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LNF-92/066 (P)
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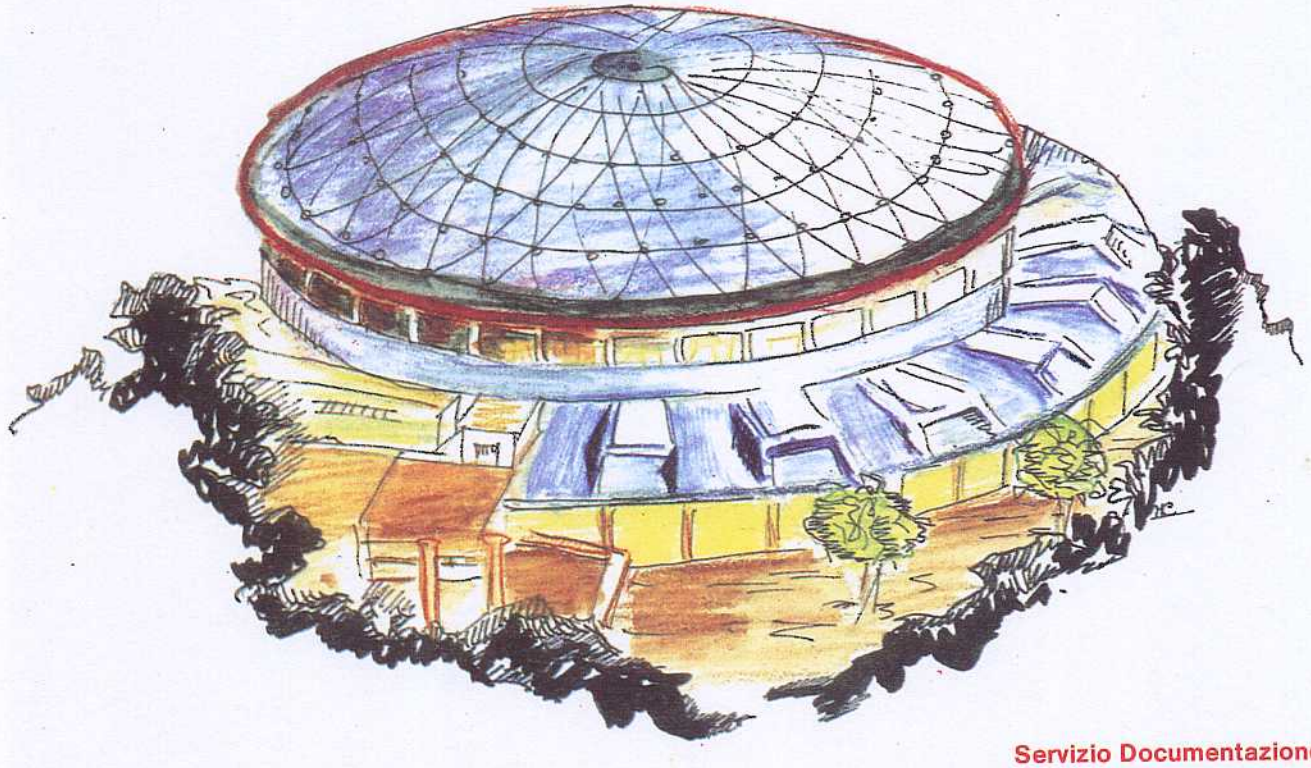
Contribution to the DAΦNE Physics Handbook



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RADIATIVE NON-LEPTONIC KAON DECAYS

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SECTION 1

1.1 Introduction

Radiative non-leptonic decays have been historically very important to understand GIM suppression in rare kaon decays [1]. They also play a crucial role in our understanding of low energy physics. Indeed, as we will see, they enable us to study phenomena which will lighten our knowledge of chiral dynamics. We overview the decays which we will be interested in table I. We have used the average experimental widths from [2]; in the channels where more recent data with smaller errors have appeared (like $K_L \to \gamma e^+ e^-$, see §5.2), these have been used. Furthermore, in the case of the internal bremsstrahlung decays where no average number was quoted in [2], we have used the experiments with the smaller error. More details on theory and experiments will be found in the relative sections. To illustrate the improvements at DAΦNE, we have assumed the following numbers of tagged decays [3]:

$$\begin{gathered} 1.1 \cdot 10^9 K_L/yr \\ 0.9 \cdot 10^{10} K^+ \quad or \quad K^-/yr \\ 1.7 \cdot 10^9 K_S/yr \end{gathered} \tag{1.1}$$

which are reasonable for a luminosity of $5 \cdot 10^{32} s^{-1} cm^{-2}$ and a machine working $10^7 s/yr$. Furthermore, to deduce the numbers of events expected at DAΦNE, for the channels where there are only experimental limits or poor statistics, we have used the theoretical predictions. We have divided the radiative non-leptonic decays in table I in the following groups:

i) two photons in the final state;
ii) "Internal Bremsstrahlung" one photon in the final state;
iii) "Direct Emission" one photon in the final state;
iv) Dalitz pair with one pion;
v) Dalitz pair with one photon.

We will give also some possible bounds that can be put at the ϕ-factory on CP violating quantities with radiative non-leptonic decays.

1.2 Chiral perturbation theory

For a general introduction on this subject we refer for instance to [4,5,6]. We just summarize the motivations of the theory and the results. Due to the non-perturbative

TABLE I

$\mathcal{L} = 5 \cdot 10^{32} cm^{-2} s^{-1}$			
channel	*Exp.BR.*	*Th.BR.*	Fluxes
$K_S \to \gamma\gamma$	$(2.4 \pm 1.2) \cdot 10^{-6}$	$2.1 \cdot 10^{-6}$	3570
$K_L \to \gamma\gamma$	$(5.7 \pm .27) \cdot 10^{-4}$	$\sim 10^{-4}$	$6.3 \cdot 10^5$
$K_L \to \pi^0\gamma\gamma$	$(2.1 \pm .6) \cdot 10^{-6}$	$\sim 1. \cdot 10^{-6}$	2300
$K_S \to \pi^0\gamma\gamma$	–	$3.8 \cdot 10^{-8}$	65
$K^+ \to \pi^+\gamma\gamma$	$< 1.5 \cdot 10^{-4}$	$6 \cdot 10^{-7}$	5400
$K_L \to \pi^0\pi^0\gamma\gamma$	–	$\sim 10^{-7}$	~ 110
Internal Bremsstrahlung			
$K_S \to \pi^+\pi^-\gamma$ $E^*_\gamma > 50MeV$	$(1.63 \pm .05) \cdot 10^{-3}$	$1.75 \cdot 10^{-3}$	$3 \cdot 10^6$
$K_L \to \pi^+\pi^-\gamma$ $E^*_\gamma > 20MeV$	$(1.32 \pm .05) \cdot 10^{-5}$	$1.4 \cdot 10^{-5}$	$1.5 \cdot 10^4$
$K^\pm \to \pi^\pm\pi^0\gamma$ $55MeV < T^*_c < 90MeV$	$(2.55 \pm .18) \cdot 10^{-4}$	$2.61 \cdot 10^{-4}$	$2.3 \cdot 10^6$
$K^+ \to \pi^+\pi^+\pi^-\gamma$ $E^*_\gamma > 11MeV$	$(1. \pm .4) \cdot 10^{-4}$	$0.61 \cdot 10^{-4}$	$5.5 \cdot 10^5$
$K^+ \to \pi^+\pi^0\pi^0\gamma$ $E^*_\gamma > 10MeV$	$(7.4^{+5.5}_{-2.9}) \cdot 10^{-6}$	$7.0 \cdot 10^{-6}$	$6 \cdot 10^4$
$K_L \to \pi^+\pi^-\pi^0\gamma$ $E^*_\gamma > 10MeV$	–	$1.5 \cdot 10^{-4}$	$1.65 \cdot 10^5$
Direct Emission			
$K_L \to \pi^+\pi^-\gamma$	$(2.95 \pm .11) \cdot 10^{-5}$	$\sim 10^{-5}$	$3.2 \cdot 10^4$
$K^\pm \to \pi^\pm\pi^0\gamma$ $55MeV < T^*_c < 90MeV$	$(1.8 \pm .4) \cdot 10^{-5}$	$\sim 10^{-5}$	$1.6 \cdot 10^5$
$K_S \to \pi^+\pi^-\gamma$ $E^*_\gamma > 50MeV$	$< .9 \cdot 10^{-4}$	$\sim 10^{-6}$	~ 1700
$K_S \to \pi^0\pi^0\gamma$	–	$\sim 10^{-8}$	~ 17
π – Dalitz pair			
$K_S \to \pi^0 e^+e^-$	$< 4.5 \cdot 10^{-5}$	$5 \cdot 10^{-9} \sim 5 \cdot 10^{-10}$	≤ 8
$K_S \to \pi^0\mu^+\mu^-$	–	$10^{-9} \sim 10^{-10}$	≤ 2
$K^\pm \to \pi^\pm e^+e^-$	$(2.7 \pm .5) \cdot 10^{-7}$	*input*	$2.4 \cdot 10^3$
$K^\pm \to \pi^\pm\mu^+\mu^-$	$< 2.3 \cdot 10^{-7}$	$(4 \sim 6.1) \cdot 10^{-8}$	≤ 550
γ – Dalitz pair			
$K_L \to \gamma e^+e^-$	$(9.1 \pm .4^{+.6}_{-.5}) \cdot 10^{-6}$	$9.1 \cdot 10^{-6}$	$1.0 \cdot 10^4$
$K_L \to \gamma\mu^+\mu^-$	$(2.8 \pm 2.8) \cdot 10^{-7}$	$3.8 \cdot 10^{-7}$	418
$K_S \to \gamma e^+e^-$	–	$3.4 \cdot 10^{-8}$	58
$K_S \to \gamma\mu^+\mu^-$	–	$7.9 \cdot 10^{-10}$	0
$K_L \to e^+e^-e^+e^-$	$(4 \pm 3) \cdot 10^{-8}$	–	44

structure of QCD at low energy, matrix elements for kaon decays should be evaluated using symmetry arguments. PCAC and soft pion theorems have been very useful in this regard. For instance, assuming that pions and kaons are the Goldstone bosons of the approximate chiral symmetry $SU(3)_L \times SU(3)_R$ broken to $SU(3)_V$, it is possible for instance to relate the $K \to 2\pi$ to the $K \to 3\pi$ amplitude

$$\lim_{p\to 0} < \pi^+(p_1)\pi^-(p_2)\pi^0(p)|H_W|K^0 >= \frac{-i}{2F_\pi} < \pi^+(p_1)\pi^-(p_2)|H_W|K^0 > \qquad (1.2)$$

Though the physical pions in the $K \to 3\pi$ decay are kinematically somewhat faraway from the pions of $K \to 2\pi$ decay, this relation holds phenomenologically at $20\% \sim 30\%$ level. The problem is then how to take care of non-vanishing four-momenta of the physical pions (or kaons).

CHPT fulfills this task, introducing a Lagrangian which satisfies symmetry requirements (the interactions have to be invariant under $SU(3)_L \times SU(3)_R$) and by having the mesons as the Goldstone bosons associated with the spontaneous breaking of the chiral symmetry of $SU(3)_L \times SU(3)_R \to SU(3)_V$. Meson masses are introduced through an explicit breaking of the symmetry. At tree level all the low energy theorems, PCAC and soft pion properties are recovered.

Furthermore, although the theory is not renormalizable we do require unitarity, which is obtained perturbatively by considering also meson loops. This can be done at the price of adding to the theory, order by order, new counterterms which have to be determined by experiments. Nevertheless CHPT furnishes a systematic way of computing physical amplitudes correcting the soft pion theorems.

Radiative non-leptonic decays are very important since, like other meson amplitudes, they will give us information on chiral dynamics. Furthermore some decays, such as $K_S \to \gamma\gamma$ and $K_L \to \pi^0\gamma\gamma$, have the property to depend only on meson loops and not on counterterms (which are vanishing for these decays), so that experiments will probe in that case an unambigous prediction of CHPT.

The Lagrangian

$$L = L_{\Delta S=0} + L_{\Delta S=1} \qquad (1.3)$$

can be expanded in the external momenta and masses. $L_{\Delta S=0}$ is the strong lagrangian including electromagnetic interactions. At order p^2 one has

$$L_{\Delta S=0} = \frac{1}{4}f^2 Tr D_\mu U D^\mu U^\dagger + \frac{f^2}{2} Tr U^\dagger \mu M + \frac{f^2}{2} Tr U \mu M \qquad (1.4)$$

where

$$U = e^{\frac{2i}{f}\pi_a T_a}, \quad D_\mu U = \partial_\mu U + ieA_\mu[Q,U] \qquad (1.5)$$

$$M = diag(m_u, m_d, m_s) \quad Q = diag(2/3, -1/3, -1/3)$$

$$Tr T_a T_b = \frac{1}{2}\delta_{ab}, \quad f \simeq F_\pi = 93.3\text{MeV} \quad T_a = \lambda_a/2 \qquad (1.6)$$

λ_a are the Gell-Mann matrices, μ is the correct factor to reproduce the observed meson masses and A_μ is the electromagnetic field†.

† The Condon-Shortley-De Swart phase convention is not satisfied.

The CP-conserving $\Delta S = 1$ weak Lagrangian consists of two pieces: the octet and the 27-plet:

$$L^{(8)}_{\Delta S=1} = \frac{1}{4} f^2 h_8 Tr\lambda_6 D_\mu U D^\mu U^\dagger \tag{1.7}$$

$$L^{(27)}_{\Delta S=1} = \frac{h_{27} f^2}{4} T^{kl}_{ij} (U D_\mu U^\dagger)^i_k (U D^\mu U^\dagger)^j_l + h.c. \tag{1.8}$$

where the tensor T is the U-spin=1, ΔS=1, ΔQ=0 element of the 27 with components:

$$T^{12}_{13} = T^{21}_{13} = T^{21}_{31} = T^{12}_{31} = \frac{3}{5} \quad T^{22}_{23} = T^{22}_{32} = T^{23}_{33} = T^{32}_{33} = -\frac{3}{10} \tag{1.9}$$

From $K \to \pi\pi$ decays we have at order p^2

$$h_8 = 3.2 \cdot 10^{-7} \quad h_{27} = -1 \cdot 10^{-8} \tag{1.10}$$

1.3 Higher dimension operators

We will consider also $o(p^4)$ operators. There are strong $o(p^4)$ counterterms described by Gasser and Leutwyler [5]. Their coefficients can be taken from the experiments, vector meson dominance [7] or $1/N_c$ model [8,9]. Weak operator for the octet and for the 27-plet operators are described by Kambor, Missimer and Wyler [10] (see also [11]). Furthermore, they have tried to fit some coefficients of these operators fitting them to all experiments on K$\to \pi\pi$ and K$\to \pi\pi\pi$ [12]. Some of these coefficients are well reproduced by VMD [13].

For electromagnetic couplings there are two kinds of operators: *i*) the ones obtained by minimal coupling $\partial_\mu U \to D_\mu U$ (thus these can be fixed also from amplitudes without photons; all $o(p^4)$ operators relevant for $K \to 2\pi$ and $K \to 3\pi$ will appear in radiative non-leptonic decays); *ii*) direct couplings in $L_{\Delta S=0}$ and $L_{\Delta S=1}$; these couplings can be determined only through amplitudes involving photons (real and virtual). In $L_{\Delta S=0}$ at $o(p^4)$ we have couplings with one photon and with two photons

$$L^{(4)}_{em} = -ieL_9 F^{\mu\nu} tr(Q D_\mu U D_\nu U^\dagger + Q D_\mu U^\dagger D_\nu U) + e^2 L_{10} F^{\mu\nu} F_{\mu\nu} tr(U Q U^\dagger Q) \quad (1.11).$$

In addition to these new operators there are meson loops, which are generally divergent and therefore will be regulated at a scale μ; such scale dependence must cancel a correspondent scale dependence in the afore-mentioned counterterms. The full physical amplitude is thus scale independent. $L_9 + L_{10}$ is scale independent and from the decay $\pi \to l\nu\gamma$ it turns out to be [5,6]

$$L_9 + L_{10} = (1.39 \pm .38) \cdot 10^{-3} \tag{1.12}$$

For the $\Delta S = 1$ direct couplings there are several operators [14,10]. Specifically the relevant ones for $K \to \pi\gamma^* \to \pi l\bar{l}$ are

$$L^{(4)}_{\Delta S=1,em} = \frac{-ieh_8}{8f^4} F^{\mu\nu}(w_1 tr(Q\lambda_{6-i7} L_\mu L_\nu) + w_2 tr(QL_\mu \lambda_{6-i7} L_\nu) + \tilde{w}_3 \epsilon_{\mu\nu\rho\sigma} tr(QL^\rho) tr(\lambda_{6-i7} L^\sigma)) + h.c. \tag{1.13}$$

$$L_\mu = if^2 U D_\mu U^\dagger \tag{1.14}.$$

It is to be remarked that new $o(p^4)$ direct counterterms will appear in $K \to \pi\pi\gamma$ decays [15], as we will describe in §3.4. The direct coupling to two photons which appears for instance in $K^\pm \to \pi^\pm\gamma\gamma$ is

$$L^{(4)}_{\Delta S=1,em^2} = \frac{e^2}{8} h_8 w_4 F^{\mu\nu} F_{\mu\nu} tr(\lambda_{6-i7} QUQU^\dagger) + h.c. \tag{1.15}.$$

Furthermore at order p^4 there is the anomalous Wess-Zumino term; the contribution linear in the meson fields is

$$L^{(4)}_{WZ,em^2} = \frac{\alpha}{8\pi f} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} (\pi^0 + \frac{\eta_8}{\sqrt{3}}) \tag{1.16}.$$

This will be relevant for $K_S \to \pi^0\gamma\gamma$, $K_L \to \gamma\gamma^* \to \gamma l^+ l^-$ and others.

SECTION 2. Radiative kaon decays with two photons in the final state

Two photons can have either $CP = +1$ or $CP = -1$. Thus in the case of two photon final state, due to gauge invariance, the amplitude will be proportional either to $F_{\mu\nu}F^{\mu\nu}$ (parallel polarization, $CP = +1$) or to $\varepsilon_{\mu\nu\rho\lambda}F^{\mu\nu}F^{\rho\lambda}$ (perpendicular polarization, $CP = -1$). In the case of more particles in the final state also other invariant amplitudes will appear.

2.1 $K_S \to \gamma\gamma$

Since K^0 is neutral, there is no tree level contribution to $K^0 \to \gamma\gamma$. We will consider the long distance contribution to the CP conserving amplitude $A(K_S \to \gamma\gamma)$ in the framework of CHPT [16]. There are no $o(p^2)$ tree diagrams, thus we have to consider $o(p^4)$ contributions: chiral meson loops (Fig. 2.1) and in principle the $o(p^4)$ counterterms (see (1.15)); but again since K^0 is neutral $o(p^4)$ counterterms are vanishing for these amplitudes.

