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A. Bramon, A. Grau, G. Pancheri:

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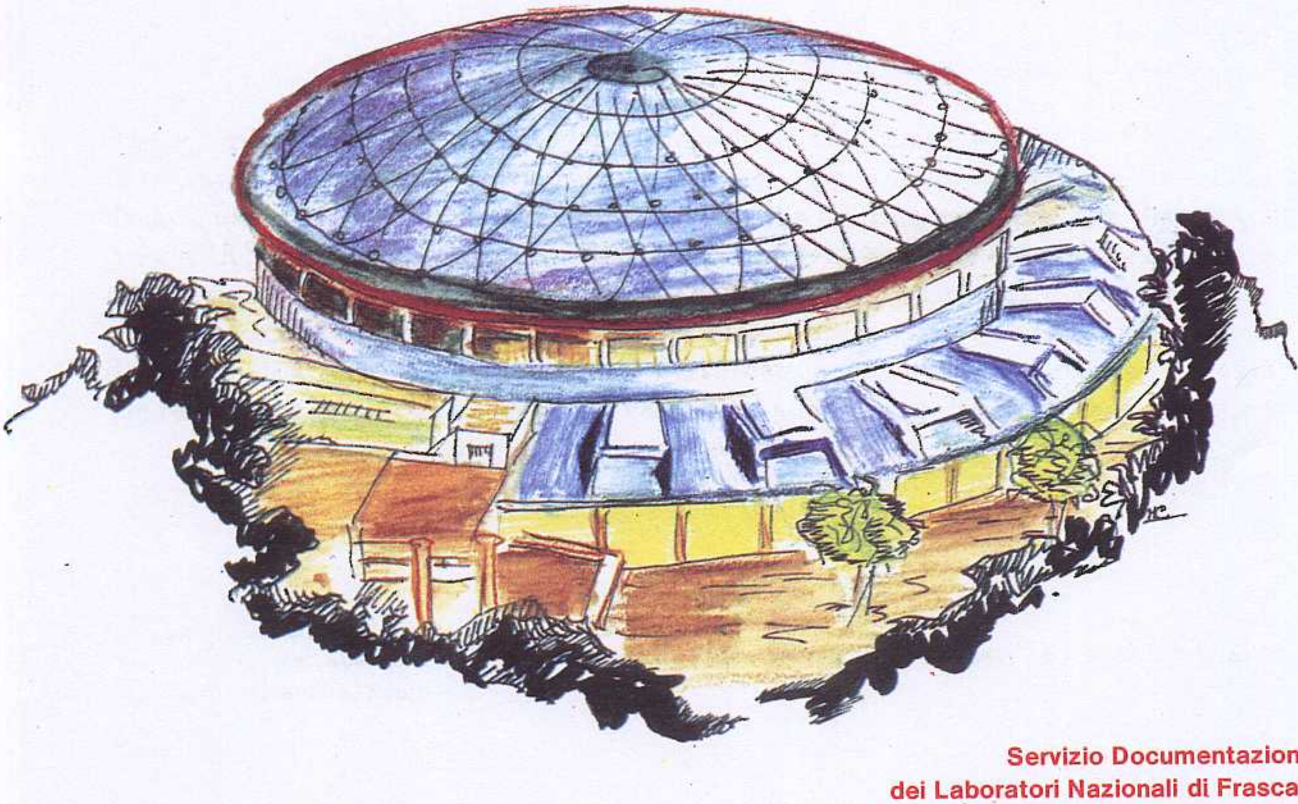
Contribution to the DAΦNE Physics Handbook



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VECTOR MESON AND CHIRAL LOOP CONTRIBUTIONS TO THE DECAYS $V^0 \to P^0 P^0 \gamma$

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Abstract

The contribution of intermediate vector mesons to the decays $V^0 \to P^0 P^0 \gamma$ is calculated and compared with previous estimates and approximate evaluations. For some decays, like ϕ or $\omega \to \pi^0 \pi^0 \gamma$ our results update the existing numbers. For the decays $\phi \to K^0 \bar{K}^0 \gamma$ and $\phi, \omega \to \pi^0 \eta \gamma$, we find values different from the ones in the literature and discuss the origin of the discrepancy. Discrepancies with approximate evaluations from product of branching ratios of vector meson decay chains are also discussed in detail. In an attempt to extend Chiral Perturbation Theory to this radiative decays the contributions of (chiral) loops are calculated and found to be important. Relation to the VMD contributions and global results are presented.

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1 Introduction

With a high-luminosity, low-energy e^+e^--machine like Daϕne, rare decays of vector mesons with branching ratios even smaller than 10^{-6} can be studied. More precisely, the Frascati Φ-Factory is expected to provide $\sim 10^{10}\phi$-decays per year thus allowing for analyses of final states such as $\pi^0\pi^0\gamma$, $\pi^0\eta\gamma$ and, in principle, $K^0\bar{K}^0\gamma$. Their dynamics includes aspects of low-energy hadron physics and, particularly, the effects of well-known vector mesons and of largely unknown scalar mesons. Among the latter, one has the $f_0(975)$ and $a_0(980)$ mesons, whose rather controversial nature could be hopefully clarified [1]. Indeed, their effects will probably manifest themselves in radiative ϕ-decays, superimposed to the well understood contributions of vector mesons. The first purpose of this contribution is to provide a rather exhaustive analysis of these contributions. Previous studies [2, 3, 4, 5] are based on old data and turn out to be rather incomplete or not free of contradictions.

Radiative decays of low-mass vector-mesons into a single photon and a pair of neutral pseudoscalars have not been detected up-to-now. Only upper limits for three branching ratios of this type of processes have been stablished (see PDG [6],[7]), namely, BR($\omega \to \pi^0\pi^0\gamma$) $\leq 4 \times 10^{-4}$, BR($\phi \to \pi^0\pi^0\gamma$) $\leq 10^{-3}$ and BR($\phi \to \pi^0\eta\gamma$) $< 2.5 \times 10^{-3}$. This contrasts with the situation concerning similar radiative decays involving a charged pseudoscalar pair or only one neutral pseudoscalar in the final state. In the first case (particularly, in the observed $\rho^0 \to \pi^+\pi^-\gamma$ process) one has to deal with a bremsstrahlung photon, thus obtaining much larger branching ratios but reducing the physical interest of such an emission. In the second one, whose best example is the long-ago detected $\omega \to \pi^0\gamma$ decay, the two-particle phase-space enlarges the rate and the absence of charged particles makes the dynamics more interesting. Indeed, many succesful theoretical ideas such as Vector-Meson-Dominance (VMD), quark model, (anomalous) effective lagrangians, etc. have been originated or tested in this kind of $V \to P\gamma$ transitions, where V and P stand for vector and pseudoscalar mesons, respectively.

In the following section, we start by presenting the amplitude for the general process $V^0 \to P^0P^0\gamma$ within the framework of Vector Meson Dominance (VMD). In section 3, the relative decay widths and branching ratios are obtained numerically and compared with previous studies and approximate evaluations from known branching ratios of vector meson decay chains. Eventual discrepancies are discussed in detail for the specific case $\phi \to \pi^0\eta\gamma$ in section 4.

