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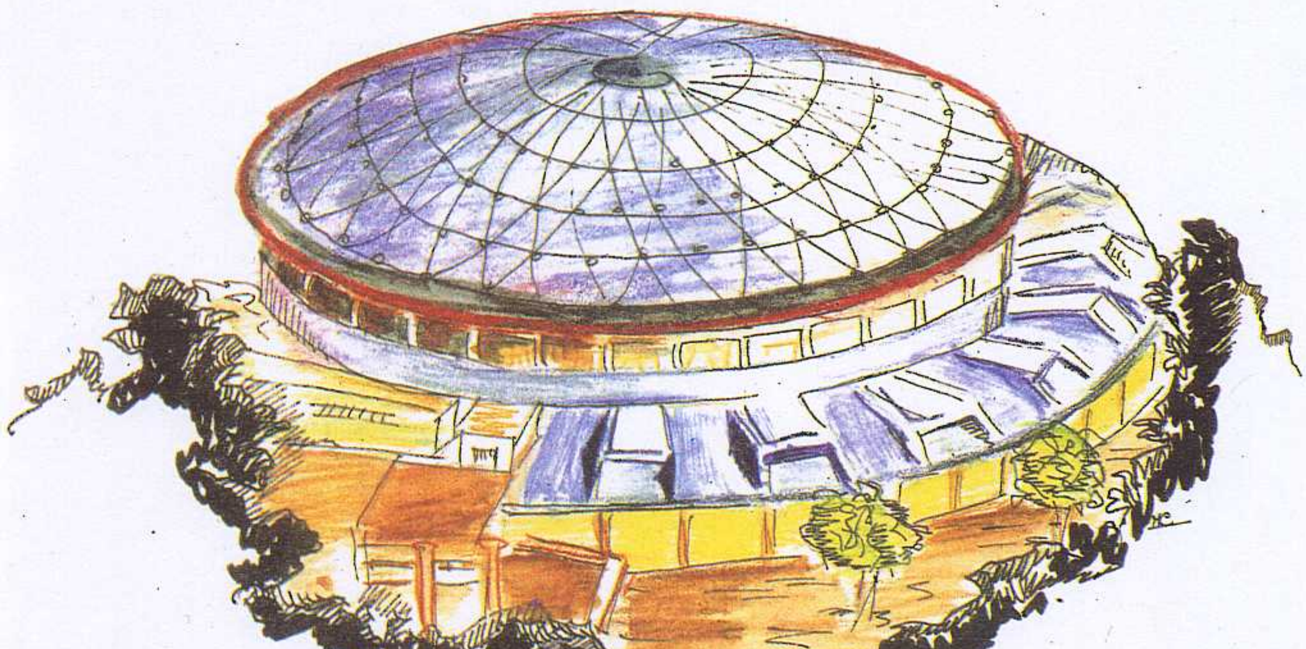
V.S. Demidov, E. Shabalin:

SYMMETRY TEST WITH K, η AND ϕ DECAYS

Contribution to the DAΦNE Physics Handbook



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Symmetry Test with K, η and ϕ Decays

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ABSTRACT

A possibility of test of CPT and T-invariance in K-decays and testing C invariance in η and ϕ decays at the Φ-factory in Frascati is considered. The most interesting and feasible experiments are discussed. Our consideration does not include a study of CP violation at a Φ-factory. This main aim of a Φ-factory is considered in details in other papers.

Contents

1 Test of CPT and T invariance in Kaon decays.

1.1 General remarks

The usual field theory of microscopic processes is built in such a way, that the lagrangian satisfies the following conditions:

a) to be Hermitian;

b) to be invariant under proper Lorentz transformation (excluding reflections);

c) to incorporate a finite number of derivatives of fields;

d) to incorporate the operators of integer-spin fields satisfying to commutation relations and operators of half-integer-spin fields obeying anti-commutation relations.

Such a field theory is automatically invariant under the product of charge conjugations (C), space reflection (P) and time reversal (T) [1]. For this reason a test of CPT invariance is really a test of correctness of description of the microscopic phenomena in terms of existing field theory.

CPT violation would mean an existence of unknown properties of the fields and their interactions which are outside the standard field theory. And if the nature is even CPT invariant but we have missed some channels of decay with new very weak interacting particles, then we also may observe a mimic CPT violation.

So, a wide search the effects of CPT violation in very different processes is desirable.

The best known consequence of CPT invariance is the equality of masses and lifetimes of a particle and its antiparticle. For a free particle and its antiparticle the equality of masses follows from a definition of charge conjugation. But for the masses of a physical particle and its antiparticle such equality is not naively obvious, because of the renormalization due to weak interaction violating C-invariance could be different for a particle and antiparticle. But in CPT invariant theory such a renormalization turns out to be the same. In CPT invariant theory, propagation of a particle and its antiparticle in momentum space is described by one and the same propagator

$$\Delta(k^2) = [k^2 - (m - \tfrac{1}{2}i\Gamma)^2]^{-1} \tag{1.1.1}$$

where k is the 4-momentum, m and Γ are the mass and the width respectively.

The quantities m and Γ appear just in such combination because a propagator is a contraction of the product $\Phi^*(\frac{x}{2})\Phi(-\frac{x}{2})$, and a wave function of an unstable particle has to depend on time as

$$\Phi \sim exp(-imt - \tfrac{\Gamma}{2}t) \tag{1.1.2}$$

In more details the question on the equality of the masses and lifetimes of a particle and its antiparticle is discussed in refs. [1, 2, 3]. It is more natural to look for CPT violation in the processes in which one of the invariances C, P, or T is violated. Such are the processes initiated by the weak interaction, in particular, the decays of K mesons [10].

The most impressive limit on mass difference between a particle and its antiparticle was obtained for the system $\{K^0, \bar{K}^0\}$. In literature [11, 12, 4], the reader can meet the estimate

$$\left| \frac{m_{K^0} - m_{\bar{K}^0}}{m_{K^0}} \right| < 5 \cdot 10^{-18} \tag{1.1.3}$$

but it does not mean that the parameters describing CPT violation are extremely small too.

First of all, the extraordinary smallness of the ratio (1.1.3) originates from the factor

$$2(m_L - m_S)/m_{K^0} \cong 1.4 \cdot 10^{-14}$$

which has nothing with CPT violation but relates the l.h.s. of the relation (1.1.3) to parameter Δ really characterizing the strength of CPT violation (see Eq.(1.2.3) below).

Secondly, the estimate (1.1.3) is not independent of some approximations and theoretical assumptions.[1] Without them, the present data on $K^0(\bar{K}^0)$ decays, according to Kobayashi and Sanda [28], allow to test CPT invariance at best with 10% accuracy.Though our analysis shows that this accuracy is 5 times better such a precision is apparently unsufficient and futher investigation of CPT violating effects would be very actual.

The CPT theorem predicts not only the equality of total widths of a particle and its antiparticle, but also the equality of partial widths if the decay goes in the channels which are not connected by strong interaction (See, for example, [3]). In accordance with CPT theorem, the following relations must take place

$$w(K^+ \to \mu^+ \nu) = w(K^- \to \mu^- \nu)$$

$$w(K^+ \to 3\pi) = w(K^- \to 3\pi) \tag{1.1.4}$$

and so on. Therefore, a precise measurement of partial widths of the charged kaons can also give a valuable information on validity of CPT theorem.

[1] Among them, the crucial assumption was used that there is no direct CPT violation, i.e., that the direct transition $K_2^0 \to 2\pi$ is impossible. The effects proportional to difference between $\eta_\pm$ η_{00} and contribution of $\Delta I = 3/2$ transition also were neglected. For details of criticizm see ref.[32]

In this chapter we analyse the possibilities of search for CPT violation in decays of kaons which will be produced at the Φ-factory in Frascati.

Besides, we shall discuss the possibilities to look for T-violating effects. Independent search for T violating effects is interesting for the following reason. In the world with broken CPT invariance a strength of T violation must not be correlated with the strength of CP violation. And then the degree of T violation may be different from the known degree of CP violation in kaon decays.

The outline of the paper is as following.

In sect. 1.2 we remind shortly the phenomenology of CPT violation in the system $\{K^0, \bar{K}^0\}$ and discuss a possibility of improvement of some parameters at a Φ-factory.

In the Sect. 1.3, the experiments on measurement of K^+ and K^- decays are considered.

The Sect. 1.4 is devoted to the discussion of search for T-odd effects at DAΦNE in kaon decays.

In our numerical estimations of possible experimental limits on one or another parameter describing CPT violation that can be achieved at DAΦNE we have used the conventional set of initial data. At the luminosity $\mathcal{L} = 5 \cdot 10^{32} s^{-1} cm^{-2}$ a one year data (for the period of $10^7 s$) would provide the following numbers of charged and neutral kaons:

$$N^0_{K^+K^-} = 1.2 \cdot 10^{10}, \qquad N^0_{K_S K_L} = 8.5 \cdot 10^9$$

The numbers of the detected kaons would be

$$N^{det}_{K^+K^-} = 9.6 \cdot 10^9, \qquad N^{det}_{K_S} = 8.5 \cdot 10^9, \qquad N^{det}_{K_L} = 2.2 \cdot 10^9,$$

and the numbers of "tagged" kaons would be

$$N^{tag}_{K^+K^-} = 9. \cdot 10^9, \qquad N^{tag}_{K_S} = 1.7 \cdot 10^9, \qquad N^{tag}_{K_L} = 1.1 \cdot 10^9.$$

In our estimates we proceeded from the statistics only and, at this stage, did not take into account the possible systematic uncertainties.

Brief summary of the possibilities to improve the limits on CPT violation at the *DAΦNE* machine is represented in the Table 1.

Table 1

Expected accuracy in measurement of the quantities discribing CPT violation in K-decays at the $DA\Phi NE$.

