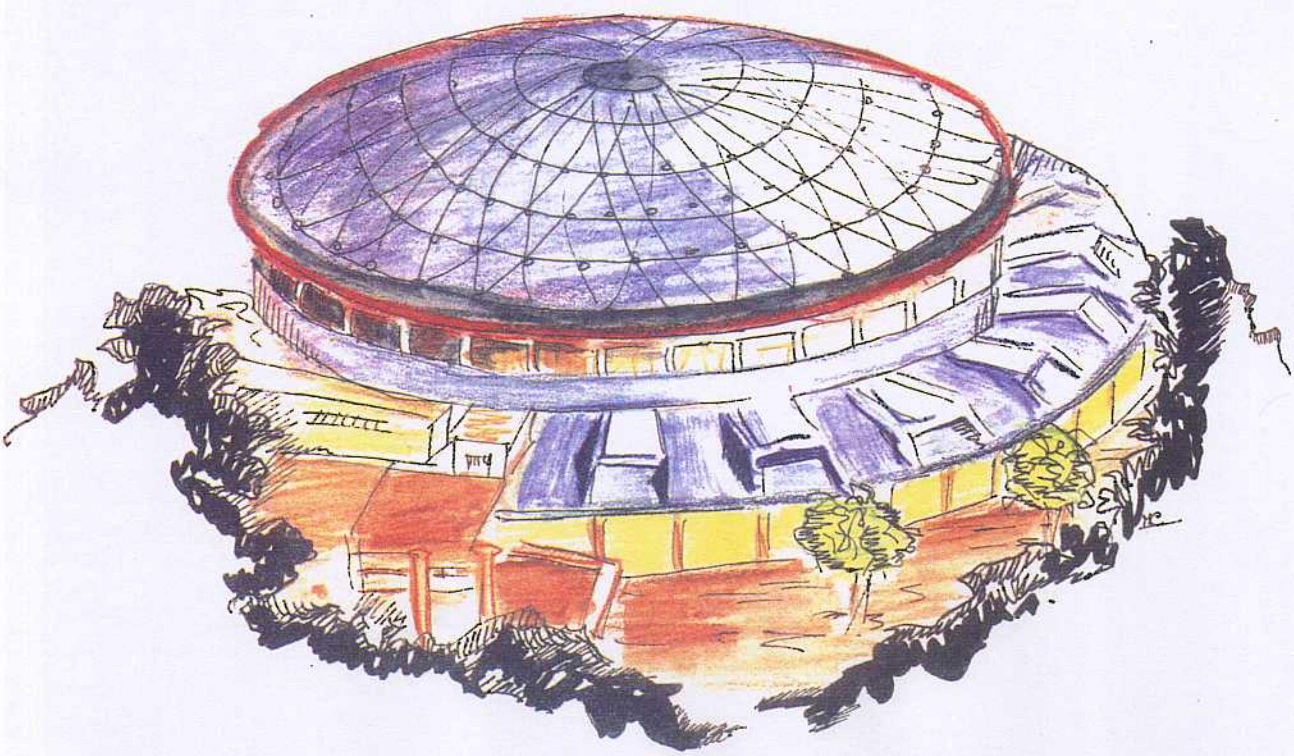
Laboratori Nazionali di Frascati

LNF-92/075 (P)
28 Settembre 1992

Giuliano Preparata:

**WHY IS GENERALLY ACCEPTED QCD WRONG?
TOWARDS THE TRUE QCD**

Invited Lecture at the
First German Polish Symposium on Particles and Fields
28 April – 1 May, 1992, Rydzyna Castle, Poland



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WHY IS GENERALLY ACCEPTED QCD WRONG? TOWARDS THE TRUE QCD

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ABSTRACT

We discuss the flaws of GAQCD, in the light of the essential instability of the QCD perturbative ground state. A new, very promising strategy to solve QCD is presented, that arises from an understanding of the causes of the instability of the perturbative vacuum.