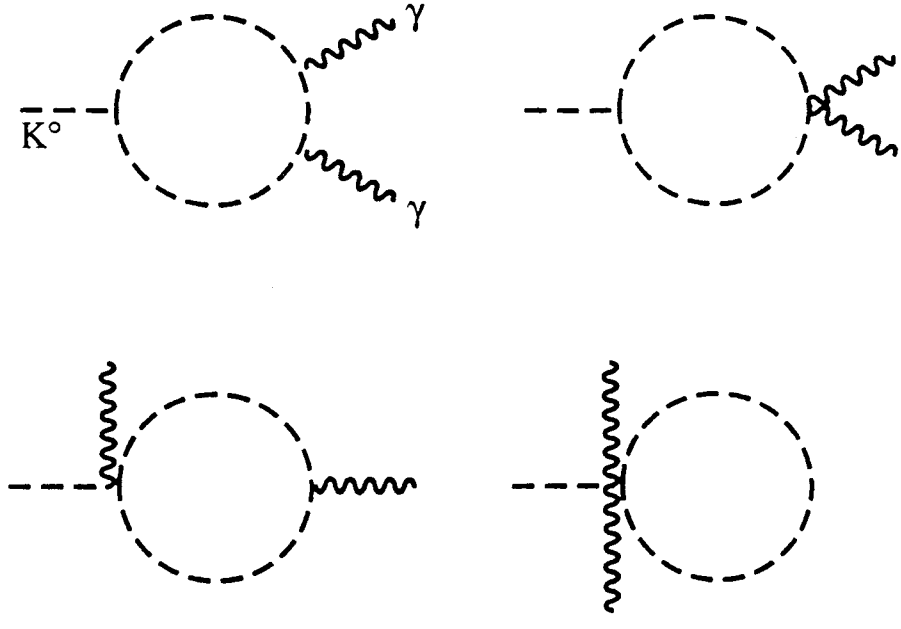

Figure 2.1 : Long distance contributions to $K^0 \to \gamma\gamma$, where the photons have parallel polarizations.

This fact has two implications:

1) the chiral meson loops are finite and so free of the ambiguity of the cut-off;
2) these are the only contributions $o(p^4)$; no dependence on unknown coupling constants of counterterms.

The finiteness of the 1-loop amplitude can be simply understood also by the fact that the superficial degree of divergence of the amplitude $\sim \Lambda^2$ (with Λ an ultraviolet cut-off) is decreased by gauge invariance together with the condition that the amplitude should be proportional to $A(K_1 \to \pi^+\pi^-)$ and consequently to $m^2_{K^0} - m^2_{\pi^+}$ (Cabibbo, Gell-Mann theorem [17]). Kaon loops, for the previous argument, are proportional to $m^2_{K^0} - m^2_{K^+}$ and thus they can be neglected.

Adding the 8 and 27 contributions one obtains [16,18]:

$$A(K_1 \to 2\gamma_{\|}) = -\frac{\alpha}{\pi m_K^2}\left(\frac{h_8}{2f} - \frac{3h_{27}}{5f}\right)(m_K^2 - m_\pi^2)\cdot$$

$$\cdot(q_1 \cdot \epsilon_2 q_2 \cdot \epsilon_1 - \epsilon_1 \cdot \epsilon_2 q_1 \cdot q_2)\left[1 + \frac{m_\pi^2}{m_K^2} ln^2 \frac{\beta - 1}{\beta + 1}\right] \tag{2.1}$$

where $q_1, \epsilon_1, q_2, \epsilon_2$ are the photon momenta and polarizations and

$$\beta = \sqrt{1 - \frac{4m_\pi^2}{m_K^2}} \tag{2.2}$$

The argument of the logarithm is negative because the two pions can be on shell. Indeed

$$1 + \frac{m_\pi^2}{m_K^2} ln^2 \frac{\beta - 1}{\beta + 1} = 1 + \frac{m_\pi^2}{m_K^2}\left(ln^2 \frac{1 - \beta}{\beta + 1} - \pi^2 + 2i\pi ln \frac{1 - \beta}{\beta + 1}\right) \tag{2.3}$$

From the amplitude in (2.1) we can calculate the rate with the result

$$\Gamma(K_S \to 2\gamma_{\|}) = \frac{(h_8 - \frac{6}{5}h_{27})^2 (m_K^2 - m_\pi^2)^2 \alpha^2}{256\pi^3 m_K f^2} \cdot \left|1 + \frac{m_\pi^2}{m_K^2} ln^2 \frac{\beta - 1}{\beta + 1}\right|^2 \tag{2.4}$$

The prediction for this decay is $\Gamma(K_S \to \gamma\gamma) = 1.52\ 10^{-11} eV$, giving a branching ratio

$$Br(K_S \to \gamma\gamma) = 2.1 \cdot 10^{-6} \tag{2.5}$$

and the ratio R

$$R \equiv \frac{\Gamma(K_S \to 2\gamma)_{th}}{\Gamma(K_L \to 2\gamma)_{exp}} \approx 2 \tag{2.6}$$

Experimentally NA31 [19] has measured the branching ratio

$$Br(K_S \to 2\gamma) = (2.4 \pm 1.2) \cdot 10^{-6} \tag{2.7}$$

It is important to remark that this is a significant test for chiral perturbation theory, since it is unambiguously predicted by this theory in terms of the pion loop with coupling given by CHPT. Indeed, the lacking of the corresponding $o(p^4)$ operator not only says the amplitude is finite but also ensures that contact terms will be order p^6 and so suppressed by $\frac{m_K^2}{\Lambda^2}$. Since the absorptive part, which is model independent, is overwhelming in this decay, using a phenomenological coupling in the pion loop model [20,21] one obtains similar results. DAΦNE is certainly suitable to improve the experimental result (2.7).

2.2 $K_L \to \pi^0\gamma\gamma$

The general amplitude for $K_L \to \pi^0\gamma\gamma$ is given by

$$M(K_L \to \pi^0(p')\gamma(q_1,\varepsilon_1)\gamma(q_2,\varepsilon_2)) = \varepsilon_{1\mu}\varepsilon_{2\nu}M^{\mu\nu}(p',q_1,q_2) \tag{2.8}$$

where ε_1, ε_2 are the photon polarizations and $M^{\mu\nu}$, if CP is conserved, is made of two invariant amplitudes:

$$M^{\mu\nu} = \frac{A(y,z)}{m_K^2}(q_2^\mu q_1^\nu - q_1\cdot q_2 g^{\mu\nu})$$

$$+\frac{2B(y,z)}{m_K^4}(-p\cdot q_1 p\cdot q_2 g^{\mu\nu} - q_1\cdot q_2 p^\mu p^\nu + p\cdot q_1 q_2^\mu p^\nu + p\cdot q_2 p^\mu q_1^\nu) \tag{2.9}$$

where

$$y = |p\cdot(q_1-q_2)|/m_K^2 \qquad z = (q_1+q_2)^2/m_K^2$$

Note that $A(y,z)$ and $B(y,z)$ are symmetric for $q_1 \leftrightarrow q_2$ as required by Bose symmetry.

The physical region in the dimensionless variables y and z is given by the inequalities

$$0 \le y \le \frac{1}{2}\lambda^{1/2}(1,r_\pi^2,z) \quad 0 \le z \le (1-r_{\pi^0})^2 \tag{2.10}$$

where

$$\lambda(1,r_\pi^2,z) = 1+z^2+r_\pi^4-2(z+zr_\pi^2+r_\pi^2) \quad \text{and} \quad r_\pi = \frac{m_{\pi^0}}{m_{K^0}} \tag{2.11}$$

From (2.10) and (2.11) we obtain the double differential rate for unpolarized photons:

$$\frac{d^2\Gamma}{dy\,dz} = \frac{m_K}{2^9\pi^3}\left\{z^2|A+B|^2 + \left[y^2 - \left(\frac{(1+r_\pi^2-z)^2}{4} - r_\pi^2\right)\right]^2 |B|^2\right\} \tag{2.12}$$

We remark that, due to the different tensor structure in (2.9), the A and B parts of the amplitude give rise to contributions to the differential decay rate which have different dependence on the two-photon invariant mass z. In particular, the second term in (2.12) gives a non vanishing contribution to $\dfrac{d\Gamma(K_L \to \pi^0\gamma\gamma)}{dz}$ in the limit $z \to 0$. Thus the kinematical region with collinear photons is important to disentangle the B amplitude. We will consider the long distance contributions to this decay in the framework of CHPT. Since K_L and π^0 are neutral there is no tree level $o(p^2)$ contribution. At $o(p^4)$ there are loops and the counterterms in (1.13). But since again K_L and π^0 are neutral the latter ones do not contribute and this implies that the 1-loop amplitude is finite [22,23]. In Fig. 2.2 are reported the relative diagrams. Actually at order p^4 the amplitude $B(y,z)$ is zero since there are not enough powers of momenta at this order. The result for the

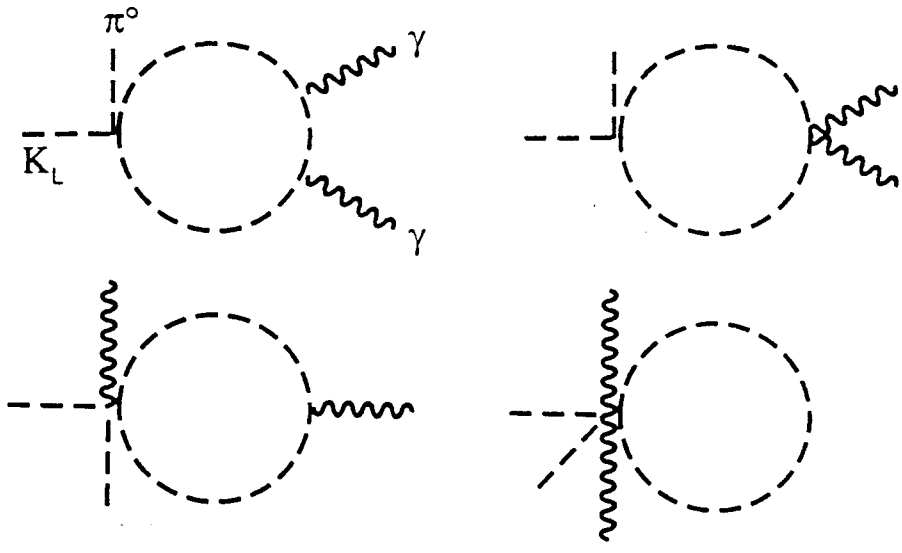

Figure 2.2 : Long distance contribution to $K_L \to \pi^0\gamma\gamma$ in the basis of [22].

amplitude $A(y,z)$ (which at this order it depends only on z), limiting to the $SU(3)_L$ octet contribution, is [22,23]:

$$A^{(8)}(y,z) = \frac{h_8 \alpha m_K^2}{4\pi f^2}\left[(1-\frac{r_\pi^2}{z})\cdot f(\frac{z}{r_\pi^2}) - (1-\frac{r_\pi^2}{z}-\frac{1}{z})f(z)\right] \tag{2.13}$$

$$f(x) = 1 + \frac{1}{x} ln^2 \frac{\beta(x)-1}{\beta(x)+1}, \qquad \beta(x) = \sqrt{1-\frac{4}{x}} \tag{2.14}$$

The function $f(x)$ is real for $x \leq 4$ and complex for $x \geq 4$. More explicitly it is written

$$f(x) = \begin{cases} 1 - \frac{4}{x}\arcsin^2\frac{\sqrt{x}}{2} & x \leq 4 \\ 1 + \frac{1}{x}[ln^2\frac{1-\beta(x)}{1+\beta(x)} - \pi^2 + 2i\pi ln\frac{1-\beta(x)}{1+\beta(x)}] & x \geq 4 \end{cases} \tag{2.15}$$

In (2.13) the contribution proportional to $f(z)$ comes from the kaon loops and so it does not have absorptive part, while the one proportional to $f(\frac{z}{r_\pi^2})$ comes from pion loops and has absorptive part, since the pions can be on shell. Correspondingly the kaon contribution is much less than the one of pions.

An interesting aspect of this decay could be the contribution of the 27. Due to the vanishing of the corresponding counterterms also this contribution is finite and unambigously predicted; for the pion loop, which give the larger contribution, one obtains [24]:

$$A_{1/2}^{(27)}(y,z) = -\frac{h_{27}\alpha m_K^2}{20\pi f^2}(1-\frac{r_\pi^2}{z})f(\frac{z}{r_\pi^2}) \tag{2.16}$$

$$A_{3/2}^{(27)} = \frac{h_{27}\alpha m_K^2}{8\pi f^2}\left[\frac{3-r_\pi^2-14r_\pi^4-(5-14r_\pi^2)z}{(1-r_\pi^2)z}\right] f(\frac{z}{r_\pi^2}) \tag{2.17}$$

Compared to the octet, there is a slight modification of the spectrum and of the width, which might be measured with high precision experiments.

The z spectrum for the y independent amplitudes (2.13), (2.16) and (2.17) is given by

$$\frac{d\Gamma}{dz} = \frac{m_K}{2^{10}\pi^3} z^2 [(1+r_\pi)^2 - z]^{\frac{1}{2}} [(1-r_\pi)^2 - z]^{\frac{1}{2}} |A(z)|^2 \qquad (2.18)$$

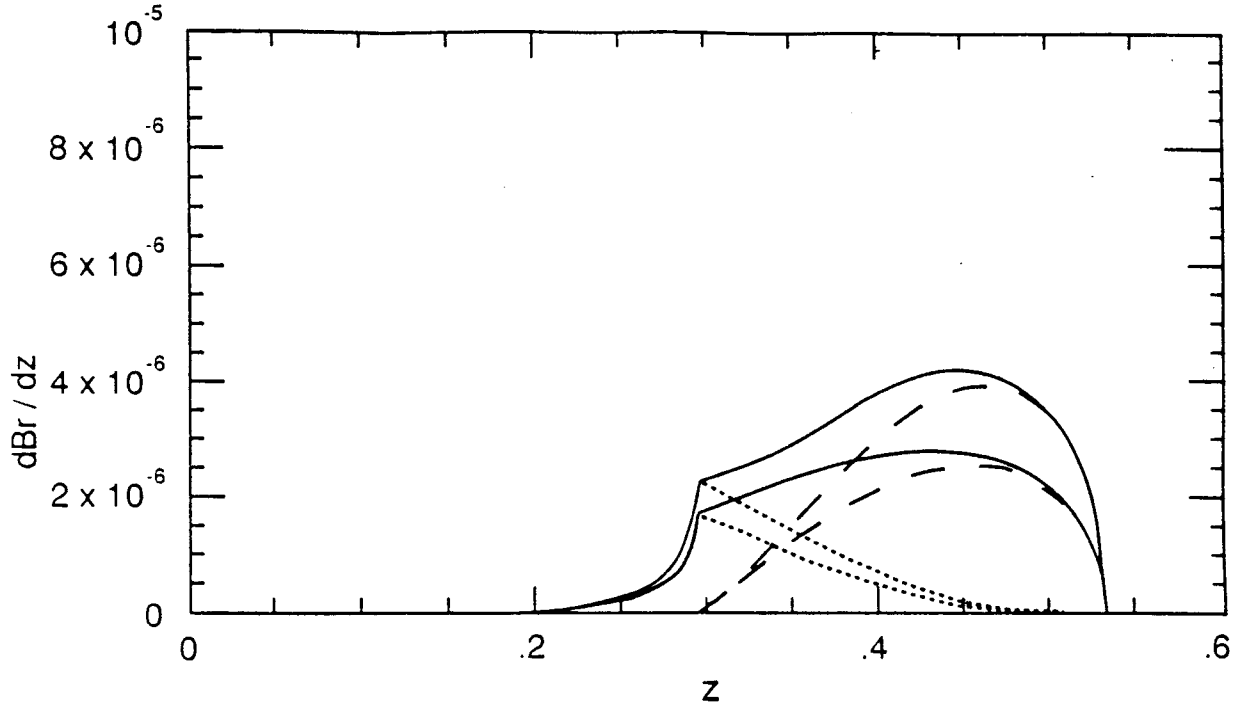

Figure 2.3 : Theoretical predictions for $\frac{d\Gamma}{dz}(K_L \to \pi^0\gamma\gamma)$. The full lines are total contributions. The lower curve is the $o(p^4)$ 8+27 contribution. The upper curve is obtained by adding all $o(p^4)$ $K \to 3\pi$ contributions in the absorptive part [24] and taking h_8 and h_{27} from α_1 and α_3 (these constants are defined for instance in [11,12]) in the dispersive part. The dashed and dotted lines are are the absorptive and dispersive contributions respectively.

The $o(p^4)$ CHPT prediction of $\frac{dBr}{dz}(K_L \to \pi^0\gamma\gamma)$ is reported in Fig. 2.3, while the prediction for the branching ratio is:

$$Br^{(8)}(K_L \to \pi^0\gamma\gamma) = .68 \cdot 10^{-6} \quad Br^{(8+27)}(K_L \to \pi^0\gamma\gamma) = .60 \cdot 10^{-6} \qquad (2.19)$$

As for the $A(K_S \to \gamma\gamma)$ this is an excellent test for chiral perturbation theory. In particular the spectrum and the width are very characteristic of CHPT; in fact the peak is due to the absorptive part. It is interesting to remark that from the amplitude (2.13) we can recover the soft pion limit [25].

NA31 and E731 have recently obtained some data in this decay. An important background for this process is $K_L \to \pi^0\pi^0$; as a consequence experiments cannot explore the

region $z \sim \frac{m_\pi^2}{m_K^2}$. In particular NA31 [26] and E731 [27] have given the result for the branching ratio

$$Br(K_L \to \pi^0\gamma\gamma) = (2.1 \pm 0.6) \cdot 10^{-6} \quad \text{for} \quad m_{\gamma\gamma} > 280 MeV \quad \text{NA31} \tag{2.20}$$

$$Br(K_L \to \pi^0\gamma\gamma) = (1.86 \pm .60 \pm .60) \cdot 10^{-6} \quad \text{for} \quad m_{\gamma\gamma} > 280 MeV \quad \text{E731} \tag{2.21}$$

where $m_{\gamma\gamma}$ is the two photon invariant mass:

$$m_{\gamma\gamma} = \sqrt{(q_1 + q_2)^2}$$

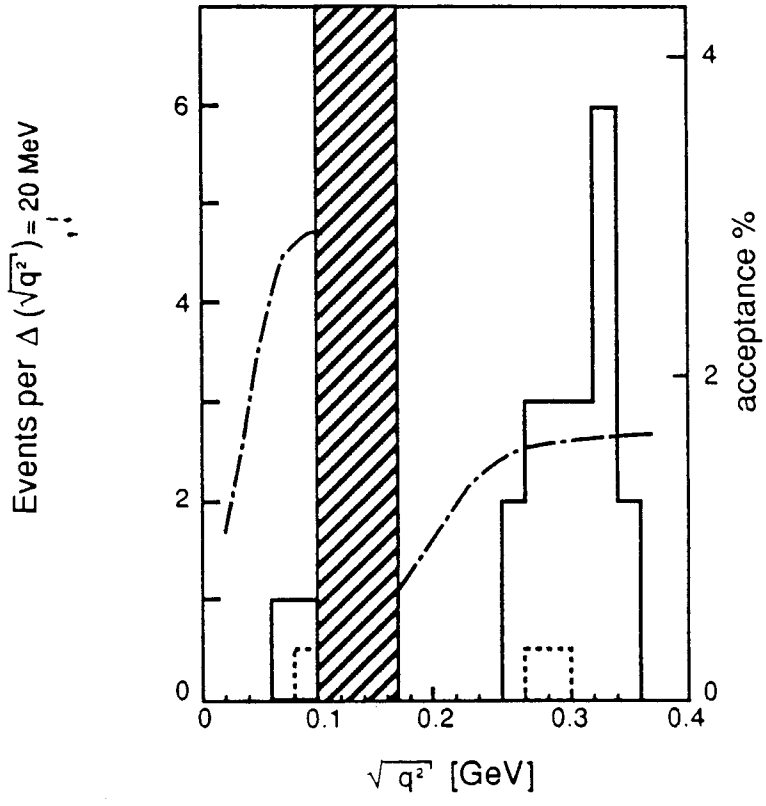

Figure 2.4 : Measured $\gamma\gamma$ invariant mass distribution [26] for $K_L \to \pi^0\gamma\gamma$ (solid histogram). The shaded area indicates the region where NA31 experiment is insensitive. The experimental acceptance is given by the dashed-dotted line. The dashed histogram corresponds to the estimated remaining background.