CPT violating effect	Present limits	References	Expected sensitivity at $DA\Phi NE$
$\mid \frac{m_{K^0}-m_{\bar{K}^0}}{m_{K^0}} \mid$	$< 3.5 \cdot 10^{-16}(90\% c.l.)$	our estimate	$7 \cdot 10^{-18}$
$(\delta_2 - \delta_0) - arg\omega$ $\mid \omega \mid^2 = \frac{2}{3} \cdot \frac{\Gamma(K^\pm \pi^\pm \pi^0)}{\Gamma(K_S \to 2\pi)}$	$7.4^0 \pm 6.7^0$	$\delta_2 - \delta_0 =$ $-45.6^0 \pm 5^0$[32] $arg\omega =$ $-53^0 \pm 4.4^0$[7]	$\pm 1^0$ $\approx 0.3^0$
Rea, $a = \frac{A(K_2 \to 2\pi, I=0)}{A(K_1 \to 2\pi, I=0)}$	$-(0.75 \pm 6)10^{-3}$	our estimate	$\pm 5 \cdot 10^{-4}$
Charge asymmetry in K^0_{l3}: $\delta_S - \delta_L$	no exp. data $\mid \delta_S - \delta_L \mid \leq 0.07$ (our indirect estimate)	$\delta_S = ?$ $\delta_L =$ $(3.27 \pm 0.12)10^{-3}$	$\approx 7 \cdot 10^{-4}$
$\frac{\tau(K^+)-\tau(K^-)}{\tau(K^+)}$	$(9 \pm 8)10^{-5}$	[4]	$\simeq 10^{-5}$
$\frac{\Gamma_i(K^+)-\Gamma_i(K^-)}{\Gamma_i(K^+)}$ for partial widths		See Table 3 below	

All these quantities will be considered in details below. In our analysis we shall use the formulae for CPT violating quantities, which were obtained in ref's [2, 5, 6, 7, 8, 9, 27].

1.2 Test of CPT-invariance in the system $K^0, \bar{K}^0$.

In this section, we shall use the notations of ref's [7] and [27].

1.2.1 Measurement of K^0, $\bar{K}^0$ mass difference

A possible violation of CPT invariance (at conserved T invariance and violated CP invariance) is described by the parameter Δ, which enters in the wave functions of K_L and K_S mesons in the following way:

$$K_S = (K_1 + \varepsilon K_2 + \Delta K_2)/\sqrt{1+ \mid \varepsilon + \Delta \mid^2}$$

$$(1.2.1)$$

$$K_L = (K_2 + \varepsilon K_1 - \Delta K_1)/\sqrt{1+ \mid \varepsilon - \Delta \mid^2}$$

where ε is the parameter describing CP violation in CPT invariant world. The parameter Δ is connected to mass difference of K^0 and $\bar{K}^0$ in the following way (see the formula (74) of ref.[7]):

$$\Delta = 1/2(< \bar{K}^0 \mid H \mid \bar{K}^0 > - < K^0 \mid H \mid K^0 >) \ / \ (m_L - m_S + i(\Gamma_S - \Gamma_L)/2) \quad (1.2.2)$$

where m_L, m_S, Γ_L and Γ_S are the masses and widths of the K_L and K_S mesons. The above equation gives:

$$\begin{aligned} m_{\bar{K}^0} - m_{K^0} &= 2(m_L - m_S)(Re\ \tilde{\Delta} + Re\ a - \cot \Phi_{sw} Im\ \tilde{\Delta}) \\ \tfrac{1}{2}(\Gamma_{K^0} - \Gamma_{\bar{K}^0}) &= 2(m_L - m_S)[Im \tilde{\Delta} + \cot \Phi_{sw} Re(\tilde{\Delta} + a)] \end{aligned} \quad (1.2.3)$$

where

$\Phi_{sw} = \arctan[2(m_L - m_S)/(\Gamma_S - \Gamma_L)]$, $\tilde{\Delta} = \Delta - a$, $Im\ a = 0$ (by definition [5, 6])

and

$$Re\ a \equiv Re\left(A(K_2 \to 2\pi, I=0)\ /\ A(K_1 \to 2\pi, I=0)\right) \tag{1.2.4}$$

As it is seen from (1.2.3), to obtain $\Delta m_{K^0\bar{K}^0}$ and $\Delta\Gamma_{K^0\bar{K}^0}$ one needs to know $Re(\tilde{\Delta}+a)$ and $Im\tilde{\Delta}$.

But these quantities can not be extracted from existing data directly. Indeed, all measured to date CP-violating (and possibly, CPT violating at the same time) effects depend either on combination of CPT violating and CPT conserving parameters, or on combination of $Re\Delta$ with new CPT violating parameter y_l and with parameters $x_l(\bar{x}_l)$ characterizing a strength of violation of the $\Delta Q = \Delta S$ rule.

So,

$$\eta_{+-} = (1+\omega)^{-1}(\varepsilon_0 + \varepsilon_2) \tag{1.2.5}$$

$$\eta_{00} = (1-2\omega)^{-1}(\varepsilon_0 - 2\varepsilon_2) \tag{1.2.6}$$

$$\delta_L = 2Re(\varepsilon - \Delta) - 2Re\ y_l - Re(x_l - \bar{x}_l) \tag{1.2.7}$$

where

$$\varepsilon_0 = \varepsilon - \Delta + a \tag{1.2.8}$$

$$\varepsilon_2 = \varepsilon' + (\varepsilon - \Delta)\omega \tag{1.2.9}$$

$$\delta_L = [\Gamma(K_L \to e^+\nu\pi) - \Gamma(K_L \to e^-\nu\pi)]/[\Gamma(K_L \to e^+\nu\pi) + \Gamma(K_L \to e^-\nu\pi)] \tag{1.2.10}$$

$$\omega = \frac{1}{\sqrt{2}} A(K_S^0 \to 2\pi, I=2) \ / \ A(K_S^0 \to 2\pi, I=0) \tag{1.2.11}$$

$$\varepsilon' = \frac{1}{\sqrt{2}} A(K_2^0 \to 2\pi, I=2) \ / \ A(K_S^0 \to 2\pi, I=0) \tag{1.2.12}$$

The parameters x_l and $\bar{x}_l$ are defined by the relations

$$x_l = < l^+\nu_l\pi^- \mid \bar{K}^0 > \ / \ < l^+\nu_l\pi^- \mid K^0 > \quad , \quad \bar{x}_l = < l^-\bar{\nu}_l\pi^+ \mid K^0 >^* \ / \ < l^-\bar{\nu}_l\pi^+ \mid \bar{K}^0 >^* \tag{1.2.13}$$

The parameter y_l describing the direct CPT violation in K_{l3}-decay amplitude is defined as follows:

$$< l^+\nu_l\pi^- \mid K^0 > \ / \ < l^+\bar{\nu}_l\pi^+ \mid \bar{K}^0 >^* = (1-y_l) \ / \ (1+y_l) \tag{1.2.14}$$

The combination

$$\frac{2}{3}\eta_{+-} + \frac{1}{3}\eta_{00} + \frac{2}{3}(\eta_{+-} - \eta_{00})\omega = \varepsilon - \tilde{\Delta} \tag{1.2.15}$$

does not contain $x_l(\bar{x}_l)$ or y_l. But it defines a difference between CPT conserving parameters ε and CPT violating parameter $\tilde{\Delta}$. For this reason, basing on smallness of η_{+-} and η_{00}, one can not declare that ε and $\tilde{\Delta}$ separately are small too.

To determine $\tilde{\Delta}$ we need a measurement of some other CP (and CPT)- violating correlations, depending on other combinations of the above parameters. In principle, these parameters may be determined from data on proper-time dependence of probability of the decays into $\pi l \nu$ for particles which were produced initially as K^0 and $\bar{K}^0$. The programm of necessary measurements was considerd by Tanner and Dalitz [27] and it may be fulfilled in experiments where $K^0(\bar{K}^0)$ are produced by $K^+(K^-)$ or in pair with $K^-(K^+)$ respectively.

A Φ-factory provides an opportunity to measure for the first time the charge assymmetry parameter for K_S

$$\delta_S = [\Gamma(K_S \to l^+\nu\pi) - \Gamma(K_S \to l^-\bar{\nu}_l\pi)]/[\Gamma(K_S \to l^+\nu\pi) + \Gamma(K_S \to l^-\bar{\nu}_l\pi)] \quad (1.2.16)$$

due to possibility to separate the $K_S \to \pi l \nu$ transitions tagged by accompaning K_L decays. This parameter depends on ε, Δ, y_l and $x_l(\bar{x}_l)$ as follows:

$$\delta_S = 2Re(\varepsilon + \Delta) - 2Re\ y_l + Re(x_l - \bar{x}_l) \quad (1.2.17)$$

Combining (1.2.15) with (1.2.7) we obtain

$$Re\ \Delta = \tfrac{1}{4}(\delta_S - \delta_L) - \tfrac{1}{2}Re(x_l - \bar{x}_l) \quad (1.2.18)$$

In the standard theory the parameter x_l must be very small because a violation of the $\Delta Q = \Delta S$ rule occurs in the second order of weak interaction and consequently $x_l \leq G_F m_K^2 \leq 2 \cdot 10^{-6}$. But the standard theory is CPT invariant one and $\Delta = 0$ in it.

So, testing CPT invariance it would not be right to affirm in advance that x_l and $\bar{x}_l$ are negligibly small.

The Particle Data Group [4] gives for x the value

$$„x" = 0.006 \pm 0.018 + i(0.003 \pm 0.026) \quad (1.2.19)$$

but this result was obtained supposing CPT invariance! If CPT does not hold the proper-time rates of decay into $\pi^+ l^- \bar{\nu}$ and $\pi^- l^+ \nu$ of the particle produced at $t = 0$ as K^0 are as follows:

$$N^-(l^-;t) \propto e^{\frac{-(\Gamma_S+\Gamma_L)}{2}t}[\cosh(\alpha t) - \cos(\beta t) - 2(Re\bar{x}\sinh(\alpha t) + Im\bar{x}\sin(\beta t))] \quad (1.2.20)$$

$$N^+(l^+;t) \propto e^{\frac{-(\Gamma_S+\Gamma_L)}{2}t}[\cosh(\alpha t) + \cos(\beta t) - 2(Re(x + 2\Delta)\sinh(\alpha t) + Im(x + 2\Delta)\sin(\beta t))] \quad (1.2.21)$$

where $\alpha = 1/2(\Gamma_S - \Gamma_L), \beta = m_L - m_S$.

Therefore studying $N^-(t)$ distribution one gets in fact the information on $\bar{x}$. The $N^+(t)$ distribution gives the information on $(x + 2\Delta)$. Unfortunately, in all experimental works (see refs. in [4]), both distributions were fitted simultaneously assuming that $\Delta = 0$ and $x = \bar{x}$.

But the fact that both distributions were described with one and the same parameter „x" means that $\bar{x}$ and $(2\Delta + x)$ do not differ one from another and from „x" within the error bars in determination of „x".