In section 5, an attempt to extend Chiral Perturbation Theory (χPT) to radiative vector-meson decays into two neutral pseudoscalars, $V^0 \to P^0P^0\gamma$, is presented. This constitutes the second purpose of this contribution. The effects of (chiral) loops are found to be important in some cases. Unambiguous predictions are given for $\phi \to \pi^0\pi^0\gamma$, $\pi^0\eta\gamma$, $\rho \to \pi^0\pi^0\gamma$ and other processes of interest. The relation of χPT

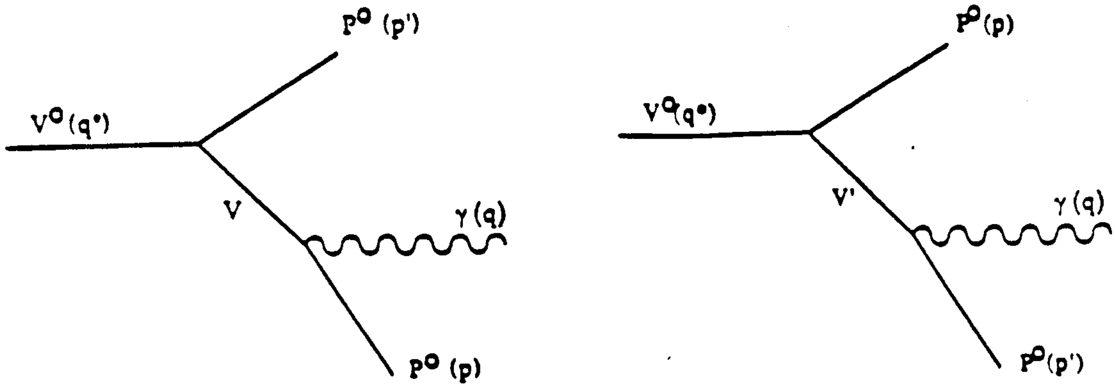

Figure 1: Feynman diagrams for the process $V^0 \to P^0 P^0 \gamma$ from Vector Meson contributions

amplitudes to the VMD ones is discussed in terms of the (VM-)resonance saturation of counterterms in the chiral lagrangian. Global results are presented in section 6.

2 Lagrangians and Amplitudes in VMD Approach

In conventional VMD models, the amplitude for the process $V^0 \to P^0 P^0 \gamma$ is obtained by calculating the Feynman diagrams shown in Fig.1. All the couplings of our amplitudes can be deduced from the two well-known lagrangians obeying the SU(3)-symmetry dictates

$$\begin{aligned} L(V\gamma) &= -2egf^2 A^\mu \; tr(QV_\mu) \\ L(VVP) &= \frac{G}{\sqrt{2}} \epsilon^{\mu\nu\alpha\beta} \; tr(\partial_\mu V_\nu \partial_\alpha V_\beta P) \end{aligned} \tag{1}$$

where A^μ is the photon field, Q is the quark charge matrix,

$$Q = \begin{pmatrix} 2/3 & & \\ & -1/3 & \\ & & -1/3 \end{pmatrix} \tag{2}$$

V stands for the vector meson nonet, i.e.

$$V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}, \tag{3}$$

and $G = 3\sqrt{2}g^2/(4\pi^2 f)$ is the $\rho^0\omega\pi^0$ coupling constant. The latter played an essential role in conventional VMD results, together with the $V\gamma$ coupling constant, f_V. Both of them have been written in terms of g and $f_\pi \equiv f$ to make contact with modern Chiral and effective lagrangian schemes. From the first eq.(1) one immediately obtains the Vγ-couplings

$$f_\rho = f_\omega/3 = -\sqrt{2}f_\phi/3 = \sqrt{2}g = 5.9 \tag{4}$$

in good agreement with the data [6]. With this value of g = 4.2 and the pion decay constant f = 132 MeV, eqs.(1) satisfactorily describe the set of data for $V \to P\gamma$. For instance, one predicts $\Gamma(\omega \to \pi^0\gamma)$=0.76 MeV quite close to the experimental value [6] of 0.72±0.05 MeV. Similarly, one can adjust the experimental result [6] $\Gamma(\phi \to \pi^0\gamma)$=5.8±0.6 KeV, assuming that the small contamination of non-strange quarks in the ϕ meson is given by

$$\epsilon = -0.059 \pm 0.004, \tag{5}$$

where the sign comes from observed $\omega - \phi$ interference effects in $e^+e^- \to \pi^+\pi^-\pi^0$ [6]. From this reaction one has also measured [6]

$$\Gamma(\phi \to \pi^+\pi^-\pi^0) = \Gamma(\phi \to \rho\pi) \times BR(\rho \to \pi\pi) = 0.57 \pm 0.05\ MeV \tag{6}$$

with ρ-dominance in the $\pi\pi$ invariant mass. These results are in good agreement with our values for G, g and ϵ. Indeed, a calculation[8] of the ρ-dominated decay chain $\phi \to \rho^+\pi^- + \rho^-\pi^+ + \rho^0\pi^0 \to \pi^+\pi^-\pi^0$ leads to

$$\Gamma(\phi \to \pi^+\pi^-\pi^0) = 0.65\ MeV \tag{7}$$

By contrast, a simple estimate in the narrow ρ-width approximation leads to

$$\Gamma(\phi \to \rho^0\pi^0) = \epsilon^2 G^2 |\vec{q}_\rho|^3/12\pi \tag{8}$$

Assuming isospin invariance and neglecting interference effects (which is justified only in the narrow width approximation), one obtains the less satisfactory value $\Gamma(\phi \to \rho\pi) = 3\ \Gamma(\phi \to \rho^0\pi^0) \simeq 0.36$ MeV, showing that interference and off-mass-shell effects are important in these ϕ-decays. In particular, we notice that

$$\Gamma(\phi \to \rho^0\pi^0) = \frac{1}{3}\left[\Gamma(\phi \to \pi^+\pi^-\pi^0) - interference\ terms\right] \tag{9}$$

where a comparison with the numbers above, shows that the interference terms is $\approx$ 50 % of the first term.

The combined use of both lagrangians (1) leads to the correct values for the axial-anomaly, *i.e.*, $\Gamma(\pi^0 \to \gamma\gamma)$=7.6 eV (experiment [6] requires 7.7±0.6 eV), and for the SU(3) rotated processes $\eta, \eta' \to \gamma\gamma$ [9] . In the last cases one needs to implement the quark content of the η and η' mesons, namely,

$$\begin{aligned} \eta &\sim \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} - s\bar{s}) \\ \eta' &\sim \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d}) + \sqrt{\frac{2}{3}} s\bar{s} \end{aligned} \tag{10}$$

All these results confirm the correctness of the lagrangians (1). Essentially, these have been a part of the traditional successes of VMD combined with simple quark model arguments. From a more modern point of view, eqs(1) are the main ingredients of the anomalous part of Chiral lagrangians [10, 11] incorporating vector mesons as gauge fields of a "hidden" symmetry [12, 13, 14] (see sect.5). In any case, the lagrangians (1) and the quoted values for their coupling constants can be (and have been) used to predict the intermediate vector-meson contributions to $V^0 \to P^0 P^0 \gamma$ to which we now turn.