In Fig. 2.4 the NA31 two-photon invariant mass histogram is reported. The shaded area is not accessible experimentally. Furthermore notice that the experimental acceptance is larger at small z. We can see that, though the rate seems underestimated, the spectrum agrees rather well, confirming in some way the loop amplitude. Actually, in this regard we would like to comment on this situation [24,28]; the spectrum results as sum of the square of dispersive and absorptive parts. The latter can be expressed in a complete model independent way in terms of the physical $A(K \to 3\pi)$ [11,12]; thus one obtains including linear and quadratic slopes from just this contribution the branching ratio:

$$Br_{abs}(K_L \to \pi^0\gamma\gamma) = .61 \cdot 10^{-6} \tag{2.22}$$

This is still far away from the experimental numbers, which accordingly would imply a large dispersive part, much larger than predicted by CHPT. In fact as a way to estimate the higher order effects coming from the pion extrapolation, we have changed the determination of h_8 and h_{27} in (2.13), (2.16) and (2.17) from $K \to 2\pi$ to that given by $K \to 3\pi$ at order p^2. One obtains the spectrum in Fig. 2.3 and the branching ratio

$$Br(K_L \to \pi^0\gamma\gamma) = .86 \cdot 10^{-6} \tag{2.23}$$

Of course a full $o(p^6)$ dispersion relation of all these contributions would be quite interesting [24].

Another point of interest of $K_L \to \pi^0\gamma\gamma$ is his role in $K_L \to \pi^0 e^+e^-$. The decay $K_L \to \pi^0 e^+e^-$ has three kinds of contributions [29,30,31,32]: direct CP violation, mass CP violation $K_L \to \pi^0\gamma^* \to \pi^0 e^+e^-$ and CP conserving $K_L \to \pi^0\gamma^*\gamma^* \to \pi^0 e^+e^-$. The CP violating contributions are expected to give a $Br(K_L \to \pi^0 e^+e^-) \simeq 10^{-11}$, while since $o(p^4)$ $K_L \to \pi^0\gamma\gamma$ gives an helicity suppressed CP conserving amplitude (from $A(y,z)$ in (2.13)), only the $o(p^6)$ contribution might give an appreciable rate, which indeed CHPT naive power counting would predict of order 10^{-14} [31,32,33]; still in this framework, $o(p^6)$ vector meson exchange diagrams [32,34,35] might enhance both $Br(K_L \to \pi^0\gamma\gamma)$ and the CP conserving $Br(K_L \to \pi^0 e^+e^-)_{CP}$, but it is difficult to reach values for these branching ratios of about $2{\cdot}10^{-6}$ and 10^{-11} respectively.

Phenomenological models with large vector [36,37] or scalar exchange [38] can obtain these values, but would alter the z-spectrum, particularly at small z, where they would predict a large increase. These models have been criticized by the authors of [32], claiming that diagrams with $o(p^4)$ direct weak vertices, $L_W^{(4)}$, have to be added to the ones in Fig. 2.5, arising from $L_S^{(4)}$, and a cancellation between these two kinds of diagrams might occur.

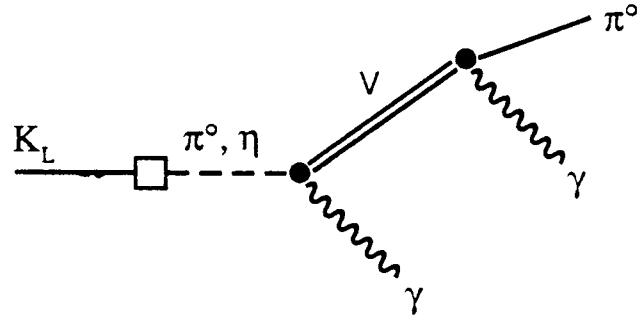

Figure 2.5 : Diagrams arising from $L_S^{(4)}$, the $o(p^4)$ strong Lagrangian, to $K_L \to \pi^0\gamma\gamma$.

In particular, they show how neglecting the $L_W^{(4)}$ term for $\Gamma(K^+ \to \pi^+ e^+e^-)$ brings to a width 30 times bigger than the experimental one. They parametrize the $o(p^6)$ contributions

to $A(K_L \to \pi^0\gamma\gamma)$, including direct weak transitions, by an effective vector coupling a_V:

$$A(y,z) = \frac{h_8 m_K^2 \alpha}{4\pi f^2} a_V (3 - z + r_\pi^2) \tag{2.24}$$

$$B(y,z) = -\frac{h_8 m_K^2 \alpha}{2\pi f^2} a_V \tag{2.25}$$

Then $Br(K_L \to \pi^0 e^+ e^-)|_{abs} = 4.4 a_V^2 \cdot 10^{-12}$. Varying a_V they obtain:

a_V	$Br(K_L \to \pi^0\gamma\gamma)\cdot 10^6$	$Br(K_L \to \pi^0 e^+ e^-)_{CP=+1}\cdot 10^{13}$
0	0.67	0.08
0.32	0.60	4.5
−0.32	0.89	4.5
1.5	1.6	100
−1.5	3.0	100

The weak deformation model predicts $a_V = -.32$ [32]. It is important to remark that eq.(2.25) will change the spectrum in the region of small z. Actually NA31, due to its larger experimental acceptance in this region, seems to exclude a large dispersive contribution at small z. So as a conclusion DAΦNE seems an ideal machine to investigate the relative roles of CHPT and VMD; in particular to assess the values of both the absorptive part and the dispersive part of these amplitude.

2.3 $K^+ \to \pi^+\gamma\gamma$

At present there is only an upper bound for the branching ratio [39], which depends upon the shape of the spectrum due to the different experimental acceptance. For a constant amplitude and for the $o(p^4)$ CHPT amplitude, which will be discussed below, the following limits are respectively obtained:

$$Br_{exp}(K^+ \to \pi^+\gamma\gamma) \leq 1.0 \cdot 10^{-6} \tag{2.26}$$

and

$$Br_{exp}(K^+ \to \pi^+\gamma\gamma) \leq 1.5 \cdot 10^{-4} \tag{2.27}$$

A cut in the two-photon invariant mass q^2 is necessary to disentangle this channel from the background $K^+ \to \pi^+\pi^0 \to \pi^+\gamma\gamma$. Gauge invariance and chiral symmetry imply that this decay can start only at order p^4 in CHPT. Two invariant amplitudes contribute at this order:

$$M(K^+(p) \to \pi^+(p')\gamma(q_1,\epsilon_1)\gamma(q_2,\epsilon_2)) =$$

$$= \epsilon_\mu(q_1)\epsilon_\nu(q_2)\left[A(y,z)\frac{(q_2^\mu q_1^\nu - q_1\cdot q_2 g^{\mu\nu})}{m_K^2} + C(y,z)\varepsilon^{\mu\nu\alpha\beta}\frac{q_{1\alpha}q_{2\beta}}{m_K^2}\right] \tag{2.28}$$

where y and z have been defined in formula (2.9). The physical region of y and z is given in (2.10) and (2.11) with

$$r_\pi = \frac{m_{\pi+}}{m_{K+}} \tag{2.29}$$

The amplitude A corresponds to a final state with $CP = +1$, while C corresponds to a state with $CP = -1$. Compared to (2.9) the new amplitude $C(y,z)$ appears, since the initial and final states have no definite CP. The theoretical estimates of A and C, reported in the following, give a dependence only on z. Therefore it is possible to integrate on all the other variables, obtaining the following expression for the two photon normalized invariant mass spectrum:

$$\frac{d\Gamma}{dz}(K^+ \to \pi^+\gamma\gamma) = \frac{m_{K^+}}{2^{10}\pi^3} z^2 [(1+r_\pi)^2 - z]^{\frac{1}{2}} [(1-r_\pi)^2 - z]^{\frac{1}{2}} (|A(z)|^2 + |C(z)|^2) \tag{2.30}$$

The calculation for this decay proceeds very similarly to the case outlined previously for $K_L \to \pi^0\gamma\gamma$. The crucial difference is that in this case the $o(p^4)$ counterterms do not vanish, since here the external kaon and pion are charged. Loops and counterterms contribute to A and the pole diagram in Fig. 2.6 contribute to C; to notice that the $\Delta I = \frac{1}{2}$ coupling $K^+\pi^-\pi^0$ is different from zero for off-shell π^0. It is possible to show that the 27 contribution is negligible for A and C in our kinematical region.

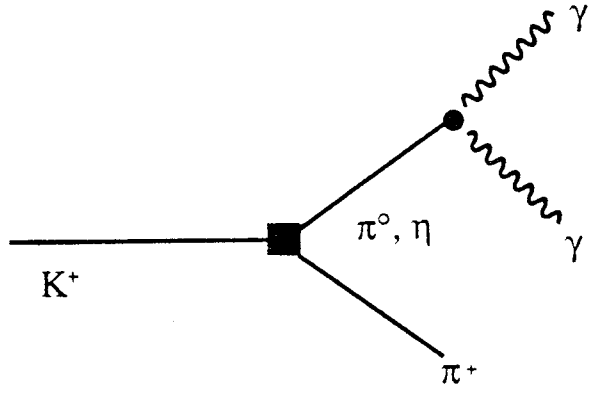

Figure 2.6: Pole diagram to $K^+ \to \pi^+\gamma\gamma$.

The loop contribution turns out to be finite and this implies that the total contribution of the counterterms have to be scale independent. One finds [33]:

$$A = \frac{h_8 m_{K^+}^2 \alpha}{8\pi f^2 z}\left[(r_\pi^2 - 1 - z) f(\frac{z}{r_\pi^2}) + (1 - r_\pi^2 - z) f(z) + \hat{c} z\right] \tag{2.31}$$

$$C = \frac{h_8 m_{K^+}^2 \alpha}{4\pi f^2}\left[\frac{z - r_\pi^2}{z - r_\pi^2 + i r_\pi \frac{\Gamma_{\pi^0}}{m_K}} - \frac{z - \frac{2+r_\pi^2}{3}}{z - r_\eta^2}\right] \tag{2.32}$$

$f(x)$ is defined in (2.14). $\hat{c}$ is an unknown complex (if there is CP violation) coupling

constant due to $o(p^4)$ counterterms (see formulae (1.12) and (1.13)):

$$\hat{c} = 32\pi^2[4(L_9 + L_{10}) - \frac{1}{3}(w_1 + 2w_2 + 2w_4)] \tag{2.33}$$

$L_9 + L_{10}$ and $w_1 + 2w_2 + 2w_4$ are separately both scale independent. Analogously to (2.13), in (2.31) the term in $f(z)$ comes from the kaon loops and so does not have absorptive part, while the term in $f(\frac{z}{r_\pi^2})$ comes from pion loops and thus has absorptive part, so that the kaon loop contribution is much smaller than the pion loop one.

Using (2.31) and (2.32) the rate depends smoothly on the cut on z and one has

$$\Gamma_A(K^+ \to \pi^+\gamma\gamma) = (2.80 + 0.87\hat{c} + 0.17\hat{c}^2) \cdot 10^{-20}\text{MeV} \tag{2.34}$$

$$\Gamma_C(K^+ \to \pi^+\gamma\gamma) = 0.26 \cdot 10^{-20}\text{MeV} \tag{2.35}$$

Since $\hat{c}$ is unknown we can deduce from (2.34) only a lower bound for the rate, which is obtained for $\hat{c} = -2.6$. One obtains

$$\Gamma(K^+ \to \pi^+\gamma\gamma) = \Gamma_A + \Gamma_C \geq 2.0 \cdot 10^{-20}\ \text{MeV} \tag{2.36}$$

or equivalently

$$Br(K^+ \to \pi^+\gamma\gamma) \geq 3.7 \cdot 10^{-7} \tag{2.37}$$

which is below the present experimental upper limit (2.27). The analytic form (2.30) of the z-spectrum is predicted up to the unknown parameter $\hat{c}$. Experiments can both test the predicted shape and constrain the possible values of $\hat{c}$ (and so the related counterterms). We observe that the shape of the spectrum is very sensitive to the value of $\hat{c}$ (see Fig. 2.7). The weak deformation model predicts $\hat{c}$=0 [32] and consequently $Br(K^+ \to \pi^+\gamma\gamma) = 5.8 \cdot 10^{-7}$.

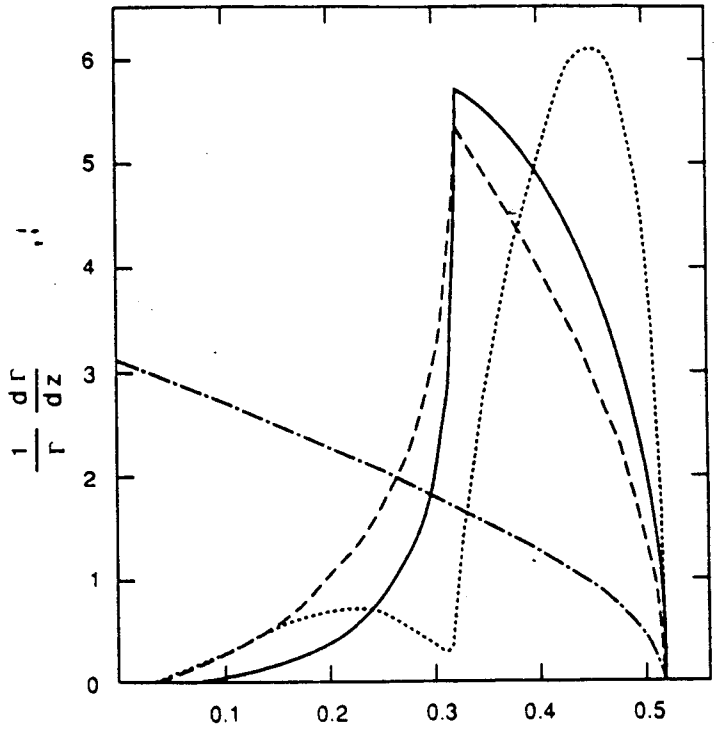

Figure 2.7 : Normalized theoretical z-distribution for $K^+ \to \pi^+\gamma\gamma$ [33] for several values of $\hat{c}$: $\hat{c} = 0$ full curve, $\hat{c} = -4$ dotted curve, $\hat{c} = 4$ dashed curve; the dashed-dotted curve is the phase space.

CP *violation*

CP violation in this channel can originate from the interference between the imaginary part of $\hat{c}$ and the absorptive part of the A amplitude. The amplitude $A(K^-(p) \to \pi^-(p')\gamma(q_1,\epsilon_1)\gamma(q_2,\epsilon_2))$ is obtained from the K^+ one replacing h_8 and $\hat{c}$ by their complex conjugates. This generates a charge asymmetry

$$\Gamma(K^+ \to \pi^+\gamma\gamma) - \Gamma(K^- \to \pi^-\gamma\gamma) =$$

$$\frac{Im\hat{c}|h_8\alpha|^2 m_{K^+}^5}{2^{14}\pi^5 f^4}\int_{4r_\pi^2}^{(1-r_\pi)^2} dz\lambda^{\frac{1}{2}}(1,z,r_\pi^2)(r_\pi^2-1-z)z\ Imf(\frac{z}{r_\pi^2}) = 1.5\cdot 10^{-20} Im\hat{c}\ \ MeV \tag{2.38}$$

The lower integration limit is not zero but $4r_\pi^2$ since the imaginary part of $f(\frac{z}{r_\pi^2})$ has the threshold in this point.

Let us comment on the formula of $\hat{c}$ (2.33). L_9 and L_{10} correspond to strong counterterms and consequently do not have CP violation. The electroweak counterterms w_1, w_2, w_4 might have CP violation. The authors of [33] claim that only w_1 has a large imaginary part since the operator corresponding to it, which transforms as an octet, takes contribution from the electromagnetic penguin quark diagram. Using the $\frac{1}{N_C}$ expansion, they obtain the following estimate:

$$Im(h_8 w_1) = \frac{4f^2 G_F}{3\sqrt{2}\pi^2} c_2 s_1 s_2 s_3 s_\delta ln\frac{m_t}{m_c} \tag{2.39}$$

Under the assumption $Imh_8 \cdot Rew_1 \ll Reh_8 \cdot Imw_1$ and taking $\frac{m_t}{m_c} = 60$ they obtain the following value:

$$|Im\hat{c}| \sim 3\cdot 10^{-3} \tag{2.40}$$

Using this value in (2.38) one obtains the following estimate of the charge asymmetry

$$|\Gamma(K^+ \to \pi^+\gamma\gamma) - \Gamma(K^- \to \pi^-\gamma\gamma)| \simeq 4\cdot 10^{-23}\ \text{MeV} \tag{2.41}$$

A slightly larger value is obtained by using in (2.38) the total absorptive part from $K \to 3\pi$: $6\cdot 10^{-23}\ MeV$ [24].

Considering the lower limit (2.36) for the rate one obtains for the asymmetry

$$\frac{|\Gamma(K^+ \to \pi^+\gamma\gamma) - \Gamma(K^- \to \pi^-\gamma\gamma)|}{\Gamma(K^+ \to \pi^+\gamma\gamma) + \Gamma(K^- \to \pi^-\gamma\gamma)} \leq 1\cdot 10^{-3} \tag{2.42}$$

To measure this asymmetry one needs approximately $10^{12} K^\pm$ and thus DAΦNE will put only a limit on it.