Then error bars for $\bar{x}$ and $(2\Delta + x)$ do not exceed $\sqrt{2}$ of error bars for the quantity „x" defined by the formula (1.2.19).

Additional information on x and $\bar{x}$ may be obtained attracting the data on $K^+ \to e^+\pi^0\nu$ and $K^+ \to e^+\nu\pi^+\pi^-$ decays.

For this we may use the relation

$$R = \frac{Rate(K_L \to e^\pm \pi^\mp \nu)}{2Rate(K^+ \to e^+ \pi^0 \nu)} = \mid f_+^{K^0\pi^-} / f_+^{K^+\pi^0} \mid^2 \frac{\rho_{K^0}}{\rho_{K^+}} (1 - Re(x + \bar{x})) \qquad (1.2.22)$$

The ratio of phase–space factors is $\rho_{K^0}/\rho_{K^+} \cong 1.02$.

The ratio of the form factors of $K_L \to \pi l \nu$ and $K^+ \to \pi l \nu$ decays according to ref. [31] is

$$\mid f_+^{K^0\pi^-} / f_+^{K^+\pi^0} \mid^2 = 0.9574$$

Using $R^{exp} = 0.9606 \pm 0.0187$ we find

$$Re(\bar{x} + x) = 0.016 \pm 0.019 \qquad (1.2.23)$$

Independently, from the experimental data on K_{e4}^+ decay [4] we conclude that

$$\mid x \mid^2 \leq 3 \cdot 10^{-4} \qquad at\ 90\%\ CL \qquad (1.2.24)$$

Consequently,

$$| Rex | \leq 0.017, \qquad | Imx | \leq 0.017 \qquad (1.2.25)$$

Therefore, for the quantity $\frac{1}{2}Re(x - \bar{x})$ entering into Eq. (1.2.18) we obtain

$$\frac{1}{2}Re(x - \bar{x}) = -0.008 \pm 0.02 \qquad (1.2.26)$$

The parameter δ_L is known with comparatively good precision:

$$< \delta_L >= (3.27 \pm 0.12)10^{-3} \qquad [4] \qquad (1.2.27)$$

If CPT holds, δ_S must be equal to δ_L.

At present, using the the Eq's (1.2.18), (1.2.21), (1.2.24) and our conclusion that the accuracy in determination of $\bar{x}$ and $(2\Delta + x)$ is not worse than $\sqrt{2}$ of accuracy in determination of "x" Eq. (1.2.19) we come to the estimate

$$| \delta_S - \delta_L | \leq 0.07 \qquad (1.2.28)$$

At the DAΦNE, the expected number of $K_S^{tag} \rightarrow \pi l \nu$ decays is $1.9 \cdot 10^6$ and expected statistical uncertainty in δ_S will be $0.7 \cdot 10^{-3}$. Therefore, the difference $\delta_S - \delta_L$ may be measured with accuracy $\approx 0.7 \cdot 10^{-3}$.

Then, to obtain the limit on $Re\Delta$ at the level of few $\cdot 10^{-4}$ it would be necessary to measure $Re(x_l - \bar{x}_l)$ with accuracy of order of $10^{-4} \div 10^{-3}$.

In the planned CP-LEAR experiment [22], the expected accuracy in measurement of Rex_l and Imx_l will be $6 \cdot 10^{-4}$ and $7 \cdot 10^{-4}$ respectively. But again these sensitivities are related to the case when $x = \bar{x}$ and $\Delta = 0$. It would be very desirable to reanalyse the question what limits on x_l and $\bar{x}_l$ may be achieved if CPT invariance is not assumed . If they will be found to be the same, then, together with expected at the $DA\Phi NE$ accuracy in definition of $(\delta_S - \delta_L) \simeq 7 \cdot 10^{-4}$ it will allow to lower the limit on $Re\Delta$ down to $| Re\Delta | \leq 4.6 \cdot 10^{-4}$. At present, combining our estimates of $Re(x + 2\Delta)$ and Rex we conclude that

$$| Re\Delta | \leq 1.5 \cdot 10^{-2} \qquad (1.2.29)$$

¿From our estimates $| Im(x+2\Delta) | \leq 0.026\sqrt{2}$ and $| Imx | \leq 0.017$, we obtain

$$| Im\Delta | \leq 2 \cdot 10^{-2} \tag{1.2.30}$$

Our limits on $Re\Delta$ and $Im\Delta$ show that CPT invariance is tested for kaon mass matrix with 2% accuracy, or by 5 times better than it was declared in ref. [28]. The same limit takes place for Rey_l. It can be obtained attracting the data on integrated proper-time distributions of $\pi^+\pi^-$ from K^0 and $\bar{K}^0$ in the process

$$p\bar{p} \rightarrow K^0(or\bar{K}0)\ K^{\pm} + \text{pions}$$

It follows from these data that

$$Re(\varepsilon - \Delta) = (2.3 \pm 6) \cdot 10^{-3} \qquad [27] \tag{1.2.31}$$

Then, using Eq. (1.2.7) and our limits on x and $\bar{x}$ we conclude that Rey_l is consistent with zero within 2% accuracy.

Using our results (1.2.29), (1.2.30) and (1.2.3) we conclude also that the most conservative limit on mass difference of K^0 and K^0 is as follows

$$| \frac{m_{K^0} - m_{\bar{K}^0}}{m_{K^0}} | < 3.5 \cdot 10^{-16} \tag{1.2.32}$$

The limits on $Im\Delta$ and $Re\Delta$ may be improved making use of the Bell-Steinberger unitarity relation [9]. In accordance with this relation

$$Im\Delta \cong \frac{1}{2}\{(1-2\mid\omega\mid^2)(fRe æ - Im æ) + \frac{\Gamma_L(\pi l\nu)}{\Gamma_S} f\delta_L +$$
$$+ \frac{\Gamma_L(\pi l\nu)}{\Gamma_S}[f\frac{\delta_S-\delta_L}{2} + Im(\bar{x}_l + x_l)] + fRe\alpha_{3\pi} - Im\alpha_{3\pi}\} \tag{1.2.33}$$

where

$$f = \frac{\Gamma_S - \Gamma_L}{\Gamma_S + \Gamma_L}\tan\Phi_{sw},$$

$$\text{æ} = (\frac{2}{3}\eta_{+-} + \frac{1}{3}\eta_{00}) + \frac{4}{3}(\eta_{+-} - \eta_{00})Re\omega + \frac{2}{3} \mid \omega \mid^2 (\eta_{+-} + 2\eta_{00})$$

$$\alpha_{3\pi} = \frac{A^*(K_S \to 3\pi)A(K_L \to 3\pi)}{\Gamma_S} \quad (1.2.35)$$

With existing data on $\alpha_{3\pi}$ (see Sec. 1.2.2) and our estimate

$$\mid \frac{\delta_S - \delta_L}{2} + Im(x + \bar{x}) \mid \leq 0.053 \quad (1.2.37)$$

following from the expression (1.2.28) and our limits on δ_S, x and $\bar{x}$, we obtain

$$Im\Delta = (-0.78 \pm 0.70)10^{-4} \quad (1.2.37)$$

To have an idea, what could be done for more precise definition of $Im\Delta$, we present the separate terms of Eq. (1.2.33) in form of Table 2, where existing data and their possible improvement are shown.

Table 2

Contributions to $Im\Delta$ from separate terms of Eq. (1.2.33)

Separate terms of Eq. (1.2.33)	Present exper. value	Expected sensitivity at $DA\Phi NE$
$\frac{1}{2}(1 - 2 \mid \omega \mid^2)(fRe\text{æ} - Im\text{æ})$	$(-0.087 \pm 0.022)10^{-3}$	
$\frac{\Gamma_L(\pi l \nu)}{2\Gamma_s} f\delta_L$	$(0.176 \pm 0.0065)10^{-5}$	Present accuracy cannot be improved at $DA\Phi NE$
$\frac{\Gamma_L(\pi l \nu)}{2\Gamma_s}(\frac{\delta_s - \delta_L}{2} + Im(x + \bar{x}))$	$\mid$ this term $\mid \leq 0.3 \cdot 10^{-4}$	$\pm 0.6 \cdot 10^{-6}$ $DA\Phi NE$ + CP-LEAR)
$\frac{1}{2}(fRe\alpha_{3\pi} - Im\alpha_{3\pi})$	$(0.11 \pm 0.60)10^{-4}$	$\pm 0.6 \cdot 10^{-5}$
$Im\Delta$	$(-0.78 \pm 0.70)10^{-4}$	$\pm 3 \cdot 10^{-6}$

The Bell-Steinberger relation allows to improve a limit on $Re\Delta$ too. This relation gives

$$Re\varepsilon = Re\varepsilon_0 + O(10^{-4}) \cong 1.5 \cdot 10^{-3} + O(10^{-4}) \quad (1.2.38)$$

Using the estimate (1.2.31) for $Re(\varepsilon - \Delta)$ we concludle that

$$Re\Delta \approx (-0.8 \pm 6) \cdot 10^{-3} \tag{1.2.39}$$

The limits (1.2.37) and (1.2.39) allow to lower the estimate of $m_{K^0} - m_{\bar{K}^0}$ down to $\mid \frac{m_{K^0}-m_{\bar{K}^0}}{m_{K^0}} \mid < 10^{-16}$ but this result already depends on assumption that CPT violation does not spoil the Unitarity necessary for foundation of the Bell-Steinberger relation [29]. If $Re\Delta$ will be determined with accuracy $4.6 \cdot 10^{-4}$ which is expected after a measurument of δ_S at $DA\Phi NE$ and $x_l(\bar{x}_l)$ in CP-LEAR experiment the value of $\mid m_{K^0} - m_{\bar{K}^0} \mid / m_{K^0}$ may be limited at the level of $7 \cdot 10^{-18}$.

This conclusion relates to the most general picture of CPT violation named by L.Maiani[77] as Scheme 3. In the case in which semileptonic amplitudes are assumed to be conserving CPT invariance and obeying to the $\Delta Q = \Delta S$ rule, the present data already give the value $(3.0 \pm 2.4) \cdot 10^{-18}$ for the ratio under consideration [77].

