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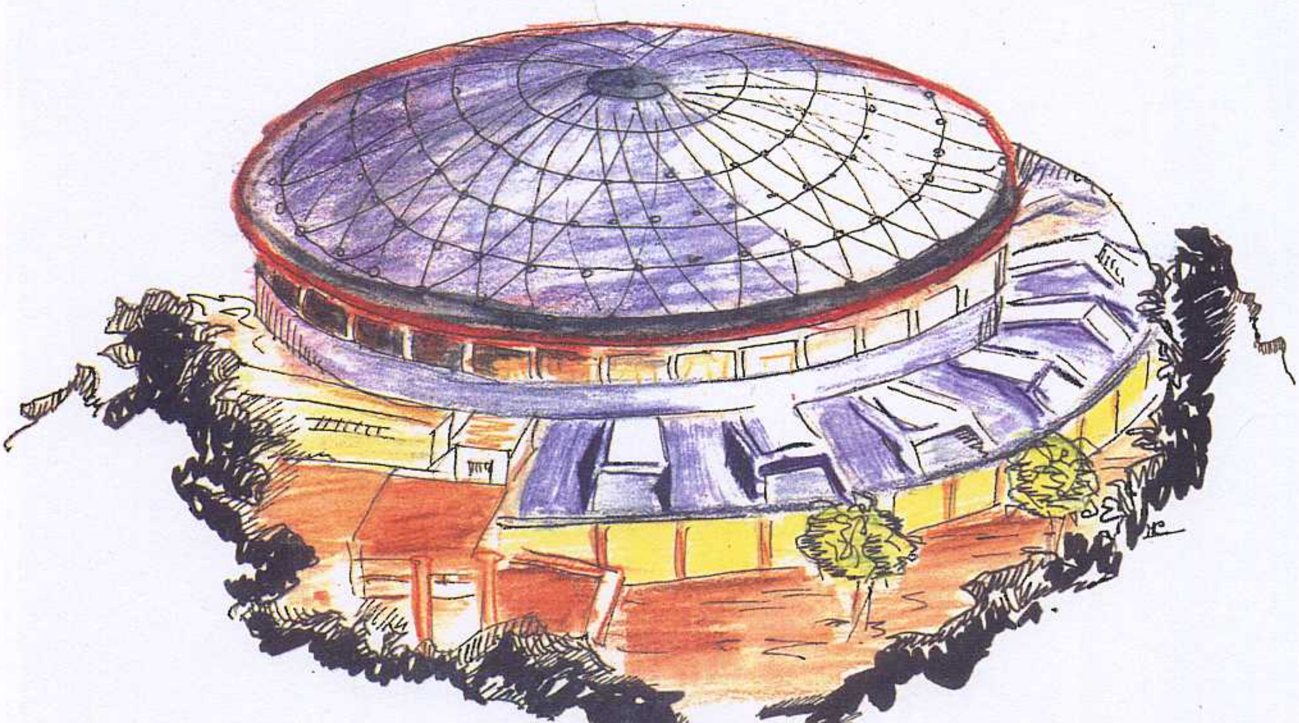

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A Possible Massive Solution to Dyson-equation with Small Gauge Coupling

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Abstract

On the "Planck lattice" possibly generated by the quantum fluctuations of gravity, we consider a "rainbow" Dyson equation for fermion self-energy function by only taking the NJL and QED interactions into account. We find a finite inhomogeneous term adding to usual Dyson-equation in continuum space-time even with Nambu-Jona-Lasinio mass vanishing (M_{NJL}). Thus, a massive solution to Dyson-equation is possible for small gauge couplings.

