



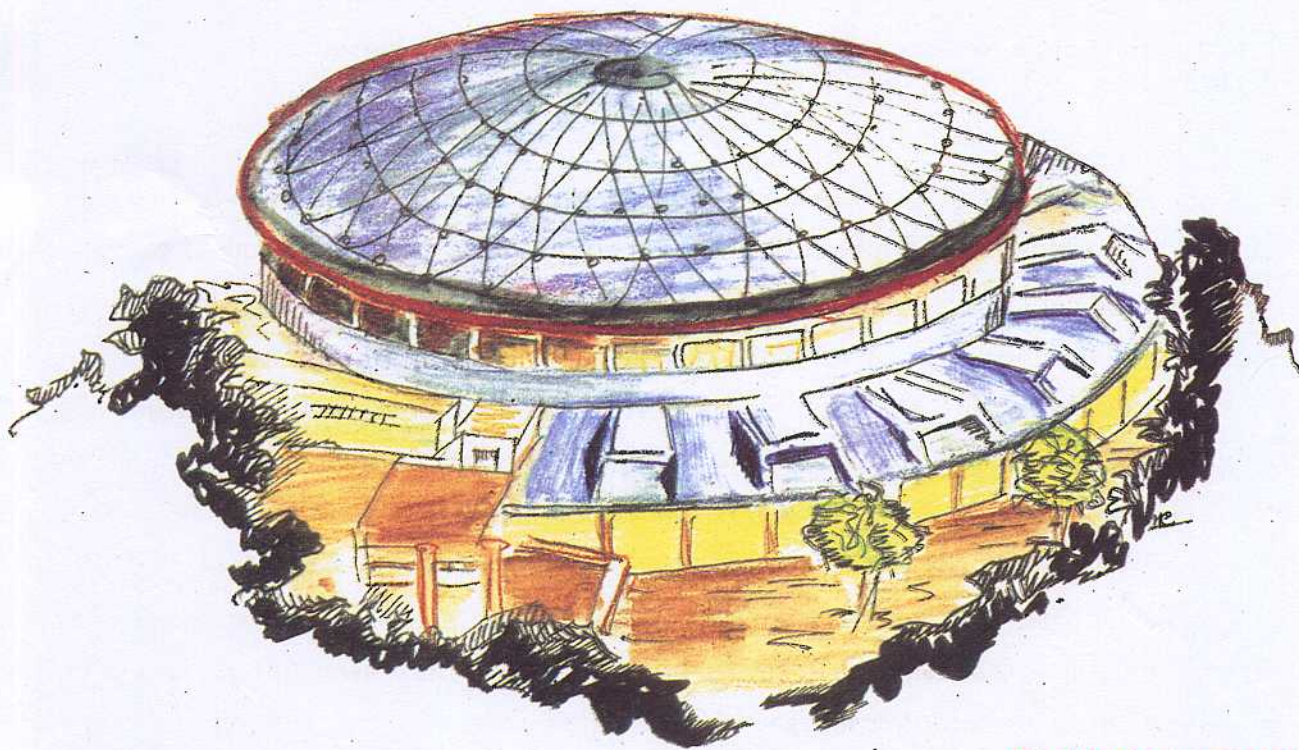
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**LOW-ENERGY KAON INTERACTIONS AND SCATTERING ON  
NUCLEONS AND LIGHT NUCLEI**

Contribution to the DAΦNE Physics Handbook



**Low-Energy Kaon Interactions and Scattering on Nucleons  
and Light Nuclei**

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We present in this contribution the basic formulæ for the analysis of low-momentum charged- and neutral-kaon interactions in hydrogen, including as well a (brief) description of the problems left open by past experiments and of the improvements DAΦNE can be expected to offer over them. Interactions in deuterium and other light nuclei will be only briefly mentioned, and only in those respects touching directly upon the more “elementary” aspects of kaon-nucleon interactions.

**1. Introduction.**

DAΦNE is expected to produce, with “standard” assumptions about luminosity and cross sections, about  $1.25 \times 10^{10}$   $K^\pm$ 's and  $8.5 \times 10^9$   $K_L$ 's per year of operation, considering a conventional “Snowmass year” of  $10^7$  s. With a detector of KLOE's size one can thus expect to observe up to millions of interactions per year, even in the light element (quite probably gaseous helium at atmospheric pressure) filling its fiducial volume.

A first question has thus to be answered: do these events contain useful physics to be worth recording and interpreting? One could even go further and envisage using DAΦNE as a source of high-resolution ( $\Delta p/p$  of respectively  $1.1 \times 10^{-2}$  for  $K^\pm$ 's and  $1.5 \times 10^{-2}$  for  $K_L$ 's), low-momentum kaons (127 MeV/c for the former, 110 MeV/c for the latter), to measure with a *dedicated* detector  $KN$  and  $\bar{K}N$  interactions in a toroidal volume filled with gaseous  $H_2$  or  $D_2$ , and possibly also interactions in heavier, gaseous elements.

The machine has thus the ability to explore a kinematical region very little investigated in the past: only few bubble-chamber  $K^-$  experiments in hydrogen (and deuterium)<sup>1,2</sup> plus very few data points on  $K_S$  regeneration<sup>3</sup> exist, all with extremely

low statistics and more than a decade old (the last experiment<sup>2</sup> to cover this region was carried out by the TST Collaboration at the hydrogen bubble chamber in NIMROD's low-energy kaon beam in the second half of the seventies). Since in dispersive calculations of low-energy parameters for  $KN$  interactions (couplings, scattering lengths, sigma terms) the bulk of the uncertainties comes from the integrals over the unphysical region, to describe which one must extrapolate down in energy from that just above threshold, DAΦNE can be expected to make substantial improvements over our present knowledge of those parameters.

The following sections are therefore dedicated to illustrating both the level and the limitations of present-day information on low-energy kaon-nucleon physics, spotlighting those points which still await being clarified, and where DAΦNE can be expected to improve. Being the phenomenology in this case more complex than in the (strictly related) pion-nucleon one, we shall start almost from scratch.