2.4 $K_S \to \pi^0\gamma\gamma$

This decay proceeds through the diagram in Fig. 2.8 [22], where the anomalous coupling π^0 (η) $\gamma\gamma$ is given by formula (1.16). We have not included the $\eta - \eta'$ mixing since this is an higher order in CHPT. Actually, as we will see, the process is sensitive only to the π^0 pole, and as result we are probing the momentum dependence of the coupling $K^0\pi^0\pi^0$, deduced by CHPT in (1.7). Due to the strong background coming from $K_S \to \pi^0\pi^0$ a cut in z has to be done.

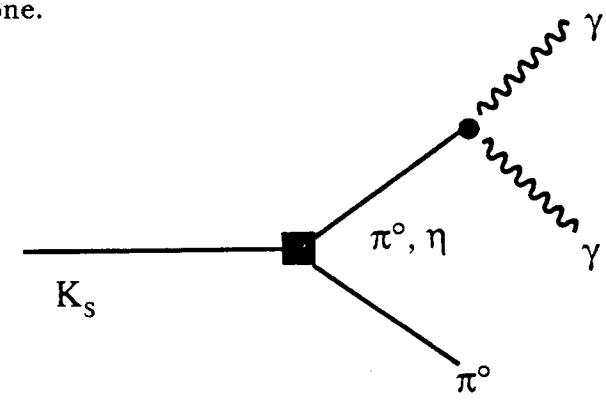

Figure 2.8 : $K_S \to \pi^0\gamma\gamma$ diagram .

The amplitude [22] is given by

$$M(K_S(p) \to \pi^0(p')\gamma(q_1,\epsilon_1)\gamma(q_2,\epsilon_2)) = C(z)\varepsilon^{\mu\nu\alpha\beta}\frac{q_{1\alpha}q_{2\beta}}{m_K^2}\epsilon_\mu(q_1)\epsilon_\nu(q_2) =$$

$$= \frac{h_8\alpha}{4\pi f^2}\varepsilon^{\mu\nu\alpha\beta}\frac{q_{1\alpha}q_{2\beta}}{m_K^2}\epsilon_\mu(q_1)\epsilon_\nu(q_2)\left[\frac{2-z-r_\pi^2}{z-r_\pi^2+ir_\pi\frac{\Gamma_{\pi^0}}{m_K}} - \frac{F_\pi(2-3z+r_\pi^2)}{3F_\eta(z-r_\eta^2+ir_\eta\frac{\Gamma_\eta}{m_K})}\right] \qquad (2.43)$$

where $r_\pi = \frac{m_{\pi^0}}{m_K}$ and $r_\eta = \frac{m_\eta}{m_K}$. F_η can be put phenomenologically different from F_π, but this does not affect the z-spectrum, which is dominated by the π^0 pole and is given by

$$\frac{d\Gamma}{dz}(K_S \to \pi^0\gamma\gamma) = \frac{m_K}{2^{10}\pi^3}z^2[(1+r_\pi)^2-z]^{\frac{1}{2}}[(1-r_\pi)^2-z]^{\frac{1}{2}}|C(z)|^2 \qquad (2.44)$$

This spectrum and the one obtained with a constant weak coupling are shown in Fig. 2.9, where the cut $z \geq 0.2$ has been made. Also the very marginal dependence on the η pole is shown. The branching ratio with this cut is

$$Br(K_S \to \pi^0\gamma\gamma)_{z\geq 0.2} = 3.8 \cdot 10^{-8} \qquad (2.45)$$

DAΦNE, see table I, should be able to see this decay, but it will be very difficult to look for the momentum dependence of the vertex.

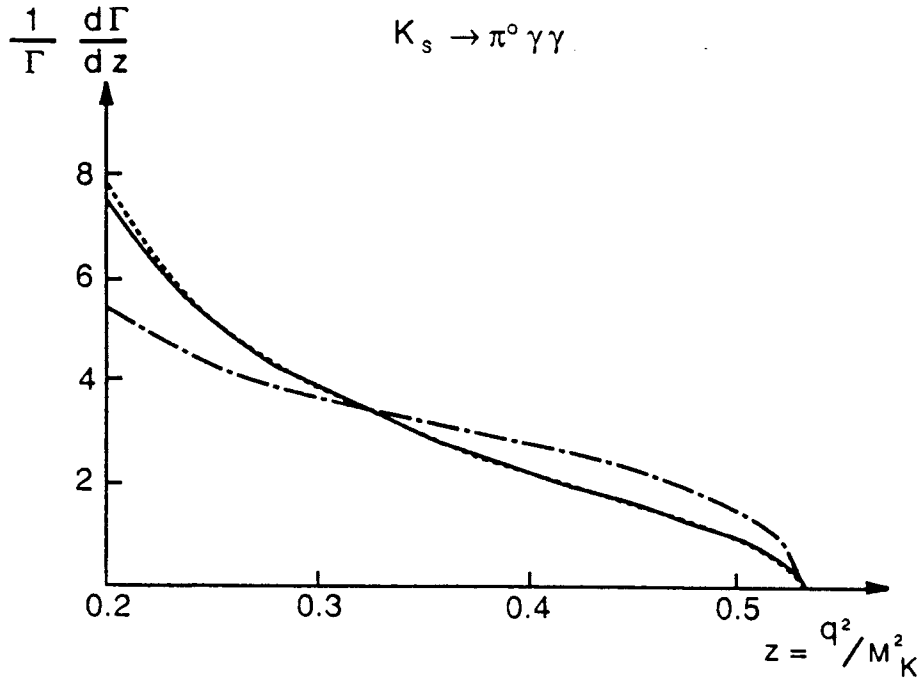

Figure 2.9 : Normalized theoretical [22] z-distribution for $K_S \to \pi^0\gamma\gamma$ in the region $.2 \leq z \leq (1-r_\pi)^2$ (full curve). The pion pole contribution alone is given by the dotted curve. The dashed curve shows the spectrum for momentum independent weak vertices.

2.5 $\mathbf{K_L \to \pi^0\pi^0\gamma\gamma}$

We just mention this decay, which can be predicted by using the leading Wess-Zumino term. The rate predicted [40] with a cut of width $2\delta m$ in the mass of the photon pair around m_π is

$$\Gamma_{cut} = \frac{3.4P}{(\delta m/MeV)} \qquad P = 4.9 \cdot 10^{-7} \cdot \Gamma_L \tag{2.46}$$

Thus for $\delta m \simeq 20MeV$ one predicts a $Br(K_L \to \pi^0\pi^0\gamma\gamma) \sim 10^{-7}$, which could be within reach for DAΦNE, thus testing the Wess-Zumino term.

2.6 $K_L \to \gamma\gamma$

If CP is conserved K_L goes into two photons with perpendicular polarizations ($2\gamma_\perp$). This decay has been historically very important to understand GIM mechanism [1]; also the interplay between the short distance contributions in Fig. 2.10 and the long distance contribution in Fig. 2.11 is matter of past and current interest [41,42,21]. The interplay becomes even more attractive in connection to the study of CP violation in this channel at LEAR [44,20,18,21,45]. Unfortunately, the theory is affected by several uncertainties for this process, as we shall see. Nevertheless the experimental width $\Gamma(K_L \to \gamma\gamma)$ can be used as input to predict direct CP violation in this channel.

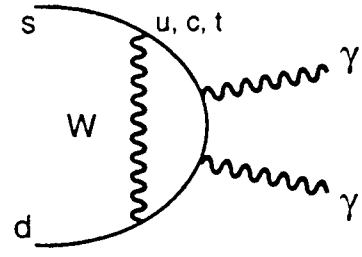

Figure 2.10 : Short distance contribution to $K^0 \to \gamma\gamma$, where the photons have perpendicular polarizations.

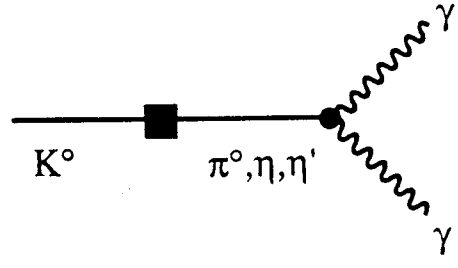

Figure 2.11 :Long distance contribution to $K^0 \to \gamma\gamma$, where the photons have perpendicular polarizations.

5

The loop integral of the short distance contributions in Fig. 2.10 is a function of $\frac{m_i^2}{m_W^2}$, where m_i is the mass of intermediate quark. The contributions for $m_i = 0$ (anomaly contributions) cancel, when we sum over all the u-like quarks (GIM mechanism). Also for $m_i \neq 0$ short distance contributions are negligible compared to the long distance ones [1,41]. Thus the main contribution is expected to come from long distance effects, which can be described in the framework of CHPT. The effective Wess-Zumino term and the $\Delta S = 1$ weak Lagrangian will generate the CP conserving amplitude. At the lowest order

in CHPT one has:

$$A(K_L \to 2\gamma_\perp)_{o(p^2)} = A(K_L \to \pi^0 \to 2\gamma_\perp) + A(K_L \to \eta_8 \to 2\gamma_\perp) =$$
$$= A(K_L \to \pi^0)A(\pi^0 \to 2\gamma_\perp)\left[\frac{1}{m_K^2 - m_\pi^2} + \frac{1}{3}\cdot\frac{1}{m_K^2 - m_8^2}\right] \propto$$
$$\propto A(K_L \to \pi^0)A(\pi^0 \to 2\gamma_\perp)\left[4m_K^2 - 3m_8^2 - m_\pi^2\right] \simeq 0 \qquad (2.47)$$

The Gell-Mann Okubo formula, which holds at this order, tells us that the amplitude (2.7) is zero at the lowest order. Going to the next order, one assumes [1,41,42,16,20,43,18] that inclusion of $SU(3)$ breaking effects, which are described by CHPT [42], $\eta - \eta'$ mixing and the $\eta'-$pole would correctly describe the decay. In other words one assumes that the higher order operators which contribute to $K_L \to \gamma\gamma$ are dominated by the η' resonances, which is called "pole model".

We briefly sketch the $\eta - \eta'$ mixing scheme; η_0 and η_8 are the strong interaction eigenstates, while η and η' are the mass eigenstates:

$$\eta = \eta_8 \cos\theta - \eta_0 \sin\theta \qquad \eta' = \eta_8 \sin\theta + \eta_0 \cos\theta \qquad (2.48)$$

The couplings $\pi^0\gamma\gamma$ and $\eta_8\gamma\gamma$ are given by the Wess-Zumino term [6] in (1.16). Nonet symmetry will give also the coupling $\eta_0 F\tilde{F}$ as:

$$L_{WZ}^{nonet} = \frac{e^2}{32\pi^2 F_\pi}\left(2\eta_0\sqrt{\frac{2}{3}}\right)F\tilde{F} \qquad F\tilde{F} = \epsilon_{\nu\rho\sigma\lambda}F^{\nu\rho}F^{\sigma\lambda} \qquad (2.49)$$

Actually to account for symmetry breaking effects, different values of decay constants F_π, F_8, F_0 are introduced in the physical amplitudes. Neglecting direct $o(p^4)$ coupling for pseudoscalars going in two photons [46], from (1.16), (2.48) and (2.49) one can write the following amplitudes

$$\frac{A(\eta \to 2\gamma)}{A(\pi^0 \to 2\gamma)} = \frac{1}{\sqrt{3}}\left(\cos\theta\frac{F_\pi}{F_8} - 2\sqrt{2}\sin\theta\frac{F_\pi}{F_0}\right) \qquad (2.50)$$

$$\frac{A(\eta' \to 2\gamma)}{A(\pi^0 \to 2\gamma)} = 2\sqrt{\frac{2}{3}}\left(\cos\theta\frac{F_\pi}{F_0} + \frac{\sin\theta}{2\sqrt{2}}\cdot\frac{F_\pi}{F_8}\right) \qquad (2.51)$$

The most recent average values of the widths [47] are:

$$\Gamma(\pi^0 \to \gamma\gamma) = (7.50 \pm .17) \quad eV$$
$$\Gamma(\eta \to \gamma\gamma) = (.526 \pm .044) \quad keV \qquad (2.52)$$
$$\Gamma(\eta' \to \gamma\gamma) = (4.29 \pm .19) \quad keV$$

From (2.50), (2.51) and (2.52) and assuming $\frac{F_\pi}{F_8} = .8$, as CHPT predicts, one obtains [47]

$$\theta = (-22.4 \pm 1.5)^0 \qquad (2.53)$$

$$\frac{F_\pi}{F_0} = .957 \pm .027 \qquad (2.54)$$

This angle is consistent with the $\frac{1}{N_c}$ prediction. Though this is a consistent scheme it is not universally accepted.

Back to $K_L \to \gamma\gamma$, assuming the pole model [41,42,20,18,48,21], one writes

$$A(K^0 \to 2\gamma_\perp) = \sum_{P=\pi^0,\eta,\eta'} \frac{A(K^0 \to P)}{m_K^2 - m_P^2} A(P \to 2\gamma_\perp) \qquad (2.55).$$

While for $< \pi^0|H_W|K_2 >$ and $< \eta_8|H_W|K_2 >$ one can use CHPT prediction [42]:

$$\frac{< \eta_8|H_W|K_2 >}{< \pi^0|H_W|K_2 >} = \frac{(1+\xi)}{\sqrt{3}} \qquad (2.56)$$

where ξ comes from the $o(p^4)$ corrections [42]. $< \eta_0|H_W|K_2 >$ cannot be related by $SU(3) \times SU(3)$ symmetry. For this matrix element the quark model will be used. Let us write the hamiltonian

$$H_W = \frac{G_F}{\sqrt{2}} \cos\theta_c \sin\theta_c \sum c_i(\mu) O_i(\mu) \qquad (2.57)$$

where $c_i(\mu)$ are numerical coefficients calculable in perturbative QCD [49,11] and $O_i(\mu)$ are local four-quark operators made of the strange, up and down quarks. For our purposes we can assume that only four operators are relevant: O_1, O_S, O_5 and O_6; the last two (penguin operators) are believed to be responsible for the $\Delta I = \frac{1}{2}$ enhancement and CP violation. Within the factorization approximation [50] one obtains

$$< \eta_0|c_1 O_1 + c_S O_S|K^0 >= \sqrt{\frac{2}{3}} < \pi^0|c_1 O_1 + c_S O_S|K^0 > \qquad (2.58)$$

$$< \eta_0|c_5 O_5 + c_6 O_6|K^0 >= -2\sqrt{\frac{2}{3}} < \pi^0|c_5 O_5 + c_6 O_6|K^0 > \qquad (2.59)$$

Consequently

$$\frac{< \eta_0|H_W|K_L >}{< \pi^0|H_W|K_L >} = -\sqrt{\frac{2}{3}}(3f - 1) \equiv -2\sqrt{\frac{2}{3}}\rho \qquad (2.60)$$

where ρ should be one if nonet symmetry would be exact and f is the penguin fraction of the amplitude $< \pi^0\pi^0|H_W|K_1 >$, due to O_5 and O_6; f should be close to 1 to explain the $\Delta I = 1/2$ rule. Using the experimental widths (2.52) and the experimental value for $\Gamma(K_L \to \gamma\gamma)$ one obtains two different values for ρ and consequently for f : $f_+ = .92$ and $f_- = .45$. The explanation of the $\Delta I = 1/2$ rule and the decay $\Gamma(K_L \to \pi^+\pi^-\gamma)$ [43] seem to indicate that the first solution is correct. This, unfortunately would indicate a small value for direct CP violation in this channel. Direct CP violation in the decays of kaons in two photons can arise from diagrams in Fig. 2.1, where the photons have parallel polarizations $(2\gamma_\parallel)$, or from Fig. 2.11, where the photons have perpendicular polarizations $(2\gamma_\perp)$. The corresponding CP violating quantities are

$$\eta_\parallel = \frac{A(K_L \to 2\gamma_\parallel)}{A(K_S \to 2\gamma_\parallel)} \qquad \eta_\perp = \frac{A(K_S \to 2\gamma_\perp)}{A(K_L \to 2\gamma_\perp)} \equiv \epsilon + \epsilon'_{\gamma\gamma}. \qquad (2.61)$$

The theoretical prediction for the first is $\eta_{||} \simeq \epsilon + \epsilon'$ [20,18], while for $\epsilon'_{\gamma\gamma}$ there are two solutions, according to the value of the penguin fraction: $\epsilon'_{\gamma\gamma} \simeq -9|\epsilon'|$ for f_+ and $70|\epsilon'|$ for f_-. However, short distance contributions might enhance somehow the direct CP violation in this channel [45]. As shown in [51], it is possible to study CP violation through time asymmetries in this channel, but statistics at DAΦNE does not seem to be large enough to see direct CP violation.

As conclusion DAΦNE could improve the present experimental width and it might be of interest to study CP violation in this channel.

SECTION 3. Radiative kaon decays with one photon in the final state

3.1 Introduction and motivation for K $\rightarrow \pi\pi\gamma$

The total amplitude for the processes [52,53,54,55,56,57,58]

$$K_{S,L}(p) \rightarrow \pi^+(p_+)\pi^-(p_-)\gamma(q,\varepsilon) \tag{3.1}$$

is a linear combination of these three Lorentz and gauge invariants:

$$B = \frac{\varepsilon \cdot p_+}{q \cdot p_+} - \frac{\varepsilon \cdot p_-}{q \cdot p_-} \tag{3.2a}$$

$$\overline{B} = \varepsilon \cdot p_+ \; q \cdot p_- \; - \; \varepsilon \cdot p_- \; q \cdot p_+ \tag{3.2b}$$

$$B_{WZ} = \epsilon_{\alpha\beta\gamma\delta} p_+^\alpha p_-^\beta q^\gamma \varepsilon^\delta \tag{3.2c}$$

which are the possible invariants up to third order in momenta. The total invariant amplitude for the process must be then a superposition of these invariants multiplied some scalar functions. B and $\overline{B}$ correspond to electric transitions, while B_{WZ} to magnetic transitions. If photon polarization is not measured there is no interference among electric and magnetic transitions. Although

$$\overline{B} = q \cdot p_+ q \cdot p_- B \tag{3.3}$$

one generally prefers to treat B abd $\overline{B}$ separately because of the different behaviour with the photon energy going to zero; in this limit Low theorem [59] establishes a correspondence among radiative and non radiative decays (and cross-sections). In particular for $K_{S,L} \rightarrow \pi\pi\gamma$ it tells us that the amplitude can be written as:

$$\lim_{E^*_\gamma \rightarrow 0} A(K_{S,L} \rightarrow \pi^+\pi^-\gamma) \simeq eBA(K_{S,L} \rightarrow \pi^+\pi^-) \equiv A_{IB}(K_{S,L} \rightarrow \pi^+\pi^-\gamma) \tag{3.4}$$

where we have defined the Internal Bremsstrahlung (IB) amplitude and E^*_γ is the photon energy in the kaon rest frame. This can be interpreted in the classical limit as radiation by the external charged particles. The photon spectrum behaves as $\frac{1}{E^*_\gamma}$ for $E^*_\gamma \rightarrow 0$:

$$\frac{d\Gamma}{dE^*_\gamma}(K_{S,L} \rightarrow \pi^+\pi^-\gamma)\Big|_{E^*_\gamma \rightarrow 0} \simeq \frac{\alpha}{\pi} \frac{\Gamma(K_{S,L} \rightarrow \pi^+\pi^-)}{E^*_\gamma} \left(\frac{1+\beta_0^2}{\beta_0} ln \frac{1+\beta_0}{1-\beta_0} - 2 \right) \tag{3.5}$$

where

$$\beta_0 = \sqrt{1 - \frac{4m_\pi^2}{m_K^2}} \tag{3.6}$$

Direct Emission (DE) amplitude A_{DE}, which is defined by subtracting A_{IB} from the total amplitude, does not have poles for $E_\gamma^* \to 0$ and thus it can be observed at large values of E_γ^*. A_{DE} depends upon the $K\pi\pi\gamma$ couplings and so upon chiral dynamics. This is different from Internal Bremsstrahlung which is predicted just by gauge invariance. Historically these contributions were studied to check the validity of the $\Delta I = \frac{1}{2}$ rule outside the area of the purely hadronic weak processes.

