



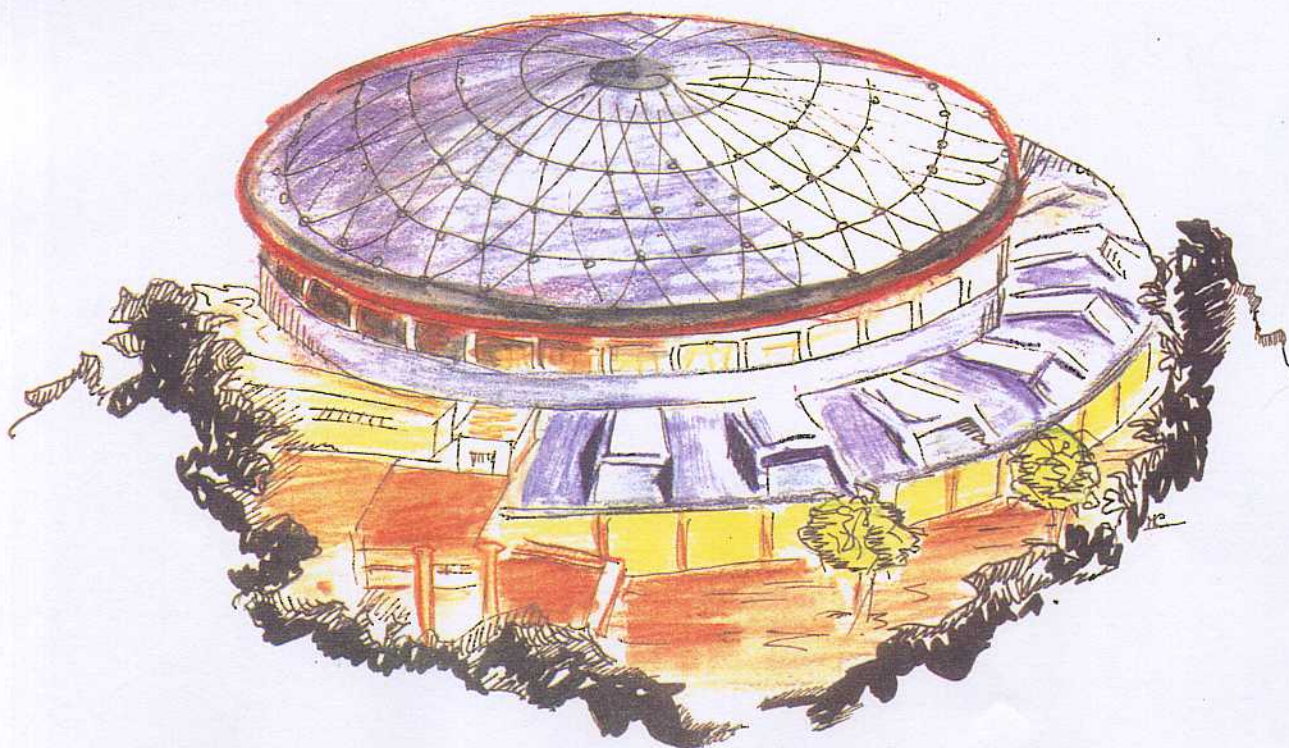
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Vector-Meson Resonances and Pion Polarizabilities

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ABSTRACT

We discuss the pion polarizabilities α_π, β_π using a vector dominance model. We find that the commonly used constraint $(\alpha+\beta)_\pi = 0$ is not admissible for the π^0 . We show that the vector-meson contribution accounts for almost 70% of the π^0 forward angle dispersion sum rule.

The electric (α) and magnetic (β) polarizabilities of any composite system are fundamental quantities that are inherently sensitive to their internal structure and, together with the charge, mass and the magnetic moment, fully determine the expression of the Compton amplitude at low energy. Their experimental determination constitute an important testing ground for any hadron model. In recent years, a great deal of interest has been expressed in the literature for the polarizabilities of the pion [1] which belongs to the pseudoscalar meson octet and thus is believed to be one of the Goldstone bosons associated with the spontaneously broken chiral symmetry.