We shall also take the liberty of not going into the details of models, in particular for the spectroscopic classification of the  $J^P = \frac{1}{2}^-$ ,  $S$ -wave resonance  $\Lambda(1405)$ : data are so scarce, for the moment being, that any interpretation of such a state is to be regarded as purely conjectural<sup>4</sup>.

## 2. Amplitude formalism for two-body $KN$ and $\bar{K}N$ interactions.

Any  $a_1(0^-, q) + B_1(\frac{1}{2}^+, p) \rightarrow a_2(0^-, q') + B_2(\frac{1}{2}^+, p')$  process is most economically described in the centre-of-mass (c.m.) frame by two amplitudes,  $G(w, \theta)$  and  $H(w, \theta)$ , when the T-matrix element  $T_{\alpha\beta}$  is expressed in terms of the two-component Pauli spinors  $\chi_\alpha$  and  $\chi_\beta$  (respectively for the final and initial  $\frac{1}{2}^+$  baryons) as  $T_{\alpha\beta} = \chi_\alpha^\dagger \mathbf{T} \chi_\beta$ , where

$$\mathbf{T} = G(w, \theta) \times \mathbf{I} + iH(w, \theta) \times (\vec{\sigma} \cdot \hat{n}) \quad (1)$$

and  $\hat{n}$  defines the normal to the scattering plane<sup>5</sup>.

These c.m. amplitudes have a simple expansion in the partial waves  $T_{\ell\pm}(w) = (\eta_{\ell\pm} e^{2i\delta_{\ell\pm}} - 1)/2iq$ , given by

$$G_N(w, \theta) = \sum_{\ell=0}^{\infty} [(\ell+1)T_{\ell+}(w) + \ell T_{\ell-}(w)] P_\ell(\cos \theta) \quad (2)$$

$$H_N(w, \theta) = \sum_{\ell=1}^{\infty} [T_{\ell+}(w) - T_{\ell-}(w)] P'_\ell(\cos \theta), \quad (3)$$

where the subscript  $N$  indicates that only the *purely nuclear* part of the interaction has been considered.

To describe adequately the data, the amplitudes must also include electromagnetism and can be rewritten as

$$G(w, \theta) = \tilde{G}_N(w, \theta) + G_C(w, \theta) \quad (4)$$

$$H(w, \theta) = \tilde{H}_N(w, \theta) + H_C(w, \theta) , \quad (5)$$

where the tilded nuclear amplitudes differ from the untilded ones only in the Coulomb shifts  $\sigma_{\ell\pm}^{\text{in(fin)}}$  (differing from zero only when both particles in the initial (final) state are charged) having been applied to each partial wave  $T_{\ell\pm}$ , namely when

$$T_{\ell\pm} \rightarrow \tilde{T}_{\ell\pm} = e^{i\sigma_{\ell\pm}^{\text{in}}} T_{\ell\pm}(w) e^{i\sigma_{\ell\pm}^{\text{fin}}} . \quad (6)$$

The one-photon-exchange amplitudes  $G_C$  and  $H_C$  (of course absent for charge- and/or strangeness-exchange processes, but not for  $K_S$  regeneration, which at  $t \neq 0$  goes also via one-photon exchange) can be expressed in terms of the Dirac nucleon form factors as<sup>5,6</sup>

$$G_C(w, \theta) = \pm e^{\pm i\phi_C} \cdot \left\{ \left( \frac{2q\gamma}{t} + \frac{\alpha}{2w} \frac{w+m}{E+m} \right) \cdot F_1(t) + \left[ w - m + \frac{t}{4(E+m)} \right] \cdot \frac{\alpha F_2(t)}{2wm} \right\} \cdot F_K(t) \quad (7)$$

and

$$H_C(w, \theta) = \pm \frac{\alpha F_K(t)}{2w \tan \frac{1}{2}\theta} \cdot \left\{ \frac{w+m}{E+m} \cdot F_1(t) + \left[ w + \frac{t}{4(E+m)} \right] \cdot \frac{F_2(t)}{m} \right\} \quad (8)$$

for the interactions of (respectively)  $K^\pm$  with nucleons, while for  $K_S$  regeneration one has to change the sign of the isovector part of the kaon form factor  $F_K(t)$ . Here  $\gamma = \alpha \cdot (w^2 - m^2 - \mu^2)/2qw$  and the Coulomb phase  $\phi_C$  is defined as

$$\phi_C = -\gamma \log(\sin^2 \frac{1}{2}\theta) + \gamma \cdot \int_{-4q^2}^0 \frac{dt}{t} \cdot [1 - F_K(t)F_1(t)] \quad (9)$$

for charged kaons scattering on protons, while it reduces to

$$\phi_C = -\gamma \int_{-4q^2}^0 \frac{dt}{t} F_K(t)F_1(t) \quad (9')$$

for processes involving  $K^0$ 's and/or neutrons.

We have denoted with  $w$  and  $\theta$  respectively the total energy and the scattering angle in the c.m. frame,  $q = [\frac{1}{4}w^2 - \frac{1}{2}(m^2 + \mu^2) + (m^2 - \mu^2)^2/4w^2]^{1/2}$  is the c.m. momentum

(in the initial state: for inelastic processes, including charge exchange, we shall indicate final-state kinematical quantities with primes),  $E$  the total energy of the baryon in the c.m. frame,  $E = (w^2 + m^2 - \mu^2)/2w$ , and  $t$  the squared momentum-transfer,  $t = m^2 + m'^2 - 2EE' + 2qq' \cos \theta$ . We shall also use the laboratory-frame, initial-meson momentum  $k = \frac{1}{2}(\omega^2 - \mu^2)^{1/2}$  and energy  $\omega$ , related to the c.m. total energy via  $\omega = (w^2 - m^2 - \mu^2)/2m$ , and, besides  $t$ , the two other Mandelstam variables  $s = w^2$  and  $u$ , the square of the c.m. total energy for the crossed channel  $\bar{a}_2(0^-) + B_1(\frac{1}{2}^+) \rightarrow \bar{a}_1(0^-) + B_2(\frac{1}{2}^+)$ , obeying on the mass shell the identity  $s + t + u = m^2 + m'^2 + \mu^2 + \mu'^2$ .