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In a recent paper [1] we have observed that a new view of the Standard Model (SM) might emerge if space-time would effectively exhibit at the Planck scale a random lattice structure: the Planck lattice. The reason why the possible foam-like structure of space-time, induced by the violent quantum fluctuations at the Planck scale [2], does change the very formulation of the SM, is a consequence of the well known "no-go" theorem of Nielsen and Ninomiya [3]. According to such theorem no consistent chiral gauge symmetry can be written on a lattice, when the lagrangian density only contains terms bilinear in the Fermi fields, as suggested by the simple transcription of the continuum lagrangian. In order to evade the "no-go" theorem in [1] we suggested adding to the usual lagrangian of the SM the simplest quadrilinear terms,

$$\begin{aligned} S^{F1}_{NJL} &= G_1 \sum_x \{\bar{\psi}^{Fi}_L(x) \cdot \psi^{Fj}_R(x) \bar{\psi}^{Fj}_R(x) \cdot \psi^{Fi}_L(x)\} \\ S^{F2}_{NJL} &= \frac{G_2}{2} \sum_{\pm\mu,x} \left[\bar{\psi}^{Fi}_L(x) G^L_\mu(x) U^c_\mu(x) \cdot \psi^{Fj}_R(x+a_\mu) \bar{\psi}^{Fj}_R(x) G^R_\mu(x) U^c_\mu(x) \cdot \psi^{Fi}_L(x+a_\mu)\right], \end{aligned} \tag{1}$$

whereF=l (F=q) denotes its lepton (quark) sector, the indices i, j denote fermion families; the Dirac indices are denoted by scalar product "$\cdot$". The gauge link $U^c_\mu(x)$ connects left-and right-handed quark fields in neighboring points so as to have the $SU_c(3)$ gauge symmetry. The chiral gauge links $G^L_\mu(x)$ ($G^R_\mu(x)$) connect left-handed (right-handed) fermion fields to enforce $SU_L(2) \otimes U_Y(1)$ chiral gauge symmetry. $G_{1,2}$ are two, yet unspecified, Fermi-type $O(a_p^2)$ coupling constants which are assumed universal for both the lepton and quark sectors. Its structure was necessary to remove, in principle, the unwanted "doublers" that appear in the low energy spectrum of that lagrangian.

A careful examination of the theory [1] has indicated that the quadrilinear terms can develop the necessary dynamical chiral symmetry breaking through the non-zero vacuum expectation values, related to fermion mass m and the Wilson parameter r [4], depending on the values of $G_{1,2}$. As discussed in ref. [1], one can "tune" G_1 in such a way that only one fermion of quark-type acquires a non-zero mass, thus giving rise to the phenomenologically appealing $\bar{t}t$-condensate model [5]. However, at this stage, even though all other fermions remain massless, the emergence of a non-zero r, equal for all fermions, is sufficient to remove from the long-wave spectrum all "doublers".

In this paper we wish to outline our strategy to proceed further in our computation of fermion masses, by considering the contributions arising from the photon γ, $W^{\pm}$, and Z° interaction that we have so far neglected. The problem we must now solve is the evaluation of the self-energy function of the fermions with the SM action and supplemented by the interaction (1) on the Planck lattice. In order to render this problem manageable we shall work in the "rainbow" approximation of the Dyson equation, which neglects all vertex and gauge propagator corrections. Thus on the Planck lattice we must solve the Dyson-Schwinger equation which we give in diagrammatic form in Fig. 1. The first diagram in the left-hand side represents the result of the first stage calculation of ref. [1], while the second is a contact interaction typical of gauge theories on a lattice. Please note that in the Dyson equation we have not included colored gluon exchanges for they are related to color confinement, producing a mass-shift of the order of the Planck mass for quarks and all colored states. We shall return on this point in a future publication; for, as a matter of fact, in this paper our sole intention is to study the structure of the solutions of eqn. (2)on a Planck lattice and compare it with the rather large amount of extant works on the continuum version of the Dyson equation. A final remark on our approximation: the smallness of the coupling constants associated with electroweak exchanges makes it quite plausible.