From the kinematical point of view these processes involve the two following amplitudes

$$\begin{aligned} \{a\} &= (\epsilon^* \cdot \epsilon)\,(q^* \cdot q) - (\epsilon^* \cdot q)\,(\epsilon \cdot q^*) \\ \{b\} &= -(\epsilon^* \cdot \epsilon)\,(q^* \cdot P)\,(q \cdot P) - (\epsilon^* \cdot P)\,(\epsilon \cdot P)\,(q^* \cdot q) \\ &\quad + (\epsilon^* \cdot q)\,(\epsilon \cdot P)\,(q^* \cdot P) + (\epsilon \cdot q^*)\,(\epsilon^* \cdot P)\,(q \cdot P) \end{aligned} \tag{11}$$

where $\epsilon(\epsilon^*)$ are the polarizations of the final photon (initial vector meson), and q ($q^* = q + p + p'$) are the corresponding four-momenta; $P = p + q$, $P' = p' + q$ are those for the virtual (intermediate) vector mesons (V and V') of the direct and crossed terms (see Fig.1). The total amplitude is then found to be

$$A(V^0 \to P^0 P^0 \gamma) = C(V^0 P^0 P^0 \gamma) \frac{G^2 e}{g\sqrt{2}} \left\{ \frac{P^2\{a\} + \{b(P)\}}{M_V^2 - P^2 - iM_V\Gamma_V} + \frac{P'^2\{a\} + \{b(P')\}}{M_{V'}^2 - P'^2 - iM_{V'}\Gamma_{V'}} \right\} \tag{12}$$

where V^0 is the decaying vector meson. The intermediate ones, V and V', can be either the ω or the ρ-mesons, with $V = V'$ in $\pi^0\pi^0\gamma$ and $V \neq V'$ in $\eta\pi^0\gamma$-decays; for $\phi \to K^0 \bar{K}^0 \gamma$ one obviously has $V = K^{*0}$ and $V' = \bar{K}^{*0}$. The coefficient C is the same for both terms (using SU(3)-symmetric couplings) and changes from process to process according to well-known quark-model or nonet-symmetry rules:

$$1 = C(\rho^0\pi^0\pi^0\gamma) = 3C(\omega\pi^0\pi^0\gamma) = \frac{3\sqrt{3}}{\sqrt{2}} C(\rho^0\pi^0\eta\gamma) = \sqrt{\frac{3}{2}} C(\omega\pi^0\eta\gamma) = -\frac{3}{\sqrt{2}} C(\phi K^0 \bar{K}^0 \gamma) \tag{13}$$

and

$$\epsilon = 3C(\phi\pi^0\pi^0\gamma) = \sqrt{\frac{3}{2}}C(\phi\pi^0\eta\gamma) \tag{14}$$

for the ϕ-decays where the Zweig-rule is operative.

From the above amplitudes, the partial widths are obtained performing a numerical integration of

$$\Gamma(V \to PP'\gamma) = \left(\frac{1}{2}\right)\frac{1}{192\pi^3 M_V}\int dE_\gamma \int dE_P \sum_{pol}|A(V \to PP'\gamma)|^2 \tag{15}$$

In eq.(15) the factor (1/2) has to be included only in $\pi^0\pi^0$ decays and the limits of the integration over the photon energy E_γ are 0 and $[M_V^2 - (m_P + m_{P'})^2]/(2M_V)$, while those for the pseudoscalar energies E_P are given by

$$\frac{1}{2}\left[(M_V - E_\gamma)\left(1 + \frac{m_P^2 - m_{P'}^2}{M_V^2 - 2M_V E_\gamma}\right) \pm E_\gamma\sqrt{1 - \frac{2(m_P^2 + m_{P'}^2)}{M_V^2 - 2M_V E_\gamma} + \left(\frac{m_{P'}^2 - m_P^2}{M_V^2 - 2M_V E_\gamma}\right)^2}\,\right].$$

Before presenting the numerical results for the various vector meson decays of interest, we shall discuss here the particular case of $\phi \to \pi^0\eta\gamma$ and the connection of our calculation with the narrow width approximation results.

3 $\phi \to \pi^0\eta\gamma$ Decay Amplitude

It is our purpose, in what follows, to clarify the connection between the full numerical evaluation of eq.(15) and the approximate one which can be obtained from a product of branching ratios in the decay chain $V \to V'P^0 \to P'^0\gamma P^0$. The full amplitude for this process is given by eqs.(11) and (12) with $V = \rho$, and $V' = \omega$. In this section and in order to simplify the argument,we shall neglect the ω-meson contribution to this decay. This is justified, since the complete calculation (whose results are reported in the next section) shows that the ω-meson contribution is negligible, relative to the ρ term.

With the above simplifying assumptions, eqs.(12), (14) give

$$\Gamma(\phi \to \pi^0\rho^0 \to \pi^0\eta\gamma) = \left(\frac{\epsilon G^2 e}{\sqrt{3}g}\right)^2 \frac{1}{192\pi^3 M_\phi}\int dE_\pi \int dE_\gamma \frac{\Sigma_{pol}[P^2\{a\} + \{b(P)\}]^2}{(P^2 - M_\rho^2)^2 + M_\rho^2\Gamma_\rho^2} \tag{16}$$

where $P^2 = M_\phi^2 + m_\pi^2 - 2M_\phi E_\pi$. After an analytical integration over E_γ we obtain

$$\Gamma(\phi \to \pi^0\rho^0 \to \pi^0\eta\gamma) = \left(\frac{\epsilon G^2 e}{\sqrt{3}g}\right)^2 \frac{1}{192\pi^3 M_\phi^3} \frac{1}{48} \cdot$$
$$\int_{m_\eta^2}^{(M_\phi - m_\pi)^2} \frac{dP^2}{P^2} \frac{(P^2 - m_\eta^2)^3 \Lambda^{3/2}(M_\phi^2, P^2, m_\pi^2)}{(P^2 - M_\rho^2)^2 + M_\rho^2\Gamma_\rho^2} \quad (17)$$

with $\Lambda^{3/2}(M_\phi^2, P^2, m_\pi^2) = \left\{\left[M_\phi^2 - \left(\sqrt{P^2} + m_\pi\right)^2\right]\left[M_\phi^2 - \left(\sqrt{P^2} - m_\pi\right)^2\right]\right\}^{3/2}$

Then eq.(17) can be written as

$$\Gamma(\phi \to \pi^0\rho^0 \to \pi^0\eta\gamma) = \frac{1}{\pi}\int ds \frac{\sqrt{s}\,\Gamma(s)_{\phi\to\rho^0\pi^0} \cdot \Gamma(s)_{\rho\to\eta\gamma}}{(s - M_\rho^2)^2 + M_\rho^2\Gamma_\rho^2} \quad (18)$$

where

$$\Gamma(s)_{\phi\to\rho^0\pi^0} = \epsilon^2 G^2 \frac{1}{12\pi}\frac{1}{8M_\phi^3}\Lambda^{3/2}(M_\phi^2, s, m_\pi^2) \quad (19)$$

and

$$\Gamma(s)_{\rho\to\eta\gamma} = \frac{e^2G^2}{3g^2}\frac{1}{12\pi}\left(\frac{s - m_\eta^2}{2\sqrt{s}}\right)^3 \quad (20)$$

If the narrow width approximation is used in eq.(18) we obtain

$$\Gamma(\phi \to \pi^0\rho^0 \to \pi^0\eta\gamma) = \Gamma(\phi \to \rho^0\pi^0) \cdot BR(\rho^0 \to \eta\gamma) \simeq 4.94 \times 10^{-5} MeV \quad (21)$$

with $\Gamma(\phi \to \rho^0\pi^0)$ and $\Gamma(\rho \to \eta\gamma)$ evaluated from eqs.(19),(20) with $s = M_\rho^2$. This result is to be contrasted with 2.45×10^{-5} obtained by numerical integration of eq.(17). Thus, the narrow width approximation alone, for the ρ-meson, overestimates the result by $\approx 50\%$. Further approximations are introduced if one uses the naive expression

$$\Gamma(\phi \to \rho^0\pi^0) = \frac{1}{3}\Gamma(\phi \to \rho\pi) \quad (22)$$

which introduces an additional overestimate, as discussed in the previous section. In conclusion, the use of the narrow width approximation in the decay chain, may result in an overestimate by almost a factor three relative to the exact numerical calculation.

4 VMD Numerical predictions and discussion

Our results using the full VMD amplitudes (12) and the three-body phase space formulae (15) are shown in the two last columns of Table 1. For comparison we

Table 1: Global contribution of intermediate vector mesons to decay rates (in eV) and branching ratios (last column) for different $V^0 \to P^0 P^0 \gamma$ transitions as predicted by several authors. Experimental upper limits are also quoted.