1.2.2 Measurement of probabilities of the $K_S \to \pi^+\pi^-\pi^0$ and $K_S \to \pi^0\pi^0\pi^0$ decays.

It follows from Tab.2 that the largest uncertainty in $Im\tilde{\Delta}$ appears from $\alpha_{3\pi}$:

$$\alpha_{3\pi} = \frac{\Gamma_L}{\Gamma_S}(B_{+-0}\eta^*_{+-0} + B_{000}\eta^*_{000}), \tag{1.2.40}$$

where B_{+-0} and B_{000} are the rates of the $K_L \to \pi^+\pi^-\pi^0$ and $K_L \to \pi^0\pi^0\pi^0$ decays,

$$\eta_{+-0} = \frac{A^-(K_S \to \pi^+\pi^-\pi^0)}{A(K_L \to \pi^+\pi^-\pi^0)} \quad ; \quad \eta_{000} = \frac{A^-(K_S \to \pi^0\pi^0\pi^0)}{A(K_L \to \pi^0\pi^0\pi^0)}.$$

and A^- is the CP violating part of total amplitude. The amplitude of $K_S \to \pi^0\pi^0\pi^0$ transition is CP violating one, but the amplitude of $K_S \to \pi^+\pi^-\pi^0$ transition consists of CP violating part A^- and CP conserving part A^+.

Experimentally,

$$\alpha_{3\pi}^{exp} = [-0.02 \pm 0.07 + i(-0.04 \pm 0.10)]10^{-3}, \qquad [7] \tag{1.2.41}$$

For the relative probability of decay $K_S \to \pi^+\pi^-\pi^0$ the chiral theory predicts [13, 14, 15]:

$$Br^+(K_S \to \pi^+\pi^-\pi^0) = (1.8 \div 3.9) \cdot 10^{-7} \quad (1.2.42)$$

In the Wolfenstein model [16] of CP violation:

$$\eta_{+-0} = \eta_{000} = \eta_{+-} = \eta_{00} = \mid \eta \mid e^{i\Phi_{sw}} \quad (1.2.43)$$

The direct violation of CP invariance in $K^0 \to 3\pi$ transition can lead to corrections to the equation (1.2.40) of order of 0.1 % [15, 16, 17, 18, 19, 20].

For the CP violating decay rates, the predicted magnitudes are as follows:

$$Br^-(K_S \to \pi^+\pi^-\pi^0) = \mid \eta \mid^2 \frac{\Gamma_L}{\Gamma_S} Br(K_L \to \pi^+\pi^-\pi^0) = 1 \cdot 10^{-9}$$

(1.2.44)

$$Br(K_S \to \pi^0\pi^0\pi^0) = \mid \eta \mid^2 \frac{\Gamma_L}{\Gamma_S} Br(K_L \to \pi^0\pi^0\pi^0) = 1.8 \cdot 10^{-9}$$

The best experimental 90% confidence level limits now are [4]:

$$Br^{exp}(K_S \to \pi^+\pi^-\pi^0) < 4.9 \cdot 10^{-5};$$

$$Br^{exp}(K_S \to \pi^0\pi^0\pi^0) < 3.7 \cdot 10^{-5}$$

. The expected number of $K_S \to \pi^+\pi^-\pi^0$ decays at Φ-factory will be:

$$N_{K_S}^{det} \cdot Br^+(K_S \to \pi^+\pi^-\pi^0) \approx 1500 \div 3100,$$

and approximately

$$N_{K_S}^{tag} \cdot Br^+(K_S \to \pi^+\pi^-\pi^0) \approx (300 \div 660)$$

events will be accompanied by detection of the decay $K_L(\pi\mu\nu, \pi e\nu, \pi^+\pi^-\pi^0)$. For the CP violating rates we may anticipate:

$$N_{K_S}^{det} \cdot Br^-(K_S \to \pi^+\pi^-\pi^0) \approx 9 events$$

$$N_{K_S}^{tag} \cdot Br^-(K_S \to \pi^+\pi^-\pi^0) \approx 2event$$

An expected number of the decays $K_S \to \pi^0\pi^0\pi^0$ will be roughly equal to 18. About five of them will be "tagged".

At $DA\Phi NE$, a magnitude of $\alpha_{3\pi}$ may be lowered to the level: [2]

$$\alpha_{3\pi}^{DA\Phi NE} < [(0.006 \pm 0.0002) \pm i(0.006 \pm 0.0002)] \cdot 10^{-3}. \tag{1.2.45}$$

Due to confident detection of K_S mesons at $DA\Phi NE$ it may occur that the decay $K_S \to 3\pi$ will be observed firstly at the Φ-factory. An expected statistics of K_S decays at FNAL E621 experiment (see Table 1 in [21]) is comparable with one at $DA\Phi NE$. It is expected that the CP-LEAR experiment [22] will excel the $DA\Phi NE$ in this field and experimental uncertainty in η_{+-0} would be decreased to the value $\sim 6 \cdot 10^{-4}$. As a result, a magnitude of $\alpha_{3\pi}$ will be probably lowered down to the level:

$$\alpha_{3\pi}^{CP-LEAR} = [0.9 \pm 0.03 + i(0.9 \pm 0.03)] \cdot 10^{-6}. \tag{1.2.46}$$

1.2.3 Parameter ω

Following the definitions of ref. [7] let us write the amplitudes of $K^0(\bar{K}^0)$ transition into 2π state with isotopic spin I as

$$A(K^0 \to 2\pi, I) = (A_I + B_I)e^{i\delta_I}$$

$$A(\bar{K}^0 \to 2\pi, I) = (A_I^* - B_I^*)e^{i\delta_I}$$

where A_I are CPT-invariant amplitudes and B_I are CPT-noninvariant ones. Then, the parameter ω defined by Eq. (1.2.11) may be written in the form

$$\omega \cong \frac{1}{\sqrt{2}} \frac{ReA_2 + iImB_2}{ReA_0 + iImB_0} e^{i(\delta_2 - \delta_0)} \tag{1.2.47}$$

[2] In this estimate we suppose that

$$\eta_{+-0} < \frac{A^-(K_S \to \pi^+\pi^-\pi^0) \; + \; A^+(K_S \to \pi^+\pi^-\pi^0)}{A(K_L \to \pi^+\pi^-\pi^0)}$$

So, the difference of arg $\cdot\omega$ from $(\delta_2 - \delta_0)$ would be a signal of CPT violation. Besides, the parameter ω enters into the formul (1.2.9), (1.2.15) and (1.2.33) expressing other parameters of CPT violation through the observables.

Experimentally, the parameter ω is determined by the equations:

$$|\,\omega\,| = \sqrt{\frac{2}{3}\frac{\Gamma(K^{\pm} \to \pi^{\pm}\pi^{0})}{\Gamma(K_S \to 2\pi)}} = (31.9 \pm 0.6)\cdot 10^{-3}$$

$$Re\ \omega = \frac{1}{12}\left(\frac{\rho_{00}}{\rho_{+-}}\frac{\Gamma(K_S\to\pi^+\pi^-)}{\Gamma(K_S\to\pi^0\pi^0)} - 2\right) = (19 \pm 2)\cdot 10^{-3} \qquad (1.2.48)$$

where $\rho_{00}/\rho_{+-} = 1.014$ [25] - is a ratio of the phase volumes of $K_S \to 2\pi$ decays. Experimentally

$$arg\ \omega = \pm \arccos[\ Re\omega\ /\ \ |\,\omega\,|\ ] = \pm(53^0 \pm 4.4^0) \qquad (1.2.49)$$

It is seen from (1.2.47) and (1.2.48) that the accuracy in ω is completely determined by the results of measurements of probability of decays $K^{\pm}$ and K_S into 2π. In spite of the fact, that these modes are basic, the world statistics accountes for only $\sim 5\cdot 10^4$ of $K^{\pm} \to \pi^{\pm}\pi^{0}$ decays and $\sim 5.5\cdot 10^4$ of $K_S \to 2\pi$ decays[4].

$DA\Phi NE$ is a unique instrument for the revision of these decays probabilities. The expected number of the "tagged" decays of $K^{\pm} \to \pi^{\pm}\pi^{0}$ make up $\approx 1.7\cdot 10^9$, the number of the "tagged" $K_S \to 2\pi$ decays is estimated to be approximately the same. Preliminary estimates show that it allows to define $|\,\omega\,|$ by two order more exactly and to measure the phase of ω with the accuracy $\sim 0.3^0$.The importance of accurate measurement of arg ω will be considered in one of the next sections of this paper.

It must be taken into account that the equality (1.2.47) is an approximate one. The contributions of the $\Delta I = 5/2$ part of the amplitude of $K \to 2\pi$ decays as well as radiative corrections [7, 2, 26] also need further theoretical studies.

1.2.4 Definition of CPT-forbidden parameter a

. A value of the parameter a defined by the formula (1.2.4) may be estimated using Eq. (1.2.15) and the estimate (1.2.31)

$$Re(\varepsilon - \tilde{\Delta}) = Re(\varepsilon - \Delta + a) = (1.55 \pm 0.03)10^{-3}$$

$$Re(\varepsilon - \Delta) = (2.3 \pm 6)10^{-3} \qquad [27]$$

leading to the result

$$Rea = (-0.75 \pm 6)10^{-3} \tag{1.2.50}$$

The limit on a may be improved when δ_S will be measured at $DA\Phi NE$ and $x_l(\bar{x}_l)$ will be measured in CP-LEAR experiment [22]. The expected accuracy in measurement of $Re\Delta$ is $4.6 \cdot 10^{-4}$. With the accuracy in definition of ε of order of 10^{-4} following from Bell- Steinberger relation it will allow to determine a with accuracy $\approx 10^{-4}$.

Completing this section we would like to mention that assuming a validity of the $\Delta Q = \Delta S$ rule it is possible to obtain from existing and expected in future data a few times lower limits on CPT violating parameters. So, Buchanan et al [74] have found that these parameters may be tested to one part in 10^{-4} in future experiments at aΦ-factory. The same possibility of Φ-factory in test of CPT invariance was revealed by Cline[75].