An analogous distinction in Internal Bremsstrahlung and Direct Emission amplitudes holds for $K^\pm \to \pi^\pm\pi^0\gamma$.

We have reported in table I the possible channels to study at DAΦNE with the relative experimental and theoretical branching ratios. As explained in §1.1, for the IB rate, since in [2] no average experimental branching is quoted (only the total branching), we have used the result from the experiment with the smaller error. The references and the details are in the relative sections. Furthermore, note that $K_S \to \pi^0\pi^0\gamma$ has no bremsstrahlung since the external particles are neutral; due to the very small branching ratio and the difficult detection, we will not discussed it.

3.2 Invariants and Low theorem

The total amplitude for the process $K_{S,L}(p) \to \pi^+(p_+)\pi^-(p_-)\gamma(q,\varepsilon)$ is a linear combination of the three Lorentz and gauge invariants $B, \overline{B}$ and B_{WZ} in (3.2). Analogously for the decay $K^+(p) \to \pi^+(p_+)\pi^0(p_0)\gamma(q,\varepsilon)$ the total amplitude is a superposition of the following invariants:

$$B^c = \frac{\varepsilon \cdot p_+}{q \cdot p_+} - \frac{\varepsilon \cdot p}{q \cdot p} \tag{3.7a}$$

$$\overline{B}^c = \varepsilon \cdot p_+ \; q \cdot p - \varepsilon \cdot p \; q \cdot p_+ \tag{3.7b}$$

$$B_{WZ}^c = \epsilon_{\alpha\beta\gamma\delta} p^\alpha p_+^\beta q^\gamma \varepsilon^\delta \tag{3.7c}$$

The amplitudes B^c and $\overline{B}^c$, analogously to B and $\overline{B}$ (see (3.3)), though not independent, are treated separately.

Consistently with Low theorem, one predicts from gauge invariance for the bremsstrahlung amplitude of a neutral scalar particle M in the charged scalar particles m_+, m_- plus n photons [15]

$$A_{IB}(M(p) \to m_+(p_+), m_-(p_-), \gamma_1(q_1, \varepsilon_1), .., \gamma_n(q_n, \varepsilon_n)) = \tag{3.8}$$

$$= A(M(p) \to m_+(p_+), m_-(p_-)) \cdot A_B(\varepsilon_a, q_a, p_i) \quad a = 1, .., n \quad i = 0, +, -$$

$A(M(p) \to m_+(p_+), m_-(p_-))$ is the on-shell amplitude for the decay of either scalar into the other two, and $A_B(\varepsilon_a, q_a, p_i)$ is the general bremsstrahlung amplitude, independent of the weak lagrangian. For the $K_{S,L} \to \pi^+\pi^-\gamma$ amplitudes one obtains

$$A_{IB}(K_{S,L} \to \pi^+\pi^-\gamma) = eBA(K_{S,L} \to \pi^+\pi^-) \tag{3.9}$$

Defining E^*_γ as the photon energy in the CMS (center of mass system or kaon rest frame) and θ as the angle between the photon and the π^+ in the di-pion rest frame and summing over the photon polarizations one has

$$\sum_{pol} |B|^2 = \frac{4 \sin^2\theta \frac{\beta^2}{E^{*2}_\gamma} \left(1 - \frac{2E^*_\gamma}{m_K}\right)}{(1 - \beta^2 \cos^2\theta)^2} \tag{3.10}$$

where

$$\beta = \sqrt{1 - \frac{4m_\pi^2}{m_K^2 - 2m_K E^*_\gamma}} \tag{3.11}$$

Since

$$\frac{d^2\Gamma(K_{S,L} \to \pi^+\pi^-\gamma)}{dE^*_\gamma d\cos\theta} = \frac{\beta E^*_\gamma}{2m_K(4\pi)^3} |A(K_{S,L} \to \pi^+\pi^-\gamma)|^2 \tag{3.12}$$

integrating one obtains

$$\frac{d\Gamma(K_{S,L} \to \pi^+\pi^-\gamma)_{IB}}{dE^*_\gamma} = \frac{\alpha}{\pi} \frac{\Gamma(K_{S,L} \to \pi^+\pi^-)}{E^*_\gamma} \left(1 - \frac{2E^*_\gamma}{m_K}\right) \frac{\beta}{\beta_0} \left(\frac{1+\beta^2}{\beta} \log\frac{1+\beta}{1-\beta} - 2\right) \tag{3.13}$$

where β_0 has been defined in (3.6). The (3.13) represents the Low theorem for radiative kaon decays. Analogously for $K^\pm \to \pi^\pm\pi^0\gamma$ one obtains for the bremsstrahlung on the external legs

$$A_{IB}(K^\pm \to \pi^\pm\pi^0\gamma) = eB^c A(K^\pm \to \pi^\pm\pi^0) \tag{3.14}$$

where B^c has been defined in (3.7) and $A_{IB}(K^\pm \to \pi^\pm\pi^0\gamma)$ is then suppressed by $\Delta I = \frac{1}{2}$ rule.

Choosing as independent variables E^*_γ, the photon energy in the K^+ rest frame and another kinematical variable, for instance the angle between the π^+ and the photon in the $\pi^+\pi^0$ rest-frame, one would recover the $\frac{1}{E^*_\gamma}$ behaviour; however for a better experimental analysis of this spectrum, as we shall see in §3.7, one prefers different kinematical variables.

3.3 Direct Emission: selection rules, multipole expansion

Direct Emission is defined by subtracting the Internal Bremsstrahlung contribution from the total amplitude. It is interesting since it tests the meson structure. In general there are three contributions: Γ_{WZ}, which comes from the Wess-Zumino term [60,6] and it does not interfere with the other amplitudes when the photon polarization is not measured, Γ_{int}, which comes from interference between B and $\overline{B}$ (or B^c and $\overline{B}^c$) and thus can be negative and there is the pure $|\overline{B}|^2$ ($|\overline{B}^c|^2$ for $K^\pm \to \pi^\pm\pi^0\gamma$) term $\Gamma_{|\overline{B}|^2}$. The $\frac{1}{E_\gamma^*}$ dependence strongly tends to enhance the IB amplitudes compared to DE ones, unless the amplitudes $A(K \to \pi\pi)$ are inhibited. Indeed $A(K_L \to \pi^+\pi^-\gamma)_{IB}$ is forbidden if CP is conserved while $A(K^+ \to \pi^+\pi^0\gamma)_{IB}$ is suppressed by the $\Delta I = \frac{1}{2}$ rule.

To understand the selection rules and the angular momenta of these radiative decays, we give sketchy and naive arguments and we refer to the literature for the complete analysis [61,57]. Let us define L the relative angular momentum of $\pi^+\pi^-$ and I their total isospin, l the relative angular momentum between the photon and the di-pion system: we mean that l and the spin $\vec{1}$ of the photon express the properties of the wave function under the rotation group. In particular, the parity of the photon will be $(-1)^{l+1}$. Since the kaons are spinless

$$\vec{l} + \vec{L} + \vec{1} = 0$$

Then the lowest angular momentum states consistent with Bose symmetry for $K \to \pi\pi\gamma$ are

L	l	I	$CP \equiv (-1)^l$
1	0	1	+1
1	1	1	−1

We have also given the CP eigenvalue of these states, which is $(-1)^l$. Note the state with $L = 0, \quad l = 1, \quad I = 0, 2 \quad (CP = -1)$ does not satisfy the transversality condition which has to be imposed on the photon and thus has been omitted. The possible values of parity in terms of the total angular momentum $\vec{J} = \vec{l} + \vec{S}$ are

$$l = J, \quad P = (-1)^{l+1} = (-1)^{J+1} \qquad\qquad l = J + 1, \quad P = (-1)^{l+1} = (-1)^J$$

Thus we can introduce the conventional terminology [61]: a photon with angular momentum J and parity $(-1)^J$ is defined electric 2^J-pole (EJ), while if parity is $(-1)^{J+1}$ it is called magnetic 2^J-pole (MJ). Consequently, the EJ transitions will have $CP = (-1)^{J+1}$ while MJ will have $CP + (-1)^J$; CP invariance would require E1,M2,E3,M4... transitions vanish for $K_L \to \pi^+\pi^-\gamma$ and M1,E2,M3,E4... for $K_S \to \pi^+\pi^-\gamma$ [53,56]. Alternatively expanding [56] the amplitudes A_E and A_M defined in

$$A(K_{S,L} \to \pi^+\pi^-\gamma) = eA(K_{S,L} \to \pi^+\pi^-)B + \overline{B}A_E((p_+ + p_-)^2, (p_+ - p_-)\cdot q) +$$

$$+B_{WZ}A_M((p_+ + p_-)^2, (p_+ - p_-)\cdot q) \tag{3.15}$$

in powers of $\dfrac{(p_+ - p_-)\cdot q}{m_K^2}$

$$A_{E,M}((p_+ + p_-)^2, (p_+ - p_-)\cdot q) \simeq A^{(1)}_{E,M}((p_+ + p_-)^2) + A^{(2)}_{E,M}((p_+ + p_-)^2)\frac{(p_+ - p_-)\cdot q}{m_K^2} \tag{3.16}$$

one defines the first term as electric or magnetic dipole moment while the second corresponds to the quadrupole one. The higher multipoles are expected to be small [55]. Due to its different physical nature, IB does not follow this amplitude expansion [59]. Analogous discussion holds for charged kaons. It is important to remark that so far there is no evidence of $B\overline{B}$ interference, which is theoretically expected and it is important for CP violation and meson dynamics. Chiral perturbation theory can try to predict this amplitude.

3.4 CHPT: $o(p^2); o(p^4)$ loops and counterterms

At the lowest order in CHPT one obtains only internal bremsstrahlung amplitudes. For instance in $K_S \to \pi^+\pi^-\gamma$ at order p^2 the diagrams in Fig. 3.1 will appear. Diagram 3.1a is needed to make the amplitude $o(p^2)$ gauge invariant. The total contribution at this order is:

$$A(K_S \to \pi^+\pi^-\gamma)^{o(p^2)} = eA(K_S \to \pi^+\pi^-)^{o(p^2)}B \tag{3.17}$$

Thus diagram 3.1a, contrary to appearance, contribute to the IB amplitude; its existence is due to the derivative couplings in CHPT. Actually we take (3.9) as a definition of the Internal Bremsstrahlung amplitudes, meaning that this relation holds order by order in CHPT.

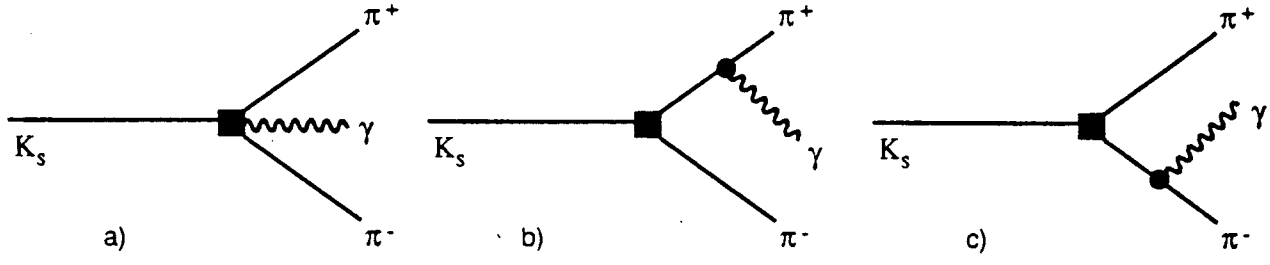

Figure 3.1 : Long distance contribution to $K_S \to \pi^+\pi^-\gamma$.

Analogous discussion holds for the other channels. At higher orders, where loops and counterterms will appear, also direct emission amplitudes, proportional to $\overline{B}$ and B_{WZ}, will be in general present. At order p^4 we will have counterterms for the IB amplitude, fixed by $K \to \pi\pi$ decays, and direct electroweak counterterms contributing to B_{WZ} and $\overline{B}$. The direct weak counterterms contributing to B_{WZ} come from the chiral anomaly, which give rise to a $\Delta S = 1$ non-leptonic $o(p^4)$ Lagrangian. For the octet one has

$$L_{anom}^{\Delta S=1} = -\frac{ieh_8}{32\pi^2 f^3}\tilde{F}_{\mu\nu}\partial^\mu\pi^0(K^+D^\nu\pi^- - \pi^-D^\nu K^+) + h.c. \tag{3.18}$$

where

$$\tilde{F}_{\mu\nu} = \epsilon_{\mu\nu\rho\lambda}F^{\rho\lambda} \tag{3.19}$$

$$D^\nu\pi^\pm = (\partial^\nu \pm ieA^\nu)\pi^\pm \tag{3.20}$$

At order p^4 the number of independent counterterms for electric transitions is decreased by the requirement that the chiral Lagrangian has to be invariant under CPS symmetry [14,62]. Indeed the $\Delta S = 1$ quark Lagrangian is invariant under a CP transformation plus an interchange of d and s quarks, even if CP is violated. Also strong and electromagnetic Lagrangian satisfies this symmetry. CPS imposes that there are only four chiral invariant $o(p^4)$ independent counterterms for $K \to \pi\gamma^*$ and $K \to \pi\pi\gamma$. Defining[†] C_i as [14]

$$\begin{aligned}
C_{1L} &= F^{\mu\nu} < Q\lambda_{6-i7}L_\mu L_\nu > \\
C_{2L} &= F^{\mu\nu} < QL_\mu\lambda_{6-i7}L_\nu > \\
C_{1R} &= F^{\mu\nu} < QU^\dagger\lambda_{6-i7}UR_\mu R_\nu > \\
C_{1\hat{R}} &= F^{\mu\nu} < QR_\mu R_\nu U^\dagger\lambda_{6-i7}U > \\
C_{2R} &= F^{\mu\nu} < QR_\mu U^\dagger\lambda_{6-i7}UR_\nu >
\end{aligned} \tag{3.21}$$

where L_μ is defined in (1.14) and

$$R_\mu = if^2U^\dagger D_\mu U$$

$$< A >= trA$$

Due to CPS symmetry only the combination $C_{1R} + C_{1\hat{R}}$ appears.

[†] Note that we have a different definition of L and R compared to [14].

In the Lagrangian

$$L_{CT} = -\frac{ieh_8}{8f^4}\left[\omega_{1L}C_{1L} + \omega_{2L}C_{2L} + \frac{\omega_{1R}+\omega_{1\hat{R}}}{2}(C_{1R}+C_{1\hat{R}}) + \omega_{2R}C_{2R}\right] + h.c. \quad (3.22)$$

ω_i are dimensionless coupling constant which could be determined from decays like $K^+ \to \pi^+e^+e^-$, $K_L \to \pi^0e^+e^-$, $K^+ \to \pi^+\gamma\gamma$, $K_S \to \pi^+\pi^-\gamma$. The coefficients appearing in (1.13) and in [15] can be written in terms of the ones in (3.22):

$$\begin{aligned} \omega_1 &= \omega_{1L} + \omega_{1R} + \omega_{1\hat{R}} & \omega_2 &= \omega_{2L} + \omega_{2R} \\ \omega_1' &= \omega_{1L} - (\omega_{1R} + \omega_{1\hat{R}}) & \omega_2' &= \omega_{2L} - \omega_{2R} \end{aligned} \quad (3.23)$$

As we shall see the same combination of counterterms will appear in electric transitions of $K_S \to \pi^+\pi^-\gamma$ [63] and $K^+ \to \pi^+\pi^0\gamma$ [15]; thus CHPT is predictive. Furthermore this combination turns out to be scale independent [10,64,65], as it has to be since the loop contribution is finite.