Following a standard formalism, the general form of the photon scattering amplitude by a pseudoscalar meson can be written in the following form (see Fig.1):

$$\mathcal{A}(\gamma\pi \rightarrow \gamma\pi) = \epsilon_1^\mu \epsilon_2^\nu M_{\mu\nu}, \quad (1)$$

$$M_{\mu\nu} = A(s, t) T_{\mu\nu}^{(1)} + B(s, t) T_{\mu\nu}^{(2)},$$

where ϵ_1, ϵ_2 are the initial and final photon polarizations and the two gauge-invariant

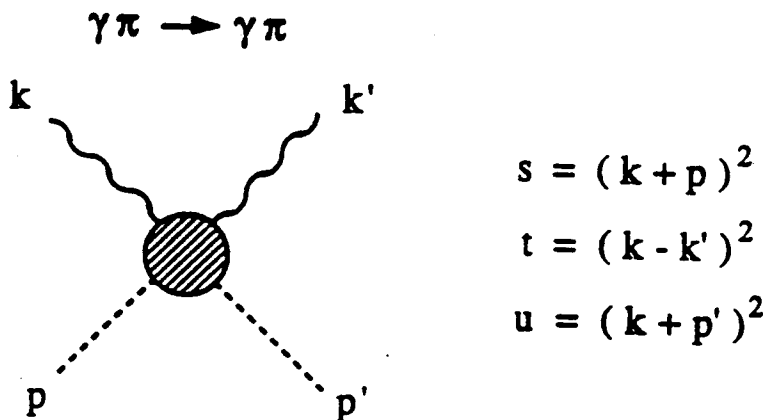


FIG. 1 - Compton scattering kinematics.

Lorentz tensors are given by :

$$T_{\mu\nu}^{(1)} = -\left(\frac{t}{2}g_{\mu\nu} + k_{2\mu}k_{1\nu}\right), \quad (2)$$

$$T_{\mu\nu}^{(2)} = \frac{1}{t} \left[\frac{1}{2}(s - m_\pi^2)(u - m_\pi^2)g_{\mu\nu} + tp_{1\mu}p_{1\nu} + (s - m_\pi^2)k_{2\mu}p_{1\nu} - (u - m_\pi^2)p_{1\mu}k_{1\nu} \right].$$

The two structure functions $A(s, t)$ and $B(s, t)$ may be decomposed in two terms:

$$A(s, t) = A_p(s, t) + A_s(s, t), \quad B(s, t) = B_p(s, t) + B_s(s, t). \quad (3)$$

The first terms (A_p, B_p) describe the scattering from a point-like charge (Born) and the second terms (A_s, B_s) arise from the inner structure of the pion. The Born terms are given by [2]:

$$\begin{aligned} A_p(s, t) &= 0, \\ B_p(s, t) &= 16\pi\alpha \frac{t}{(s - m_\pi^2)(u - m_\pi^2)} |q|, \end{aligned} \quad (4)$$

where α is the fine structure constant and $|q| = 1(0)$ for the charged (neutral) pion. The two structure functions $A_s(s, t)$ and $B_s(s, t)$ contain the dynamics of the process and resemble very closely the role that the hadron form factors have in electron scattering. The non-relativistic limit for the Compton scattering amplitude at low energy can be written as follows:

$$\mathcal{A}^{NR}(\gamma\pi \rightarrow \gamma\pi) = \left[-\frac{\alpha}{m_\pi} + \alpha_\pi \omega \omega' \right] (\hat{\epsilon} \cdot \hat{\epsilon}') + \beta_\pi \omega \omega' (\hat{\epsilon} \wedge \hat{k}) \cdot (\hat{\epsilon}' \wedge \vec{k}'), \quad (5)$$