Let's now turn to eqn. (2), where, for illustration purposes, we keep only the exchange of a massive vector gauge-field of mass m_v coupled to a vector current. We can write it as:

$$\Sigma_{PL}(p) = W_r(p) + \bar{g}^2 \int_{-\frac{\pi}{a_p}}^{\frac{\pi}{a_p}} \frac{d^4q}{(2\pi)^4} \frac{1}{S(p-q)^2 + m_v^2}$$
$$\left(\delta_{\mu\nu} - \xi \frac{S_\mu(p-q) S_\nu(p-q)}{S(p-q)^2 + m_v^2}\right) \left[V_{\mu\nu}^{(2)}(p,p) - V_\mu^{(1)}(p,q) \frac{1}{\gamma_\rho D_\rho(q) + \Sigma_{PL}(q)} V_\nu^{(1)}(p,q)\right], \tag{3}$$

where $S_\mu(l) = \frac{2}{a_p} \sin \frac{l_\mu a_p}{2}$, $D_\mu(l) = \frac{1}{a_p} \sin(l_\mu a_p)$ and $\bar{g}^2 = e^2 (\frac{g^2(N^2-1)}{2N})$ for $U_{em}(1)(SU(N))$ gauge group. The $W_r(p)$ is the self-energy of the fermion generated through the Nambu-Jona Lasinio mechanism [6]. As discussed in ref. [1]for the one-loop contribution, $W_r(p) = m_t + M_0 + \frac{2r}{a_p} \sum_\mu \sin^2 \frac{p_\mu a_p}{2}$; m_t is non-vanishing for the top-quark only and the "bare" mass term $M_0 = \frac{cr}{a_p}$ together with the Wilson term $\frac{2r}{a_p} \sum_\mu \sin^2 \frac{p_\mu a_p}{2}$ are different from zero for all fermions. The vertices [7] are ($k_\mu = \frac{(p_\mu + q_\mu) a_p}{2}$)

$$V_\mu^{(1)}(p,q) = (\gamma_\mu \cos k_\mu + r \sin k_\mu); \quad V_{\mu\nu}^{(2)}(p,q) = a_p (-\gamma_\mu \sin k_\mu + r \cos k_\mu) \delta_{\mu\nu}. \tag{4}$$

We limit our attention to fermions other than the top quark and to values of the external momentum p such that $pa_p \ll 1$: in this kinematical region the self energy function $\Sigma_{PL}(p)$ should not and does not differ from its continuum limit version $\Sigma_c(p)$. One of the main novelties of (3) is the non trivial interplay between the continuum-limit region, i.e. for momenta ($qa_p \ll 1$), and the truly discrete region, which is probed for momenta $qa_p \simeq 1$. In order to study such interplay it is important to introduce a "dividing scale" ϵ, such that $pa_p \ll \epsilon \ll \pi$. Separating the integration region in (3) into two regions $[0,\epsilon]^4$ and $[\epsilon,\pi]^4$, we may write our integral equations as

$$\Sigma_c(p) = M_0 + \frac{\lambda}{4\pi^2}\int_{\Lambda=\epsilon\Lambda_p} d^4q \frac{1}{(p-q)^2+m_v^2}\frac{\Sigma_c(q)}{q^2+m^2} + \delta_{PL}(r,\epsilon), \tag{5}$$

where $\lambda = \frac{3\bar{g}^2}{4\pi^2}$, and the continuum-limit integral equation, following common practice, has been linearized, i.e. in the denominator one has set $\Sigma_c(q) \simeq m$, the"physical" mass of the fermion, and the Landau gauge $\xi = 1$, in which the self-energy function for $m_v = 0$ contains a mass renormalization only, is chosen for simplicity. As for $\delta_{PL}(r,\epsilon)$, the contribution to the integral equation from the discrete lattice region, we may write it as ($l_\mu = q_\mu a_p$):

$$\begin{aligned}\delta_{PL}(r,\epsilon) \simeq & \frac{\lambda r}{a_p}\int_{[\epsilon,\pi]^4} \frac{d^4l}{12\pi^2}\frac{1}{(4\sin^2(\frac{l_\mu}{2}))}\left[\frac{1}{2} - \frac{\frac{1}{r}G_{PL}(l)(-\cos^2(\frac{l_\mu}{2})+r^2\sin^2(\frac{l_\mu}{2}))+\sin^2(l_\mu)}{G_{PL}(l)^2+\sin^2(l_\mu)}\right] \\ & -\lambda\int_{[\epsilon,\pi]^4}\frac{d^4l}{16\pi^2}\frac{\Sigma_c(l)}{(4\sin^2(\frac{l_\mu}{2}))}\left[\frac{-\cos^2(\frac{l_\mu}{2})+r^2\sin^2(\frac{l_\mu}{2})}{G_{PL}(l)^2+\sin^2(l_\mu)}\right],\end{aligned} \tag{6}$$