1.2.5 CPT-invariance and phases

It was shown by authors of many papers (for example see refs.[2, 7, 8]) that CPT-theorem leads to the simple form of dependence between 3 angles: arguments of two complex parameters - ω and ε' describing decays of neutral kaons into 2π and phase difference $(\delta_2 - \delta_0)$ of $\pi\pi$ -scattering with isospin = 2 and 0:

$$arg\omega = (\delta_2 - \delta_0) \quad or \quad (\delta_2 - \delta_0 + \pi) \tag{1.2.51}$$
$$arg\varepsilon' = \delta_2 - \delta_0 \ \pm \ \frac{\pi}{2} \tag{1.2.52}$$

At present time the values of $\delta_2 - \delta_0$ received from different experiments are in poor agreement (see, for example, table in [28]).Moreover, the fitting performed considering all the experimental data related to the problem results in different values of $\delta_2 - \delta_0$ [15, 32, 33]. We shall use the result of paper [32]: [3]

[3] Uncertainties in these values are explained mainly by inconsistency of the experimental results of δ_0^0 phase at $E_{\pi\pi} \approx m_K$. At $DA\Phi NE$ δ_0^0 can be measured with greater accuracy ($\approx 1^0$) in the region $E_{\pi\pi}$ from 0 to $\sim$ 380 Mev using $K^\pm \to e^\pm \pi^+\pi^-\nu$ decays. The expected number of "tagged" decays $K_{l4}^\pm$ is $\approx 2.9 \cdot 10^5$, that is one order greater than in [34], The accuracy of δ_0^0 in [34] was reported to be $\pm 3^0$ at each point.

$$(\delta_2 - \delta_0)^{exp} = -45.6^0 \pm 5^0 \tag{1.2.53}$$

The phase $arg\omega$ is determined from the equality (1.2.49) except for sign. However, taking into account the arguments produced in [7] sign "-" must be chosen:

$$arg\omega = -53^0 \pm 4.4^0 \tag{1.2.54}$$

Of 3 angles mentioned above the arg ω will be measured at $DA\Phi NE$ with the best accuracy. Therefore it is convenient to change the form of relations (1.2.51) and (1.2.52) resulting from CPT-theorem using new angles φ_1, φ_2 and φ_2':

$$\varphi_1 = (\delta_2 - \delta_0) - \arg\omega \tag{1.2.55}$$
$$\varphi_2 = arg\varepsilon' - arg\omega + \frac{\pi}{2} \tag{1.2.56}$$
$$\varphi_2' = arg\varepsilon' - arg\omega - \frac{\pi}{2} \tag{1.2.57}$$

The CPT-invariance requires $\varphi_1 = 0$ and also one of the angles φ_2 or φ_2' be equal to zero.

It is easy to see from (1.2.53) and (1.2.54) that φ_1 is in a satisfactory agreement with CPT-theorem: $\varphi_1 = 7.4^0 \pm 6.7^0$ (see also fig.1). At $DA\Phi NE$ the angle φ_1 may be measured with the precusion $\approx 1^0$.

Angles φ_2 and φ_2' can be considered only if $\mid \varepsilon' \mid \neq 0$, because in opposite case the phase $arg\omega$ may not be defined and equalities (1.2.56) and (1.2.57) become senseless. At present time only in NA31 experiment the effect of direct CPT-violation is steadily observed [4, 31, 35, 36]. The analysis conducted for the first run of this experiment shows that the values of both angles φ_2 and φ_2' differ from those predicted by CPT theorem more then 1 standard deviation. 68% confidence intervals for these angles are shown in fig.1.

Since the arg ε' is defined not only by the phase difference $arg\eta_{00} - arg\eta_{+-}$, but also by the ratio $\mid \varepsilon'/\varepsilon \mid$, the latter value measured with better accuracy at $DA\Phi NE$ and other experiments as well as the expected results of phase difference measurements at CP-LEAR, E731, NA31 allows to test the predictions of CPT theorem for phases φ_1, φ_2 with accuracy of about 5^0.

1.2.6 Test of CPT-invariance in $K_{L,S} \to \pi l \nu$ decays.

It seems that experiments with semileptonic decays of neutral kaons at $DA\Phi NE$ allow not only to register CP-invariance violation in $K_S \to \pi e(\mu)\nu$ decays for the first time and to measure the values of asymmetry parameters $\delta_{L,S}^{e,\mu}$ (1.2.21), but also to estimate the possible of CPT-violation in these decays. The difference between charge asymmetry parameters for K_L and K_S mesons is connected with CPT-invariance violation parameter Δ (1.2.1) as well as with the $\Delta Q = \Delta S$ rule violation parameter in K^0 , $\bar{K}^0$ decays by the following expression [30]:

$$\delta_S - \delta_L = 4Re\ \Delta + 2Re(x - \bar{x}) \tag{1.2.58}$$

It is shown in section 1.2.2 that this difference seems to be obtained at $DA\Phi NE$ with the accuracy of $\sim 7 \cdot 10^{-4}$. Measurement of parameters x and $\bar{x}$ at CP-LEAR or at any other experiment by that time will allow to estimate $Re\ \Delta$ with the precision $\sim 10^{-3}$.

If it will appear that $\delta_S^{e,\mu} \neq \delta_L^{e,\mu}$, it would mean either anomally strong CPT-violation or violation of the rule $\Delta Q = \Delta S$.

1.3 Test of CPT-invariance in charged kaon decays

As it was mentioned in section 1.1, CPT-invariance demands the equalities of masses and lifetimes of K^+ and K^- mesons and an equality of some partial widths, six of which will be considered in this section.

1.3.1 Measurement of mass diffference for charged K (and π) mesons

To date the mass difference of charged kaons is measured for $2 \times 1.5 \cdot 10^6$ decays $K^\pm \to \pi^+\pi^-\pi^\pm$ [4, 37]:

$$m_{K^+} - m_{K^-} = -0.032 \pm 0.09 MeV/c^2.$$

About $5 \cdot 10^8$ "tagged" decays $K^+ \to \pi^+\pi^+\pi^-$ (and the same number of K^- decays) are expected at $DA\Phi NE$. If the effective mass resolution would be $\Delta(M_{3\pi}) \leq 10 MeV/c^2$ [38, 39], the masses of K^+ and K^- seem to be measured with the statistical accuracy of about $\leq 7 \cdot 10^{-4} MeV/c^2$. It is necessary to take into account, that using the measured K^+ and K^- effective mass difference we obtain really the value

$$(m_{K^+} - m_{K^-}) + (m_{\pi^+} - m_{\pi^-}).$$

Charged pions mass difference, which is CPT-noninvariant also, is defined now with the accuracy of $0.07 MeV/c^2$ [40]. As it is expected, about $9.6 \cdot 10^9$ charged pions per year will be produced at $DA\Phi NE$ only from kaon decays into three and more particles [41]. 10% of them will decay inside detector. The problem of measurement of $\Delta m_{\pi^+\pi^-}$ has to be considered in details.

1.3.2 More accurate definition of lifetimes of K^+ and K^-

The best accuracy in measurement of the relative difference

$$d\tau = 2(\tau^+ - \tau^-)/(\tau^+ + \tau^-) = (0.09 \pm 0.08) \cdot 10^{-3}$$

was reached in [42] using $2 \times 5 \cdot 10^7$ $K^\pm$ decays. Paying no attention to the methodical possibilities of $DA\Phi NE$ experiment, let us underline the fact that the expected number of "tagged" K^+ and K^- with all their decay modes, registred by detector, will reach $\approx 9 \cdot 10^9$ while the expected statistical error will be $\sim 1 \cdot 10^{-5}$ (rough estimate).

1.3.3 Search for CPT violation in partial widths of koans

Table 3 lists charged kaons decay channels for which the equality of partial widths for K^+ and K^- is predicted by CPT theorem. The accuracy of some of them can be improved at $DA\Phi NE$, the others can be measured for the first time. First column summarizes

K^+ , K^- decay channels "i", the second one - the number of events observed in the world ($N_{K^+} = N_{K^-}$) , the result for measured value

$$2(\Gamma_i^+ - \Gamma_i^-)/(\Gamma_i^+ + \Gamma_i^-)$$

(where $\Gamma_i^\pm$ is the rate of $K^\pm$ decaying into the channel "i") and reference to the paper with the highest statistics. In the third column the expected statistics and the expected statistical errors of the result are presented.

Table 3

Decay modes of charged kaons, which may be investigated at the $DA\Phi NE$ for search for CPT violation.

Decay mode,i	To date (PDG-90): 1) $2(\Gamma_i^+ - \Gamma_i^-)/(\Gamma_i^+ + \Gamma_i^-)$ 2) N_{K^+}, N_{K^-}	$DA\Phi NE$: 1)expected sensitivity 2)expected $N_{K^+}^{tag}$
$\mu^\pm\nu$	$(-5.4 \pm 4.1) \cdot 10^{-3}$ $2 \cdot 10^7$ [43]	$\pm 0.3 \cdot 10^{-4}$ $\approx 5.6 \cdot 10^9$
$\pi^\pm\pi^0$	$(8 \pm 12) \cdot 10^{-3}$ $2.5 \cdot 10^4$ [44]	$\pm 0.45 \cdot 10^{-4}$ $\approx 1.7 \cdot 10^9$
$\mu^\pm\pi^0\nu$	no data	$\pm 1.3 \cdot 10^{-4}$ $\approx 3 \cdot 10^8$
$e^\pm\pi^0\nu$	no data	$\pm 1.3 \cdot 10^{-4}$ $\approx 4.1 \cdot 10^8$
$e^\pm\nu$	no data	$\pm 5 \cdot 10^{-3}$ $\approx 1.4 \cdot 10^5$
$(\pi^+\pi^+\pi^-)+$ $(\pi^+\pi^0\pi^0)$	no data	$\pm 0.8 \cdot 10^{-4}$ $\approx 6.5 \cdot 10^8$

1.4 Search for T-violating effects in kaon decays.

The direct test of T invariance implicates a comarison between the direct and reversed reactions. Such check-up is practically impossible for the kaon decays. For this reason we can only investigate the effects appearing when the requirement of T invariance is not imposed on behaviour of the form factors of the amplitude under consideration. If T-invariance does not take place, the form factors of the transition into different dinamical states of final particles (which would be pure real or pure imaginary values in T invariant theory) become complex values. As a result, the possible T violating correlations between the momenta of particles or between the momenta and spin of particles turns out to be proportional to $\sin\chi$, where χ is the difference between the phases of the form factors of the amplitude. A test of T invariance becomes more complicated if the final-state particles are the stong-interacting ones. In this case the final-state interaction (even the T invariant one) also leads to appearence of nonzero phases and T-odd correlation turns out to be proportional to

$$sin(\delta_i - \delta_j + \chi_{ij})$$

where δ_i and δ_j are the phases of scattering in the channels of i and j.

