$\phi\rightarrow\pi^+\pi^-\gamma$ at Φ-factories

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The possible detection of the reaction φ→f_0(975)γ→π⁺π⁻γ at Daφne is discussed together with the initial and final state background radiation. The size of the effect is shown to depend crucially on the relative sign of the e⁺e⁻→ρ→π⁺π⁻γ and e⁺e⁻→φ→f_0γ→π⁺π⁻γ amplitudes. Some numerical results are also given.

Projected high-luminosity, low-energy e⁺e⁻ machines, such as the Daφne Φ-factory, will allow for the detection and measurement of rare decay modes of the well-known, low-lying vector mesons. The φ→π⁺π⁻γ decay, whose branching ratio is known to be smaller than 7×10^{-3} [1], will probably be studied in the near future due to the (relatively) clean signature of a charged pion pair with a photon of energy larger than some 20 MeV. In this case, not only the properties of the φ-meson will be explored but also those of the final pion-pair and, in particular, their resonant states such as the controversial f_0-scalar meson at 975 MeV [2]. The purpose of this communication is to discuss the possible detection of such an f_0-signal under a considerably large background. The main two sources for the latter are expected to be initial-state bremsstrahlung and ρ-formation followed by its (off-shell) decay into π⁺π⁻γ. In the first case, the pion pair is in a negative charge-conjugation state, while the opposite is true for the second as well as for the pion pair in any genuine φ→π⁺π⁻γ radiative decay. Interference effects will be important between these two latter ($C = +$) amplitudes, but those with the first one ($C = -$) will disappear when integrating over pion angles disregarding their charges [3].

The most obvious and important background comes from initial-state hard photon radiation. One obtains [4]

$$\frac{d\sigma}{dE}\bigg|_{in} = 2\sigma_0(s(1-x))H(x, s, \theta_{min})/\sqrt{s}, \qquad (1)$$

where σ_0 stands for the non-radiative e⁺e⁻→ρ→π⁺π⁻ cross-section

$$\sigma_0(s) = \frac{\pi}{3}\frac{\alpha^2}{s}|F_\rho(s)|^2(1-\xi)^{3/2}, \qquad (2)$$

with x related to the photon energy E through $x \equiv 2E/\sqrt{s}$, $\xi \equiv 4m_\pi^2/s$,

$$H(x, s, \theta_{min}) = \frac{\alpha}{\pi}\frac{2(1-x)+x^2}{x}$$

$$\times\left[\ln\left(\frac{1+\beta\cos\theta_{min}}{1-\beta\cos\theta_{in}}\right) - \frac{1}{\gamma^2}\frac{\cos\theta_{min}}{1-\beta^2\cos^2\theta_{min}}\right]. \qquad (3)$$

In eq. (3), θ_{min} is the minimal angle (with respect to the beam direction) allowed for the photons to be detected. The radiator $H(x, s, \theta_{min})$ is obtained from the angular distribution $d\sigma/d\cos\theta_\gamma \simeq \sin^2\theta_\gamma/(1-\beta^2\cos\theta_\gamma)^2$ and reduces to the well-known integrated radiator

$$H(x, s) = \frac{\alpha}{\pi}\left[\frac{2(1-x)+x^2}{x}\left(\ln\frac{s}{m_e^2}-1\right)\right], \qquad (4)$$

for $\theta_{min} = 0$. Finally $F_\rho(s)$ in eq. (2) stands for the ρ-dominated pion form factor [#1]

[#1] The use of a more sophisticated version of the pion form factor, as in ref. [5], in the φ-region, introduces numerically irrelevant effects.
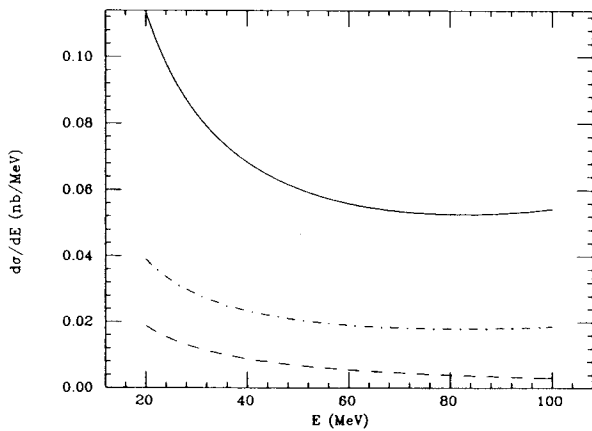
Fig. 1. Differential cross-sections as a function of the photon energy E for initial state, hard photon radiation, $e^+e^-\to\rho^0\gamma\to\pi^+\pi^-\gamma$ (solid line for $\theta_{\min}=0°$ and dot-dashed line for $\theta_{\min}=10°$) and for off-shell ρ-formation and decay $e^+e^-\to\rho\to\pi^+\pi^-\gamma$ (dashed line).