There is a ghost that is wandering around in the world of high energy particle physics, that since 1985 threatens the generally accepted picture of hadrons and their interactions (Generally Accepted QCD, GAQCD): how many high energy physicists know of his existence? How many really understand how strong and deep are the foundations of GAQCD, that this ghost is undermining? This lecture is about GAQCD and this ghost, that we shall call the "Essential Instability of the Perturbative Ground State (PGS)", and of the change in our views that must follow the collapse of GAQCD that will result from the inevitable realization that this ghost has got flesh and bones.

There must have been, and indeed there were many good exorcists that in their different capacities as polemists, referees of prestigious journals, conference organizers, funding agencies' officials etc. have done a very good job to keep this ghost away from the view of especially the young, and to this aim the flexible tools of GAQCD, an incredibly effective laboratory for computer assisted epicycles, have also played a fundamental role. Be that as it may, my invincible optimism and my deep belief in the final triumph of scientific truth (even

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though possibly remote) induces me to carry the very heavy burden to defy such a well oiled, effective machine that has been put in place to protect the generally accepted view, or better ideology, from the devious practices of the ghost. This lecture is one of the many, so far not very successful, trials to carry out my task.

1. - GAQCD: A Sketch

The Generally Accepted Quantum Chromo Dynamics is not, contrary to common belief, a well founded (if not rigorous, as anybody will have no difficulty to admit) consequence of the Quantum Field Theory (QFT) system known as QCD, but it is rather an ideology developed through the last twenty years, motivated by both experimental and theoretical observations, that rests upon two pillars:

- Perturbative QCD (PQCD) at short space–time distances,
- Lattice Gauge Theories (LGT's) at long space–time distances.

These approximations, that allow one to make well defined inferences about a theory otherwise impossible to handle, are as noticed above a fundamental part of the GAQCD ideology and have their theoretical motivation in the early discovery[1] that QCD is "asymptotically free", i.e. the effective running coupling constant $g(\mu)$ (μ is the mass scale at which one is computing the theory) vanishes when $\mu \to \infty$. However, for practical applications $\mu \to \infty$ is hardly a meaningful statement, there must exist a finite well defined scale parameter, called Λ_{QCD}, that acts as a "divide" between the region where the effective coupling constant is large ($\mu \lesssim \Lambda_{QCD}$) and the region ($\mu \gg \Lambda_{QCD}$) where the coupling constant is small. If this happens, and this is by no means a trivial occurrence, then GAQCD stipulates that:

$$\text{GAQCD} \underset{\substack{\mu \gg \Lambda_{QCD} \\ \text{short distances}}}{\Rightarrow} \text{PQCD} \quad ; \tag{1a}$$

$$\text{GAQCD} \underset{\substack{\mu \lesssim \Lambda_{QCD} \\ \text{long distances}}}{\Rightarrow} \text{LGT} \quad , \tag{1b}$$

i.e. for scales μ much larger than Λ_{QCD}, that "experimentally" has been "determined" to be

$$\Lambda_{QCD} \simeq (100 \div 200)\ \text{MeV}, \tag{2}$$

due to the smallness of the effective coupling constant $g(\mu)$ one can use Perturbation Theory (PQCD); while for $\mu \lesssim \Lambda_{QCD}$, the true hadronic scale, QCD can well be approximated by a LGT on a rather coarse lattice.

GAQCD rests on a series of theoretical "miracles" (which, of course, are not absolutely impossible), namely:

(i) the existence of a finite scale (Λ_{uv} is the ultraviolet cut–off)

$$\Lambda_{QCD} = \Lambda_{uv} \exp - \frac{c}{g(\Lambda_{uv})^2} \quad , \tag{3}$$

obeying the renormalization group equations with the perturbative β–function, yielding a well defined coefficient c, solely dependent on the gauge group and on the number of fermionic "flavours";

(ii) the validity in the limit of small coupling constant of perturbation theory even for a non–abelian gauge–theory, where the gauge fields are coupled directly, and not through charged fermionic matter as in the case of abelian gauge theories like QED;

(iii) the possibility of describing the physics at large distances (the physics of hadrons) discretizing space–time as a lattice structure with a lattice constant a $\simeq$ .1 F, not too much smaller than the typical hadron size.