Decay rates (in eV)	EXP [6, 7]	Ref. [4]	Ref. [5]	Ref. [2]	This calculation Γ	This calculation B.R.
$\Gamma(\rho \to \pi^0\pi^0\gamma)$	—	$(2.5)1.6 \cdot 10^3$	—	$4.3 \cdot 10^3$	$1.6 \cdot 10^3$	$1.1 \cdot 10^{-5}$
$\Gamma(\rho \to \pi^0\eta\gamma)$	—	—	—	593	0.061	$4 \cdot 10^{-10}$
$\Gamma(\omega \to \pi^0\pi^0\gamma)$	$< 3.4 \cdot 10^3$	(350) 227	—	690	235	$2.8 \cdot 10^{-5}$
$\Gamma(\omega \to \pi^0\eta\gamma)$	—	—	—	53	1.39	$1.6 \cdot 10^{-7}$
$\Gamma(\phi \to \pi^0\pi^0\gamma)$	$< 4.4 \cdot 10^3$	(250) 54	45	153	51	$1.2 \cdot 10^{-5}$
$\Gamma(\phi \to \pi^0\eta\gamma)$	$< 11 \cdot 10^3$	—	35	228	23.9	$5.4 \cdot 10^{-6}$
$\Gamma(\phi \to K^0\bar{K}^0\gamma)$	—	—	—	0.18	$1.2 \cdot 10^{-5}$	$2.7 \cdot 10^{-12}$

also include (first column) the upper limit for the three experimentally studied decay rates [6, 7] and the predictions of other authors [2, 4, 5]who have worked in our same context. Our results are not incompatible with those by Singer [3], who first noticed the simple relation $\Gamma(V^0 \to \pi^+\pi^-\gamma) = 2\Gamma(V^0 \to \pi^0\pi^0\gamma)$ for the VMD part of the rate. This relation allows for a comparison of our results with those by Renard [4], quoted (in parenthesis) in the second column of Table 1. The accompanying values are the original ones [4] corrected by the present-day data for $\Gamma(\omega \to \pi^0\gamma)$ and ϵ, and turn out to be in excellent agreement with our predictions. The agreement with ref.[5] is somewhat less satisfactory. Finally, we disagree in the complete list of numerical predictions quoted in ref.[2] even if the initial expressions for the lagrangians are the same (notice that our coupling constant g has been defined as 1/2 of that in ref.[2]).

Concentrating on ϕ-decays one first observes that our vector-meson dominated mechanism predicts a completely negligible $\Gamma(\phi \to K^0\bar{K}^0\gamma)$, contrasting with the (four orders of magnitude larger) prediction from ref.[2]. We have carefully analyzed our calculation and, for this channel containing exclusively soft photons ($E \lesssim 25$ MeV), an analytic expression for the amplitude in this low-E limit has been obtained. One has

$$A(\phi \to K^0\bar{K}^0\gamma) \simeq \frac{eG^2}{3g(M_{K^*}^2 - m_{K^0}^2 - iM_{K^*}\Gamma_{K^*})}\Big[(p \cdot p' - m_{K^0}^2)\{a\} + \epsilon^* \cdot (p - p')\,[(\epsilon \cdot p)\,(q \cdot p') - (\epsilon \cdot p')\,(q \cdot p)]\Big] \quad (23)$$

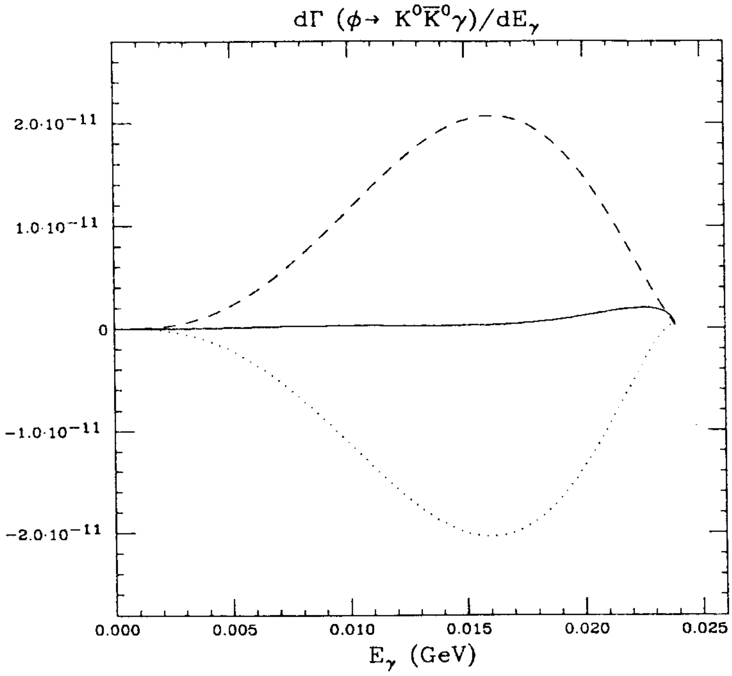

Figure 2: Photonic spectrum generated by intermediate vector-mesons in $\phi \to K^0\bar{K}^0\gamma$ (solid line). Dashed and dotted lines correspond to twice the contribution of a single diagram and their interference, respectively.

where p, p' are the pseudoscalar four-momenta, and

$$\sum |A(\phi \to K^0\bar{K}^0\gamma)|^2 = \frac{e^2G^4}{9g^2}\frac{(p\cdot p' - m^2_{K^0})}{(M^2_{K^*} - m^2_{K^0})^2 + M^2_{K^*}\Gamma^2_{K^*}}\cdot \left[2(p\cdot p')\,(q^*\cdot q)^2 - (p\cdot p' + m^2_{K^0})\,4(q\cdot p)\,(q\cdot p')\right] \tag{24}$$

accidentally containing the small numerical factor $(p\cdot p' - m^2_{K^0}) = (M^2_\phi - 4m^2_{K^0})/2$ [6]. In other words, the $\phi \to K^0\bar{K}^0\gamma$ decay is predicted to be exceptionally suppressed not only by the obviously scarce available phase-space but also due to an almost complete destructive interference in the amplitude, as explicitly shown in Fig.2. (Reversing the sign of the interference term would enlarge the width by 2 orders of magnitude).

If $\phi \to K^0\bar{K}^0\gamma$ is detected in Daϕne this would signal decay mechanisms [15, 16] different from ours, such as $\phi \to f_0, a_0\gamma \to K^0\bar{K}^0\gamma$ or other final-state interactions in $\phi \to K^+K^-\gamma \to K^0\bar{K}^0\gamma$. Similarly, only this kind of alternative mechanisms could produce sizable even-wave $K^0\bar{K}^0$ contaminations to the p-wave $\phi \to K^0\bar{K}^0$ decay (pure in K_L-K_S) thus disturbing CP-violation experiments [17].

On the contrary, our mechanism predicts sizable contributions to $\phi \to \pi^0\pi^0\gamma$ and $\eta\pi^0\gamma$ decays. The corresponding photonic spectra are shown in Figs.3 and 4, where the interference effects have again been separated.