where we have decomposed $\Sigma_{PL}(l)$ as $a_p\Sigma_{PL}(l) = a_p\Sigma_c(l) + G_{PL}(l)$. In the one-loop calculation of (1), $G_{PL}(l)$ is $2r\sum_\mu \sin^2\frac{l_\mu a_p}{2}$, which is non zero only in the discrete lattice region, i.e. for $l \in [\epsilon,\pi]^4$. It will be clear soon that a consistent cancellation of the $\frac{1}{a_p}$ divergence, which stems from the lattice region, is enforced by fine-tuning the bare mass term M_0. The dependence on the external momentum pa_p is omitted in $\delta_{PL}(\epsilon,r)$ because of $pa_p \ll l \in [\epsilon,\pi]^4$. Note that (i) there is no dependence on the gauge parameter ξ for there is a perfect cancellation between the contact and the "rainbow" diagrams, that is guaranteed by Ward's identities; (ii) $\delta_{PL}(\epsilon,r)$ cannot vanish if r, as in our case, does not vanish. Thus, on the Planck lattice, the self energy integral equation (5) acquires an inhomogeneous term even for $m_t = 0$. This implies the very important consequence that (5) admits only massive solutions provided $\lambda > 0$ [8]. The most appealing aspect of this result is that on the Planck lattice mass gets generated for all fermions, but the top-quark, without the appearance of Goldstone bosons: the reason for this fact, extremely important phenomenologically, is the connection of the inhomogeneous term $\delta_{PL}(\epsilon, r)$ with a non zero r-value that, as emphasized, already breaks the chiral symmetry of the four-fermion interaction.

We know from our mathematical analysis that the fermion mass $\Sigma_c(p) \simeq m$ is certainly non-zero, how do we know that it is not proportional to the Planck mass $\Lambda_p = \frac{\pi}{a_p}$? In order to answer this important question we note that if $M_0 = \frac{cr}{a_p}$ were such that the terms in eqn. (5), that are in principle proportional to $\frac{1}{a_p}$, vanish then $\delta_{PL}(\epsilon, r)$ would automatically be finite, thus ensuring fermion masses much smaller than the Planck mass. A sufficient condition for this to happen is the vanishing of the expression

$$\frac{cr}{a_p} + \frac{\lambda r}{a_p} \int \frac{d^4 l}{12\pi^2} \frac{1}{(4\sin^2(\frac{l_\mu}{2}))} \left[\frac{1}{2} - \frac{\frac{1}{r} G_{PL}(l)(-\cos^2(\frac{l_\mu}{2}) + r^2 \sin^2(\frac{l_\mu}{2})) + \sin^2(l_\mu)}{G_{PL}(l)^2 + \sin^2(l_\mu)} \right] = 0, \quad (7)$$

as for the coefficient of the "bare" mass term, c is the very small value $c \simeq 0.07\lambda$ [9]. We can thus write the finite part of $\delta_{PL}(r, \epsilon)$ is

$$\delta^f_{PL}(r, \epsilon) = -\int_{[\epsilon,\pi]^4} \frac{d^4 l}{16\pi^2} \frac{\Sigma_c(l)}{(4\sin^2(\frac{l_\mu}{2}) + (a_p m_v)^2)} \left[\frac{-\cos^2(\frac{l_\mu}{2}) + r^2 \sin^2(\frac{l_\mu}{2})}{G_{PL}(l)^2 + \sin^2(l_\mu)} \right]. \quad (8)$$

