im L_i \mathcal{A}_i + \sin(\delta_{1S} - \delta_2) \frac{1}{a_C} \sum_{2,3,8} Im L_i \mathcal{B}_i, \qquad (2.22)$$

with:

$$\mathcal{A}_2 = \left(1 + \frac{9}{4\sqrt{2}} \omega \Omega_{IB} \right) \frac{a_C}{b_C} + \frac{1}{3} \frac{b_C - b_N}{b_C} \simeq +7.67 \times 10^{-2} \qquad (2.23)$$

---

* Normalizations are such that c_2, $\frac{10}{3} c_3$ and c_8 are equal to the coefficients c_2, c_3 and c_8 of Ref.[7], multiplied by $\frac{F_K}{F_\pi}$.



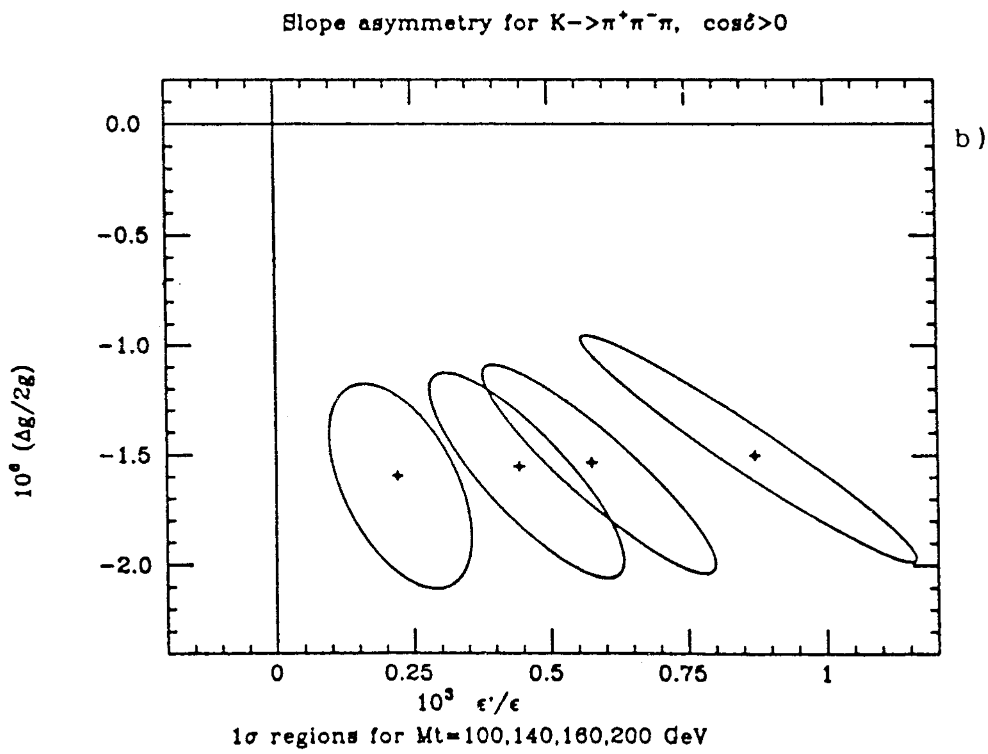


Figure 5: Combined behaviour of the slope asymmetry for $K^{\pm} \to \pi^{\pm}\pi^{\pm}\pi^{\mp}$ decays and of ϵ'/ϵ as functions of the t quark mass.

$$\mathcal{A}_3 = \frac{21}{20}\frac{a_C}{b_C} - \frac{2}{5}\frac{b_C - b_N}{b_C} \simeq -1.70 \tag{2.24}$$

$$\mathcal{A}_8 = \frac{9}{4}\frac{a_C}{b_C} + 3\frac{b_C - b_N}{b_C} \simeq +5.58 \tag{2.25}$$

$$\mathcal{B}_2 = -\frac{9}{4\sqrt{2}}\omega\Omega_{IB}\frac{a_C}{b_C} + \frac{1}{3}\frac{b_C + b_N}{b_C} \simeq -0.12 \tag{2.26}$$

$$\mathcal{B}_3 = \frac{27}{4}\frac{a_C}{b_C} - \frac{2}{5}\frac{b_C + b_N}{b_C} \simeq -4.61 \tag{2.27}$$

$$\mathcal{B}_8 = -\frac{9}{4}\frac{a_C}{b_C} + 3\frac{b_C + b_N}{b_C} \simeq +0.42 \tag{2.28}$$

In Table VII we report the values of $\frac{ImL_i}{a_C}$ obtained in [7] by combining the calculation of the short-distance coefficients performed in [43] ($m_t = 140\ GeV$), with the B-factors computed in Lattice QCD. The two lines correspond to the two possible solutions for $\cos\delta$ [42], where δ is the imaginary phase of the CKM matrix. In Table VI are reported the corresponding results for the CP violating asymmetries, with the $\pm$ representing the 1σ interval (while in Table VII only the central values are shown as an indication). Also, on the same Table is reported, for a comparison, the asymmetry resulting in the $1/N_C$ treatment of [37], by using there as an input the same value of ϵ'/ϵ (for $m_t = 140\ GeV$, $\cos\delta > 0$) as used in [7]. Finally, Figure 5 shows the combined behaviour of the Dalitz plot asymmetry for the τ mode and of ϵ'/ϵ as functions of the t quark mass.