The latter contribute to enhance the peak at high E in the $\phi \to \pi^0\pi^0\gamma$ spectrum. Roughly one-half of this decay contains a photon with an energy E in the

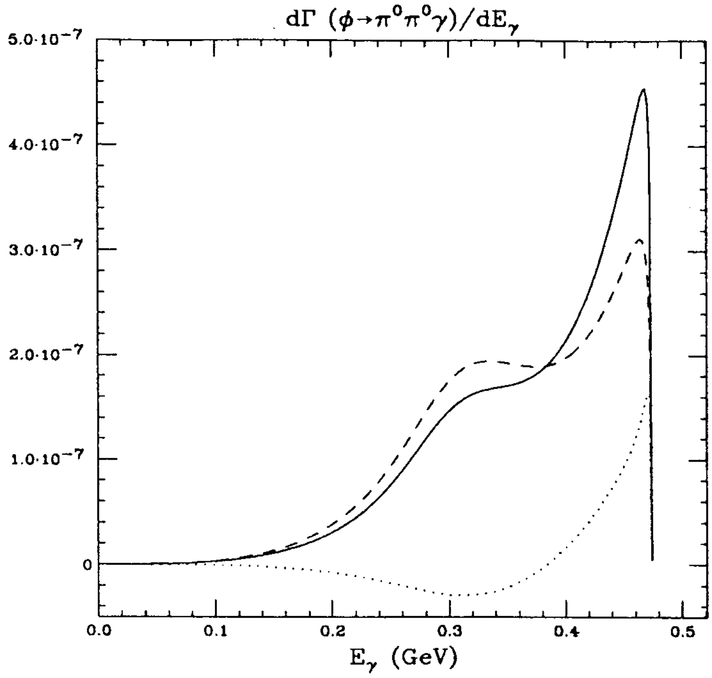

Figure 3: Photonic spectrum in $\phi \rightarrow \pi^0\pi^0\gamma$ with conventions as in Fig.2.

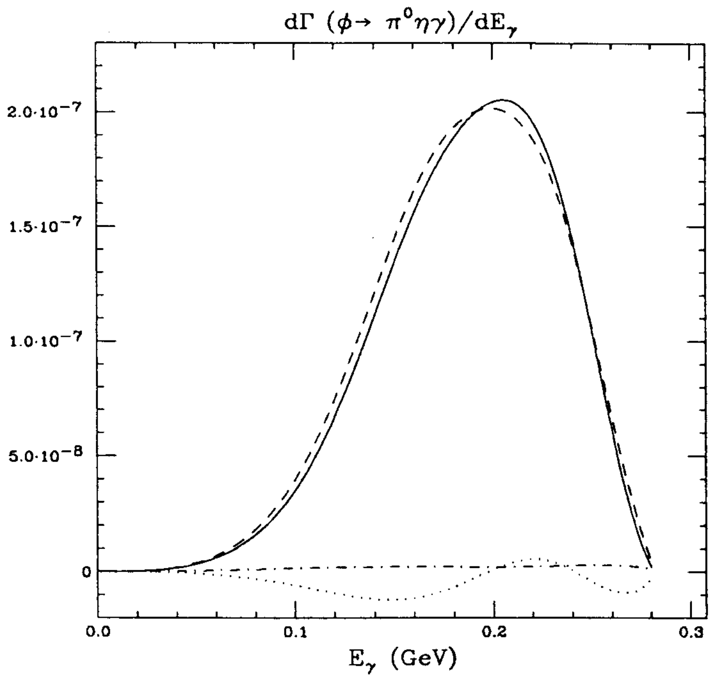

Figure 4: Photonic spectrum in $\phi \rightarrow \pi^0\eta\gamma$. Dashed and dot-dashed lines are the contributions of each diagram and the dotted line corresponds to their interference.

narrow range 400 MeV $\lesssim E \lesssim$ 470 MeV. Alternative mechanisms as the above mentioned are expected to produce exclusively lower energy photons, thus minimizing the interferences and allowing for separated analyses, particularly in $\phi \to \pi^0\pi^0\gamma$. We also notice that our predictions for this (and $\phi \to \pi^0\eta\gamma$) decay include events with the $\pi^0\gamma$ and $\eta\gamma$ invariant mass on the ρ-peak. As discussed previously when dealing with $\phi \to \pi^+\pi^-\pi^0$, our calculation thus already contain –and should be prefered to– simpler estimates in terms of two-body branching ratios. The latter imply [6] $BR(\phi \to \pi^0\pi^0\gamma) = \frac{1}{3}BR(\phi \to \rho\pi)BR(\rho \to \pi\gamma) \simeq 34 \times 10^{-6}$ and $BR(\phi \to \pi^0\eta\gamma) = \frac{1}{3}BR(\phi \to \rho\pi)BR(\rho \to \eta\gamma) \simeq 16.4 \times 10^{-6}$, only in marginal agreement with our tabulated results because important interference and off-mass-shell effects have been neglected. However, these rough estimates are usefull and allow for a numerical check of our predictions. Indeed, by artificially reducing the ρ-width in our complete (three-body) calculation one recovers the expected agreement with the above simpler (two-body) estimates discussed at the end of section 3.

5 Chiral loop contributions

Strong, electromagnetic and weak interactions of pseudoscalar mesons P at low energies are known to be well described by effective chiral lagrangians. More precisely, Chiral Perturbation Theory (χPT)[18] offers an accurate description of the whole set of existing data in terms of a systematic expansion in powers of external momenta p or pseudoscalar masses m_P. At lowest order (tree-level), the χPT lagrangian contains an ordinary and an anomalous sector, which are of order p^2 and order p^4, respectively. The non-renormalizability of the theory implies that higher order contributions – generated by the so-called chiral loops– may contain a divergent part which has to be eliminated by appropriated, higher-order counterterms. The finite part of the latter (the so-called low-energy constants) can be fixed at a given energy from experimental data[18] and, hopefully, from QCD first principles[19]. Somehow in between these two extreme possibilities one can also expect relating the remaining, finite part of the counterterms with the parameters of the well-known meson-resonances. Indeed, the latter do not strictly appear in the χPT lagrangian in spite of providing the more prominent effects in the low-energy region where χPT is confidently operative. The assumption that the finite part of the counterterms is dominated (or saturated) by the contribution of meson-resonances was already advanced by Gasser and Leutwyler[18], successfully developed and tested by Ecker et al.[20] and further confirmed by other authors in both the ordinary[21] and the anomalous[22, 23, 8, 13] sectors. In other words, implementing the strict χPT lagrangian –dealing solely with pseudoscalar and electroweak currents– with the effects of meson-resonances leads to a more complete and realistic scheme with a largely increased predictive power. In particular, it can incorporate and improve most of the VMD results so far discussed.

For concreteness, let us consider $\phi \to \pi^0\pi^0\gamma$ decay for which a rather low branching ratio should be expected (see Table 1). There is a two-fold reason for that: neutral particles cannot radiate copiously (bremsstrahlung) photons and, moreover, the Zweig rule supress ϕ-decays into pions. In the χPT context this double suppression is at once avoided through the contributions of charged-kaon loops. If so, the smallness of the $\phi \to \pi^0\pi^0\gamma$ branching ratio will no longer hold and the analysis of this and related decays could evidentiate the effects of the (otherwise ellusive) chiral loops. Notice, however, that we are pushing χPT somewhat outside its original context which did not allow for the inclusion of ϕ and other resonances. Our purpose is to compute some consequences of this extended version of χPT to allow for future comparison with experimental data.

To this aim the closely related (both are Zweig forbidden) $\phi \to \pi^0\pi^0\gamma$ and $\phi \to \pi^0\eta\gamma$ decays will be discussed, along with the (Zweig allowed) $\phi \to K^0\bar{K}^0\gamma$ decay mode. Φ-factories are expected to provide valuable data on these K-loop dominated processes in a near future. In these cases the relevant range of energies extends from the lowest limits (where χPT can safely be trusted) up to the ϕ-mass, where the perturbative series could lose its convergence (Notice, however, that χPT successfully explains $\eta'(960)$ decays in this energy region). For all these reasons we also consider lower mass ρ and ω decays, such as $\rho \to \pi^0\pi^0\gamma$ (proceding mainly through charged-pion loops) and $\omega \to \pi^0\pi^0\gamma$, $\omega \to \pi^0\eta\gamma$ and $\rho \to \pi^0\eta\gamma$ (with only K-loops in the good $SU(2)$ limit). An important, common feature of all these ϕ, ρ and ω radiative decays into uncharged pseudoscalars is that no-counterterms are required. As we shall see, one loop contributions are always finite and well definite thus allowing for a clear cut comparison with future data when available.