The shifts  $\sigma_{\ell\pm}$  have been computed accurately by Tromborg, Waldenström and Øverbø<sup>6</sup> in a dispersive formalism for the  $\pi N$  case: the same formalism<sup>7</sup> could be extended (but has not been up to now) also to the  $KN$  and  $\bar{K}N$  ones. Minor corrections remain to be applied to the phases  $\delta_{\ell\pm}$  and to the elasticities  $\eta_{\ell\pm}$ , to extract their purely nuclear parts<sup>5</sup>; a way of removing them efficiently has been devised by the Karlsruhe-Helsinki group for  $\pi N$  amplitudes: it consists in starting from a (preliminary) set of phase shifts, calculating from them the corrections in the above-mentioned dispersive formalism<sup>6,7</sup>, then the changes in the observables brought about by these latter, and finally correcting the data for these effects and starting the phase-shift analysis all over again, this time from the “corrected data”. This procedure has turned out to be both self-consistent and fast<sup>5</sup>.

In terms of the amplitudes  $G$  and  $H$  the c.m. differential cross sections for an unpolarized target (which will most surely be the case in DAΦNE’s almost  $4\pi$  geometry) take the simple form

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} \sum_{\alpha,\beta} |T_{\alpha\beta}|^2 = |G|^2 + |H|^2. \quad (10)$$

The other observables possibly accessible at DAΦNE, in the strangeness-exchange processes  $\bar{K}N \rightarrow \pi\Lambda$  and  $\bar{K}N \rightarrow \pi\Sigma$ , are the polarizations  $P_Y$  ( $Y = \Lambda$  or  $\Sigma$ ) of the final hyperons, measurable through the asymmetries  $\alpha$  of their weak nonleptonic decays  $\Lambda \rightarrow \pi^- p$  or  $\pi^0 n$ , for both of which we have an asymmetry  $\alpha \simeq 0.64$ , and  $\Sigma^+ \rightarrow \pi^0 p$  for which the asymmetry is  $\alpha \simeq -0.98$ , while there is very little chance to be able to use the neutron decay modes  $\Sigma^\pm \rightarrow \pi^\pm n$ , which have the very small asymmetries  $\alpha \simeq \pm 0.068$ ; we have for these quantities

$$P_Y \cdot \left( \frac{d\sigma}{d\Omega} \right) = 2 \operatorname{Im} (GH^*). \quad (11)$$

Note that, for an ( $S+P$ )-wave parametrization (fully adequate at such low momenta), while the *integrated* cross sections depend only *quadratically* on the  $P$ -waves, both the

first Legendre coefficients of the differential cross sections

$$L_1 = \frac{1}{2} \int_{-1}^{+1} \cos \theta \left( \frac{d\sigma}{d\Omega} \right) d \cos \theta = \frac{2}{3} \operatorname{Re} [T_{0+}(2T_{1+} + T_{1-})^* + \dots] \quad (12)$$

and the polarizations

$$P_Y \cdot \left( \frac{d\sigma}{d\Omega} \right) = 2 \operatorname{Im} [T_{0+}(T_{1+} - T_{1-})^* + 3T_{1-}T_{1+}^* \cos \theta + \dots] \sin \theta \quad (13)$$

are essentially *linear* in the small  $P$ -wave contributions, and give complementary information on these latter. It is perhaps not useless to remind the reader that the low statistics of the experiments, performed *only* up to the late seventies, have not been enough to determine any of these parameters, putting only rather generous (and utterly useless) upper bounds<sup>2</sup> on  $|L_1|$  for the hyperon production channels.

We shall now devote the last part of this section to show explicitly why this absence of *direct* information on the low-energy behaviour of the  $P$ -waves has been a serious shortcoming for  $\bar{K}N$  amplitude analyses. Remember that we know, from *production* experiments, that the  $I = 1, S = -1$   $T_{1+}$  partial wave resonates *below* threshold at a c.m. energy around  $w = 1385 \text{ MeV}$ , the mass of the isovector member of the  $J^P = \frac{3}{2}^+$  decuplet.

One has to turn from the Pauli amplitudes  $G$  and  $H$  to the invariant amplitudes  $A(s, t)$  and  $B(s, t)$ , defined in term of four-component Dirac spinors as

$$2\pi w T_{\alpha\beta} = \bar{u}_\alpha(p') [A(s, t) + B(s, t) \cdot \gamma^\mu Q_\mu] u_\beta(p) , \quad (14)$$

where  $Q = \frac{1}{2}(q + q')$ , the average between incoming- and outgoing-meson c.m. four-momenta: these amplitudes obey simple crossing relations and are free of kinematical singularities, so that they are the ones to be used, rather than  $G$  and  $H$ , for any analytical extrapolation purpose; it is also customary to use the combination  $D(\nu, t) = A(\nu, t) + \nu \cdot B(\nu, t)$ , where  $\nu = (s - u)/2(mm')^{1/2}$ , which has the same properties as  $A(\nu, t)$  under crossing, and furthermore, for elastic scattering, obeys the optical theorem in the simple form

$$\operatorname{Im} D(\nu, t = 0) = k \cdot \sigma_{tot} , \quad (15)$$

where of course all electromagnetic effects must be subtracted on *both* sides.