$$F_\rho(s) = \frac{M_\rho^2}{M_\rho^2 - s - i\sqrt{s}\,\Gamma_\rho}\,.$$ (5)

The resulting differential cross-section for initial-state radiation is shown in fig. 1 for $\theta_{\min}=0°$ (solid line) and for $\theta_{\min}=10°$ (dot-dashed line).

The second source of background comes from off-shell ρ-formation followed by radiative decay into a $C=+$ pion pair. The corresponding, gauge-invariant amplitude is given by

$$A(\rho\to\pi^+\pi^-\gamma) = 2\sqrt{2}\,eg\left(\frac{\epsilon p_+}{qp_+}\,\epsilon^*(p_- - \tfrac{1}{2}q^*)\right.$$

$$\left. + \frac{\epsilon p_-}{qp_-}\,\epsilon^*(p_+ - \tfrac{1}{2}q^*) + \epsilon\epsilon^*\right),$$ (6)

where $g=4.2$ comes from the total ρ-width of 149 MeV, p_+, p_- and q are the pion and photon four-momenta, and ϵ, ϵ^* are the polarizations of the photon and the ρ. The correctness of this amplitude can easily be tested by comparing with the measured decay rate of on-shell ρ-mesons into a pion pair and a photon of energy E larger than 50 MeV. One immediately obtains a partial width of 1.62 MeV in good agreement with the observed value of 1.48 ± 0.24 MeV ([1,6], see also ref. [7] [#2]). The effect of the

[#2] Notice that the $\rho\to\pi^+\pi^-\gamma$ amplitude appearing in this work apparently is not gauge invariant.

above amplitude around the ϕ-peak requires the introduction of the ρ-dominated form factor $F_\rho(s)$. One easily obtains the following differential cross-section:

$$\left.\frac{d\sigma}{dE}\right|_\rho = 2\sigma_0(s)F(x,s)/\sqrt{s}\,,$$ (7)

with

$$F(x,s)$$

$$= \frac{\alpha}{\pi}\,\frac{2}{(1-\xi)^{3/2}}\left[\left(x-(1-\xi)\frac{1-x}{x}\right)\sqrt{1-\frac{\xi}{1-x}}\right.$$

$$\left. + (1-\xi)(1-x-\tfrac{1}{2}\xi)\frac{1}{x}\ln\left|\frac{1+\sqrt{1-\xi/(1-x)}}{1-\sqrt{1-\xi/(1-x)}}\right|\right],$$ (8)

where small corrections coming from higher order terms in x have been neglected. For s on the ϕ-peak, the values of the above expression are shown (dashed line) in fig. 1. Typically, they are one order of magnitude below the previously considered background coming from the initial-state radiation.

We now turn to the ϕ-signal as produced through the $\phi\to f_0\gamma\to\pi^+\pi^-\gamma$ decay chain. Amplitudes and couplings for each step are defined according to

$$A(\phi\to f_0\gamma) = eG_s[(\epsilon^*\epsilon)(q^*q) - (\epsilon^*q)(\epsilon q^*)]$$

$$\equiv eG_s\{a\}\,,$$

$$A(f_0\to\pi^+\pi^-) = g_s\,.$$ (9)

where now ϵ^* and q^* stand for the ϕ-polarization and four-momentum. These amplitudes lead to the following decay rates:

$$\Gamma(\phi\to f_0\gamma) = \frac{1}{12\pi}\,G_s^2 e^2 |\boldsymbol{q}_\gamma|^3\,,$$

$$\Gamma(f_0\to\pi^+\pi^-) = \tfrac{2}{3}\Gamma(f_0\to\pi\pi) = \frac{1}{8\pi}g_s^2\,\frac{|\boldsymbol{p}_+|}{m_f^2}\,.$$ (10)

From these expressions and the experimental values quoted in ref. [1] one obtains the modulus of the product of the relevant coupling constants in terms of the unknown branching ratio $BR(\phi\to f_0\gamma)$:

$$|G_s g_s| = (144\pm15)\,[BR(\phi\to f_0\gamma)]^{1/2}\,.$$ (11)