3.5 $K_S \to \pi^+\pi^-\gamma$

If we neglect CP violation we can write the total amplitude for the process $K_S(p) \to \pi^+(p_+)\pi^-(p_-)\gamma(q,\varepsilon)$ as

$$A(K_S \to \pi^+\pi^-\gamma) = eBA(K_S \to \pi^+\pi^-) + \frac{e\overline{B}}{(4\pi f)^2 m_K^2} f_{DE}(E_\gamma^*, \cos\theta) \quad (3.24)$$

where $f_{DE}(E_\gamma^*, \cos\theta)$ is the structure dependent amplitude, E_γ^* is the photon energy in the K_S rest frame, θ is the angle between the photon and the π^+ in the dipion frame, and B and $\overline{B}$ have been defined in (3.2). Magnetic transitions proportional to B_{WZ} have been neglected since suppressed by CP violation and/or angular momentum barrier. One obtains for the double differential decay width for unpolarized photon

$$\begin{aligned} \frac{d^2\Gamma(K_S \to \pi^+\pi^-\gamma)}{dE_\gamma^* d\cos\theta} &= \frac{2\alpha\beta^3}{\pi\beta_0}\left(1 - \frac{2E_\gamma^*}{m_K}\right)\sin^2\theta\,\Gamma(K_S \to \pi^+\pi^-)\times \\ \times\Bigg[\frac{1}{E_\gamma^*(1-\beta^2\cos^2\theta)^2} &+ \frac{E_\gamma^*\mathrm{Re}[f_{DE}(E_\gamma^*,\cos\theta)A^*(K_S \to \pi^+\pi^-)]}{2(4\pi f)^2(1-\beta^2\cos^2\theta)|A(K_S \to \pi^+\pi^-)|^2} + \\ &+ \frac{|f_{DE}(E_\gamma^*,\cos\theta)|^2 E_\gamma^{*3}}{16(4\pi f)^4|A(K_S \to \pi^+\pi^-)|^2}\Bigg] \end{aligned} \quad (3.25)$$

$$E_{\gamma_{min}}^* \le E_\gamma^* \le \frac{m_K^2 - 4m_\pi^2}{2m_K} \quad \text{and} \quad -1 \le \cos\theta \le 1$$

β and β_0 have been defined in (3.11) and (3.6) respectively. The first term is the Internal Bremsstrahlung, the second the interference term between B and $\overline{B}$; the third is the pure Direct Emission rate. $f_{DE}(E^*_\gamma, \cos\theta)$ can be computed in chiral perturbation theory. At the lowest order in the chiral expansion only Internal Bremsstrahlung appears. At order p^4, where chiral loops and counterterms contribute, one has IB, which satisfies (3.9), and DE amplitudes. The counterterms from (3.21) give the following combination:

$$A(K_S \to \pi^+\pi^-\gamma) = \frac{eh_8}{4f^3}\overline{B}(\omega_1 + 2\omega_2 - \omega_1' + 2\omega_2') \tag{3.26}$$

We notice that this give a θ independent contribution to $f_{DE}(E^*_\gamma, \cos\theta)$. These coefficients have not been determined yet, but this combination is known to be scale independent. This implies that the loop contribution is finite. Indeed this has been computed and $f_{DE}(E^*_\gamma, \cos\theta)$ turns out also to be independent of $\cos\theta$ at this order [63]. Thus one can integrate the (3.25):

$$\frac{d\Gamma}{dE^*_\gamma}(K_S \to \pi^+\pi^-\gamma) = \frac{2\alpha}{\pi}\frac{\Gamma(K_S \to \pi^+\pi^-)\beta^3}{\beta_0}\left(1 - \frac{2E^*_\gamma}{m_K}\right) \times$$

$$\times\{\frac{1}{E^*_\gamma}\left[\frac{1+\beta^2}{2\beta^3} ln\frac{1+\beta}{1-\beta} - \frac{1}{\beta^2}\right] +$$

$$+\frac{E^*_\gamma \mathrm{Re}[f_{DE}(E^*_\gamma)A^*(K_S \to \pi^+\pi^-)]}{2(4\pi f)^2|A(K_S \to \pi^+\pi^-)|^2}\left[\frac{2}{\beta^2} - \frac{1-\beta^2}{\beta^3} ln\frac{1+\beta}{1-\beta}\right]\} \tag{3.27}$$

where we have neglected the term proportional to $|f_{DE}|^2$ since it is small and in CHPT is of the same order of an interference between the IB amplitude and a two loop DE amplitude, which has not been computed yet. We remark that the interference for $E^*_\gamma \to 0$ goes as constant; this is due to loop contribution, while the counterterms give a contribution which goes as E^*_γ; this implies that looking at the spectrum it could be possible to disentangle between these effects. The predicted theoretical Internal Bremsstrahlung branching ratio depends upon the photon energy cut:

$$Br(K_S \to \pi^+\pi^-\gamma)_{E^*_\gamma > 50MeV} = 1.75 \cdot 10^{-3}$$

$$Br(K_S \to \pi^+\pi^-\gamma)_{E^*_\gamma > 20MeV} = 4.80 \cdot 10^{-3}$$

The interferencial photon energy spectrum and width depend upon the unknown values of the counterterms; furthermore there is the ambiguity of the value to choose for $A(K_S \to \pi^+\pi^-)$: $o(p^2)$ CHPT, $o(p^4)$ CHPT, or the physical value [63]. Anyway varying $\omega = \omega_1 + 2\omega_2 - \omega_1' + 2\omega_2'$ appearing in the counterterm amplitude (3.26) from 0 to $8L_9$ one obtains for the interferencial $Br(K_S \to \pi^+\pi^-\gamma)_{E^*_\gamma > 20MeV}$ values between 10^{-6} and 10^{-5}, which is statistically within reach at DAΦNE, and it would be very interesting to observe. Indeed it would fix the value of the counterterm combination, testing theoretical models [32], like the weak deformation model, which try to predict the counterterms.

Experimental results

We have two experiments for $K_S \to \pi^+\pi^-\gamma$: [66] and E731 [67]. In table I for IB we have reported the data from E731; here we give the details on the two experiments which have both 4000 events. Authors of [66] looking the photon spectrum have been able to measure the Γ_{IB} and to put limits on the interference term $\Gamma_{Interf.}$ and on the pure DE emission width Γ_{DE}. [67] reports only the total width, but the theoretical prediction for $\Gamma_{Interf.}$ and $\Gamma_{(DE)}$ is a value smaller than the experimental error; so in the following table and in table I to summarize the results we will assume that E731 is measuring really Γ_{IB}. The results for IB are summarized below.

Experiments	$\frac{BR(K_S\to\pi^+\pi^-\gamma)_{(IB)}}{BR(K_S\to\pi^+\pi^-)}$	cut on E^*_γ
[66]	$(2.68 \pm 0.15) \times 10^{-3}$	$> 50MeV.$
[67]	$(2.38 \pm 0.06 \pm 0.04) \times 10^{-3}$	$> 50MeV.$
[67]	$(6.36 \pm 0.09 \pm 0.05) \times 10^{-3}$	$> 20MeV.$
Theory	2.55×10^{-3}	$> 50MeV.$
Theory	7.00×10^{-3}	$> 20MeV.$

It has to be noticed that E731 gives the results with two photon energy cuts, with different relative errors.

The bounds of Taureg et al. [66] for DE and interference terms are:

$$Br_{DE,E^*_\gamma>50MeV} < 0.06 \times 10^{-3}$$

$$|Br_{Interf.,E^*_\gamma>50MeV}| < 0.09 \times 10^{-3}$$

In principle Φ-factories have more statistics (table I). The potential backgrounds are:
i) $K_S \to \pi^+\pi^-$ plus accidental photons.
ii)$K^0 \to \pi^+\pi^-\pi^0$ where one photon is missed.
iii) $K_L \to \pi^+\pi^-\gamma$

3.6 $\mathbf{K_L \to \pi^+\pi^-\gamma}$

The amplitude for the decay $K_L(p) \to \pi^+(p_+)\pi^-(p_-)\gamma(q,\varepsilon)$ is defined analogously to K_S (see (3.1)). The IB amplitude

$$A_{IB}(K_L \to \pi^+\pi^-\gamma) = eBA(K_L \to \pi^+\pi^-) \tag{3.28}$$

is suppressed by CP violation. The term $\overline{B}$ gives rise to CP violating amplitudes for odd multipole electric transitions (E1,E3...) and CP conserving amplitudes for even multipoles (E2,E4...). B_{WZ} will contribute to CP conserving amplitudes for odd multipole magnetic transitions (M1,M3...) and CP violating amplitudes for even multipoles (M2,M4...). The spectrum for the IB contribution is given in (3.13).

If the photon polarization is not measured, there is no interference among electric and magnetic transitions. Interference amplitudes are not enhanced by the $\frac{1}{E_\gamma^*}$ behaviour; this implies that interference between the CP violating IB amplitude and a CP conserving E2 amplitude is very suppressed (10^{-2} compared to IB rate) due also to the high multipolarity state. Also interference between CP violating IB amplitudes and CP violating E1 are suppressed by a factor 10^{-3} at least. Thus the non-IB contributions are dominated by the magnetic transition M1, which is not suppressed by CP violation; in CHPT this is generated by the chiral Wess-Zumino [60,15,6] term. Very similarly to $K_L \to \gamma\gamma$, the amplitude $A(K_L \to \pi^+\pi^-\gamma)_{WZ}$ can be understood in terms of pole diagrams (Fig. 3.2). The $o(p^4)$ CHPT amplitude, which has only π^0 and η_8 poles vanishes exactly. At the next order the η' pole, the mixing angle ϑ, the SU(3) breaking, nonet symmetry breaking become relevant and the prediction for the width is model dependent [58,43,48,68,69,15].

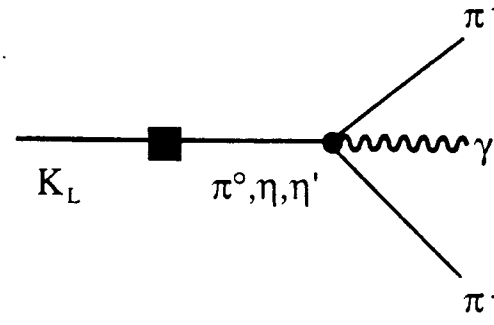

Figure 3.2 : Long distance contribution to $K_L \to \pi^+\pi^-\gamma$.

Using the same notation as for $K_L \to \gamma\gamma$ (see §2.6) one has, neglecting form factors

$$A(K_L \to \pi^+\pi^-\gamma)_{WZ} = \hat{A} B_{WZ} \tag{3.29}$$

$$\hat{A} = A(K_L \to \pi^0)\frac{1}{m_K^2 - m_\pi^2}\frac{e}{4\pi^2 F_\pi^3}\times$$

$$\{1 + \frac{m_K^2 - m_\pi^2}{m_K^2 - m_\eta^2}\left[(1+\xi)\frac{\cos\vartheta}{\sqrt{3}} + 2\sqrt{\frac{2}{3}}\sin\vartheta\right]\times$$

$$\times\left[\left(\frac{F_\pi}{F_8}\right)^3\frac{\cos\vartheta}{\sqrt{3}} - \sqrt{\frac{2}{3}}\left(\frac{F_\pi}{F_0}\right)^3\sin\vartheta\right] +$$

$$\frac{m_K^2 - m_\pi^2}{m_K^2 - m_{\eta'}^2}\left[(1+\xi)\frac{\sin\vartheta}{\sqrt{3}} - 2\sqrt{\frac{2}{3}}\cos\vartheta\right]\cdot\left[\left(\frac{F_\pi}{F_8}\right)^3\frac{\sin\vartheta}{\sqrt{3}} + \sqrt{\frac{2}{3}}\left(\frac{F_\pi}{F_0}\right)^3\cos\vartheta\right]\} \tag{3.30}$$

where B_{WZ} has been defined in (3.2c). $K_L \to \gamma\gamma$ is slightly different compared to this amplitude: the former has a linear dependence on $\frac{F_\pi}{F_8}$ and $\frac{F_\pi}{F_0}$, while the latter has a cubic dependence, which makes it even more sensitive to breaking symmetry corrections.

Thus this amplitude is strongly dependent on the theoretical uncertainties, making it unpredictable.

The double differential spectrum for the magnetic transition (3.29) is given by

$$\frac{d^2\Gamma}{dE_\gamma^* d\cos\theta} = \frac{\beta^3}{8(4\pi)^3} E_\gamma^{*3} \sin^2\theta \left(1 - \frac{2E_\gamma^*}{m_K}\right) m_K^3 |\hat{A}|^2 \tag{3.31}$$

where β is defined in (3.11) and the kinematical limits are given in (3.25).

Experimental situation

After the $K_L \to \pi^+\pi^-\gamma$ events have been selected one generally fits the photon energy spectrum to a linear combination of the IB and the DE theoretical amplitudes. Alternatively one could measure the DE by subtracting the IB from the total rate. There have been two experiments which have measured $K_L \to \pi^+\pi^-\gamma$. The IB contribution, which is proportional to $|\eta_{+-}|^2$ is found in agreement with the theoretical prediction. In table I the results from [67] has been used, while the complete list of experimental results is

	$\frac{Br(K_L\to\pi^+\pi^-\gamma)_{IB,E_\gamma^*>20MeV}}{Br(K_L\to\pi^+\pi^-)}$	$\#events$
[70]	$(7.5 \pm .8)\cdot 10^{-3}$	516 ± 31
[67]	$(6.49 \pm .17 \pm .20)\cdot 10^{-3}$	1453 ± 38
Theory	$7.00 \cdot 10^{-3}$	/
DAΦNE	/	$1.6 \cdot 10^4$

For the DE these are the experimental values

	$Br(K_L \to \pi^+\pi^-\gamma)_{DE}$	$\#events$
[70]	$(2.89 \pm .28)\cdot 10^{-5}$	546 ± 32
[67]	$(2.95 \pm .06 \pm .09)\cdot 10^{-5}$	2363
DAΦNE	/	$3.2 \cdot 10^4$

Of course, since kinematical cuts have to be made, the number of events at DAΦNE will be much less of the ones in the tables. Furthermore we remind that the separation of the events in IB and DE has only a statistical meaning. Due to the theoretical uncertainties there is no clear prediction for the total rate. The photon spectrum is predicted by the pseudoscalar pole model, i.e. $\hat{A}$ constant. [70] shows possible evidence for a form factor in (3.29), which could be explained by VMD [58]. However the experiment in [67] does not seem to confirm such a form factor, which is not theoretically established [15]. DAΦNE should allow important progress at this issue.

3.7 $K^{\pm} \to \pi^{\pm}\pi^0\gamma$

The Internal Bremsstrahlung amplitude for the decay $K^{\pm}(p) \to \pi^{\pm}(p_{\pm})\pi^0(p_0)\gamma(q,\varepsilon)$ is suppressed by $\Delta I = \frac{1}{2}$ rule.

$$A(K^{\pm} \to \pi^{\pm}\pi^0\gamma)_{IB} = \pm eB^c A(K^{\pm} \to \pi^{\pm}\pi^0) = \pm eB^c A_W(K^{\pm} \to \pi^{\pm}\pi^0)e^{i\delta_0^2} \qquad (3.32)$$

$$B^c = \left(\frac{\varepsilon \cdot p_{\pm}}{p_{\pm} \cdot q} - \frac{\varepsilon \cdot p}{p \cdot q}\right) \qquad (3.33)$$

where it has been shown the explicit dependence upon the δ_L^I phases: I is the isospin of the two pions and L their relative angular momentum. Direct Emission amplitudes, which can be electric or magnetic, do not need to be suppressed by the $\Delta I = \frac{1}{2}$ rule. Actually, as it can be seen in table I, the branching ratios for IB and DE are almost comparable, since the factor 22^2 of $\Delta I = \frac{1}{2}$ enhancement competes with the 10^{-3} suppression of DE amplitudes, which do not have the $\frac{1}{E_\gamma^*}$ pole. T_c^* which appears in table I is the kinetic energy of the charged pion in the CMS.

Particularly interesting at DAΦNE could be the measurement of the charge asymmetry

$$\frac{\Gamma(K^+ \to \pi^+\pi^0\gamma) - \Gamma(K^- \to \pi^-\pi^0\gamma)}{\Gamma(K^+ \to \pi^+\pi^0\gamma) + \Gamma(K^- \to \pi^-\pi^0\gamma)} \qquad (3.34)$$

which could arise from interference of the E1 transitions (which is not suppressed by the $\Delta I = \frac{1}{2}$ rule) with the IB amplitude. The total amplitude is the sum of the IB in (3.32), electric and magnetic amplitudes, which we write in the following way:

$$A(K^{\pm} \to \pi^{\pm}\pi^0\gamma)_E = \pm eA_W(K^{\pm} \to \pi^{\pm}\pi^0)X_E e^{i(\delta_1^1 \pm \phi_E)}(p \cdot \varepsilon p_{\pm} \cdot q - p \cdot q p_{\pm} \cdot \varepsilon) \qquad (3.35)$$

$$A(K^{\pm} \to \pi^{\pm}\pi^0\gamma)_M = \pm eA_W(K^{\pm} \to \pi^{\pm}\pi^0)X_M e^{i(\delta_1^1 \pm \phi_M)}\epsilon_{\mu\nu\alpha\beta}p_{\pm}^{\mu}p^{\nu}q^{\alpha}\varepsilon^{\beta} \qquad (3.36)$$

where X_E and X_M are in general functions of the kinematical variables. ϕ_E and ϕ_M are the CP violating phases. As for $K_S \to \pi^+\pi^-\gamma$, at $o(p^2)$ in CHPT there is no DE amplitude, but just IB (without strong interaction phases). At the next order $o(p^4)$, the chiral anomaly generates (3.36), while loops and $o(p^4)$ counterterms contribute to both (3.32) and (3.35). There is no interference among electric (including IB) and magnetic transitions if photon polarization is not measured. Experimentally this process requires the detection of three photons, consequently there is an ambiguity on the photon energy spectrum. The kinetic energy of the charged pion (T_c^* in the CMS) is not affected by this problem. The second variable is chosen as the angle θ between π^+ and γ in the $\pi^0 - \gamma$

rest-frame. The resulting Dalitz plot is rectangular

$$-1 \leq \cos\theta \leq 1$$

$$0 \leq T_c^* \leq \frac{(m_{K^+} - m_{\pi^+})^2 - m_{\pi^0}^2}{2m_{K^+}} \tag{3.37}$$

By choosing these variables the Low theorem is not manifest and the differential rate is written as

$$\frac{d^2\Gamma(K^\pm \to \pi^\pm\pi^0\gamma)}{dT_c^* d\cos\theta} = \frac{|\vec{p}_\pm^*|q_0}{(4\pi)^3 m_K \sqrt{(p-p_\pm)^2}} \sum_{pol} |A(K^\pm \to \pi^\pm\pi^0\gamma)|^2 \tag{3.38}$$

where $|\vec{p}_\pm^*|$ is the modulus of the $\pi^\pm$ three-momentum in the CMS and q_0 is the photon momentum in the $\pi^0 - \gamma$ rest frame.