One can rewrite  $A$  and  $B$  in terms of  $G$  and  $H$ , and thus reexpress them through the partial waves  $T_{\ell\pm}$ , by projecting eq. (14) on the different spin states (polarized *perpendicularly* to the scattering plane) and obtain in the most general kinematics



$$A(\nu, t) = \frac{4\pi}{(E+m)^{1/2}(E'+m')^{1/2}} \left\{ \left[ w + \frac{1}{2}(m+m') \right] G(w, \theta) + \right. \\ \left. + [(E+m)(E'+m') \left\{ w - \frac{1}{2}(m+m') \right\} + \left\{ \frac{1}{2}t + EE' - \frac{1}{2}(m^2+m'^2) \right\} \left\{ w + \frac{1}{2}(m+m') \right\}] \cdot \right. \\ \left. \frac{H(w, \theta)}{qq' \sin \theta} \right\}, \quad (16)$$

and

$$B(\nu, t) = \frac{4\pi}{(E+m)^{1/2}(E'+m')^{1/2}} \left\{ G(w, \theta) - \right. \\ \left. - [(E+m)(E'+m') - \frac{1}{2}t - EE' + \frac{1}{2}(m^2+m'^2)] \frac{H(w, \theta)}{qq' \sin \theta} \right\}. \quad (17)$$

Considering for sake of simplicity forward elastic scattering only, the amplitudes become, leaving out  $D$ - and higher waves,

$$D(\nu, 0) = \frac{4\pi w}{m} [T_{0+} + 2T_{1+} + T_{1-} + \dots] \quad (18)$$

and

$$B(\nu, 0) = \frac{4\pi w}{mq^2} [(E-m)T_{0+} - 2(2m-E)T_{1+} + (E+m)T_{1-} + \dots]; \quad (19)$$

introducing the (complex) scattering lengths  $a_{\ell\pm}$  and (complex) effective ranges  $r_{\ell\pm}$  one can expand up to  $O(q^2)$  the partial waves close to threshold, and obtain for the forward  $D$  amplitudes

$$D(q, 0) = 4\pi \left(1 + \frac{\mu}{m}\right) \left\{ a_{0+} + ia_{0+}^2 q + [2a_{1+} + a_{1-} - (a_{0+} + \frac{1}{2}r_{0+})a_{0+}^2 - \frac{a_{0+}}{2m\mu}] q^2 + \dots \right\}, \quad (20)$$

dominated by the  $S$ -waves, while for the  $B$  amplitudes the same approximations give

$$B(q, 0) = \frac{2\pi}{m} \left(1 + \frac{\mu}{m}\right) [a_{0+} - 4m^2(a_{1+} - a_{1-}) + ia_{0+}^2 q + \dots], \quad (21)$$

where the factor  $4m^2 \simeq 90 fm^{-2}$  enhances considerably the contributions by the low-energy  $P$ -waves (virtually unknown), rendering practically useless the unsubtracted dispersion relation for the better converging  $B$  amplitudes, so important for the  $\pi N$  case in fixing accurately the values of the coupling constant  $f^2$  and of the  $S$ -wave scattering lengths<sup>5</sup>.

### 3. Open channels and baryon spectroscopy at DAΦNE.

As mentioned above, in the momentum region which could be explored by the kaons coming from the decays of a  $\phi$ -resonance formed at rest in an  $e^+e^-$  collision, we have only

data from low-statistics experiments, mostly hydrogen bubble-chamber ones on  $K^-p$  (and  $K^-d$ ) interactions<sup>1,2</sup> (dating from the early sixties through the late seventies), plus scant data from  $K_L$  interactions and  $K_S$  regeneration, mostly on hydrogen<sup>3</sup>.

The channels, open at a laboratory energy  $\omega = \frac{1}{2}M_\phi$  (for  $K^\pm$ 's to obtain the exact value of  $\omega$  one has to include their energy losses through ionization as well), are tabulated below for interactions with free protons and neutrons, together with their threshold energies  $E_{thr}$  (in  $MeV$ ), strangeness and isospin(s). We do not list  $K^+$ -initiated processes, which are (apart from charge exchange) purely elastic in this energy region.

Table I

Channel	$E_{thr}/MeV$	$S$	$I$
$K^-p, K_L^0 n \rightarrow \pi^0 \Lambda$	1250.6	-1	1
$K^-p, K_L^0 n \rightarrow \pi^0 \Sigma^0$	1327.5	-1	0
$K^-p, K_L^0 n \rightarrow \pi^- \Sigma^+$	1328.9	-1	0,1
$K^-p, K_L^0 n \rightarrow \pi^+ \Sigma^-$	1337.0	-1	0,1
$K^-p, K_L^0 n \rightarrow \pi^0 \pi^0 \Lambda$	1385.6	-1	0
$K^-p, K_L^0 n \rightarrow \pi^+ \pi^- \Lambda$	1394.8	-1	0,1
$K^-p, K_L^0 n \rightarrow K^-p$	1431.9	-1	0,1
$K^-p, K_L^0 n \rightarrow K_S^0 n$	1437.2	-1	0,1
” ”	”	+1	1 <sup>†</sup>

†) This amplitude only appears in the regeneration process  $K_L^0 n \rightarrow K_S^0 n$ .

Table II

Channel	$E_{thr}/MeV$	$S$	$I$
$K^-n \rightarrow \pi^- \Lambda$	1255.2	-1	1
$K^-n \rightarrow \pi^- \Sigma^0$	1332.1	-1	1
$K^-n \rightarrow \pi^0 \Sigma^-$	1332.1	-1	1
$K^-n \rightarrow \pi^0 \pi^- \Lambda$	1388.2	-1	1
$K^-n \rightarrow K^-n$	1433.2	-1	1



Table III

Channel	$E_{thr}/MeV$	$S$	$I$
$K_L^0 p \rightarrow \pi^+ \Lambda$	1255.2	-1	1
$K_L^0 p \rightarrow \pi^0 \Sigma^+$	1324.3	-1	1
$K_L^0 p \rightarrow \pi^+ \Sigma^0$	1332.1	-1	1
$K_L^0 p \rightarrow \pi^0 \pi^+ \Lambda$	1388.2	-1	1
$K_L^0 p \rightarrow K^+ n$	1433.2	+1	0,1
$K_L^0 p \rightarrow K_S^0 p$	1435.9	+1	0,1
" "	"	-1	1

For interactions in hydrogen, the c.m. energy available for each final state is limited by momentum conservation to the initial total c.m. energy, equal (neglecting energy losses) to  $w = (m_p^2 + \mu_K^2 + m_p M_\phi)^{1/2}$ , or 1442.4 MeV for incident  $K^\pm$ 's and 1443.8 MeV for incident  $K_L$ 's. Energy losses for charged kaons can be exploited (using the inner parts of the detector as a "moderator") to explore  $K^-p$  interactions in a *limited* momentum range, down to the charge-exchange threshold at  $w = 1437.2$  MeV, corresponding to a  $K^-$  laboratory momentum of about 90 MeV/c.