The amplitude for the ϕ-decay into $f_0\gamma\to\pi^+\pi^-\gamma$ then follows:

$$A(\phi\to\pi^+\pi^-\gamma) = eg_s G_s P_f(s')\{a\}\,,$$ (12)

with

$$P_f(s') = \frac{1}{m_f^2 - s' - i m_f \Gamma_f} . \tag{13}$$

$\{a\}$ as in eq. (9), $s' = s(1-x)$ and $s = q^{*2} = M_\phi^2$ on the ϕ-peak.

As previously stated, the off-shell ρ-dominated amplitude in an e^+e^- experiment interferes with the just derived (near on-shell) one for the ϕ. The total amplitude (with $C = +$) may be written as

$$A^+(\gamma^* \to \pi^+\pi^-\gamma)$$

$$= \frac{e}{f_\rho} F_\rho(s) A(\rho \to \pi^+\pi^-\gamma) + \frac{e}{f_\phi} F_\phi(s) A(\phi \to \pi^+\pi^-\gamma), \tag{14}$$

where $f_o = -3f_\rho/\sqrt{2} = -3g$, $F_{\rho,\phi}(s)$ are form factors and $A(\rho, \phi \to \pi^+\pi^+\gamma)$ are given in eqs. (6) and (12).

Integrating over the pion energies the C-even amplitude (14) leads to the differential cross-section

$$\frac{d\sigma}{dE}\bigg|_{\rho+o} = \frac{d\sigma}{dE}\bigg|_\rho + \frac{d\sigma}{dE}\bigg|_o + \frac{d\sigma}{dE}\bigg|_{int}, \tag{15}$$

where the ρ-term has been given in eq. (7), and the ϕ- and interference-terms (the ϕ-signal) are given by

$$\frac{d\sigma}{dE}\bigg|_o = \frac{2\alpha^3}{3s}\left(\frac{g_sG_s}{3g}\right)^2 |F_\phi(s)P_f(s(1-x))|^2$$

$$\times E^3 \sqrt{1 - \frac{\xi}{1-x}}, \tag{16}$$

$$\frac{d\sigma}{dE}\bigg|_{int} = \frac{-4\alpha^3}{3s}\frac{g_sG_s}{3g} \mathrm{Re}(F_\rho^*(s)F_o(s)P_f(s'))$$

$$\times E\left(\sqrt{1 - \frac{\xi}{1-x}} + \tfrac{1}{2}\xi \ln\left|\frac{1 - \sqrt{1 - \xi/(1-x)}}{1 + \sqrt{1 - \xi/(1-x)}}\right|\right). \tag{17}$$

Eqs. (16) and (17) contain the unknown product g_sG_s, i.e., relevant information on the f_0-meson and, more specifically, on the $BR(\phi \to f_0\gamma)$ as indicated in eq. (11). Theoretical estimates of this branching ratio range from 10^{-3} to 10^{-6}. Recent analyses by Brown and Close [2] indicate $BR(\phi \to f_0\gamma) = 1.36 \times 10^{-4}$ (if a calculation along the lines of ref. [8] is performed), 10^{-4} (if the f_0 is a $q\bar{q}q\bar{q}$ system) or 10^{-5} (if the f_0 were pure $s\bar{s}$). Since the last possibility is not entirely reasonable for an f_0-meson decaying mainly in two pions, smaller values for the branching

ratio should also be considered. For all these reasons we present our results in figs. 2 and 3 for three values of $BR(\phi \to f_0\gamma)$ in eq. (11), namely, 10^{-4} (solid lines), 10^{-5} (dashed lines) and 10^{-6} (dotted lines). Fig. 2 shows the photonic spectra coming from the terms quoted in eq. (16) (positive curves) and eq. (17) (negative curves). This corresponds to choosing the positive sign for g_sG_s in eq. (11). In this case the effects of the ϕ-decay tend to cancel when interfering with the off-shell $\rho \to \pi^+\pi^-\gamma$- decay. Fig. 3a, where the values for $1 + d\sigma(\text{signal})/d\sigma(\text{background})$ have been plotted, shows the remaining effects of the order of a few percent. Choosing the negative sign for g_sG_s reverses the sign of the interference term shown in the lower half of fig. 2, thus enhancing the effect. This is clearly seen in fig. 3b where now the ratio between the signal and the background can reach a 50%. In all these curves we have assumed $\theta_{min} = 10°$, which seems reasonable for a ϕ-factory [9]. If we had integrated over the whole solid angle, the signal over background ratio would have reduced by a factor of 2.5.