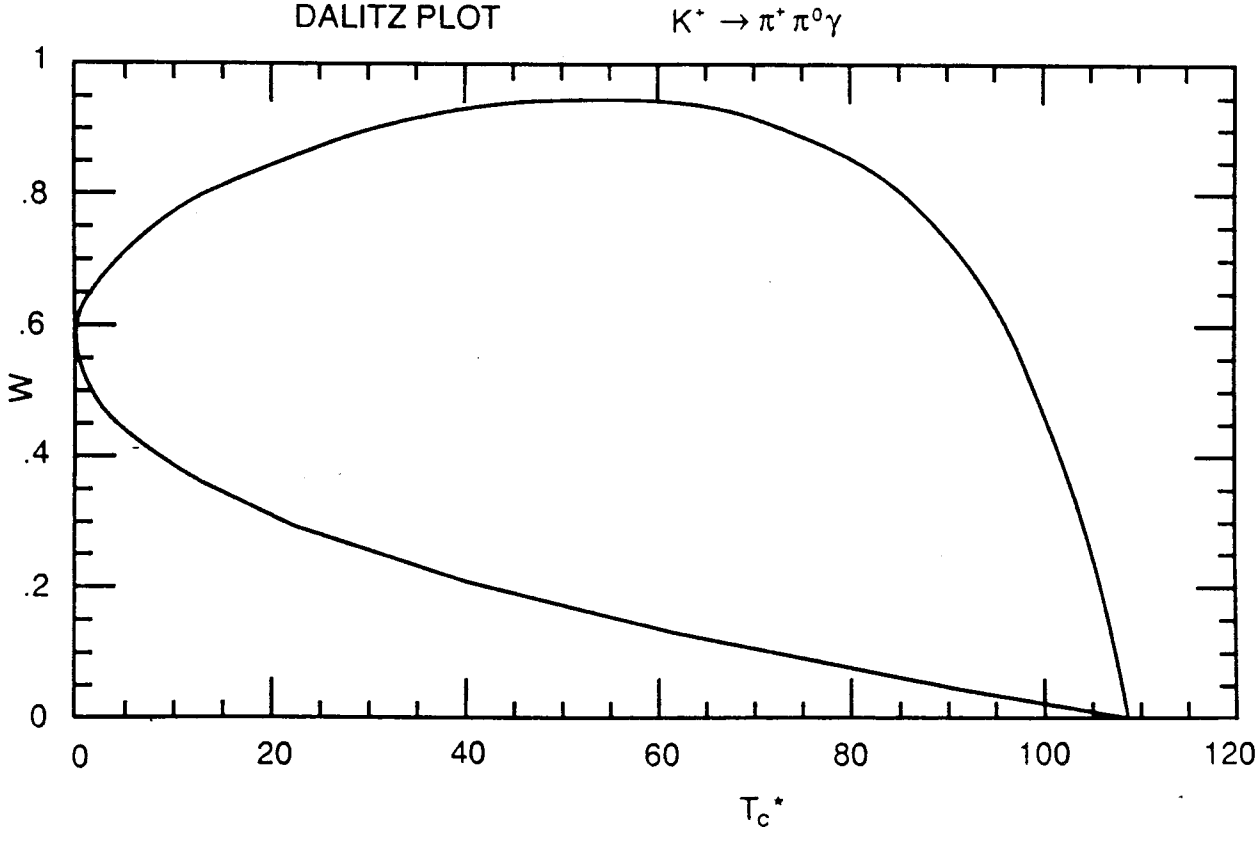

Figure 3.3 : $T_c^* - W$ Dalitz plot for the decay $K^\pm \to \pi^\pm\pi^0\gamma$.

$$|\vec{p}^{*}_{\pm}| = \sqrt{T^{*}_{c}(T^{*}_{c} + 2m_{\pi+})} \qquad q_0 = \frac{(p - p_{\pm})^2 - m^2_{\pi^0}}{2\sqrt{(p - p_{\pm})^2}} \tag{3.39}$$

Since from (3.32), (3.35) and (3.36) one has

$$\sum_{pol} |A(K^{\pm} \to \pi^{\pm}\pi^0\gamma)|^2 = e^2 |A_W(K^{\pm} \to \pi^{\pm}\pi^0)|^2 \left[\frac{2p \cdot p_{\pm}}{p \cdot q p_{\pm} \cdot q} - \frac{m^2_{K+}}{(p \cdot q)^2} - \frac{m^2_{\pi+}}{(p_{\pm} \cdot q)^2}\right] \times$$

$$\left[1 + 2X_E m^2_{\pi+} m^2_{K+} W^2 \cos(\delta^1_1 - \delta^2_0 \pm \phi_E) + (X^2_E + X^2_M) m^4_{\pi+} m^4_{K+} W^4\right] \tag{3.40}$$

where

$$W^2 = \frac{p \cdot q \; p_{\pm} \cdot q}{m^2_{\pi+} m^2_{K+}} \tag{3.41}$$

it is convenient to choose T^{*}_{c} and W as independent variables. The Dalitz plot for these variables is shown in Fig. 3.3. Thus the (3.32), (3.35) and (3.36) can be disentangled by looking at the different behaviour in W^2.

If the strong phases, X_E and X_M are constant over the Dalitz plot the differential branching ratio is

$$\frac{dBr(K^{\pm} \to \pi^{\pm}\pi^0\gamma)}{dW} = \frac{dBr(K^{\pm} \to \pi^{\pm}\pi^0\gamma)_{IB}}{dW}[1 + 2X_E m^2_{\pi+} m^2_{K+} W^2 \cos(\delta^1_1 - \delta^2_0 \pm \phi_E) +$$

$$+ (X^2_E + X^2_M) m^4_{\pi+} m^4_{K+} W^4] \tag{3.42}$$

where $\dfrac{dB(K^{\pm} \to \pi^{\pm}\pi^0\gamma)_{IB}}{dW}$ is the IB spectrum.

Thus one obtains for the asymmetry and the sum of the branching ratios

$$\frac{dB^+}{dW} - \frac{dB^-}{dW} = -4 \cdot \frac{dB(K^{\pm} \to \pi^{\pm}\pi^0\gamma)_{IB}}{dW} \cdot X_E m^2_{\pi+} m^2_{K+} W^2 \sin(\delta^1_1 - \delta^2_0) \sin\phi_E \tag{3.43}$$

$$\frac{dB^+}{dW} + \frac{dB^-}{dW} = 2 \cdot \frac{dB(K^{\pm} \to \pi^{\pm}\pi^0\gamma)_{IB}}{dW}[1 + 2X_E m^2_{\pi+} m^2_{K+} W^2 \cos(\delta^1_1 - \delta^2_0) \cos\phi_E +$$

$$+ (X^2_E + X^2_M) m^4_{\pi+} m^4_{K+} W^4] \tag{3.44}$$

For a complete analysis of all physical parameters the sum and the asymmetry have to be measured. For experimental reasons the region $55MeV \leq T^{*}_{c} \leq 90MeV$ has to be chosen. To decrease the number of parameters to be determined experimentally, the average over the Dalitz plot for $(\delta^1_1 - \delta^2_0)$ has been taken [71,72]: 10^o [73]. Then one tries to fit (3.43) and (3.44) to the experimental data. As it can be seen in Fig. 3.4, there is some evidence that the DE is different from zero. Furthermore a pure magnetic transition

(no interference term) seems favoured (Fig. 3.5). The experimental situation for IB and DE is:

	$Br(K^{\pm} \to \pi^{\pm}\pi^0\gamma)_{IB}$ $55MeV \leq T_c^* \leq 90MeV$	$Br(K^{\pm} \to \pi^{\pm}\pi^0\gamma)_{DE}$ $55MeV \leq T_c^* \leq 90MeV$
[71]	$(2.55 \pm .18) \cdot 10^{-4}$	$(1.56 \pm .35 \pm .5) \cdot 10^{-5}$
[72]	/	$(2.3 \pm 3.2) \cdot 10^{-5}$
[74]	$(2.50 \pm .44) \cdot 10^{-4}$	$(2.05 \pm .46 \, {}^{+.39}_{-.23}) \cdot 10^{-5}$
Theory	$2.61 \cdot 10^{-4}$	$(.8 \; to \; 1.1) \cdot 10^{-5}$

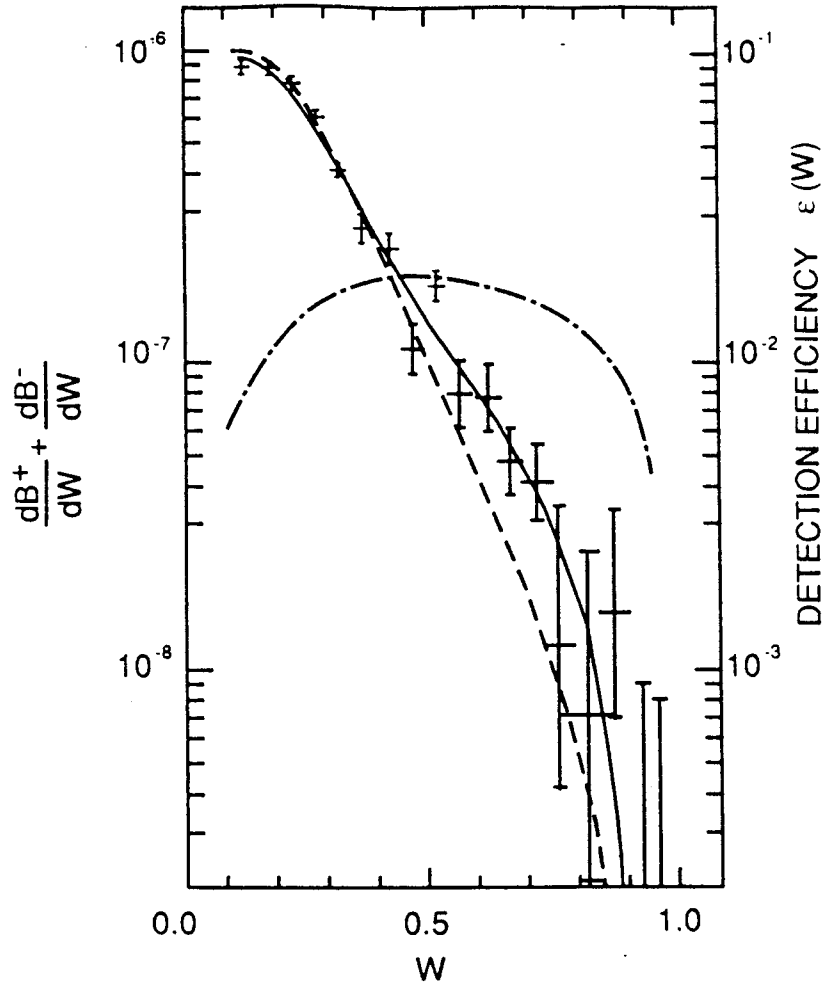

Figure 3.4 : Results for $\frac{d(B^+ + B^-)}{dW}$ (defined in (3.44)) as a function of W [71]. The solid curve is the best fit to the sum of IB and DE; the dashed curves are the best fits to IB alone. Also shown, as dot-dashed curve, is the detection efficiency.

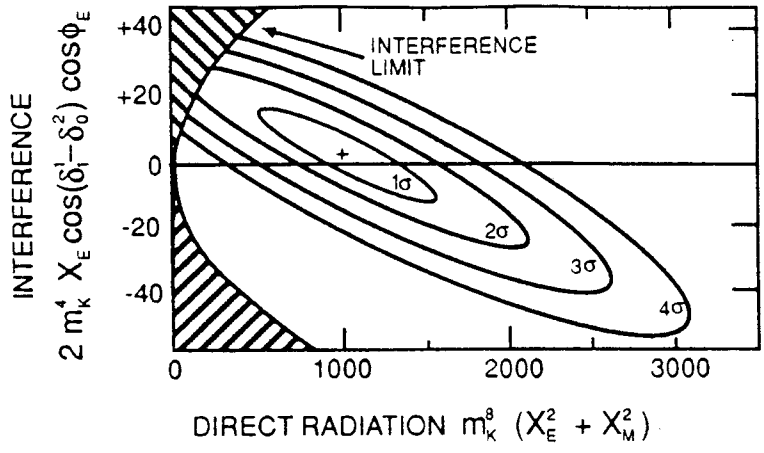

Figure 3.5 Results for fitting [71] data to the sum of IB, DE and interference term; the axis are defined in (3.44). The best fit is the cross and the contours are shown for 1 to 4 standard deviations. The interference limit is obtained by setting $X_E = 0$ and $\cos(\delta_1^1 - \delta_0^0)\cos\phi = 1$.

The quoted theoretical prediction for the DE regards only the contribution from magnetic transitions, which experiments seem to indicate to be the relevant one; furthermore for this contribution only the leading contribution in CHPT [15] has been considered in the quoted prediction. Other predictions are available in the literature [75,68,76,77] which might reproduce the experimental average value $(1.8 \pm .4) \cdot 10^{-5}$. The inclusion of higher dimension operators in CHPT [77] seems to be important, as shown by $\frac{1}{N_c}$ models [76]. E1 electric transitions arise from the loop amplitude and from the counterterms in (3.21). The counterterm combination for this decay is the same which appears in $A(K_S \to \pi^+\pi^-\gamma)_{DE}$; thus the measurement of the interference of bremsstrahlung with E1 transitions in both channels is very important for our knowledge of these counterterms.

CP violation

Experiment in [72] quotes the following value for the asymmetry A

$$A = \frac{\Gamma(K^+ \to \pi^+\pi^0\gamma) - \Gamma(K^- \to \pi^-\pi^0\gamma)}{\Gamma(K^+ \to \pi^+\pi^0\gamma) + \Gamma(K^- \to \pi^-\pi^0\gamma)} \simeq (.4 \pm 2.9) \cdot 10^{-2} \tag{3.45}$$

from which

$$X_E \sin\phi = (-167 \pm 294)\ GeV^{-4} \tag{3.46}$$

which is consistent with zero. DAΦNE would improve the limit (3.45) from $2.9 \cdot 10^{-2}$ to 10^{-3}. Theoretically there is the optimistic limit [78] which puts a bound on this asymmetry

$$A \leq 9 \cdot 10^{-4}$$

which is at the limit of the capability for DAΦNE.

3.8 K $\to 3\pi\gamma$

For completeness we mention this decay, which have been analyzed in the non relativistic approximation [79,80] and in the $\frac{1}{N_c}$ limit [81]. Due to the smallness of phase space N.R. approximation seems justified and DE amplitudes should be suppressed [79]. However recently the authors of [81] have claimed that DE amplitudes are significant. We report below the two predictions for total rates and the experimental status [2,82]

	[79,80]	[81]	experiments
$K^+ \to \pi^+\pi^+\pi^-\gamma$	$.61 \cdot 10^{-4}$	$.6 \cdot 10^{-4}$	$(1.0 \pm .4) \cdot 10^{-4}$
$E^*_\gamma > 11 MeV$			
$K^+ \to \pi^+\pi^0\pi^0\gamma$	$7.0 \cdot 10^{-6}$	$2.2 \cdot 10^{-6}$	$(7.4^{+5.5}_{-2.9}) \cdot 10^{-6}$
$E^*_\gamma > 10 MeV$			
$K_L \to \pi^+\pi^-\pi^0\gamma$	$1.5 \cdot 10^{-4}$	$5.9 \cdot 10^{-5}$	
$E^*_\gamma > 10 MeV$			

3.9 Improvements at DAΦNE for radiative decays with one photon in the final state

We think DAΦNE will be able to see an interference between IB and electric transitions, probably in both channels $K^+ \to \pi^+\pi^0\gamma$ and $K_S \to \pi^+\pi^-\gamma$; furthermore it will establish the relative roles of electric and magnetic transitions in $K^+ \to \pi^+\pi^0\gamma$ and in this last channel will improve the present experimental limit on the charge asymmetry. Finally, DAΦNE could clarify the role of vector mesons in $K_L \to \pi^+\pi^-\gamma$.

SECTION 4. π-Dalitz pairs: $K \to \pi l^+ l^-$

$K \to \pi\gamma$ with γ a real photon is forbidden by gauge invariance. The π-Dalitz pair decays go through virtual photon ($K \to \pi\gamma^*$) and in CHPT at the lowest order there is a mismatch between the number of derivatives and powers in masses required by chiral symmetry and gauge invariance, so these decays start at order p^4. At this order there are contributions of the loops and contributions of the counterterms which depend on the coefficients w_1, w_2 defined in (1.13). The determination of these coefficients is one of the goals of the study of these decays.

4.1 Kinematics

The general amplitude for

$$K(p) \to \pi(p') + \gamma^*(q) \to \pi(p') l^+(k') \bar{l}(k) \qquad p^2 = m_K^2 \quad (p')^2 = m_\pi^2 \quad q^2 \neq 0, \tag{4.1}$$

can be written in the form dictated by Lorentz and gauge invariance

$$A(K \to \pi l^+ l^-) = \frac{-ie^2 h_8}{4f^2} V_\mu(p,q) \frac{-ig^{\mu\nu}}{q^2 + i\epsilon} \bar{u}(k)\gamma_\nu v(k'), \tag{4.2}$$

where

$$V_\mu(p,q) = [q^2(p+p')_\mu - (m_K^2 - m_\pi^2)q_\mu]\phi(\frac{q^2}{m_K^2}, \frac{m_\pi^2}{m_K^2}). \tag{4.3}$$

In (4.3) ϕ is the dynamical contribution which we will take from $o(p^4)$ CHPT [14]. Using equation (4.2) one obtains for the total decay rate

$$\begin{aligned}\Gamma(K \to \pi l^+ l^-) = &\frac{h_8^2 \alpha^2 m_K^3}{192\pi f^4} \int_{4m_l^2}^{(m_K - m_\pi)^2} dq^2 \lambda^{3/2}(1, \frac{q^2}{m_K^2}, \frac{m_\pi^2}{m_K^2}) \\ &\cdot (1 - \frac{4m_l^2}{q^2})^{1/2} (1 + \frac{2m_l^2}{q^2}) |\phi(\frac{q^2}{m_K^2}, \frac{m_\pi^2}{m_K^2})|^2,\end{aligned} \tag{4.4}$$

where $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2(ab + bc + ca)$.

4.2 Theoretical predictions

Historically [1] short distance diagrams were expected to dominate these decays. These diagrams are only logarithmically GIM suppressed and they reconstruct the effective Gilman-Wise quark operator:

$$Q_7 = \frac{e^2}{4\pi}\bar{s}\gamma_\mu(1-\gamma_5)d\bar{e}\gamma^\mu e \tag{4.5}$$

Later it was shown that strong interaction corrections are important and the decays could not be described by only short distance contributions. Long distance contributions can be taken into account in CHPT framework.

The o(p^4) contributions have been calculated in ref. [14], where one can find the analytic expression of the loop amplitudes; these satisfy the following relation:

$$A(K^0 \to \pi^0\gamma^*)|_{loop} = -\frac{1}{\sqrt{2}}A(K^+ \to \pi^+\gamma^*)|_{loop} \tag{4.6}$$

For the counterterms they obtain:

$$\begin{aligned} A(K^+ \to \pi^+\gamma^*)|_{counter.} &= \frac{h_8 e}{12f^2}[w_1 - w_2 + 3(w_2 - 4L_9)]q^2\epsilon^\mu(p+p')_\mu, \\ A(K^0 \to \pi^0\gamma^*)|_{counter.} &= -\frac{h_8 e}{12\sqrt{2}f^2}[w_1 - w_2]q^2\epsilon^\mu(p+p')_\mu, \end{aligned} \tag{4.7}$$

where the L_9 contribution comes from pole diagrams. We remind that the coefficients w_1, w_2 and L_9 have a divergent part which is compensated by the divergent part of the loops. Both loops and counterterm coefficients depend on a renormalization scale μ but the sum (the physical amplitude) is μ independent.

Since the loop contributions satisfy formula (4.6) for the divergent part, the following relation must hold:

$$w_2^{div} = 4L_9^{div}. \tag{4.8}$$

If one would find that the relation (4.8) holds also for the finite part of the counterterms, i.e.

$$w_2^{finite} = 4L_9^{finite}, \tag{4.9}$$

this would imply that the physical amplitudes respect the symmetry imposed by the Gilman-Wise operator (4.5).

The coefficient L_9 can be taken from the measurement of the pion charge radius [6]. At the renormalization scale $\mu = M_\eta$ one has

$$L_9(\mu = M_\eta) = (7.34 \pm 0.30)\cdot 10^{-3}. \tag{4.10}$$

From the measured value of $\Gamma(K^+ \to \pi^+e^+e^-)$, assuming the relation (4.9), authors of ref.[14] obtain two solutions for $w_1^r(\mu = M_\eta)$:

$$\begin{aligned} w_1^r &= (1.67 \pm 0.19)\cdot 10^{-2} \\ w_1^r &= (4.95 \pm 0.19)\cdot 10^{-2}. \end{aligned} \tag{4.11}$$

We report in table II the predicted branching ratios [14] for the other channels for the two solutions (4.11). $\frac{1}{N_c}$ models favour the first solution [76]. Furthermore also looking at the spectrum in $K^+ \to \pi^+\mu^+\mu^-$ can give independent informations.