As we shall see in a moment all such miracles demand that the Perturbative Ground State (PGS) of QCD is a good approximation of the Real Ground State (RGS) when $\mu \gg \Lambda_{QCD}$. And even though, in the light of the overwhelming evidence that at short space–time distances "Nature reads the free–field theory book" (as indicated by the successes of the Parton–Model), this conjecture appears innocuous and most probably true, in the rest of this Lecture I shall endeavour to show that it is definitely false, and shall give also a physical explanation why our intuition fares so poorly. The falsity of the conjecture evidently implies the falsity of GAQCD.

2. – PROBING FOR Λ_{QCD}: THE SAVVIDY APPROACH

The question that we wish to answer is: does Λ_{QCD} exist?, which is equivalent to: is the PGS at short space–time distances ($d \leq \Lambda_{QCD}^{-1}$) the state of lowest energy (density)?

The idea of a possible way to make such a test belongs to Savvidy [2], who in 1977 proposed to study the stability of gauge field configurations fluctuating quantum mechanically around a class of classical solutions of the Yang–Mills equations that describe (mod. gauge–tranformations) a constant chromomagnetic field pointing in a fixed direction in space and isospace. Such solutions can be expressed as:

$$A_k^c = \frac{1}{2} \epsilon_{krs}\, x_r\, u_s\, \alpha^c\, B \quad , \tag{4}$$

where $\vec{u}$ is a space direction and α^c represents an isospace direction; the energy density of the configuration (4) is, as well known,

$$E_{cl} = \frac{1}{2} B^2 . \tag{5}$$

To second order in the small "bare" coupling constant g_o ($g_o = g(\Lambda)$) Savvidy gets for the density E(B) in SU(2):

$$E(B) = E(O) + \frac{B^2}{2} - (g_o B)^2 \frac{11}{48\pi^2} \ln\left(\frac{\Lambda^2}{g_o B}\right) + O\left[g_o^4 B^2 \ln\left(\frac{\Lambda^2}{g_o B}\right)\right] , \tag{6}$$

where the minus sign in front of the logarithmic terms is most remarkable, and the structure of the fourth order correction has been subsequently determined by Abbott [3]. The importance of the result (6) for answering our questions lies in the fact that E(B) exhibits a minimum for

$$g_o B^* = \Lambda^2 \exp - \frac{24\pi^2}{11 g_o^2} = \Lambda_{QCD}^2 , \tag{7}$$

that coincides with the well known formula for the "running" coupling constant

$$g(\Lambda)^2 = \frac{24\pi^2}{11 \ln\left(\frac{\Lambda^2}{\Lambda_{QCD}^2}\right)} . \tag{8}$$

Thus the existence of a minimum for E(B) at $g_o B^*$ given by (7), which can be identified with the finite scale Λ_{QCD}^2, that in turn sets the scale for the "running" of the coupling constant to zero for very large mass scales Λ, shows that the answer that Savvidy calculation gives to our question is definitely positive: according to him GAQCD passes the test!

But one year later, in 1978, Nielsen and Olesen [4] discovered a flaw in Savvidy's calculation, for if one writes the vector potential (we shall also work in SU(2))

$$A_\mu^c = A_{\mu,cl}^c + \eta_\mu^c , \tag{9}$$

where $A_{\mu,cl}^c$ is the classical solution of the Yang–Mills equations corresponding to a constant chromomagnetic field, and η_μ^c the field of quantum fluctuations around that classical configuration. One knows that $A_{\mu,cl}^c$ is stable for small η_μ^c (as implicitly assumed in Savvidy's perturbative calculation) if on expanding the action

$$S[A] = S[A_{cl}] + \frac{1}{2} \int \frac{\delta^2 S}{\delta\eta_\mu^c \delta\eta_\nu^d} \eta_\mu^c \eta_\nu^d + \ldots\ldots \quad , \tag{10}$$

the operator

$$O_{\mu\nu}^{cd} = \frac{1}{2} \frac{\delta^2 S}{\delta\eta_\mu^c \delta\eta_\nu^d} \quad , \tag{11}$$

has a positive semidefinite spectrum.