The lowest order term (p^2) of the χPT lagrangian is

$$L_2 = \frac{1}{8} f^2 \; tr\left(D_\mu\Sigma \; D^\mu\Sigma^\dagger + \chi\Sigma^\dagger + \chi^\dagger\Sigma\right) \tag{25}$$

where $f = 132$ MeV is the π decay constant and the covariant derivative $D_\mu\Sigma = \partial_\mu\Sigma + ieA_\mu [Q,\Sigma]$ contains the photon field A_μ and the quark charge matrix Q =diag $(2/3, -1/3, -1/3)$. The pseudoscalar nonet P is the usual $SU(3)$ matrix appearing in $\Sigma = \exp(2iP/f)$ with masses given in terms of the quark mass matrix $\mathcal{M}$= diag (m_u, m_d, m_s) contained in the last χ-terms of eq.(25). Vector-mesons can be incorporated along the lines of the work by Bando et al.[12] or other (for our purposes) equivalent contexts [25]. The whole (ideally mixed) nonet appears in the usual $SU(3)$ matrix V whose diagonal elements are $(\rho^0 + \omega)/\sqrt{2}$, $(-\rho^0 + \omega)/\sqrt{2}$ and ϕ as given in eq.(3). Their $SU(3)$-symmetric couplings to P-pairs and to the photon are given by the conventional lagrangians

$$L_{VPP} = ig \; tr(V_\mu \; P \; \partial^\mu P - V_\mu \; \partial^\mu P \; P) \tag{26}$$

$$L_{V\gamma} = -2egf^2 A^\mu \; tr(QV_\mu) \tag{27}$$

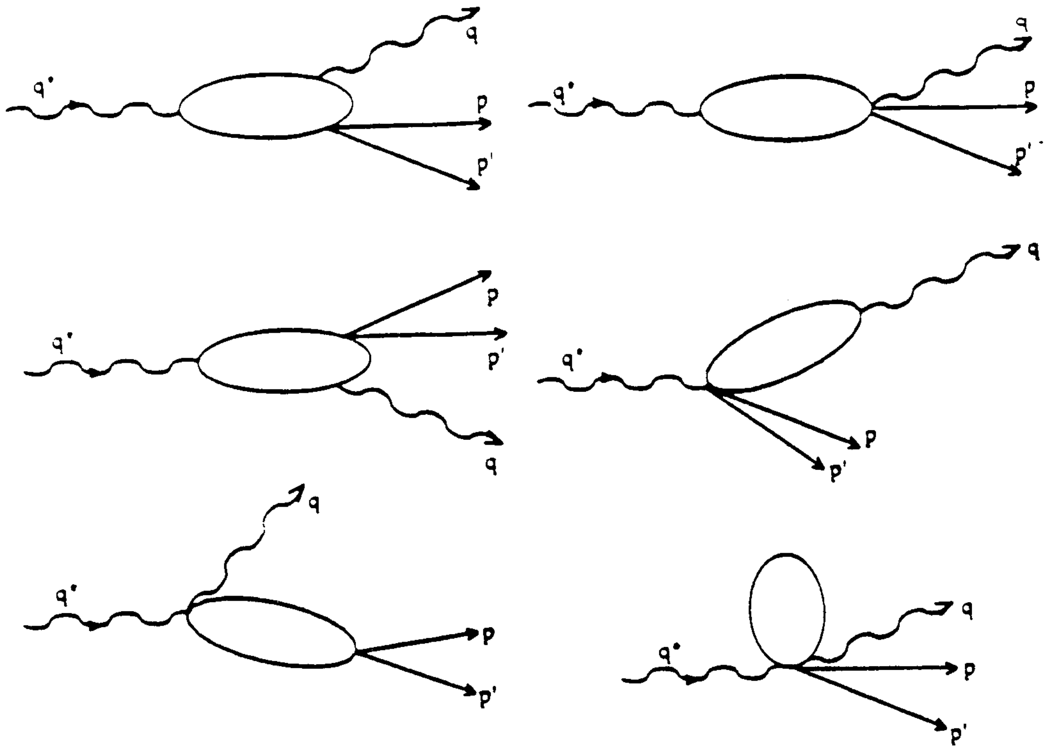

Figure 5: One loop diagrams for $V^0 \to P^0P^0\gamma$ decays.

with $\sqrt{2}gf = M_V$, the V-meson mass, or $g \simeq 4.2$ as discussed in section 2. With these values , eqs.(26) and (27) reproduce the P-photon vertices of eq.(25) and their vector-meson dominated (VMD) form-factors.

χPT-amplitudes for the decay processes $V^0 \to P^0P^0\gamma$ can now be deduced from the above lagrangians. There is no tree-level contribution and at the one-loop level one requires computing the set of diagrams shown in Fig.5.

This leads to the following finite amplitudes $A(V^0 \to P^0P^0\gamma)_P$, where the subscript $P = K$ or π indicates that charged-kaons or pions circulate along the loop and $q^*(\epsilon^*)$ and q (ϵ) stand for the initial vector-meson and photon four-momenta (polarizations), respectively,

$$\begin{aligned}
A(\rho^0 \to \pi^0\pi^0\gamma)_K &= A(\omega \to \pi^0\pi^0\gamma)_K = \frac{-1}{\sqrt{2}}A(\phi \to \pi^0\pi^0\gamma)_K = \frac{-1}{\sqrt{2}}A(\phi \to K^0\bar{K}^0\gamma)_K \\
&= \frac{-eg}{4\sqrt{2}\pi^2 f^2}\,(q^{*2} - 2\,q^*q)\,\{a\}\,\bar{J}_K \qquad (28)\\
A(\rho^0 \to \pi^0\pi^0\gamma)_\pi &= \frac{-\sqrt{2}eg}{\pi^2 f^2}\,(q^{*2} - 2\,q^*q - m_\pi^2)\,\{a\}\,\bar{J}_\pi \qquad (29)\\
A(\rho^0 \to \pi^0\eta\gamma)_K &= A(\omega \to \pi^0\eta\gamma)_K = \frac{-1}{\sqrt{2}}A(\phi \to \pi^0\eta\gamma)_K = \frac{-2}{3}A(\phi \to \pi^0\eta_8\gamma)_K \\
&= \frac{-eg}{6\sqrt{3}\pi^2 f^2}\,(3q^{*2} - 6\,q^*q - 4m_K^2)\,\{a\}\,\bar{J}_K \qquad (30)
\end{aligned}$$