where $\omega(\omega')$, $\hat{k}(\hat{k}')$ and $\hat{\epsilon}(\hat{\epsilon}')$ are the energy, momentum and polarization vectors of the incoming and outgoing photons respectively. It can be easily verified that the low energy limit of Eq.(1) reproduces exactly Eq.(5) if the two structure constants α_π and β_π of Eq.(5) are identified as:

$$\begin{aligned} \alpha_\pi &= -\frac{1}{8\pi m_\pi} \lim_{s \rightarrow m_\pi^2, t \rightarrow 0} \left[A_s(s, t) + \frac{(s - 3m_\pi^2)}{t} B_s(s, t) \right], \\ \beta_\pi &= \frac{1}{8\pi m_\pi} \lim_{s \rightarrow m_\pi^2, t \rightarrow 0} \left[A_s(s, t) + \frac{(s - m_\pi^2)}{t} B_s(s, t) \right], \\ (\alpha + \beta)_\pi &= \frac{m_\pi}{4\pi} \lim_{s \rightarrow m_\pi^2, t \rightarrow 0} \frac{1}{t} B_s(s, t). \end{aligned} \quad (6)$$

Thus, the static electric and magnetic polarizabilities that characterize the pion-photon coupling in the non-relativistic limit, are defined by the low energy limit of the two structure functions $A_s(s, t)$ and $B_s(s, t)$ of Eqs.(3).

Chiral perturbation theory (χ PT) [3] appears today as one of the most successful ideas in describing the electromagnetic interactions of pions. Its approach is to describe the interactions of the Goldstone bosons at low energy in terms of a so called chiral Lagrangian which stems directly from QCD with the only assumptions of chiral symmetry $SU(3)_L \times SU(3)_R$, Lorentz invariance and low momentum transfer. In particular, with a perturbative expansions of this effective Lagrangian limited to terms of increasing order in the external momenta and quark masses, the method is capable of establishing a network of relationships between different processes in terms of a common set of renormalized parameters L_i^r (tree level coefficients). At $O(p^4)$ -level, the perturbative expansion is truncated at terms quartic in the photon momentum and 12-coupling constants are needed. The $O(p^4)$ expressions for the structure functions of Eqs.(3) have been calculated by different authors [4],[5] and can be written as

$$A_s^{(\pm)}(s, t) = -16\pi\alpha \left\{ \frac{2}{F_\pi^2} (L_9^r + L_{10}^r) - \frac{1}{32\pi^2 F_\pi^2} \left[\frac{3}{2} + \frac{1}{t_\pi} \ln^2 Q_\pi(t) + \frac{1}{2t_K} \ln^2 Q_K(t) \right] \right\}, \quad (7)$$

$$B_s^{(\pm)}(s, t) = 0,$$

for the charged pion, and

$$A_s^{(0)}(s, t) = \frac{\alpha}{4\pi F_\pi^2} \left\{ 4 \left(1 - \frac{1}{t_\pi}\right) \left[1 + \frac{1}{t_\pi} \ln^2 Q_\pi(t)\right] + \left[1 + \frac{1}{t_K} \ln^2 Q_K(t)\right] \right\}, \quad (8)$$

$$B_s^{(0)}(s, t) = 0,$$

for the neutral pion. In Eqs.(7,8) both the contribution from pion and kaon loops have been considered, and

$$t_i = \frac{t}{m_i^2}, \quad Q_i = \frac{\sqrt{t_i - 4} + \sqrt{t_i}}{\sqrt{t_i - 4} - \sqrt{t_i}}, \quad (i = \pi, K). \quad (9)$$