The diagonalization of this operator has been performed in 1931 by Landau, who showed that the eigenvalues of this operator are

$$E_{np}^2 = p^2 + g_o B\,(2n+1) - 2\, g_o B S_3 \quad , \tag{12}$$

where p is the momentum of the massless particle (the gluon) along the direction $\vec{u}$ of the constant chromomagnetic field, and n is a non–negative integer which labels the quantized levels of a two–dimensional harmonic oscillator representing the dynamics of the quantum mechanical motion transverse to $\vec{u}$. Please note the Zeeman splitting with a relativistic gyromagnetic ratio of 2.

It is a remarkable aspect of (12) that for particles of spin $|\vec{S}| > \frac{1}{2}$ there is always a portion of the spectrum that is negative: in our case $|\vec{S}| = 1$ (the gluons are spin–1 particles) and for

$$n = 0 \text{ and } |\, p \,| \leq (g_o B)^{1/2} \quad , \tag{13}$$

the action is a maximum and not a minimum. This means that in the sector (13) the quantum fluctuations cannot be small, as required by any perturbative calculation. Thus Savvidy calculation, being perturbative, cannot give the correct answer.

What is the correct answer? Will GAQCD survive it? These questions could not be answered by Nielsen and Olesen, who lacked both the tools and the expectations to get to the correct answer, which had to wait until 1985 to finally surface out.

3. – CORRECTING SAVVIDY: THE ESSENTIAL INSTABILITY OF THE PGS OF QCD

What is the effect on E(B) of the inevitable large fluctuations that affect the modes in the Sector (13), the Unstable Modes? As alluded to above the answer to this question required first the consciousness of the physical relevance of the question and then the development of gauge invariant variational techniques [5], to deal with non-perturbative quantum fluctuations. The final answer [6] is both appalling and very simple:

$$E(B) = E(0) + a\,(g_0B)^2 - \frac{11}{48\pi^2}\,(g_0B)^2 \ln\left(\frac{\Lambda^2}{g_0\,B}\right) + O\left(g_0^4\,B^2 \ln\left(\frac{\Lambda^2}{g_0\,B}\right)\right). \quad (14)$$

Note the complete disappearance of the classical term $\frac{B^2}{2}$, which gets replaced by the quantum-fluctuation term a $(g_0\,B)^2$. The actual value of a could not be computed precisely by variational methods and had to be determined for both SU(2)[7] and SU(3)[8] through lattice Monte Carlo simulations.

The minimum is now instead of (7)

$$g_0B^* = K\,\Lambda^2 \quad , \qquad (15)$$

where K is a finite number depending on the constant a, and Λ is the ultraviolet cut-off. This means that there is no way to tune the coupling constant $g_0 = g\,(\Lambda)$ so as to render g_0B^* finite and identify it with Λ^2_{QCD}; Λ_{QCD} simply does not exist. Also the difference between the energy density $E(B^*)$ and the PGS energy density E(0) (h is a finite number)

$$E(B^*) - E(0) = -\,h\,\Lambda^4 \qquad (16)$$

cannot be made finite, and this implies that the PGS is essentially unstable, i.e. it is unstable at all scales up to the ultraviolet (absolute) cut-off Λ. This result, having been confirmed by lattice calculations in both SU(2)[7] and SU(3)[8], must be considered as completely truthful. The lattice calculation, as remarked above, could also supply an important piece of information, namely a, in Eq. (14), which in turn allows us to determine K in Eq. (15). One gets:

$$K \simeq 10^{-5} \qquad \text{for SU(2)} \;, \qquad (17a)$$
$$\simeq 10^{-8} \qquad \text{for SU(3)} \;. \qquad (17b)$$

Before spelling out in detail the consequences for GAQCD of Eqs. (14), (15), (16) and (17), let me briefly discuss the physical origin of the essential instability of the PGS.