All the above amplitudes are gauge-invariant through the factor (see the VMD amplitudes (11))

$$\{a\} = (\epsilon^*\epsilon)(q^*q) - (\epsilon^*q)(\epsilon\, q^*) \tag{31}$$

and in eq.(30) we have used an η-η' mixing angle $\theta_P = \arcsin(-1/3) \simeq -19.5^\circ$ leading to the quark content shown in eq.(10) in agreement with recent phenomenological estimates. The Feynman integral $\bar{J}_P$ is purely real and given by

$$\begin{aligned} J_P &= m_P^2 \bar{J}_P = \int_0^1 dx \int_0^{1-x} dy \, \frac{xy}{1+\alpha xy - \beta y(1-y)} \\ &= \frac{1}{2\alpha} + \frac{2}{\alpha^2} \left\{ \left(\arcsin \frac{\sqrt{\beta-\alpha}}{2} \right)^2 - \left(\arcsin \frac{\sqrt{\beta}}{2} \right)^2 \right\} + \\ & \frac{\beta}{\alpha^2} \left\{ \sqrt{\frac{4}{\beta-\alpha} - 1} \, \arctan \left(\frac{4}{\beta-\alpha} - 1 \right)^{-1/2} - \sqrt{\frac{4}{\beta} - 1} \, \arctan \left(\frac{4}{\beta} - 1 \right)^{-1/2} \right\} \end{aligned} \tag{32}$$

for $\beta \equiv q^{*2}/m_P^2 \leq 4$ and $\beta - \alpha \leq 4$, with $\alpha \equiv 2\, q^*q/m_P^2$. Otherwise, J_P becomes complex and can be deduced from (32) making the substitutions

$$\begin{aligned} \arcsin \frac{\xi}{2} &\rightarrow \frac{\pi}{2} + \frac{i}{2} \ln \frac{1+\sqrt{1-4/\xi^2}}{1-\sqrt{1-4/\xi^2}} \\ \sqrt{\frac{4}{\xi^2} - 1} \, \arctan \left(\frac{4}{\xi^2} - 1 \right)^{-1/2} &\rightarrow \frac{1}{2} \sqrt{1 - \frac{4}{\xi^2}} \left\{ \ln \frac{1+\sqrt{1-4/\xi^2}}{1-\sqrt{1-4/\xi^2}} - i\pi \right\} \end{aligned} \tag{33}$$

whenever $\xi \geq 2$, with ξ standing for $\sqrt{\beta}$ and/or $\sqrt{\beta-\alpha}$.

Two checks of our above results have been performed. The first consists in recovering the amplitude for $\eta \to \pi^0\gamma\gamma$ deduced in ref.[26] using crossing and our amplitudes (30) for ρ, ω, $\phi \to \pi^0\eta\gamma$ followed by the $V\gamma$ conversion in eq.(27). The second comes from comparing our results with those by Lucio and Pestieau [16] for the $\phi \to K^0\bar{K}^0\gamma$ amplitude which these authors assume to be dominated by the scalar mesons $f_0(975)$ or $a_0(980)$ in the $K\bar{K}$ channel. Although some dynamical aspects of the two models are different they coincide in the factors $\{a\}$, eq.(31), and J_K, eqs.(32) and (33).

Decay amplitudes at the one-loop and order-p^4 level for the processes $V^0 \to P^0P^0\gamma$ can be obtained from eqs.(28) to (30) . For ϕ-decays , one obtains

$$\begin{aligned} \Gamma(\phi \to \pi^0\eta\gamma)_K &= 131\ eV, \quad \Gamma(\phi \to \pi^0\pi^0\gamma)_K = 224\ eV \\ \Gamma(\phi \to K^0\bar{K}^0\gamma)_K &= 0.033\ eV \end{aligned} \tag{34}$$

where the subscript K reminds us that only charged-kaon loops are operative. By contrast, these K-loops give a contribution to $\rho^0 \to \pi^0\pi^0\gamma$ which is 10^3 times smaller than that due to charged-pion loops, namely,

Table 2: Contribution of Chiral loops and intermediate vector mesons to decay rates (in eV) and branching ratios (last column) for different $V^0 \to P^0 P^0 \gamma$ transitions .

Decay rates (in eV)	χ-loops	VMD	Total	B.R.
$\Gamma(\rho \to \pi^0\pi^0\gamma)$	1.42×10^3	1.62×10^3	3.88×10^3	26×10^{-6}
$\Gamma(\rho \to \pi^0\eta\gamma)$	0.006	0.061	$\simeq$VMD	VMD
$\Gamma(\omega \to \pi^0\pi^0\gamma)$	1.8	235	$\simeq$VMD	VMD
$\Gamma(\omega \to \pi^0\eta\gamma)$	0.013	1.39	$\simeq$ VMD	VMD
$\Gamma(\phi \to \pi^0\pi^0\gamma)$	224	51	281	64×10^{-6}
$\Gamma(\phi \to \pi^0\eta\gamma)$	131	23.9	151	34×10^{-6}
$\Gamma(\phi \to K^0\bar{K}^0\gamma)$	0.033	1.2×10^{-5}	$\simeq \chi$- loops	7.6×10^{-9}

$$\Gamma(\rho^0 \to \pi^0\pi^0\gamma)_\pi = 1.42 \times 10^3 eV \tag{35}$$

Finally, one also gets

$$\begin{aligned} \Gamma(\rho^0 \to \pi^0\eta\gamma)_K &= 0.006\ eV \\ \Gamma(\omega \to \pi^0\eta\gamma)_K &= 0.013\ eV\ , \quad \Gamma(\omega \to \pi^0\pi^0\gamma)_K = 1.8\ eV \end{aligned} \tag{36}$$

with π-loop contributions vanishing in the good isospin limit.

6 Chiral Loops and VMD Contributions

The relative weight of the two contributions so far discussed –the finite chiral loops of eqs.(28) to (30) versus the VMD amplitudes (12)– depends crucially on the decay mode. Since VMD amplitudes can be interpreted as saturating the χPT counterterms, the above mentioned contributions have to be added to obtain the whole χPT amplitude.

Let us first discuss $\rho^0 \to \pi^0\pi^0\gamma$ whose VMD contribution is given by eqs.(12) and (11) with the ω mass and width in both propagators. One easily obtains[27]

$$\Gamma(\rho^0 \to \pi^0\pi^0\gamma)_{VMD} = 1.62 \times 10^3 eV \tag{37}$$

which is of the same order of magnitude as the pion-loop contribution quoted in eq.(35). The global $\rho^0 \to \pi^0\pi^0\gamma$ decay width is therefore given by the sum of the two amplitudes leading separately to eqs.(35) and (37). One obtains

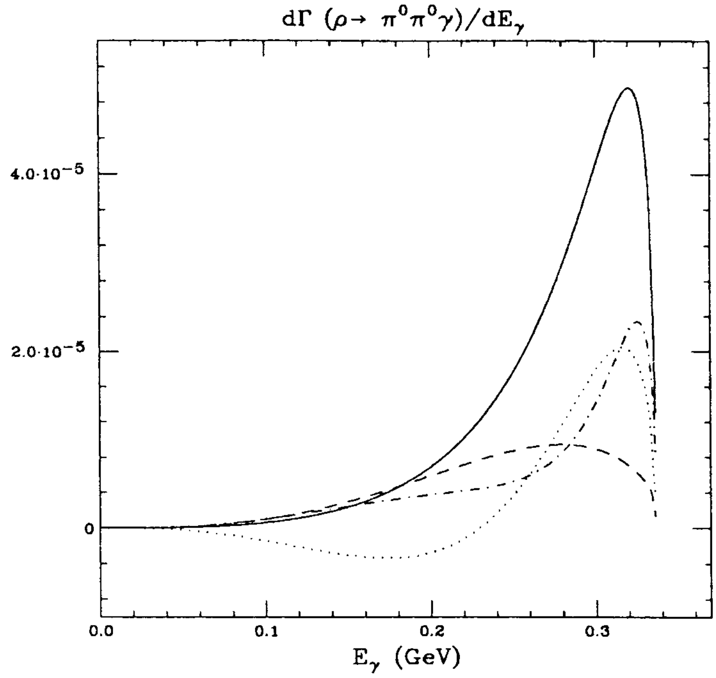

Figure 6: Photonic spectrum in $\rho^0 \to \pi^0\pi^0\gamma$ (solid line). Dashed line corresponds to the contribution of pion-loops, dotdashed line is the VMD contribution, and dotted line is their interference.

$$
\begin{aligned}
\Gamma(\rho^0 \to \pi^0\pi^0\gamma) &= 3.88 \times 10^3 eV \\
BR(\rho^0 \to \pi^0\pi^0\gamma) &= 26 \times 10^{-6}
\end{aligned}
\qquad (38)
$$

and the photonic spectrum shown (solid line) in Fig.6 clearly peaked at higher energies E_γ. The separated contributions from pion-loops and from VMD, as well as their interference, are also shown in Fig.6 (dashed, dotdashed and dotted lines, respectively).