As one can see from Table VI, total width asymmetries are estimated to be much smaller than slope asymmetries.

Eq.(2.10) also allows a simple discussion of possible higher order χPT effects, which have been advocated in [38] as the source of a two orders of magnitude enhancement of the asymmetry. Essentially, we follow here the discussion of Ref.[44].

Higher order chiral corrections can be important, because they can introduce further octet effective operators, to give independent phases to the amplitudes of the two I=1 states. CP violating interference of these two amplitudes would avoid the suppression factor ω, Eq.(2.11). Therefore one could naively hope to gain an enhancement factor $\mathcal{F} \propto \frac{1}{\omega}$.

Indeed, with reference to Eq.(2.10), we consider the first term only (the second one may be neglected as it involves the $\Delta I = 3/2$ Hamiltonian in an essential way), and after using Eq.(2.7) we rewrite it as:

$$\left(\frac{\Delta g}{2g}\right)_\tau \cong \frac{1}{a_C}\sin(\delta_{1S} - \delta_{1M})\left[\left(\frac{a_C}{b_C} + \frac{2}{3}\right) Imb_C - \left(\frac{2}{3}Imb_C + Ima_C\right)\right] \tag{2.29}$$

The first term in the square bracket is anyway suppressed by the combination of experimental amplitudes (see Table IV), which give:

$$\left(\frac{a_C}{b_C}+\frac{2}{3}\right)\simeq 0.04\simeq\omega .$$

We can expand the second term according to χPT as:

$$\left(\frac{2}{3}Imb_C+Ima_C\right)=\left(\frac{2}{3}Imb_C+Ima_C\right)^{(2)}+\left(\frac{2}{3}Imb_C+Ima_C\right)^{(4)}+\cdots$$

Therefore, the relative size of the correction is represented to a good approximation by the factor $\mathcal{F}$:

$$\mathcal{F}\simeq\frac{\left(\frac{2}{3}Imb_C+Ima_C\right)^{(4)}}{\left(\frac{2}{3}Imb_C+Ima_C\right)^{(2)}} \tag{2.30}$$

Coefficients are such that:

$$\left(\frac{2}{3}Imb_C+Ima_C\right)^{(2)}\simeq\omega\,(Ima_C)^{(2)},$$

and therefore one derives:

$$\mathcal{F}\simeq\frac{(Ima_C)^{(4)}}{\omega\,(Ima_C)^{(2)}}. \tag{2.31}$$

More precisely, it can be seen that in the decomposition (1.26) of the $O(p^4)$ $K\to 3\pi$ amplitudes, only weak counterterms are relevant to (2.31).

Chiral corrections are seen to be quite reasonable for CP conserving amplitudes, introducing in that case effects of the order of 40% or less (Table IV). We can make the assumption that the chiral expansion similarly works also for the CP violating amplitudes (although not yet verified experimentally, this assumption seems quite plausible). Thus, the order p^4 enhancement factor in (2.31) should be expected to be of the order of $\mathcal{F}\sim 10-20$ or so (the upper figure corresponding to the extreme case $(Ima_C)^{(4)}=(Ima_C)^{(2)}$), which would bring the predictions in Table VI from the 10^{-6} level to the 10^{-5} level at most.

K^0 decays

The $\pi^+\pi^-\pi^0$ decay of the K_S is in principle not forbidden by CP conservation. It is so, however, at the centre of the Dalitz plot, where symmetry in pion momenta forbids the appearance of odd values of the relative angular momenta. $3\pi^0$ decay of K_S is altogether CP violating.

Taking into account the mass-matrix mixing effect, one writes:

$$\eta_{+-0} = \frac{A(K_S \to \pi^+\pi^-\pi^0;\ I=1)}{A(K_L \to \pi^+\pi^-\pi^0)} = \epsilon + \epsilon'_{+-0}, \tag{2.32}$$

and similarly for the decay $K_S \to 3\pi^0$, with an analogous definition of η_{000} and ϵ'_{000}.