For interactions in deuterium (or in heavier nuclei), momentum can be carried away by "spectator" nucleons, and one can explore each inelastic channel from the highest available energy down to threshold. The possibility of reaching energies below the  $\bar{K}N$  threshold is particularly desirable, since the  $\bar{K}N$  unphysical region contains two resonances, the  $I = 0$ ,  $S$ -wave  $\Lambda(1405)$  and the  $I = 1$ ,  $J^P = \frac{3}{2}^+$   $P$ -wave  $\Sigma(1385)$ , observed mostly in production experiments (and, in the first case, in very limited statistics ones<sup>8</sup>), so that the information on their couplings to the  $\bar{K}N$  channel relies *entirely* on extrapolations below threshold of the analyses of the low-energy data. The coupling of the  $\Sigma(1385)$  to the  $\bar{K}N$  channel, for instance, can at present be determined only via forward dispersion relations involving the *total sum* of data collected at  $t \simeq 0$ , but with uncertainties which are, at *their best*, still of the order of 50% of the value expected from flavour- $SU(3)$  symmetry<sup>9</sup>; as for the  $\Lambda(1405)$ , even its spectroscopic classification is still an open problem, *vis-à-vis* the paucity and (lack of) quality of the *best available data*<sup>4,10</sup>.

A *formation* experiment on *bound* nucleons in an (almost)  $4\pi$  apparatus with good efficiency and resolution for low-momentum  $\gamma$ 's (such as KLOE) can reconstruct and measure a channel such as  $K^-p \rightarrow \pi^0 \Sigma^0$  (only above the  $\bar{K}N$  threshold) or  $K^-d \rightarrow$

$\pi^0\Sigma^0n_s$  (both *above* and *below* threshold), which is pure  $I = 0$ : up to now all analyses on the  $\Lambda(1405)$  have been limited to charged channels<sup>8</sup>, being forced to assume the  $I = 1$  contamination in their samples to be either negligible or smooth and not interfering with the resonance signal. This situation is particularly unsatisfactory, in view of the fact that the various spectroscopic models proposed for the classification of the  $\Lambda(1405)$  differ mostly in the detailed resonance *shape*, rather than in its couplings: now, it is precisely the shape which could be drastically changed even by a moderate amount of interference with an  $I = 1$  “background”. Note also that, having in the same apparatus and at almost the same energy *tagged*  $K^-$  and  $K_L$  produced at the same point, one can separate  $I = 0$  and  $I = 1$  channels with a minimum of systematic uncertainties, by measuring all channels  $K_L p \rightarrow \pi^0\Sigma^+$ ,  $\pi^+\Sigma^0$  and  $K^- p \rightarrow \pi^-\Sigma^+$ ,  $\pi^+\Sigma^-$ , besides, of course, the above-mentioned, pure  $I = 0$ ,  $K^- p \rightarrow \pi^0\Sigma^0$  one.

Another class of inelastic processes, which are expected to be produced (even if at a much smaller rate) by DAΦNE’s  $\bar{K}$ ’s, is radiative capture, leading in hydrogen to the final states  $\gamma\Lambda$  and  $\gamma\Sigma^0$  for incident  $K^-$ ’s, and, for incident  $K_L$ ’s, to the final state  $\gamma\Sigma^+$ : in deuterium, one expects to observe the capture processes by neutrons,  $K^-d \rightarrow \gamma\Sigma^-p_s$  and  $K_Ld \rightarrow \gamma\Sigma^0p_s$ ,  $\gamma\Lambda p_s$ , as well. Observation of these processes has been limited up to now to searches for photons emitted after capture of  $K^-$ ’s stopped in liquid hydrogen (and deuterium): alas, the spectra in these experiments are dominated by photons from unreconstructed  $\pi^0$  and  $\Sigma^0$  decays<sup>11</sup>. This poses serious difficulties already at the level of separation of signals from backgrounds, since (in  $K^-p$  capture at rest) only the photon line from the first final state falls *just above* the endpoint of the photons from decays of the  $\pi^0$ ’s in the  $\pi^0\Lambda$  final state, while that from the second falls right on top of this latter: indeed, these experiments were able to produce, within quite large errors, only an estimate of the respective branching ratios.

The  $4\pi$  geometry possible at DAΦNE, combined with the “transparency” of a KLOE-like apparatus, its high efficiency for photon detection and its good resolution for spatial reconstruction of the events, should make possible the full identification of the final states and therefore the measurement of the absolute cross sections for these processes, although in flight and not at rest.

This difference can be appreciated when comparing with theoretical predictions: the main contributions to radiative captures are commonly thought to come from radiative decays of resonant levels in the  $\bar{K}N$  system<sup>12</sup>, while the total hyperon production cross

section is expected to come from both resonant and non-resonant intermediate states. An estimate of the branching ratios would therefore be quite sensitive to the latter, while a prediction of the absolute cross sections should not.

Data<sup>11</sup> are presently indicating branching ratios around  $0.9 \times 10^{-3}$  for  $K^-p \rightarrow \gamma\Lambda$  and  $1.4 \times 10^{-3}$  for  $K^-p \rightarrow \gamma\Sigma^0$ , with errors of the order of 15% on both rates: most theoretical models<sup>13</sup> tend to give the first rate larger than the second, with both values consistently higher than the observed ones. Only a cloudy-bag-model estimate<sup>14</sup> exhibits the trend appearing (although only at a  $2\sigma$  level, and therefore waiting for confirmation by better data) from the first experimental determinations, but this is the only respect in which this model agrees with the data, still giving branching ratios larger than observations by a factor two.