Of course, these phases can be determined from the other processes, but to date the necessary accuracy (fractions of a degree) can not be achieved in the scattering experiments. In such a situation, to test T invariance, we can use only the decay of kaons, where the final-state interaction between the particles is very small or it can be calculated within the framework of usual perturbation theory. These are

$$K \to \pi\mu\nu \quad , \quad K \to \mu\nu\gamma \qquad (1.4.1)$$

$$K \to \pi l\nu\gamma \quad (l = e, \mu) \qquad (1.4.2)$$

For the decays (1.4.1), the T-odd correlations

$$\vec{\sigma}_\mu[\vec{P}_\mu \times \vec{P}_\pi] \quad [45] \qquad (1.4.3)$$

and

$$\vec{\sigma}_\mu[\vec{P}_\mu \times \vec{P}_\gamma] \quad [46] \qquad (1.4.4)$$

lead to appearence of muon transverse polarization. But the measurement of its values at a Φ-factory is connected with some difficulties because of the necessity to use the polarimeters for muon spin direction measurement.

For the decay (1.4.2), the situation is more easy, because it would be enough to measure the T-odd correlation

$$\vec{P}_\pi[\vec{P}_e \times \vec{P}_\gamma] \quad [46] \tag{1.4.5}$$

Let's consider some details necessary for understanding of the problem.

1.4.1 Decay $K \to \mu\nu\pi$

The hadron part of matrix element of this decay contains two form factors f_+ and f_-:

$$< \pi(p_\pi) \mid j_\tau^W \mid K(p_K) >= f_+(q^2)(p_K + p_\pi)_\tau + f_-(q^2)(p_K - p_\pi)_\tau, \tag{1.4.6}$$

where $q^2 = (p_K - p_\pi)^2$.

Appearence of the T-odd correlation (6.3) is possible if the parameter

$$\xi(q^2) \equiv \frac{f_-(q^2)}{f_+(q^2)}$$

has the imaginary part. This part arises not only due to T violation but due to T invariant electromagnetic interaction of final particles also. As it was noted in ref.[47], the transversal polarisation of muons arising due to final-state interaction in the decays $K^0 \to \pi^-\mu^+\nu$ and $\bar{K}^0 \to \pi^+\mu^-\nu$ must be of the same sign for both decays. The true T-violation would lead to polarization $P_{\mu\perp}$ of the opposite signs in these decays.

Dependence of the differential probability of the decay on $P_{\mu\perp}$ and its magnitude as a function of $Im\xi$ is calculated in refs. [48, 49, 47]. As for $Im\xi$ itself, the contribution of the electromagnetic final-state interaction to its value in the case of the decay $K_L^0 \to \pi^\pm\mu^\mp\nu$ has the value [50]:

$$Im\ \xi^{e.-m.} \simeq 0.008. \tag{1.4.7}$$

Experimental value:

$$Im\ \xi^{exp} = 0.009 \pm 0.030 \ [51] \tag{1.4.8}$$

is obtained on the statistics of $1.2 \cdot 10^7$ events. So, the experimental accuracy is not enough now to draw a definite conclusion on the presence of true T-violation in the $K_L \to \pi^\pm \mu^\mp \nu$ decays.

For the decay $K^\pm \to \pi^0 \mu^\pm \nu$ experimentally [4] :

$$Im\xi^{exp} = -0.017 \pm 0.025$$

The latter result is obtained on the statistics of $\sim 1 \cdot 10^7$ events.

The mimic T-noninvariant part of $Im\xi$ arising due to final-state electromagnetic interaction in this case must be very small. It can lead to $P_{\mu\perp} \sim 10^{-6}$ [52]. This magnitude is far from possibility of measurement now. Therefore, an observation of larger value of $P_{\mu\perp}$ would mean an existance of true T-violation.

To have an idea, what value of $Im\ \xi$ would be possible in CPT invariant, but CP- and T-noninvariant theory, let's refer to the estimate of Cheng [53] obtined for the Weinberg model of CP violation

$$Im\xi^{m.w.} < 5 \cdot 10^{-4}$$

Some remarks must be done on the value of $Re\ \xi$ entering into the theoretical formula for $P_{\mu\perp}$ (see, for example ref. [49]). To date this value is very uncertain.

Experimentally [4]

$$Re\xi(0) = \begin{array}{ll} -0.35 \pm 0.15 & \text{from } K^+_{\mu_3} \\ -0.11 \pm 0.09 & \text{from } K^0_{\mu_3} \end{array}$$

For this reason one of the tasks of $DA\Phi NE$ experiments must be more accurate definition of $Re\xi$. Let's note that modern chiral theory predicts for $\xi(0)$ the value

$$\xi(0) \simeq F_K/F_\pi - 1 - (p_K^2 - p_\pi^2)/m_\rho^2 - (F_K/F_\pi - 1)^2 \qquad [54]$$

that is

$$\xi(0) \simeq -0.21 \quad \text{for} \quad F_K/F_\pi = 1.22 \pm 0.01 \qquad [55]$$

In future experiments at $DA\Phi NE$, it is expected that a number of $K_{\mu 3}$ decays will exeed the existing world statistics. Namely, at $DA\Phi NE$

$$N^{tag}(K^\pm \to \pi\mu\nu) \approx 2.9 \cdot 10^8; \qquad N^{tag}(K_L^0 \to \pi\mu\nu) \approx 3 \cdot 10^8.$$

Therefore it would be possible to improve considerably our knowlege on the form factors f_+ and f_-. Though the measurement of $P_{\mu\perp}$ at $DA\Phi NE$ looks as a very difficult problem, it would be possible to estimate $\mid Im\ \xi \mid$ using the dependence of Dalitz-plot density on $Re\xi$ and $\mid \xi \mid^2$. This method was used early in ref.[56]. To recommend this way for $DA\Phi NE$ experimentators some additional analysis is necessary.

1.4.2 Measurement of T-odd momenta correlation in the decays $K^{\pm} \rightarrow \pi l \nu \gamma$

The experimental data on T-odd correlation (1.4.4) in the decay (1.4.2) are very poor. This correlation was searched in $K^{\pm} \rightarrow e\pi^0\nu\gamma$ decays [57, 58] on samples of 16 and 192 events respectively. The table 4 shows the types of decays where a search for T-odd correlation (1.4.4) would give an unique information.

Table 4

Kaon decays, in which T-odd correlations (1.4.5) can be examined at $DA\Phi NE$.

Decay mode	1) Numb. of events in experiment with highest statistics 2) results	1) Number of "tagged" events at $DA\Phi NE$ 2) expected precision
$K^{\pm} \rightarrow e\pi^0\nu\gamma$	192, [58] (0.03 ± 0.08)	$2.4 \cdot 10^6 (E^*_\gamma > 10 MeV)$ $\pm 3 \cdot 10^{-4}$
$K^{\pm} \rightarrow \mu\pi^0\nu\gamma$	no data	$\sim 0.24 \cdot 10^6 (E^*_\gamma > 10 MeV)$ $\pm 1 \cdot 10^{-3}$
$K_L \rightarrow \pi^{\pm} e^{\mp}\nu\gamma$	no data	$(19 \pm 12) \cdot 10^6 (E^*_\gamma > 15 MeV)$ $\pm 1.5 \cdot 10^{-4}$
$K_L \rightarrow \pi^{\pm}\mu^{\pm}\nu\gamma$	no data	$\sim 7 \cdot 10^6 (E^*_\gamma > 15 MeV)$ $\pm 2.5 \cdot 10^{-4}$

Unfortunately, we have no information on the theoretical works where the question on possible value of the correlation (6.4) was discussed. It is evident that a search for the T-odd correlation (1.4.4) at $DA\Phi NE$ would lead to necessary theoretical investigations.

1.4.3 Search for asymmetry in the proper-time difference distributions resulting from decays of the system $\{K^0, \bar{K}^0\}$

T-noninvariance leads to difference between the direct and reversed amplitudes, in particular, between the amplitudes

$$< K^0 \mid H^W \mid \bar{K}^0 > \quad \text{and} \quad < \bar{K}^0 \mid H^W_T \mid K^0 >$$

This difference, in its turn, leads to the proper-time difference asymmetry in distribution of semileptonic decays of the system $\{K^0, \bar{K}^0\}$ [71] formed in the decay $\phi \to K^0\bar{K}^0$.

$$A_T(\Delta t) = \frac{N(\bar{K}^0 \to K^0; \Delta t) - N(K^0 \to \bar{K}^0; \Delta t)}{N(\bar{K}^0 \to K^0; \Delta t) + N(K^0 \to \bar{K}^0; \Delta t)} = 4Re\epsilon - 4Re\ (y_l)\ ^4$$

where $N(\bar{K}^0 \to K^0; \Delta t)$ is the number of events where the first kaon is detected as K^0 observing its decay into $\pi^- l^+ \nu$ and the other one also detected as K^0 originated from transition $< K^0 \mid H^W \mid \bar{K}^0 >$ at the proper time interval Δt after the first kaon decay. The details related to the theoretical formulae for different $N(\Delta t)$ may be found in ref. [30] and in the paper of Patera V. and Pugliese A.[73].

CPT invariance predicts:

$$A_T(\Delta t) = 4Re\epsilon$$

Therefore a distinction of A_T from the above value could be considered as indication of a direct CPT violation in the decay amplitude. CPT violation in the mass-matrix leads to other asymmetry:

$$A_{CPT}(\Delta t) = \frac{N^-(\Delta t) - N^+(\Delta t)}{N^-(\Delta t) + N^+(\Delta t)}$$

where

$$N^-(\Delta t) = N(\bar{K}^0 \to \bar{K}^0; \Delta t > 0) + N(K^0 \to K^0; \Delta t < 0);$$
$$N^+(\Delta t) = N(\bar{K}^0 \to \bar{K}^0; \Delta t < 0) + N(K^0 \to K^0; \Delta t > 0);$$

The details concerning such an asymmetry may be found in ref's [30, 73]. To date the above mentioned asymmetries have not been investigated experimentally.

2 Test of C invariance in $\eta(\pi^0)$ and ϕ decays

2.1 General remarks

C invariance is violated by the weak interaction with a strength of P violation. This follows from the fact that CP invariance is approximately conserved[5]. For this reason, only the observation of the C-odd effects with a strength considerably larger than $G_F E^2_{eff}$ could be treated as the evidence for new physics in the elementary particle interaction.