In summary, the possibilities of detecting and analysing the decay chain $\phi \to f_0\gamma \to \pi^+\pi^-\gamma$ depend rather crucially on the relative sign between this amplitude and the one describing off-shell $\rho \to \pi^+\pi^-\gamma$ decays. The latter are only a minor part of the whole background dominated by non-interfering radiation from the

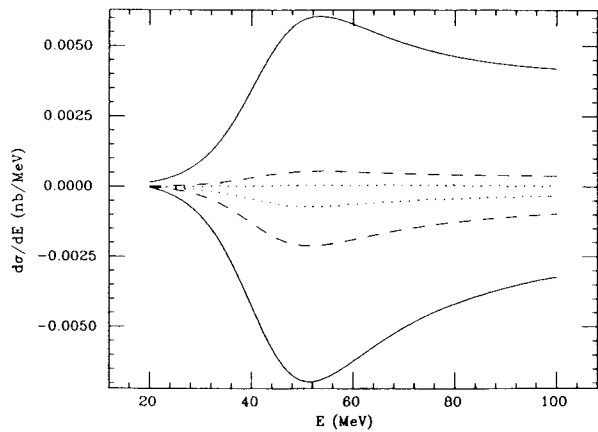Fig. 2. Contributions to the differential cross-section for $e^+e^- \to \pi^+\pi^-\gamma$ on the ϕ-peaks as a function of the photon energy E coming from the ϕ-terms in eq. (16) (upper curves) and the interference term in eq. (17) (lower curves). G_sg_s is taken from the positive root in eq. (10) for $BR(\phi \to f_0\gamma) = 10^{-4}$, 10^{-5} and 10^{-6} (solid, dashed and dotted lines, respectively). $\theta_{min} = 10°$.
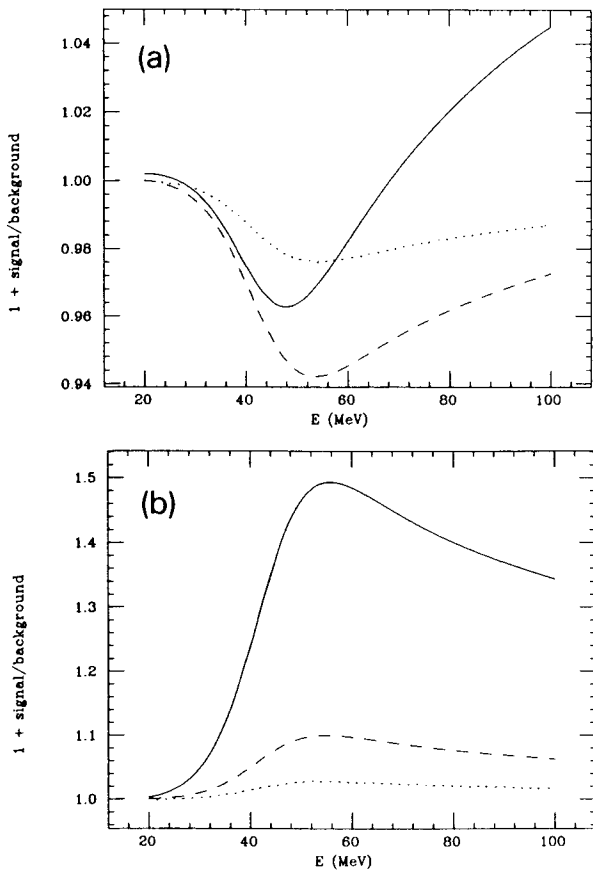
Fig. 3. Ratio between the $\phi \to f_0 \gamma \to \pi^+ \pi^- \gamma$ signal and the background as a function of the photon energy E. Solid, dashed and dotted curves correspond to $BR(\phi \to f_0 \gamma) = 10^{-4}$, 10^{-5} and 10^{-6} in eq. (10) with (a) the positive root (destructive interference) and (b) the negative (constructive) one. $\theta_{min} = 10°$.

$e^+ e^-$ initial state. In spite of this, a rather clean effect is predicted if the ϕ- and ρ-amplitudes are on-phase, while the effect tends to disappear in the (also possible) opposite case.

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