Data are also interpretable in terms of  $\Lambda(1405)$  electromagnetic transition moments<sup>12</sup>: this interpretation of measurements taken at a single energy, or over a limited interval, is clearly subject to the effect of the interference between this state and all other contributions, such as the  $\Lambda$ - and  $\Sigma$ -hyperon poles and other resonant states such as the  $\Sigma(1385)$  and the  $\Lambda(1520)$ , not to mention  $t$ -channel exchanges (since at least  $K$ -exchange has to be included, to ensure gauge invariance of the Born approximation). An extraction of the  $\Lambda(1405)$  moments, relatively freer of these uncertainties, requires measurements of the final states  $\gamma\Lambda$  and  $\gamma\Sigma$  (if possible, in different charge states) over the *unphysical region*, using (gaseous) deuterium or helium as a "target". Rates are expected to be only of the order of  $10^4$  events/y, but it must be kept in mind that such a low rate corresponds already to statistics even better than those of the *best* experiment performed till now on the shape of the  $\Lambda(1405) \rightarrow \pi\Sigma$  decay spectrum<sup>8</sup>.

#### 4. The K-matrix (or M-matrix) formalism.

An adequate description of the low-energy  $\bar{K}N$  partial waves must couple at least the dominant, two-body inelastic channels to each other and to the elastic one; the three-body channel  $\pi\pi\Lambda$  is expected to be suppressed, for  $J^P = \frac{1}{2}^-$ , by the angular momentum barrier, but it could contribute appreciably to the  $I = 0$ ,  $J^P = \frac{1}{2}^+$   $P$ -wave, due to the strong final-state interaction of two pions in an  $I = 0$   $S$ -wave. Note that most bubble chamber experiments were unable to fully reconstruct the events at the lowest momenta, and therefore often assumed all *directly* produced  $\Lambda$ 's to come from the  $\pi\Lambda$  channel *alone*, neglecting altogether the small  $\pi\pi\Lambda$  contribution.

The appropriate formalism is to introduce a K-matrix description (sometimes it is convenient to use, instead of the K-matrix, its inverse, also known as the M-matrix), defined in the isospin eigenchannel notation as

$$\mathbf{K}_{\ell\pm}^{-1} = \mathbf{M}_{\ell\pm} = \mathbf{T}_{\ell\pm}^{-1} + i \mathbf{Q}^{2\ell+1}, \quad (22)$$

for both  $I = 0, 1$   $S$ -waves (and perhaps also for the four  $P$ -waves as well). The K-matrices, assuming  $SU(2)$  symmetry, describe the  $S$ -wave data at a given energy in terms of *nine* real parameters (six for  $I = 1$  and three for  $I = 0$ ), while the experimentally accessible processes can be described, assuming pure  $S$ -waves in the same symmetry limit, by only six independent parameters, which can be chosen to be the two (complex) amplitudes  $A_0, A_1$  for the  $\bar{K}N \rightarrow \bar{K}N$  channel, the phase difference  $\phi$  between the  $I = 0$  and  $I = 1$   $\pi\Sigma$  production amplitudes, and the ratio  $\epsilon$  between the  $\pi\Lambda$  production cross section and that for total hyperon production in an  $I = 1$  state<sup>15</sup>.

Thus a single-energy measurement does not allow a complete determination of the K-matrix elements at that energy. Using high-statistics measurements at different momenta, and assuming either *constant* K-matrices or (if more complexity were needed) *effective-range* M-matrices could *in principle* fully determine the matrix elements: but for this to be possible one has to be able to subtract out the (small)  $P$ -wave contributions to the integrated cross sections

$$\sigma = 4\pi L_0 = 2\pi \int_{-1}^{+1} \left( \frac{d\sigma}{d\Omega} \right) d\cos\theta = 4\pi[|T_{0+}|^2 + 2|T_{1+}|^2 + |T_{1-}|^2 + \dots], \quad (23)$$

which could be obtained either from  $L_1$  alone, for the elastic and charge-exchange channels, or from both  $L_1$  and  $P_Y$ , which give *complementary* information, for the hyperon production channels. None of these quantities has been measured up to now: the TST Collaboration tried to extract  $L_1$  from some of their low-statistics data, but found results consistent with zero within their obviously very large errors<sup>2</sup>. At the same level of accuracy, one should also be able to isolate and separate out the  $\pi\pi\Lambda$  channel contribution as well.

Remember that an accurate analysis has also to include the *complete* e.m. corrections<sup>6,7</sup>: up to now all  $\bar{K}N$  analyses have relied on the old, approximate formulæ derived by Dalitz and Tuan for a pure  $S$ -wave scattering<sup>16</sup>.

To fix the *redundant* K-matrix parameters, different authors have tried different methods: some have used the data on the shape of the  $\pi\Sigma$  spectrum from production

experiments<sup>17</sup>, others have constrained the amplitudes in the unphysical region by imposing consistency with dispersion relations for the amplitudes  $D$  for both  $K^{\pm p}$  and  $K^{\pm n}$  forward elastic scattering<sup>18–20</sup>, relying on the existence of accurate data on the total cross sections at higher energies. More recently, some attempts have been made to combine both constraints into a “global” analysis, but with no better results than each of them taken separately<sup>21</sup>.

Unfortunately, neither of these methods has been very powerful, because of the low statistics of the  $\pi\Sigma$  production data on one side, and on the other because of the need to use for the dispersion relations the often not very accurate information (and particularly so for the  $K^{\pm n}$  amplitudes) on the real-to-imaginary-part ratios.