Theoretical analyses of the direct CP violation parameters, $\epsilon'_{3\pi}$, can be found in [6,36,37], and they proceed along much similar lines to those discussed previously for charged kaon decays. In lowest order χPT, one finds:

$$\epsilon'_{+-0} = \epsilon'_{000} = -2\epsilon'_{2\pi}. \tag{2.33}$$

The more recent discussions of [36,37,45] include the electroweak penguin contributions, as well as higher order chiral corrections. Analogously to the charged kaon case, it seems unlikely that the lowest order prediction (2.33) can be increased by more than one order of magnitude. In this case, the $K_S \to 3\pi^0$ branching ratio is predicted to be:

$$BR(K_S \to 3\pi^0) = |\epsilon_S|^2 \frac{\tau_S}{\tau_L} BR(K_L \to 3\pi^0) = 2.52 \times 10^{-9}\ |\frac{\epsilon_S}{\epsilon}|^2 \tag{2.34}$$

For $K_S \to \pi^+\pi^-\pi^0$, the problem is the presence of the CP conserving amplitude which can obscure the CP violating one. Since CP conserving and CP violating amplitudes do not interfere in the total rate due to their different dependences on the kinematical variables, the effect of (mass-mixing) CP violation in the rate for this channel is of order $|\epsilon|^2$, bringing it down to the level of $\sim 1.2 \times 10^{-9}$.

A possible strategy to search for this effect at the ϕ-factory should be the measurement of the interference between K_S and K_L amplitudes *via* the proper time distributions of combined decays of K^0 and $\bar{K}^0$ into 3π and $\pi l \nu$, at negative time differences Δt. This kind of approach was introduced in the previous section with regard to the determination of the CP conserving amplitude. As emphasized there, the advantage would be that for $\Delta t < 0$ there is enhanced sensitivity to the K_S, in contrast to fixed-target experiments. The expected statistical sensitivity on $\eta_{3\pi}$ at the ϕ-factory can be assessed by making appropriate changes in Eq.(1.39), and the relevant formulae can be found in Ref.[31]. It turns out the statistical sensitivity to $\eta_{3\pi}$ at the ϕ-factory is of the order of $(7-8) \times 10^{-3}$, which is rather far from the expected value $|\eta_{3\pi}| \simeq |\epsilon| \simeq 2.3 \times 10^{-3}$.* Nevertheless, it should still be worthwhile

* Actually, this is not so far from the sensitivity to $\eta_{3\pi}$, of the order of 5×10^{-3}, which should be reached at the FNAL $E621$ experiment [46].

to make an attempt to improve by this method the experimental upper limits on $\eta_{3\pi}$, which presently are $|\eta_{+-0}| < 0.35$ and $|\eta_{000}| < 0.32$ [1].

In the decay $K_L \to \pi^+\pi^-\pi^0$ the mass-mixing CP violating amplitude is suppressed by an angular momentum barrier and by the $\Delta I = 1/2$ rule, analogously to the CP conserving amplitude of the K_S decay into the same channel. This CP violation should manifest itself in the coefficient j of the Dalitz plot (1.10), arising from the interference between CP-even and CP-odd $K_L \to \pi^+\pi^-\pi^0$ amplitudes. According to the estimate of Ref.[47], the order of magnitude should be $j \simeq 1.2 \times 10^{-4}$, while the present determination is $j = (0.11 \pm 0.08) \times 10^{-2}$ [1]. Thus, the statistics at Daϕne would be somewhat marginal to the predicted value for this parameter. Nonetheless, a significant improvement of the experimental value should be possible.

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Table captions

Table I: Branching ratios of $K \to 3\pi$ decays, and number of decays at Daϕne (in 10^7 s).

Table II: Experimental value of Dalitz plot parameters in Eq.(1.10) (from Ref.[1]).

Table III: Isospin amplitudes for $K \to 3\pi$ (Eq.(1.11)).

Table IV: Experimental values of isospin amplitudes for $K \to 3\pi$, and comparison with theoretical predictions from chiral perturbation theory (χPT).

Table V: Values of isospin amplitudes for $K \to 2\pi$.

Table VI: Experimental values and theoretical predictions for the CP violating asymmetries in τ and τ' decay modes.

Table VII: Theoretical predictions (from Ref.[7]) for the CP violating parameters defined in Eqs.(2.19)-(2.21).