The measured value of the pion decay constant $F_\pi = (93.15 \pm 0.11)$ MeV is taken from Ref. [6]. Moreover Eqs.(7) show that, at $O(p^4)$, only the combination $(L_9^r + L_{10}^r)$ intervenes in the expression of the Compton amplitude. This is the same combination

that intervenes in the pion radiative decay where the ratio of the vector and axial vector coupling constants is related to $(L_9^r + L_{10}^r)$ by the well known expression [1, 7]

$$L_9^r + L_{10}^r = \frac{1}{32\pi^2} \frac{F_A}{F_V}. \quad (10)$$

Using the measured value of the axial vector coupling constant $F_A = 0.0116 \pm 0.0016$ and the CVC value of the vector coupling constant $F_V = 0.0259 \pm 0.0005$ in the radiative pion decay [6], we obtain

$$\frac{F_A}{F_V} = 0.45 \pm 0.07 \quad (11)$$

Here we use the CVC prediction rather than the measured value $F_V = 0.017 \pm 0.008$ because the data on F_A have been extracted in Ref. [6] assuming the CVC value for F_V . From Eqs.(10,11) we obtain $L_9^r + L_{10}^r = (1.42 \pm 0.22) 10^{-3}$.

On the contrary, Eqs.(8) are parameter free predictions for the neutral pion. Thus, following Eq.(6), the $O(p^4)$ expressions for the pion polarizabilities are ¹:

$$\begin{aligned} \alpha_{\pi^\pm} &= \frac{4\alpha}{m_\pi F_\pi^2} (L_9^r + L_{10}^r) = 2.68 \pm 0.42, \\ \alpha_{\pi^0} &= -\frac{\alpha}{96\pi^2 m_\pi F_\pi^2} = -0.498 \pm 0.001. \end{aligned} \quad (12)$$

where we assumed $m_\pi = 137$ MeV. Here the errors reflect only the uncertainties on the measured values of F_π , F_A and the CVC value of F_V . In the exact chiral limit where the pion mass vanishes, one has [1]:

$$(\alpha + \beta)_\pi = 0, \quad (13)$$

and from Eqs.(6,7,8) the same result holds in χ PT up to the $O(p^4)$ -level. This is not surprising because a strong cancellation effect in $(\alpha + \beta)$ can indeed be expected from

¹In the present paper the polarizabilities are expressed in Gaussian units of 10^{-43} cm³

classical considerations based on the Lorentz invariance of the interaction hamiltonian. Potential problems with this conclusion can arise from the finite sizes of the forward angle dispersion sum rules

$$(\alpha + \beta)_{\pi^\pm} = 0.39 \pm 0.04, \quad (\alpha + \beta)_{\pi^0} = 1.04 \pm 0.07, \quad (14)$$

which stem directly from the optical theorem and have been evaluated in a model dependent way in Ref. [8]. According to χ PT, these sum rules express only the contributions that come from the $O(p^6)$ (and higher) corrections to the lowest order result of Eq.(12). However, these higher order corrections have never been fully calculated and thus a cross-check between the full chiral predictions and Eqs.(14) has never been done.

In a recent paper [5] we have discussed in some detail the experimental knowledge that is presently available on the pion polarizabilities. In spite of Eqs.(14), practically all the data have been analyzed with the constraint of Eq.(13). This can be easily criticized because, in principle, can lead to erroneous results. The extent of the higher order corrections to $(\alpha + \beta)$ is not expected to be negligible: according to [1] $(\alpha + \beta)/\alpha$ is estimated of the order of 25% for the charged pion. The effect appears to be even larger for the π^0 if we compare the values of Eqs.(12) and (14). On the other hand the quality of all the examined data does not allow for an independent and reasonable determination of $(\alpha + \beta)$. It is true that the second analysis of the Serpukov experiment [14] shows that, by releasing the constraint of Eq.(12), $(\alpha + \beta)$ results largely consistent with zero. But the consequence is that the statistical errors in the determination of α and β worsen so much that the issue loses most of its significance. In conclusion, the present determinations suffer from the limitation imposed by Eq.(13) and the question of the experimental test of Eqs.(14) is left up to the next generation experiments.