We list below (without errors, often meaningless since the parameters are strongly correlated, and therefore not even quoted by some of the authors) the constant  $K$ -matrices found by Chao *et al.* using the first method<sup>17</sup> (which *did not* include the TST Collaboration data), and the more complex parametrization found by A.D. Martin using the second<sup>19,20</sup> (and including the *preliminary* TST data). Note that to describe the data for  $I = 0$  both above and below threshold A.D. Martin was forced to introduce a “constant-effective-range”  $M$ -matrix, where  $\mathbf{M}^{(0)} = (\mathbf{K}^{(0)})^{-1} = \mathbf{A} + \mathbf{R}k^2$ , with three more “effective range” parameters, so that to make the two analyses comparable we list separately his *threshold*  $K$ -matrix values.

The purpose of this table is to show that there is considerable uncertainty even on the value of the  $K_{NN}^{(I)}$  elements of the  $K$ -matrices (the real parts of the corresponding scattering lengths): the data have been re-analyzed by Dalitz *et al.*<sup>21</sup>, using *both* sets of constraints with different weights and different parametrizations, and yielding a variety of fits, all of them of about the same overall quality and none of them improving very much over the above ones.

Just to highlight the difficulties met in describing the data (probably plagued by inconsistencies between different experiments and BY large systematic uncertainties), we point out that A.D. Martin himself<sup>19</sup> found that including in his analysis a  $\Sigma(1385)$  resonance at the right position, with the width given by the production experiments (and listed in the Particle Data Group tables) and the coupling to the  $\bar{K}N$  channel dictated by flavour- $SU(3)$  symmetry, was worsening rather than improving the fits obtained *neglecting* it altogether: his analysis therefore considers the  $\Sigma$  Born-term contribution a “superposition” of the former and of that of the  $P$ -wave resonance, a rather unsavoury situation

considering the different  $J^P$  quantum numbers of the two states, which may raise questions about the applicability of his analysis away from  $t \simeq 0$ . Note that a similar superposition has to be considered in the  $K^\pm p$  dispersion relations for the  $\Sigma^-$  and  $\Lambda^-$ -pole contributions, which can not be separated from each other, but here the two states contribute to the same partial wave, and the  $\Sigma^-$ -pole can be extracted independently from  $K^\pm n$  scattering (or  $K_S$  regeneration on protons) data<sup>22</sup>.

Table IV

<i>Chao et al.</i>		<i>A. D. Martin</i>	
$K_{NN}^{(0)} = -1.56 fm$	$A_{NN} = -0.07 fm^{-1}$	$R_{NN} = +0.18 fm$	$K_{NN}^{(0)}(0) = -1.65 fm$
$K_{N\Sigma}^{(0)} = -0.92 fm$	$A_{N\Sigma} = -1.02 fm^{-1}$	$R_{N\Sigma} = +0.19 fm$	$K_{N\Sigma}^{(0)}(0) = +0.16 fm$
$K_{\Sigma\Sigma}^{(0)} = +0.07 fm$	$A_{\Sigma\Sigma} = +1.94 fm^{-1}$	$R_{\Sigma\Sigma} = -1.09 fm$	$K_{\Sigma\Sigma}^{(0)}(0) = -0.15 fm$
$K_{NN}^{(1)} = +0.76 fm$			$K_{NN}^{(1)} = +1.07 fm$
$K_{N\Sigma}^{(1)} = -0.97 fm$			$K_{N\Sigma}^{(1)} = -1.32 fm$
$K_{N\Lambda}^{(1)} = -0.66 fm$			$K_{N\Lambda}^{(1)} = -0.30 fm$
$K_{\Sigma\Sigma}^{(1)} = +0.86 fm$			$K_{\Sigma\Sigma}^{(1)} = +0.27 fm$
$K_{\Sigma\Lambda}^{(1)} = +0.51 fm$			$K_{\Sigma\Lambda}^{(1)} = +1.54 fm$
$K_{\Lambda\Lambda}^{(1)} = +0.04 fm$			$K_{\Lambda\Lambda}^{(1)} = -1.02 fm$

In the analysis of the low-energy data collected in the past on these processes, one of the main difficulties comes from the large spread in momentum of the typical low-energy kaon beams, for  $K^\pm$ 's because of the degrading in a "moderator" of the higher-energy beams needed to transport the kaons away from their production target, for  $K_L$ 's because of the large apertures needed to achieve satisfactory rates in the targets (typically bubble chambers): this made unrealistic the proposals (advanced from the early seventies) of better determining the low-energy  $K$ -matrices by studying the behaviour of the cross sections for  $K^-p$ -initiated processes at the  $\bar{K}^0 n$  charge-exchange threshold<sup>23</sup>. The high momentum resolution available at DAΦNE will instead make such a goal a realistically achievable one.

In this case one can no longer assume  $SU(2)$  to be a good symmetry of the amplitudes:

under the (reasonable) assumption that the forces are still  $SU(2)$ -symmetric, one can however still retain the previous  $K$ -matrix formalism, but one can no longer decouple the different isospin eigenchannels<sup>24</sup>. Introducing the orthogonal matrix  $\mathbf{R}$ , which transforms the six isospin eigenchannels for  $\bar{K}N$  ( $I = 0, 1$ ),  $\pi\Lambda$  ( $I = 1$  only) and  $\pi\Sigma$  ( $I = 0, 1, 2$ ) into the six physical charge channels  $K^-p$ ,  $\bar{K}^0n$ ,  $\pi^0\Lambda$ ,  $\pi^-\Sigma^+$ ,  $\pi^0\Sigma^0$  and  $\pi^+\Sigma^-$ , and the diagonal matrix  $\mathbf{Q}_c$  of the c.m. momenta for these latter, one can rewrite the  $T$ -matrix for the  $S$ -waves in the isospin-eigenchannel space as

$$\mathbf{T}_I^{-1} = \mathbf{K}_I^{-1} - i\mathbf{R}^{-1}\mathbf{Q}_c\mathbf{R}, \quad (24)$$

where  $\mathbf{K}_I$  is a box matrix with zero elements between channels of different isospin, and  $\mathbf{R}^{-1}\mathbf{Q}_c\mathbf{R}$  is of course no longer diagonal.