The situation changes quite clearly when turning to the other decay modes like $\rho,\ \omega \to \pi^0\eta\gamma$ and $\omega \to \pi^0\pi^0\gamma$. We find that the previously evaluated kaon-loop contributions (36) are one or two orders of magnitude smaller. The physical reason for this suppression is that the usually dominant pion-loops are isospin-forbidden in these decays. More accurate estimates and the shape of the photonic spectra seem unnecessary due to the smallness of the corresponding branching ratios (only the third one, $BR(\omega \to \pi^0\pi^0\gamma) \simeq 28 \times 10^{-6}$, could reasonably allow for detection) and also to the fact that these decay modes are dominated by the well-understood (see [27]) but less-interesting VMD contribution.

By contrast the latter VMD-contribution is expected to be much smaller in $\phi \to \pi^0\eta\gamma$ and $\pi^0\pi^0\gamma$ decays due to the Zweig rule, as shown in Table 1, well below the kaon-loop contributions quoted in eq.(34). Proceeding as before and adding the corresponding amplitudes with the appropriate phases leads to

$$\Gamma(\phi \to \pi^0\eta\gamma) = 151\ eV \qquad \Gamma(\phi \to \pi^0\pi^0\gamma) = 281\ eV$$

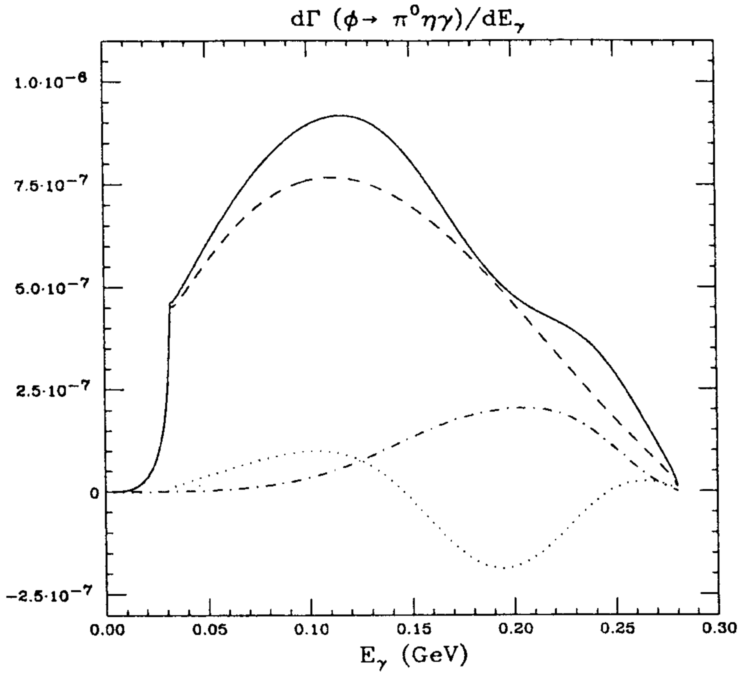

Figure 7: Photonic spectrum in $\phi \to \pi^0\eta\gamma$ (solid line). Dashed line corresponds to the contribution of kaon loops, dotdashed line is the VMD contribution, and dotted line is their interference.

$$BR(\phi \to \pi^0\eta\gamma) \;=\; 34 \times 10^{-6} \qquad BR(\phi \to \pi^0\pi^0\gamma) = 64 \times 10^{-6} \tag{39}$$

and the photonic spectra shown in Fig.7 and 8.

The Zweig allowed kaon-loops are seen to dominate both spectra and decay rates and the predicted branching ratios are large enough to allow for detection and analyses in future ϕ-factories. For completeness we have also computed $\Gamma(\phi \to K^0\bar{K}^0\gamma) \simeq \Gamma(\phi \to K^0\bar{K}^0\gamma)_K \simeq 0.033$ eV, with $BR(\phi \to K^0\bar{K}^0\gamma) \simeq 7.6 \times 10^{-9}$, again dominated by kaon-loops (due to the smallness of the VMD contribution discussed in detail in section 4 and ref. [27]).

7 Conclusions

In summary, the well understood contributions of intermediate vector mesons in $V^0 \to P^0P^0\gamma$ decays have been discussed. Vector Meson Dominance alone predicts $BR(\phi \to \pi^0\pi^0\gamma) = 12 \times 10^{-6}$ and $BR(\phi \to \pi^0\eta\gamma) = 5.4 \times 10^{-6}$, and a characteristic photonic spectrum (peaked at higher energies) in the first decay. Similarly, an exceptionally small contribution is predicted (and its physical origin understood) for the branching ratio $BR(\phi \to K^0\bar{K}^0\gamma)$, namely, $\sim 2.7 \times 10^{-12}$. Other VMD predictions are $BR(\omega \to \pi^0\pi^0\gamma) \simeq 28 \times 10^{-6}$ and $BR(\rho^0 \to \pi^0\pi^0\gamma) \simeq 11 \times 10^{-6}$.

We also find that some vector meson decays into two neutral pseudoscalars and a photon could receive important contributions from chiral loops if Chiral Perturbation Theory (χPT) is extended in the plausible and well defined way proposed

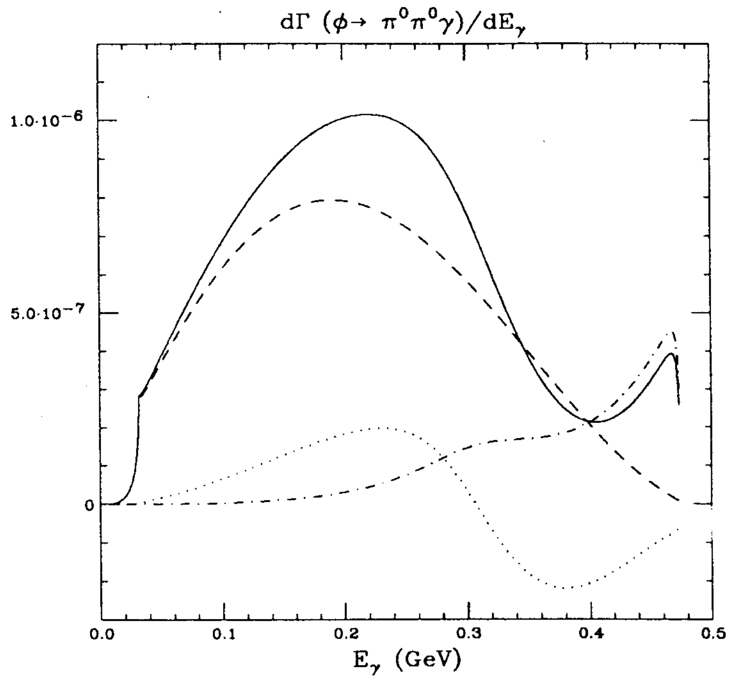

Figure 8: Photonic spectrum in $\phi \rightarrow \pi^0\pi^0\gamma$ with conventions as in Fig.7.

here. Some consequences of this extension –the relevance of pion-loops in $\rho \rightarrow \pi^0\pi^0\gamma$ and the dominance of kaon-loops in $\phi \rightarrow \pi^0\eta\gamma$, $\pi^0\pi^0\gamma$– have been unambigously predicted thus allowing for future comparison with data. If the latter turn out to confirm our predictions the domain of applicability of χPT and their relevance would be considerably increased. In case of disagreement it is not χPT itself, but rather our mechanisms to introduce extra resonances – thus incorporating the previously computed VMD contributions – which should be suspicious.

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