In view of these circumstances, it is natural to look for C violation in the processes where P-parity conserves, in particular, in the strong and electromagnetic decays of the neutral mesons:

$$\pi^0 \to 3\gamma$$

[4] For the definition of y_l see ref.[72]

[5] See also ref.[76]

$$\eta^0 \rightarrow 3\gamma$$
$$\phi \rightarrow \omega(\rho^0) + \gamma$$
$$\omega \rightarrow \rho^0 + \gamma$$
$$\eta \rightarrow \pi^0 + l^+ + l^-$$
$$\eta \rightarrow \pi^+ + \pi^- + \pi^0$$
$$\eta \rightarrow \pi^+ + \pi^- + \gamma$$
$$\phi \rightarrow \pi^+ + \pi^- + \gamma$$

The first four processes are forbidden by C invariance. This invariance forbids the fifth one occuring through one-photon intermediate state but allows it if it goes through the two-photon exchange.

At the $DA\Phi NE$ factory, at the luminosity $\mathcal{L} = 5{\cdot}10^{32}s^{-1}cm^{-2}$ the number of ϕ mesons is expected to be

$$N_\phi = 2.5 \cdot 10^{10}/year$$

and the number of η mesons

$$N_\eta = 3.2 \cdot 10^8/year$$

The latter are produced in the decay $\phi \rightarrow \gamma\eta$ with $B.R(1.28 \pm 0.06)\%$[4].

Before the discussion of the experiments with ϕ and η mesons it is useful to compare the possibilities of the Φ factory with the opportunities of other accelerator centers. In 1991, the η factory has already begun to work in Sacle at the synchrotron Saturn. They have about 10^{12} of η mesons per year, obtained in the reaction

$$p + d \rightarrow He^3 + \eta$$

near threshold of production[59, 60].

The experiments with η mesons are planned also at the setup WASA (accelerator CELSIUS) in Sweden and at the ITEP detector.

The main parameters of these detectors are shown in the Table 5.

Table 5

Significant features of the operating and planned η factories

Accelerator Start of work	Experimental set-up	Number of η per year	Accep- tance of apparatus	Number of observed η per year
Saturn (France) 1991	Spectrometer SPES2 for measurement of He^3 momentum 2-arms spectrometer for registration η-meson decay product. Identification of muons.	10^{12}	$\sim 3\%$	$\sim 3 \cdot 10^{10}$
CELSIUS (Sweden) 1995	WASA. Superconducting magnetic solenoid, e, γ calorimeter, μ identification. Forward γ detector.	$3 \cdot 10^9$	$\sim 75\%$	$\sim 2.2 \cdot 10^9$
ITEP (Russia) 1996	Warm or superconducting solenoid, detection and identification of e, μ, γ.	$(0.3 - 3) \cdot 10^9$	$\sim 90\%$	$\sim (0.27 \div 2.7) \cdot 10^9$
$DA\Phi NE$ (Italy) 1995	4π magnetic spectrometer. e, γ, μ detection. Measurement of their energies	$3.2 \cdot 10^8$ (at the luminosity of $5 \cdot 10^{32}$ $s^{-1} cm^{-2}$	$\sim 98\%$	$\sim 3.2 \cdot 10^8$

In spite of smaller luminosity than at the other η factories the $DA\Phi NE$ factory has an advantage in study of many-particle decays of η mesons. First of all, it follows from very low hadronic background at e^+e^- machine. Besides, the η factory in Sacle is intended mainly for investigation of the rare two (or three)-body decays of η with two of particles being charged. For these reasons the $DA\Phi NE$ may compete with other η factories in study of the neutral channels of η decay and η decays going to many-particle state (with $n > 3$) at the level $\geq 2 \cdot 10^{-8}$.

And, of course, a Φ factory represents the unique opportunity for search for C violating effects in the decays of ϕ meson.

Below we consider the processes allowing to search for C violation in ϕ and η decays.

2.2 C violating decays of $\eta(\pi^0)$ and ϕ mesons

2.2.1 Decays $\eta \to 3\gamma$ and $\pi^0 \to 3\gamma$

C noninvariance can lead to 3γ decay of pseudoscalar meson φ with the amplitude

$$A(\varphi \to 3\gamma) = \frac{eg_{3\gamma}}{m_\pi^2} \varphi F_{\mu\nu} F_{\mu\tau} \tilde{F}_{\nu\tau}$$

where $\varphi = \eta$ or π^0. Then, for the rate of such a transition we obtain the estimate

$$\frac{Rate(\varphi \to 3\gamma)}{Rate(\varphi \to 2\gamma)} \sim \frac{e^2}{32\pi^2} (\frac{m_\varphi}{m_\pi})^2 (\frac{\tilde{k}}{m_\pi})^2 \frac{g_{3\gamma}^2}{g_{2\gamma}^2}$$

where $g_{2\gamma}$ is the coupling constant of $\varphi\gamma\gamma$ vertex

$$A(\varphi \to 2\gamma) = g_{2\gamma} \varphi F_{\mu\nu} \tilde{F}_{\mu\nu}$$

and $\tilde{k}$ is the average momentum of the photon.

In the case of the π^0 decay this estimate gives

$$R_\pi = \frac{Rate(\pi^0 \to 3\gamma)}{Rate(\pi^0 \to 2\gamma)} \sim 3 \cdot 10^{-5} g_{\pi 3\gamma}^2 / g_{\pi 2\gamma}^2$$

¿From the experimental value $R_\pi^{exp} < 3.1 \cdot 10^{-8}$ one can obtain

$$| g_{\pi 3\gamma} / g_{\pi 2\gamma} | \leq 0.03$$

For the η meson, the estimate gives

$$R_\eta = \frac{Rate(\eta \to 3\gamma)}{Rate(\eta \to 2\gamma)} \sim 8 \cdot 10^{-3} g_{\eta 3\gamma}^2 / g_{\eta 2\gamma}^2$$

Experimentally, $R_\eta^{exp} < 1.3 \cdot 10^{-4}$ and consequently

$$| g_{\eta 3\gamma}/g_{\eta 2\gamma} | \leq 0.4$$

So, the existing limitations on $g_{\varphi 3\gamma}$ do not look to be impessive.

To date $B.R.(\eta \to 3\gamma) < 5 \cdot 10^{-4}$ and this limit may be lowered by three orders at the $DA\Phi NE$.

2.2.2 The decay $\phi \to \omega(\rho^0) + \gamma$

If C invariance violates, the general form of matrix element of transition

$$A(1^-) \to B(1^-) + \gamma$$

is the following [69]:

$$A(A(p) \to B(p') + \gamma(q)) =$$
$$ie\epsilon_\mu^\lambda(p')\epsilon_\nu^{\lambda'}(p) \cdot \epsilon_\tau^{\lambda''}(q) \cdot \{a[(p+p')_\tau q_\mu q_\nu - \tfrac{m_A^2 - m_B^2}{2}(\delta_{\tau\nu}q_\mu + \delta_{\tau\mu}q_\nu)] +$$
$$b(\delta_{\tau\nu}q_\mu - \delta_{\tau\mu}q_\nu)\}$$

The rate of the above reaction is given by

$$Rate(A \to B + \gamma) = \frac{\alpha(m_A^2 + m_B^2)(m_A^2 - m_B^2)^3}{24 m_A^5 m_B^2} g^2,$$

where

$$g^2 = \frac{1}{4}(m_A^2 - m_B^2)^2 \mid a \mid^2 + \mid b \mid^2 - \frac{1}{2}(m_A^2 + m_B^2)^{-1}(m_A^2 - m_B^2)^2(a^*b + ab^*).$$

Experimentally, $\mathrm{Br}(\phi \to \omega\gamma) < 5\%$ and $\mathrm{Br}(\phi \to \rho\gamma) < 2\%$ [70].

Then $g_{\phi\omega\gamma} < 2$ and $g_{\phi\rho\gamma} < 1.1$. But these limits were obtained in experiment where the number of observed K^+K^- pairs from ϕ decays was about 250. At the $DA\phi NE$ the number of K^+K^- pairs is expected to be $1.2 \cdot 10^{10}$. So, the limits on $g_{\phi\omega\gamma}$ and $g_{\phi\rho\gamma}$ may be decreased by a few orders.

2.2.3 Decays $\eta \to \pi^0 e^+ e^-$ and $\eta \to \pi^0 \mu^+ \mu^-$

These decays can occur in second order of the electromagnetic interaction without C violation (see the diagram in Fig.2). In this case, according to ref.[63] :

$$\Gamma(\eta \to \pi^0 e^+ e^-)/\Gamma(\eta \to \pi^0 \gamma\gamma) = (2.1 \pm 0.5)10^{-8}$$

or, using for $\Gamma(\eta \to \pi^0\gamma\gamma)$ the experimental value from [4] one should get :

$$\Gamma(\eta \to \pi^0 e^+ e^-)/\Gamma(\eta \to all) \approx (1.5 \pm 0.4)10^{-11}$$

But according to Cheng [64] this ratio may be considerably larger. Using VDM he obtained

$$\Gamma(\eta \to \pi^0 e^+ e^-) \approx 1.3 \cdot 10^{-5} eV$$

At $\Gamma^{exp}(\eta) = 1.19 \cdot 10^3 eV$ it leads to the rate

$$\Gamma(\eta \to \pi^0 e^+ e^-)/\Gamma(\eta \to all) \approx 1.1 \cdot 10^{-8}$$

The observation of such a decay at the considerably larger level than these estimates could be treated as existance of the one-photon exchange between e^+e^- pair and hadrons that would mean in its turn that C invariance is violated in this process.

Experimentally $B.R.(\eta \to \pi^0 e^+ e^-) < 4 \cdot 10^{-5}$ and $B.R.(\eta \to \pi^0 \mu^+ \mu^-) < 5 \cdot 10^{-6}$.

At the $DA\Phi NE$ these limits may be lowered by one order for decay with creation of muon pair and by two orders for decay $\eta \to \pi^0 e^+ e^-$.

2.2.4 $\eta \to \pi^+\pi^-\pi^0$. Charge asymmetry.