On the theoretical side, a complete $O(p^6)$ calculation is still lacking. However, the t-channel vector meson contributions to the photon-photon interaction that have been

calculated [17, 18, 19] in connection with the Crystal Ball data are $O(p^6)$ contributions. We know that they cannot be ignored for energies above 0.5 GeV even though they are not sufficient to give a satisfactory description of the Crystal Ball data (see Fig.2). We also know that they are negligible in the threshold region. Nevertheless, they can provide a sizeable contribution in the crossed channel reaction (Compton scattering) at $t = 0$ where the $O(p^4)$ contribution to the cross section vanishes both for the charged and neutral pion. As a matter of fact, this effect has to be expected because a substantial contribution to the sum rules of Eqs.(13) comes from the vector meson photoproduction and this necessarily has to affect the forward Compton amplitude.

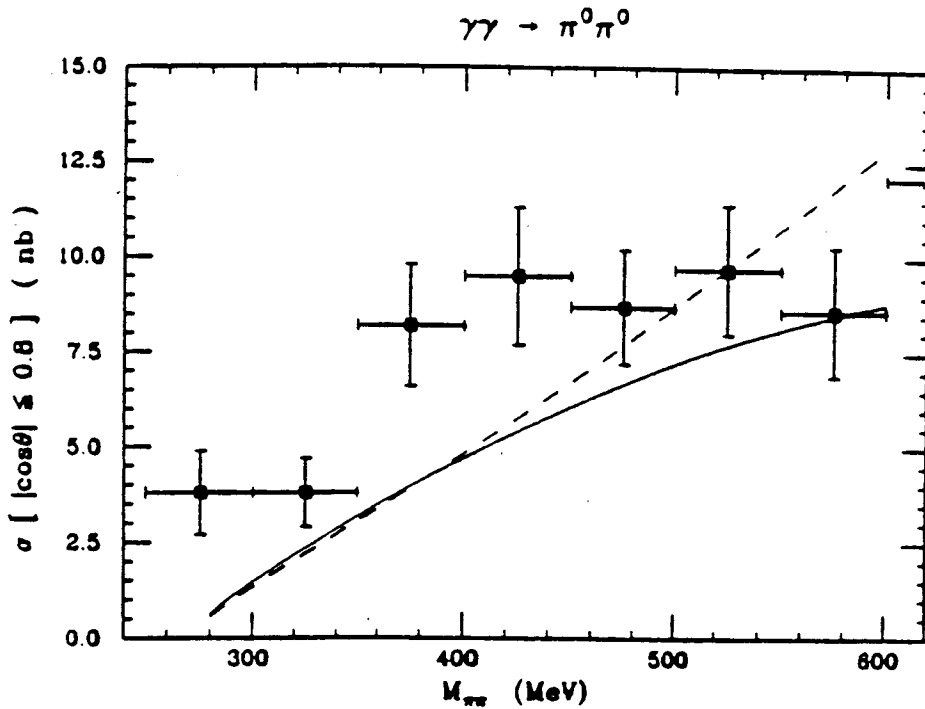


FIG. 2 - The cross-section for $\gamma\gamma \rightarrow \pi^0\pi^0$ including : i) both the 1-loop diagrams and the $O(p^6)$ contribution due to the vector-meson resonance exchange in the t-channel (full line) ; ii) the 1-loop contribution only (dashed line). The data are taken from the Crystal Ball experiment.

This effect can be fully calculated following Ref. [17] and using crossing symmetry. The result indicates that the two structure functions of Eqs.(3) acquire indeed extra contributions. By neglecting the ϕ -meson, these come from the ρ and ω -exchanges in the case of the π^0

$$A_s^{(0)}(s, t) = A_s^{(0)}(O(p^4)) - \frac{1}{2} \sum_V G_V \left(\frac{s + m_\pi^2}{s - M_V^2} + \frac{u + m_\pi^2}{u - M_V^2} \right), \quad (15)$$

$$B_s^{(0)}(s, t) = B_s^{(0)}(O(p^4)) - \frac{1}{2} \sum_V G_V t \left(\frac{1}{s - M_V^2} + \frac{1}{u - M_V^2} \right), \quad (16)$$