TABLE II

channel	$\omega_1^r = (1.67 \pm .19)\cdot 10^{-2}$	$\omega_1^r = (4.95 \pm .19)\cdot 10^{-2}$
$K^+ \to \pi^+\mu^+\mu^-$	$(6.1\ ^{+1.1}_{-1.0})\cdot 10^{-8}$	$(4.5\ ^{+1.0}_{-.8})\cdot 10^{-8}$
$K_S \to \pi^0 e^+ e^-$	$(4.8\ ^{+2.0}_{-1.7})\cdot 10^{-10}$	$(4.9 \pm 0.6)\cdot 10^{-9}$
$K_S \to \pi^0\mu^+\mu^-$	$(1.0 \pm 0.4)\cdot 10^{-10}$	$(1.0 \pm 0.1)\cdot 10^{-9}$

SECTION 5. γ-Dalitz pairs: $K_{L/S} \to \gamma l^+ l^-$

These decays proceed through $K \to \gamma\gamma^*$ and thus their study will give information on the form factor of this vertex. This information is of interest for the decays of $K_{L/S} \to l^+l^-$.

From the kinematics it is clear that the form factor can be better studied when the lepton pair is a μ-pair, but this is unfortunately more suppressed by the phase space. As can be seen from table I, only the K_L decays can be detected at the ϕ—Factory; nevertheless, good limits can be put for K_S decays.

5.1 Kinematics

The general amplitude for

$$K(p) \to \gamma(q_1) + \gamma^*(q_2) \qquad p^2 = m_K^2 \quad q_1^2 = 0 \quad q_2^2 \neq 0, \tag{5.1}$$

dictated by Lorentz invariance is

$$A(K \to \gamma l^+ l^-) = \frac{e^2}{q_2^2 + i\epsilon} M^{\mu\nu}(q_1, q_2)\epsilon_\mu(q_1)\bar{u}(k)\gamma_\nu v(k'), \tag{5.2}$$

where from gauge invariance $M^{\mu\nu}$ has the form [33]

$$M^{\mu\nu} = b(0, q_2^2)[-g^{\mu\nu} q_1 \cdot q_2 + q_2^\mu q_1^\nu] + c(0, q_2^2)[\epsilon^{\mu\nu\rho\sigma} q_\rho^1 q_\sigma^2]. \tag{5.3}$$

$b(0, q_2^2)$ is the CP-odd invariant amplitude which contributes to K_S decays with one photon, while $c(0, q_2^2)$ contributes to the K_L ones.

5.2 Theoretical predictions

For the theoretical predictions the starting points are the on-shell decays $K_{L,S} \to \gamma\gamma$, which depend respectively from $c(0,0)$ and $b(0,0)$, thus we can generally write the width in the form

$$\frac{d\Gamma(K_{L/S} \to \gamma l^+ l^-)}{dz} = \Gamma(K_{L/S} \to \gamma\gamma) \left| \frac{H_{L/S}(z)}{H_{L/S}(0)} \right|^2 \frac{2\alpha}{3\pi} \frac{(1-z)^3}{z} \left(1 + \frac{2r_l^2}{z}\right) \sqrt{1 - \frac{4r_l^2}{z}}, \tag{5.4}$$

where

$$z = \frac{q_2^2}{m_K^2} \qquad r_l = \frac{m_l}{m_K} \qquad (4r_l^2 \leq z \leq 1)$$

and $H(z)$ is the form factor which is model dependent. For a rough prediction we can set $H(z) = H(0)$ in eq.(5.4). This estimate works better for K_S decays since, as in $K_S \to \gamma\gamma$, short distance contributions should be suppressed. Furthermore, this is also a good approximation when the lepton pair is an $e^+ - e^-$ pair, because the spectrum is peaked near $z = 4m_l^2/m_K^2$. In table I this approximation has been used.

In CHPT the expression for $H_S(z)$ has been calculated by [33]. DAΦNE will not distinguish among the different model but it will only improve the existing bound. For this reason we omit the explicit expression of $H_S(z)$ which can be found in [33].

For the K_L decays the expression for $H_L(z)$ has been calculated in a vector meson dominance (VMD) model in ref.[83]. Their expression, putting the numerical values of the vector meson masses, has the form

$$\frac{H_L(z)}{H_L(0)} = \frac{1}{1-0.418z} + \frac{c\alpha_{K^*}}{1-0.311z}\left[\frac{4}{3} - \frac{1}{1-0.418z} - \frac{1}{9(1-0.405z)} - \frac{2}{9(1-0.238z)}\right]. \tag{5.5}$$

where c has been evaluated in terms of known coupling constants to be $c = 2.5$, and α_{K^*} parametrizes the unknown electroweak coupling $K^* - \gamma^*$. Using the vacuum insertion approximation with the effective QCD corrected hamiltonian one obtains $|\alpha_{K^*}| \simeq 0.2 \sim 0.3$ (this prediction is free from the delicate calculation of the penguin operator matrix elements).

Two recent experiments with about 10^3 $K_L \to \gamma e^+ e^-$ events [84,85]:

$$Br(K_L \to e^+ e^- \gamma)_{\rm exp} = \begin{cases} (9.2 \pm .5 \pm .5) \cdot 10^{-6} & \text{NA31} \quad [84] \\ (9.1 \pm .4 \,^{+.6}_{-.5}) \cdot 10^{-6} & [85] \end{cases}$$

have confirmed that the approximation $H_L(z) = H_L(0)$ works well for the estimate of the width (see table I) and furthermore looking at the electron spectrum has confirmed the VMD model, giving the value $\alpha_{K^*} = -0.28 \pm 0.12$. One should notice that although

the uncertainty is large the unknown sign of α_{K^*} has been fixed. DAΦNE will have more statistics on $K_L \to \gamma e^+ e^-$ and thus will be able to measure $K_L \to \gamma \mu^+ \mu^-$, where the dependence from the form factor is strong enough to substantially change the width. From both these measurements DAΦNE will be able to check the prediction

$$\frac{\Gamma(K_L \to \gamma e^+ e^-)}{\Gamma(K_L \to \gamma \mu^+ \mu^-)} \simeq 24 \tag{5.6}$$

based on the VMD model of [86] with $\alpha_{K^*} \simeq -0.28$. An analogous value for (5.6) has been obtained using a different VMD model in [87].

$$K_L \to e^+ e^- e^+ e^-$$

The decay $K_L \to e^+ e^- e^+ e^-$ goes through two off-shell photons, so it depends from $c(q_1^2, q_2^2)$ in eq. (5.3). There is no explicit theoretical calculation but, due to the smallness of q_1^2 and q_2^2, one can argue that the width has little dependence on the form factor. DAΦNE will improve the existing results

$$Br(K_L \to e^+ e^- e^+ e^-)_{\rm exp} = \begin{cases} (4 \pm 3) \cdot 10^{-8} & \text{NA31} \quad [88] \\ (5 \pm 2 \pm 3) \cdot 10^{-8} & \text{AGS E845} \quad [89] \end{cases}$$

and test the $K_L \to \gamma^* \gamma^*$ coupling constant.

Acknowledgements

We would like to to thank Gino Isidori for very useful discussions and suggestions. We thank also the organizers and the members of the working group for the very interesting and constructive discussions during the workshops, especially G. Ecker, J. Gasser, L. Maiani and N. Paver. We aknowledge also discussions and collaborations with L. Cappiello and F. Sannino.

References

[1] M.K. Gaillard and B.W. Lee, Phys.Rev. **D 10** (1974) 897;
[2] Review of Particle Properties, Phys. Lett. **B 239** (1990).
[3] P. Franzini, private communications ;
[4] S.Weinberg, Physica **96 A** (1979) 327 ;
A.Manohar and H.Georgi, Nucl. Phys. **B 234** (1984) 189;
H.Georgi, *"Weak Interactions and Modern Particle Theory"*, Benjamin/Commings, Menlo Park 1984.
[5] J. Gasser and H. Leutwyler, Ann. Phys. **158** (1984) 142; Nucl. Phys. **B 250** (1985) 465.
[6] J. Bijnens, G. Ecker and J. Gasser, "Introduction to chiral symmetry" in this report; and references therein.
[7] G. Ecker, J. Gasser, A. Pich and E. de Rafael Nucl. Phys. **B 321**, (1989) 311; G. Ecker, J. Gasser, H. Leutwyler, A. Pich and E. de Rafael Phys. Lett. **B 223** (1989) 425; J.F. Donoghue, C. Ramirez and G. Valencia, Phys. Rev. **D 39** (1989) 1947; M. Praszalowicz and G. Valencia, Nucl. Phys. **B 341** (1990) 27.
[8] A. Pich and E. de Rafael, Nucl. Phys. **B 358** (1991) 311.
[9] H.Y. Cheng, Phys. Lett. **B 238** (1990) 399; H.Y. Cheng, Phys. Rev. **D 43** (1991) 1579 and references therein.
[10] J. Kambor, J. Missimer and D. Wyler, Nucl. Phys. **B 346** (1990) 17.
[11] L. Maiani and N. Paver; "Non-leptonic $K \to 3\pi$ decays" , in this report; and references therein.
[12] J. Kambor, J. Missimer and D. Wyler, Phys. Lett. **B 261** (1991) 496.
[13] G. Isidori and A. Pugliese, Rome preprint **n.849** (1991).
[14] G. Ecker, A. Pich and E. de Rafael, Nucl. Phys. **B 291** (1987) 692.
[15] G. Ecker, H. Neufeld and A. Pich, Phys. Lett. **B 278** (1992) 337.
[16] G. D'Ambrosio and D. Espriu, Phys. Lett. **B 175** (1986) 237;
J.I. Goity, Z. Phys. **C 34** (1987) 341.
[17] N. Cabibbo, Phys. Rev. Lett. **12** (1964) 62; M. Gell-Mann, Phys. Rev. Lett. **12** (1964) 155.
[18] F. Buccella, G. D'Ambrosio and M. Miragliuolo, Il Nuovo Cimento **104 A** (1991) 777.
[19] H. Burkhardt et al. (NA31), Phys. Lett. **B 199** (1987) 139.
[20] L. L. Chau and H. Y. Cheng, Phys. Rev. Lett. **54** (1985) 1768; Phys. Lett. **B** 195 (1987) 275.
[21] Z.E.S. Uy, Phys. Rev. **D 29** (1984) 574; Z.E.S. Uy, Phys. Rev. **D 37** (1988) 2684; Z.E.S. Uy, Phys. Rev. **D 43** (1991) 802; Z.E.S. Uy, Phys. Rev. **D 43** (1991) 1572.
[22] G. Ecker, A. Pich and E. de Rafael, Phys. Lett. **B 189** (1987) 363.
[23] L. Cappiello and G. D'Ambrosio, Il Nuovo Cimento **99 A** (1988) 155.
[24] L. Cappiello, G. D'Ambrosio and M. Miragliuolo, Napoli preprint INFNNA IV 92-10.
[25] A. Della Selva, A. De Rujula and M. Mateev, Phys. Lett. **B 24** (1967) 468; L.M. Sehgal, Phys. Rev. **D 6** (1972) 367.
[26] G. Barr et al. (NA31), Phys. Lett. **B 242** (1990) 523.

[27] V. Papadimitriou et al. (E731), Phys. Rev. **D 44** (1991) 573.
[28] J. Donoghue in *"Introduction to non linear field theory"*, presented at International Workshop on Effective Field Theory, Budapest, August 1991, World Scientific, 1992.
[29] J. F. Donoghue, B. R. Holstein and G. Valencia, Phys. Rev. **D 35**, (1987) 2769;
[30] C.O. Dib, I. Dunietz and F. J. Gilman, Phys. Lett. **B 218**, (1989) 487; Phys. Rev. **D 39** (1989) 2639;
[31] J. Flynn and L. Randall, Nucl. Phys. **B 326** (1989) 31.
[32] G. Ecker, A. Pich and E. de Rafael, Phys. Lett. **B 237** (1990) 481, and references therein.
[33] G. Ecker, A. Pich and E. de Rafael, Nucl. Phys. **B 303** (1988) 665.
[34] P. Ko, Phys. Rev. **D 41** (1990) 1531.
[35] J. Bijnens, S. Dawson and G. Valencia, Phys. Rev. **D 44** (1991) 3555.
[36] T. Morozumi and H. Iwasaki, Progr. Theor. Phys. **82** (1989) 371.
[37] L.M.Sehgal, Phys. Rev. **D 38** (1988) 808; Phys. Rev. **D 41** (1990) 1611.
[38] M. K. Volkov and M. Nagy, JINR Rapid Commun. **N. 6 [45]-90**
[39] M.S. Atiya et al. Experiment E787, Phys. Rev. Lett. **65** (1990) 1188.
[40] H. Dykstra, J.M. Flynn and L. Randall, Phys. Lett. **B 270** (1991) 45.
[41] E. Ma and A. Pramudita, Phys. Rev. **D 24** (1984) 2476.
[42] J. F. Donoghue, B. R. Holstein and Y-C. R. Lin, Nucl. Phys. **B 277** (1986) 651.
[43] H.Y. Cheng, Phys. Lett. **B 245** (1990) 122.
[44] B.R. Martin and E. de Rafael, Nucl. Phys. **B 8** (1968) 131; P. Pavlopoulos, in: *Flavour Mixing in Weak Interactions*, edited by L. L. Chau. Series editor: A. Zichichi, (1984); R. Decker, P. Pavlopoulos and G. Zoupanos, Z. Phys. **C 28** (1985) 117.
[45] J.O. Eeg and I. Picek, Phys. Lett. **B 196** (1987) 391; J.O. Eeg, B. Nizic and I. Picek, Phys. Lett. **B 244** (1990) 513.
[46] J. Bijnens, A. Bramon and F. Cornet, Phys. Rev. Lett. **61** (1988) 1453.
[47] S. Kawabata (KEK) "Physics of two photon interaction" , in *Proceedings of the Joint International Lepton-Photon Symposium and Europhysics Conference on High Energy Physics*, Geneva 91, World Scientific.
[48] P. Ko and T.N. Truong, Phys. Rev. **D 43** (1991) R4.
[49] A. I. Vainshtein, V. I. Zakharov, and M. A. Shifman, Sov. Phys. JETP **45** (1977) 670.
[50] J. Donoghue and B. Holstein, Phys. Rev. **D 29** (1984) 2088.
[51] M. Fukawa et al., *Physics at ϕ Factory*, KEK Report 90-12 (1990).
[52] V.V. Solov'ev and M.V. Terent'ev, JETP Lett., **2** (1965) 336.
[53] T.D.Lee and C.S. Wu, Annu. Rev. Nucl. Sci. **16** (1966) 471; **16** (1966) 511.
[54] A.D. Dolgov and L.A. Ponomarev, Soviet Phys. JNP **4** (1967) 262.
[55] L. Sehgal and L. Wolfenstein, Phys. Rev. **162** (1967) 1362.
[56] G. Costa and P.K. Kabir, Nuovo Cimento **A 51** (1967) 564.
[57] M. McGuigan and A.I. Sanda, Phys. Rev. **D 36** (1987) 1413.
[58] Y.C.R. Lin and G. Valencia, Phys. Rev. **D 37** (1988) 143.
[59] F.E. Low, Phys. Rev. 110(1958)974.
[60] J. Wess and B. Zumino, Phys. Lett. B 37 (1971) 95.

[61] L. Landau, *"Course of Theoretical Physics"* , Vol. 4, part 1, Pergamon Press; A. Messiah *"Quantum Mechanics"* , Vol. II, North-Holland, Publishing Company.
[62] C. Bernard, et al., Phys. Rev. **D 32** (1985) 2343.
[63] G.D'Ambrosio, M. Miragliuolo and F. Sannino, in progress.
[64] G. Ecker, Geometrical aspects of the non-leptonic weak interactions of mesons, in Proc. IX Int. Conf. on the *Problems of Quantum Field Theory*, M.K. Volkov (ed.), Dubna, 1990.
[65] G. Esposito-Farese, Z. Phys. **C 50** (1991) 255.
[66] H. Taureg et al., Phys. Lett. **B 65** (1976) 92.
[67] A.R. Barker, Fermilab E731 Collaboration, in *Proceedings SLAC Summer Institute in Particle Physics* (1990), SLAC-378 EFI 91-04 (1991) E732(1991).
[68] S. Fajfer, Z. Phys. **C 45** (1989) 293.
[69] C.E. Picciotto, University of Victoria Preprint TRI-PP-90-89; M.K. Volkov and A.A. Osipov, JINR Rapid Communications **3** (1991) 67; C.E. Picciotto, Saclay preprint LNS/Ph/91-15.
[70] A. S. Carrol et al., Phys. Rev. Lett. **44** (1980) 529.
[71] R. J. Abrams et al. , Phys. Rev. Lett. **29** (1972) 1118.
[72] K. M. Smith et al. , Nucl. Phys. **B 109** (1976) 173.
[73] J.L. Basdevant, C.D. Froggat and J.L. Petersen, Nucl. Phys. **B 72** (1974) 413; see also J. Bijnens, G. Ecker and J. Gasser, "Semileptonic Kaon Decays", in this report.
[74] V. N. Bolotov et al. , Sov. J. Nucl. Phys. **45** (1987) 1023.
[75] M. Moshe and P. Singer, Phys. Lett. **B 51** (1974) 367; J.L. Lucio, Phys. Rev. **D 24** (1981) 2457; A.A. Bel'kov, Yu. L. Kalinowskii, V.N. Pervushin and N.A. Sarikov, Yad. Fiz. **44** (1986) 690 [Sov. J. Nucl. Phys. **44** (1986) 448].
[76] H. Y. Cheng, Phys. Rev. **D 42** (1990) 72.
[77] J. Bijnens, G. Ecker and A. Pich, CERN preprint CERN-TH. 6444/92; to appear in Phys. Lett. B.
[78] C.O. Dib and R. D. Peccei, Phys. Lett. **B 249** (1990) 325.
[79] R.H.Dalitz, Phys. Rev. **99** (1955) 915.
[80] G.D'Ambrosio, M. Miragliuolo and F. Sannino, in progress.
[81] S. Fajfer, K. Suruliz and R.J. Oakes, Phys. Rev. **D 42** (1990) 3875.
[82] V.N. Bolotov et al., JETPL **42** (1985) 481, translated from ZETFP **42** (1985) 390.
[83] L. Bergström, E. Massó and P. Singer, Phys. Lett. **B 131** (1983) 229.
[84] G.D. Barr et al., Phys. Lett. **B 240** (1990) 283.
[85] K.E. Ohl et al., Phys. Rev. Lett. **65** (1990) 1407.
[86] L. Bergström, E. Massó and P. Singer, Phys. Lett. **B 249** (1990) 141.
[87] P. Ko, Chicago University preprint EFI 90-66 (1990).
[88] NA31 Collaboration, G.D. Barr et al., Phys. Lett. **B 259** (1991) 389.
[89] AGS E845 Collaboration, W.M. Morse, talk presented at the *Conference on the Intersection Between Particle and Nuclear Physics*, Tucson (Arizona), May 1991, preprint BNL-46282 (1991).