The general expectation that the PGS is a good representation of the RGS at short space-time distances was clearly based on the phenomenological successes of the Parton Model, for, as mentioned above, from the QFT point of view the short distance stability of the PGS appears really miraculous, due to both the direct coupling of the gauge-fields and their bosonic character that allow them to "condense" with very large, coherent amplitudes, should such configurations be energically fovourable. The existence of Unstable Modes in a constant magnetic field, implied by (12) is just a very clear signal that such condensation indeed occurs, for in so doing the gauge–system lowers the enormous ($O(\Lambda^4)$) zero point energy density of the PGS by the large amount (16). One may add that a similar phenomenon is seen to occur in condensed matter and leads to the dramatic phenomena of ferromagnetism [9].

4. – THE COLLAPSE OF GAQCD

The main result of our analysis Eq. (15), as already stressed, implies a fundamental failure of the general applicability of the perturbative renormalization group. The non–existence in general of a finite Λ_{QCD} shows that there is no guarantee that the calculations of perturbation theory (PT) may yield a faithful representation of the short–distance dynamics of QCD. Actually PT, as a general dynamical approximation for the (small) quantum fluctuations around the PGS, shares the fate of dynamical irrelevance of the PGS. In this way we see that a pillar (PQCD) of GAQCD can't but collapse, thus impairing its short distance strategies. But the ruin engendered by (15) and (17) is not restricted to short distance physics only, it extends also to the long distance approach (LGT) of GAQCD.

Indeed the very small value of K for SU(3) [Eq. (17b)] makes also the lattice strategy impracticable, and this can be simply appreciated in the following way. From (17b) we have that in SU(3):

$$(gB^*)^{1/2} \simeq 10^{-4}\,\Lambda = 10^{-4}\frac{\pi}{a} \quad , \tag{18}$$

where the last equality derives from the fact that in a lattice the momentum cut–off lies at the boundary of the first Brillouin zone $\frac{\pi}{a}$ (a is the lattice constant). Let's take a hadronic universe of the minimum size of $d \simeq 1F$ (necessary to contain at least one hadron), then the minimum number L of sites per dimension is $L = \frac{d}{a}$. If we now wish that the chromomagnetic condensate be at least comparable with a typical hadronic scale μ, which we may choose as the "string tension" ($\mu \simeq .45$ GeV, from onium–spectroscopy), then we have

$$(gB^*)^{1/2} \simeq \mu \simeq 10^{-4}\frac{\pi}{a} \Rightarrow a \simeq 10^{-4}\frac{\pi}{\mu} \quad , \tag{19}$$

which implies

$$L \simeq \frac{d\mu}{\pi}\,10^4 \simeq 10^4 \,. \tag{20}$$

Considering that the largest lattices one can work with today have $L \simeq 30$, Eq. (20) means that in order to simulate the dynamics of the condensation of a classical magnetic field of the size of at least the "string tension" one needs computers $(\frac{L}{30})^4 \simeq 10^{10}$ times larger than presently available. An obviously impossible dream!

To summarize, GAQCD is wrong because:

(a) the PGS of QCD (B = 0) is essentially unstable, thus preempting Asymptotic Freedom at short distances;

(b) lattice calculations cannot hope to reproduce the violent magnetic condensation of QCD unless one is able to handle lattices with $L \gtrsim 10^4$.

GAQCD is thus seen to have no logically consistent foundations.

5. – WHAT COMES NEXT? TOWARD THE TRUE QCD

GAQCD, at least logically, is thus seen to collapse in both its strongholds. What should one do? One must remember the admonition of the great spanish philosopher Josè Ortega y Gasset: «El hombre pio y honrado contrae, cuando niega, la obligaciòn de una nueva afirmacion» [10] that reminds the pious and honourable man that when he demolishes he assumes the moral responsibility to build something new.

This I set out to do just at the time when the essential instability of the PGS was discovered [11], by analysing the main features of the configuration of the non–abelian gauge field fluctuating, with minimum energy density, around a constant chromomagnetic field. Our variational and lattice calculations have shown that such state of minimum energy which we shall call the Savvidy state, is much lower in energy density than the PGS but it is by no means the state of minimum energy density, i.e. the RGS. Thus in order to have some idea about the RGS it appeared fruitful to investigate in detail the Savvidy state, and this is just what was done in Ref. [11].