C violation leads to appearance of four kinds of asymmetry in distribution of events in the Dalitz plot [65, 66, 67].

a) The left-right asymmetry

$$A_{l-r} = \frac{N_+ - N_-}{N_+ + N_-}$$

where N_+ is the number of events with $T_{\pi^+} > T_{\pi^-}$, N - is the number of events with $T_{\pi^-} > T_{\pi^+}$ and T is the kinetic energy of pion.

b) The sextant asymmetry

$$A_S = \frac{N_1 + N_3 + N_5 - N_2 - N_4 - N_6}{N_1 + N_2 + N_3 + N_4 + N_5 + N_6}$$

where N_i is the number of events observed in i-th sector of the Dalitz plot. This asymmetry is sensitive to C violation in the system of 3π with isotopic spin $I = 0$[66].

c) The quadrant asymmetry

$$A_q = \frac{N_1 + N_3 - N_2 - N_4}{N_1 + N_2 + N_3 + N_4}$$

appears if C violation takes place in the system of 3π with isospin $I = 2$.

d) Evidence for C violation in $\eta \to \pi^+\pi^-\pi^0$ decay would follow also from the difference of the coefficient C from zero in the formula

$$\mid M(x,y) \mid^2 \sim (1 + ay + by^2 + cx + dx^2 + exy)$$

approximating the distribution of events in the Dalitz plot where $x = \sqrt{3}(T_{\pi^+} - T_{\pi^-})/Q$, $y = 3T_{\pi^0}/Q - 1$ and $Q = T_{\pi^+} + T_{\pi^-} + T_{\pi^0}$.

Existing experimental data on asymmetries are shown in Table 6.

2.2.5 $\eta \to \pi^+\pi^-\gamma$

To illustrate the situation with possible C noninvariant effects in this decay let's keep the lowest possible angular-momentum states. The system of two pions can be either in the $I = 1p$ state, produced by C conserving interaction, or in the $I = 0d$ state produced by C-violating interaction with the amplitudes

$$\sqrt{2}a(\vec{p}\vec{H}) \qquad \text{and} \qquad i\sqrt{3}b(\vec{p}\vec{H})(\vec{p}\vec{K})$$

respectively. The rate of $\eta \to \pi^+\pi^-\gamma$ decay is given by

$$Rate(\eta \to \pi^+\pi^-\gamma) = \vec{k}^2[p^2 - (\vec{p}\vec{n})^2] \cdot [| a |^2 + | b |^2 (\vec{p}\vec{k})^2 + i(a^*b - b^*a)(\vec{p}\vec{k})]d\rho[69]$$

where $\vec{k}$ is the photon momentum, $\vec{n} = \vec{k}/ | \vec{k} |$, $\vec{p} = \vec{p}_{\pi^+} - \vec{p}_{\pi^-}$ and $d\rho$ - is the invariant phase space. The relative phase between a and b is $(\delta_p - \delta_d)$ or $(\pi + \delta_p - \delta_d)$.

It follows from the above formula that C violation leads to asymmetry in distribution of events with $E_{\pi^+} > E_{\pi^-}$ in the Dalitz plot:

$$A_{l-r} = \frac{N_+ - N_-}{N_+ + N_-}$$

where N_+ is the number of events with $E_{\pi^+} > E_{\pi^-}$ and N_- is the number of events with $E_{\pi^-} > E_{\pi^+}$.

The asymmetry A_{l-r} was measured in three experiments. One of them [68] gave the nonzero value

$$A_{l-r} = (1.2 \pm 0.6)10^{-2}$$

The average value of A_{l-r} following from all data is in worse agreement with C-conservation because of

$$A_{l-r} = (0.9 \pm 0.4)10^{-2}[4].$$

At the $DA\Phi NE$, it is possible to have the statistics of $7 \cdot 10^6$ $\eta \to \pi^+\pi^-\gamma$ decays per year, which is by two orders greater than the world statistics.

It will allow to solve the question on C violation in this decay at the level $\sim 10^{-3}$.

Besides an appearance of charge asymmetry in the decay $\eta \to \pi^+\pi^-\gamma$ the C violation makes possible the decay

$$\eta \to \pi^0 + \pi^0 + \gamma$$

(forbidden by C invariance) with the rate

$$Rate(\eta \to \pi^0\pi^0\gamma) = \vec{k}^2(\vec{p}\vec{n})^2 | b |^2 [p^2 - (\vec{p}\vec{n})^2]d\rho$$

where $p = p_{\pi_2} - p_{\pi_1}$.

Experimental data on this decay are absent.

2.2.6 $\phi \to \pi^+\pi^-\gamma$

Due to C violation the charge asymmetry appears in the π^+ and π^- distribution

$$dN(E_{\pi^+} = E_1, E_{\pi^-} = E_2) \neq dN(E_{\pi^+} = E_2, E_{\pi^-} = E_1)$$

Keeping only the lowest possible angular-momentum states one could find that in addition to $I = 0s$-state of (2π) system produced by C-even interaction the state $I = 1$ p may appear if C violates. The amplitudes for these states are respectively

$$\sqrt{3}a\vec{s}\vec{E} \qquad \text{and} \qquad i\sqrt{2}b\vec{s}[\vec{p}\vec{H}]$$

where $p = p_{\pi^+} - p_{\pi^-}$ and the rate is

$$\vec{k}^2\{|\,a\,|^2 + \frac{1}{2}\,|\,b\,|^2\,[p^2 + (\vec{p}\cdot\vec{n})^2] - i(a^*b - ab^*)(\vec{p}\vec{n})\}d\rho$$

where $\vec{k}$ is the photon momentum, $\vec{n} = \vec{k}/\,|\,\vec{k}\,|$. The relative phase between a and b is given by $(\delta_s - \delta_p)$ or $(\delta_s - \delta_p + \pi)$.

2.3 Summary on search for C violation.

The summary of the opportunities of styding the C-violating effects in η and ϕ decays are presented in the table 6.

Table 6

The decays where a progress can be achieved in search for C violation.

Decay mode	To date [4]	Limits that can be achieved at $DA\Phi NE$
$\eta \to 3\gamma$	Br$< 5 \cdot 10^{-4}$	$< 1.4 \cdot 10^{-8}$
$\eta \to \omega\gamma$	Br$< 5 \cdot 10^{-2}$	$< 10^{-9}$
$\phi \to \rho\gamma$	Br$< 2 \cdot 10^{-2}$	$< 10^{-9}$
$\phi \to \pi^+\pi^-\gamma$	Br$< 7 \cdot 10^{-3}$	$< 10^{-9}$
$\eta \to \pi^0 e^+ e^-$	Br$< 4 \cdot 10^{-5}$	$< 1.4 \cdot 10^{-8}$
$\eta \to \pi^0 \mu^+ \mu^-$	Br$< 5 \cdot 10^{-6}$	$< 1.4 \cdot 10^{-8}$
$\eta \to \pi^0\pi^+\pi^-$ Asymmetry in Dalitz plot	$A_{l-r} = (0.09 \pm 0.17)10^{-2}$ $A_S = (0.18 \pm 0.16)10^{-2}$ $A_q = (0.17 \pm 0.17)10^{-2}$	Limits may be lowered by one order
$\eta \to \pi^+\pi^-\gamma$	$A_{l-r} = (0.9 \pm 0.4)10^{-2}$	— " —

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3 Appendix

The Bell-Steinberger (B-S) relation is of the form:

$$(i(m_L - m_S) + \tfrac{1}{2}(\Gamma_L + \Gamma_S)) < K_S \mid K_L >= \sum_f A^*(K_S \to f)A(K_L \to f) \qquad \text{(A.1)}$$

where f numerates the states $(2\pi, I = 0), (2\pi, I = 2), (\pi l \nu), (3\pi)$ etc.
In the linear approximation in ε and Δ

$$< K_S \mid K_L >= \varepsilon_L + \varepsilon_S^* = 2Re\ \varepsilon - 2iIm\ \Delta \qquad \text{(A.2)}$$

The separate $(2\pi, I = 0)$ and $(2\pi, I = 2)$ contributions to r.h.s. of (A.1) are

$$A^*(K_S \to 2\pi, I = 0)A(K_L \to 2\pi, I = 0) = \Gamma_S \varepsilon_0 \left[1 - 2 \mid \omega \mid^2\right],$$

$$A^*(K_S \to 2\pi, I = 2)A(K_L \to 2\pi, I = 2) = \Gamma_S 2\omega^* \varepsilon_2 (1 - 2 \mid \omega \mid^2),$$

where $\varepsilon_0 = \varepsilon - \Delta + a$ and $\varepsilon_2 = \varepsilon' + (\varepsilon - \Delta)\omega$.
The leptonic contributions are

$$< l^+ \nu \pi^- \mid K_L >< l^+ \nu \pi^- \mid K_S >^* = \Gamma(K_L \to l^+ \nu \pi^-)(1 + \varepsilon_S^* + x_l^*)(1 + \varepsilon_L^* - x_l^*)^{-1}$$

$$< l^- \bar{\nu} \pi^+ \mid K_L >< l^- \bar{\nu} \pi^+ \mid K_S >^* = -\Gamma(K_L \to l^- \bar{\nu} \pi^+)(1 - \varepsilon_S^* + \bar{x}_l)(1 - \varepsilon_L^* - \bar{x}_l)^{-1}$$

where $\varepsilon_S = \varepsilon + \Delta$ and $\varepsilon_L = \varepsilon - \Delta$.
In derivation of the leptonic contributions we used the notations

$$< l^+ \nu \pi^- \mid K^0 >= f_l, \qquad < l^- \bar{\nu} \pi^+ \mid K^0 >= \bar{x}_l^* \bar{f}_l^*$$

$$< l^+ \nu \pi^- \mid \bar{K}^0 >= x_l f_l, \qquad < l^- \bar{\nu} \pi^+ \mid \bar{K}^0 >= \bar{f}_l^*$$

Then the B-S relation may be written in the form

$$(2i(m_L - m_S) + (\Gamma_L + \Gamma_S))(Re\ \varepsilon - i\ Im\ \Delta) = \Gamma_S(1 - 2 \mid \omega \mid^2$$
$$)æ + (\delta_L + 2\Delta^* + x^* - \bar{x})\Gamma(K_L \to \pi l \nu) + \Gamma_S \alpha_{3\pi}$$

where æ and $\alpha_{3\pi}$ are defined by Eq. (1.2.35).

Considering the real and imaginary parts of this equation and excluding $Re\ \varepsilon$ we obtain the Eq. (1.2.33) where the factor $\frac{1}{2}$ before curly brackets was placed instead of the exact factor

$$(1 + \Gamma_L/\Gamma_S + \frac{(\Gamma_S - \Gamma_L)^2}{\Gamma_S(\Gamma_S + \Gamma_L)} \tan^2 \Phi_{sw} - 2\frac{\Gamma_L(\pi l \nu)}{\Gamma_S})^{-1}.$$