Apparently this involves one more parameter, since it also contains the element  $K_{\Sigma\Sigma}^{(2)}$ : in practice, if one is interested in the behaviour of the cross sections in the neighbourhood of the  $\bar{K}N$  charge-exchange threshold, one can take the c.m. momenta in the three  $\pi\Sigma$  channels as equal, so that the  $I = 2$ ,  $\pi\Sigma$  channel decouples from the  $I = 0, 1$  ones, since the “rotated” matrix  $\mathbf{R}^{-1}\mathbf{Q}_c\mathbf{R}$  has now only two non-zero, off-diagonal elements, equal to  $\frac{1}{2}(q_0 - q_-)$  (where the subscripts refer to the kaon charges), between the  $I = 0$  and  $I = 1$   $\bar{K}N$  channels, the diagonal ones being the same as in the fully  $SU(2)$ -symmetric case, if one substitutes for the  $\bar{K}N$  channel momentum  $q$  the average over the two charge states  $\frac{1}{2}(q_0 + q_-)$ .  $K_{\Sigma\Sigma}^{(2)}$  would however be important for describing accurate experiments on  $\pi\Sigma$  and  $\pi\Lambda$  mass spectra in the unphysical region below the  $\bar{K}N$  threshold without recourse to the  $SU(2)$ -symmetry limit: but the state-of-the-art of our understanding of wave-functions even for the lightest nuclei is not such as to make these isotopic-symmetry-breaking corrections relevant.

## 5. Low-energy $K^+$ scattering is important, too.

Better information on the  $S = +1$  system is also essential in several cases. We limit ourselves to mention only two of the problems coming to our mind. Isospin symmetry, as can be seen from the previous section, is an essential ingredient in the phenomenological analysis of the  $KN$  system, apart from obvious mass-difference effects, apparent only in the close proximity of the thresholds, which one can describe by modifying the  $K$ -matrix formalism as outlined above<sup>24</sup>.

One way to check isospin symmetry is to relate the amplitudes derived from charged kaon scattering to the data from  $K_S$  regeneration. Since isospin relates the scattering of



charged kaons on protons to the regeneration on neutrons (and vice versa), the test is better performed on an isoscalar nuclear target, such as deuterium or  $^4\text{He}$ . We should have indeed, apart from kinematical corrections and CP-violation effects,

$$\begin{aligned} T(K_L p \rightarrow K_S p) &= \frac{1}{2}[T(K^0 p \rightarrow K^0 p) - T(\bar{K}^0 p \rightarrow \bar{K}^0 p)] = \\ &= \frac{1}{2}[T(K^+ n \rightarrow K^+ n) - T(K^- n \rightarrow K^- n)] \end{aligned} \quad (25)$$

and

$$\begin{aligned} T(K_L n \rightarrow K_S n) &= \frac{1}{2}[T(K^0 n \rightarrow K^0 n) - T(\bar{K}^0 n \rightarrow \bar{K}^0 n)] = \\ &= \frac{1}{2}[T(K^+ p \rightarrow K^+ p) - T(K^- p \rightarrow K^- p)] ; \end{aligned} \quad (26)$$

when we introduce these equalities in a nuclear scattering calculation, as in e.g. a Glauber model, all elastic multiple scattering effects should apply equally to both the right- and left-hand sides of the equalities for an isoscalar nucleus, protecting the identity from a large fraction of the “nuclear” effects<sup>25</sup>.

Such tests could have been possible up to now only at higher momenta, where the opening of inelastic channels in the  $S = +1$  systems complicates calculations further: a test performed in the elastic region of this system should make things simpler and clearer.

The second problem, related in many theoretical analyses to observations from inelastic electron and muon scattering on nuclei, namely to changes in the electromagnetic properties and in the deep-inelastic structure functions of nucleons bound in nuclei with respect to the free ones, is the “antishadowing” effect observed at momenta around  $800 \text{ MeV}/c$  for  $K^+$  scattering on nuclear targets<sup>26</sup>. Conventional Glauber-model calculations<sup>27</sup> led to expect a ratio  $(2\sigma_A)/(A\sigma_D)$  slightly less than unity and decreasing with both the kaon momentum and the target mass number  $A$ , while the measured values were larger than unity and increasing with momentum. This led to think, as an explanation of this and of the aforementioned electromagnetic phenomena, of a “swelling” of the bound nucleons with respect to free ones, in line with some of the explanations put forward for the “nuclear” EMC effect, though at a much higher energy scale<sup>28</sup>.

New data have recently confirmed this trend<sup>29</sup>, but only for momenta higher than approximately  $600 \text{ MeV}/c$ ; a possibility coming to mind is that the opening of inelastic channels, such as  $\pi KN$  (or more simply quasi-two-body ones as  $KN^*$  and  $K^*N$ ), might necessitate the introduction of inelastic intermediate states absent in a conventional Glauber-model calculation, phenomenon analogous to the need to introduce inelastic

diffraction in the intermediate steps of a multiple-scattering formalism to explain diffractive processes on nuclei at much higher energies: thus the data would be just showing the opening of the threshold for such a phenomenon, particularly visible in the  $K^+$ -scattering case because of the extremely long mean-free path of this hadron in nuclear matter (about 7 fm).

Measurements of the  $K^+$  cross sections on different nuclei in DAΦNE's kinematical region, where  $K^+N$  interactions are purely elastic, should help close the issue when compared with accurate Glauber-model calculations<sup>27</sup>.

We would like to close reminding the reader that information on the  $S = +1$ ,  $I = 0$  channel in this energy region is coming *entirely* from extrapolations from higher-momentum data, since  $K^+$ -scattering (and regeneration) data on deuterium are *not* available at momenta lower than about 300 MeV/c: at present we have only a generic idea about the order of magnitude of the *absolute value* of the  $KN$   $I = 0$  scattering length, expected to be of the order of some times  $10^{-2}$  fm from forward dispersion relations and the lowest-momentum regeneration data<sup>18-20</sup>. An accurate measurement of the cross sections for  $K^+$  *incoherent* scattering on deuterium, possible at DAΦNE over a wide angular range, would thus give us the first *direct* measurement of this quantity.

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