What is then the physics of the Savvidy state? One can show [11] that in this state

(i) a fully gauge covariant magnetic field condenses with strength:

$$< F^b_{ik} > \sim \epsilon_{ikr}\, u_r\, \delta^{ba}\, g_0^2\, B^* \sim O(\Lambda^2)\,; \tag{21}$$

(ii) the lines of force of the chromoelectric field around a colour charge point in the direction $\vec{u}$ of the condensed chromomagnetic field $\vec{B}^*$;

(iii) colour is confined; i.e. quantum states with non–zero charge carry an energy $O(\Lambda)$;

(iv) the dynamics of the gauge–field is restricted to a set of finite energy "Landau states";

(a) with spectrum

$$\omega(p) = \sqrt{p^2 + \mu_g^2}\ , \tag{22a}$$

$$\mu_g^2 = 4 g_0 B^* \exp - \frac{8\pi^2}{g_0^2}\ ; \tag{22b}$$

(b) polarized perpendicular to $\vec{u}$;
(c) directed in colour space orthogonally to η^a;

(v) rotational invariance is broken.

While (i), ... , (iv) are quite good and desirable, (v) is definitely bad. So, either the Savvidy state is not the RGS, or QCD and the beautiful gauge principle on which it is based must be dismissed.

However, a detailed study of the mechanism by which a Savvidy state lowers its energy with respect to the PGS shows that in order to achieve the drastic energy reduction (16) we do not need to condense a spatially constant chromomagnetic field, but only one that is constant within a CROMOMAGNETIC NEEDLE, i.e. a spatial domain that is needle–shaped with its axis pointing in the $\vec{u}$–direction and length $l \simeq \frac{1}{\mu_g}$ – the Compton wave–length of the surviving gluons (see Eqs. (22)) – and of transverse area $A \simeq \frac{2\pi}{gB^*} \simeq O(\Lambda^{-2})$.

It is then possible to show [12] that by allowing space to break up into a dense liquid of such chromomagnetic needles, thus obtaining a

CHROMOMAGNETIC LIQUID (CML),

one can gain an extra energy density

$$\Delta E_{CML} \simeq -\mu_g^2 \left(\frac{gB^*}{2}\right) . \tag{24}$$

In the CML (i), ... , (iv) remain valid, for they hold within each chromomagnetic needle, but instead of (v) one now gets:

(v') rotational and Lorentz invariance are restored over times and spaces larger than the typical hadronic space–times (10^{-13} cm, 10^{-23} sec).

From the analysis I have just reported one may conclude that the CML is most likely the RGS of QCD, and, if nothing else, it may certainly go in the Guinness Book of Records, for it is the QCD state that by far has the low energy density discovered up to now!

If the CML is the RGS of QCD, the hadronic world consists of QCD fluctuations around the CML. What do they look like? An extensive analysis [12] shows that their dynamics is governed by the Lagrangian of Anisotropic Chromo Dynamics (ACD), which I introduced more than ten years ago [13] to incorporate phenomenologically into a gauge field theory (though in an extended space–time, the anisotropic space–time) the fundamental property of colour: confinement. We may thus express all this by:

$$\text{QCD} \underset{\text{CML}}{\Rightarrow} \text{ACD}, \tag{25}$$

which shows the intimate relation that on the CML ACD has with the fundamental QCD Lagrangian.

A complete strategy can be formulated [12] to solve ACD and according to (25) QCD, to any desired accuracy. The results so far look very promising [14].

As with all things of this world progress and growth require the collapse and destruction of old and inadequate structures. It is the lesson and the law of Evolution, I do not see why High Energy Physics of the end of this millenium should disregard them.

I thank most warmly the organizers of this Symposium, Profs. Doebner and Raczka, for having offered me one of the rare opportunities given to me to remind my colleagues that the ghost still wanders among us.

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