$(V \equiv \omega, \rho)$

and from the ρ -exchange only for the charged pion

$$A_s^{(\pm)}(s, t) = A_s^{(\pm)}(O(p^4)) - \frac{1}{2} G_\rho \left(\frac{s + m_\pi^2}{s - M_\rho^2} + \frac{u + m_\pi^2}{u - M_\rho^2} \right), \quad (17)$$

$$B_s^{(\pm)}(s, t) = B_s^{(\pm)}(O(p^4)) - \frac{1}{2} G_\rho t \left(\frac{1}{s - M_\rho^2} + \frac{1}{u - M_\rho^2} \right), \quad (18)$$

where M_V is the mass of the vector resonance and

$$G_V = 96\pi M_V^3 \frac{\Gamma(V \rightarrow \pi\gamma)}{(M_V^2 - m_\pi^2)^3}. \quad (19)$$

Using Eqs.(15,16) and following Eq.(6) one can see that the α -values remain unaffected and the β -values become ²:

$$\beta_{\pi^\pm} = -\alpha_{\pi^\pm} + \frac{m_\pi}{4\pi} \frac{G_\rho}{M_\rho^2 - m_\pi^2} = -2.61, \quad (20)$$

$$\beta_{\pi^0} = -\alpha_{\pi^0} + \frac{m_\pi}{4\pi} \sum_V \frac{G_V}{M_V^2 - m_\pi^2} = 1.26.$$

²Equations (15,16) include terms of order higher than $O(p^6)$. The chiral invariant procedure changes very little the numerical results [18, 19].

Consequently, the new estimated values for the sum rules are:

$$(\alpha + \beta)_{\pi^\pm} = 0.064 , \tag{21}$$

$$(\alpha + \beta)_{\pi^0} = 0.76 .$$

These numerical results have been obtained assuming the following experimental values for the $\omega^0\pi^0\gamma$ and $\rho\pi\gamma$ coupling constants [6]

$$G_\omega = 0.495 \text{ GeV}^{-2}, \quad G_\rho = 0.044 \text{ GeV}^{-2}, \tag{22}$$

as deduced using Eq.(17). Here we disagree with the numerical estimates of these constants quoted by Ko in Ref. [17]. Ko uses $G_\omega = 0.593 \text{ GeV}^{-2}$ and $G_\rho = 0.066 \text{ GeV}^{-2}$.

In a recent paper, Ivanov and Mizutani [20] presented a full calculation of the pion polarizabilities in the framework of the Dubna Quark Confinement Model (DCQM) with the explicit inclusion of scalar, vector and axial-vector mesons exchanges. They find that the axial-vector contributions are always almost negligible and can be safely omitted. The major effects come from the scalar exchanges which contribute with opposite signs to α and β and thus cancel out in the sum. Therefore the only contributions to the sum rules come from the vector exchanges that are found to be small and affecting only the β -values. Qualitatively, these results are perfectly consistent with our claim that the vector-meson exchanges could be the dominant $O(p^6)$ contributions to the sum rules. However, from the set of values they found:

$$(\alpha + \beta)_{\pi^\pm} = 0.22 , \quad \alpha_{\pi^\pm} = 3.63 , \tag{23}$$

$$(\alpha + \beta)_{\pi^0} = 0.44 , \quad \alpha_{\pi^0} = 0.74 ,$$

the value for α_{π^\pm} results higher than the chiral prediction and α_{π^0} has the right magnitude but opposite sign. Moreover both the sum rules appear underestimated by approximatively a factor two with respect to Eqs.(14). Our numerical evaluations for $(\alpha + \beta)$ differ from the values of Eq.(25) : we do a better job for the π^0 while

Ivanov and Mizutani obtain a result closer than ours to the sum rule estimate for the π^\pm .

In conclusion the vector-meson contributions are certainly important corrections to the $O(p^4)$ results but do not help much in reproducing the sum rule for the charged pion. On the contrary they account for almost 70% of the sum rule for the neutral pions. This is quite remarkable if one considers that they contribute very little to the cross section in the threshold region for the $\gamma\gamma \rightarrow \pi^0\pi^0$ channel [17, 18, 19].

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