TABLE I

Integrated Luminosity : $5 \times 10^{32} cm^{-2} s^{-1} \times 10^7 s$			
channel	BR	1 yr Daϕne	present statistics
$K^{\pm} \to \pi^{\pm}\pi^{\pm}\pi^{\mp}$	$5.59 \pm .05\%$	5.0×10^8	10^6
$K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$	$1.73 \pm .04\%$	1.5×10^8	5×10^4
$K_L \to \pi^+\pi^-\pi^0$	$12.38 \pm .21\%$	1.4×10^8	5×10^5
$K_L \to \pi^0\pi^0\pi^0$	$21.6 \pm .08\%$	2.4×10^8	10^3
$K_S \to \pi^+\pi^-\pi^0$	$< 4.9 \times 10^{-5}$	$(3.4 - 6.8) \times 10^2$	6×10^2

TABLE II

channel	$(++-)$	$(+00)$	$(+-0)$	(000)
$\Gamma(10^6 s^{-1})$	4.52 ± 0.04	1.40 ± 0.03	2.39 ± 0.04	4.19 ± 0.16
g	$-.215 \pm .0035$	$.594 \pm .019$	$.67 \pm .014$	–
h	$.012 \pm .008$	$.035 \pm .015$	$.008 \pm .007$	–
k	$.01 \pm .003$	–	$.01 \pm .002$	–

TABLE III

	constant	linear	quadratic
$\Delta I = 1/2$	α_1	β_1	ξ_1, ζ_1
$\Delta I = 3/2$	α_3	β_3, γ_3	ξ_3, ξ_3', ζ_3

TABLE IV

Units 10^{-8}							
	exp. fit Ref.[10]	χPT O(p^2) Eqs.(1.22),(1.23)	χPT O(p^4) Ref.[10]	χPT O(p^4) Ref.[23]	1/N O(p^4) Ref.[24]	1/N O(p^4) Ref.[25]	VMD O(p^4) Ref.[18]
α_1	$91.7 \pm .3$	74.0	91.8	*input*	88.8	92	$68 \sim 65$
β_1	$-25.7 \pm .3$	-16.5	-25.6	*input*	-26.5	-26	$-24 \sim -26$
ζ_1	$-.47 \pm .15$	–	$-.6$	$-.47 \pm .18$	$-.2$	$-.2$	$-.2 \sim .0$
ξ_1	$-1.5 \pm .3$	–	-1.5	$-1.58 \pm .19$	$-.9$	$-.8$	$-1.3 \sim -.9$
α_3	$-7.4 \pm -.5$	-4.1	-7.6	*input*	-5.6	-5.9	–
β_3	$-2.4 \pm .4$	-1.0	-2.5	*input*	-1.9	-1.4	–
γ_3	$2.3 \pm .3$	1.8	2.5	*input*	2.5	2.4	–
ζ_3	$-.21 \pm .08$	–	$-.02$	$-.011 \pm .006$	$-.01$	$-.01$	–
ξ_3	$-.1 \pm .2$	–	$-.05$	$.092 \pm .030$	.02	.00	–
ξ_3'	$-.2 \pm .5$	–	$-.08$	$-.033 \pm .077$	.08	–	–

TABLE V

	exp. fit	χPT O(p^2)	χPT O(p^4)*Ref.*[10]	*Units*
$a_{1/2}$	$.4699 \pm .0012$	.4698	.4698	*KeV*
$a_{3/2}$	$.0211 \pm .0001$	.0211	.0211	*KeV*
$\delta_2 - \delta_0$	-61.5 ± 4	0	-29	*degrees*
c_2/F_π^2		.95	$.662 \pm .005$	10^{-7}
c_3/F_π^2		$-.009$	$-.0083 \pm .0002$	10^{-7}

TABLE VI

	exp. data	Ref.[7] $\cos(\delta) < 0$	Ref.[7] $\cos(\delta) > 0$	Ref.[37]
$(\Delta g/2g)\|_{\tau}$	$(-1.40 \pm 1.06) \times 10^{-2}$	$(-.8 \pm .3) \times 10^{-6}$	$(-1.5 \pm .5) \times 10^{-6}$	$(-1.8 \pm .6) \times 10^{-6}$
$(\Delta\Gamma/2\Gamma)\|_{\tau}$	$(.07 \pm .12) \times 10^{-2}$	$(-2.7 \pm 1.0) \times 10^{-8}$	$(-5 \pm 2) \times 10^{-8}$	–
$(\Delta g/2g)\|_{\tau'}$	–	$(.5 \pm .2) \times 10^{-6}$	$(1.0 \pm .4) \times 10^{-6}$	–
$(\Delta\Gamma/2\Gamma)\|_{\tau'}$	$(0.0 \pm 0.6) \times 10^{-2}$	$(1.1 \pm .4) \times 10^{-7}$	$(2.0 \pm .8) \times 10^{-7}$	–

TABLE VII

	ImL_2/a_C	ImL_3/a_C	ImL_8/a_C	$(\Delta g/2g)\|_{\tau}$
$\cos(\delta) < 0$	3.1×10^{-5}	1.9×10^{-7}	-2.2×10^{-7}	$-.8 \times 10^{-6}$
$\cos(\delta) > 0$	6.1×10^{-5}	3.8×10^{-7}	-4.3×10^{-7}	-1.5×10^{-6}