Let's now address the important question of the ϵ-independence of our result. The introduction of the "dividing scale" in (2), apart from the requirement $pa_p \ll \epsilon \ll \pi$, is rather arbitrary; thus no dependence on ϵ should appear in our final results. In order for such independence to occur, as it must occur, it is clear that the ϵ-dependence from the continuum integral in eqn. (5), that is clearly logarithmic, must be compensated by an analogous logarithmic term arising in the calculation of $\delta_{PL}(r, \epsilon)$. Thus segregating the $\ell n\, \epsilon$-term in $\delta^f_{PL}(r, \epsilon)$, we may write

$$\delta^f_{PL}(r, \epsilon) = -\frac{\lambda\bar{\Sigma}}{2} \ell n\, \epsilon + \lambda\bar{\Sigma}\delta^0_{PL}(r), \quad (9)$$

where $\delta^0_{PL}(r)$ is independent of ϵ, $\bar{\Sigma}$ is the asymptotic value of $\Sigma_c(l)$ and the numerical evaluation of $\lambda\delta^0_{PL}(r)$ is reported in fig. 2. Defining

$$\Delta(p) = \Sigma_c(p) - \bar{\Sigma}, \quad (10)$$

eqn. (5)becomes

$$\bar{\Sigma} + \Delta(p) = \frac{\lambda}{4\pi^2} \int d^4q \frac{1}{(p-q)^2 + m_v^2} \frac{\Delta(q)}{q^2 + m^2} + \bar{\Sigma}\left[\frac{\lambda}{2}\ell n\left(\frac{\Lambda_P}{m}\right) + \lambda\delta^{\circ}_{PL}(r)\right] \qquad (11)$$

where in view of the asymptotic vanishing of $\Delta(q)$ the momentum integral is cut-off clearly independent. For $m_v^2 = 0$, the case of the e.m. interaction, a standard analysis easily shows that the integral equation (11) admits trivial a solution for $\Delta(p)$, as a result of the boundary condition $\Delta(\infty) = 0$. Thus, for consistency, from (11) one derives the "gap" equation

$$1 = \frac{\lambda}{2}\ell n\left(\frac{\Lambda_P}{m}\right) + \lambda\delta^{\circ}_{PL}(r), \qquad (12)$$

where in the spirit of our rainbow approximation λ is given the bare coupling constant $g_{\circ}$. In view of the extreme sensitivity of the gap equation to the actual value of λ, and of our neglect of all other gauge interactions, $SU_c(3)$ and $SU_L(2)$, we do not think a numerical estimate of the size of masses that may result from it to be worthwhile.

We end this paper with the observation that, when formulated on the Planck lattice, also the self-energy problem of fermions that did not receive any mass through the NJL-mechanism inherent in the SM on a Planck lattice appears to have a solution completely different from the one encounters in the continuum. One obtains for all charged fermions a non-zero mass, whose size depends (only and very sensitively) from the bare gauge coupling constants at the Planck scale. And due to the intrinsic violation of chiral symmetry of the NJL-mechanism, through a non-zero Wilson's r-parameter, no additional Goldstone bosons appear in the spectrum of fundamental particles.

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[9] We note here that the basic and well known "fine-tuning" inherent in NJL-interaction demands the development of all the lattice "counterterms" needed to protect the fermion masses from being $O(\frac{1}{a_p})$.

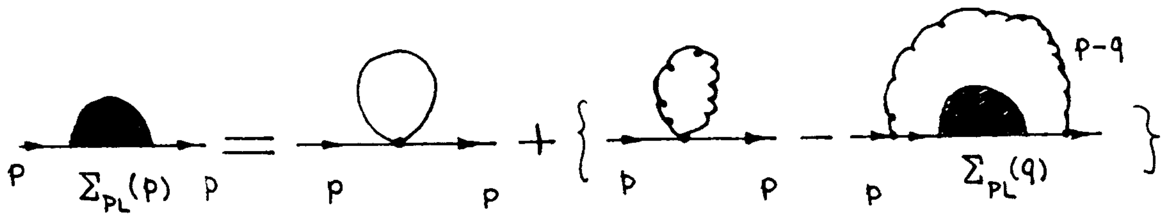

Fig. 1 The diagrammatic form of Dyson-Schwinger equation

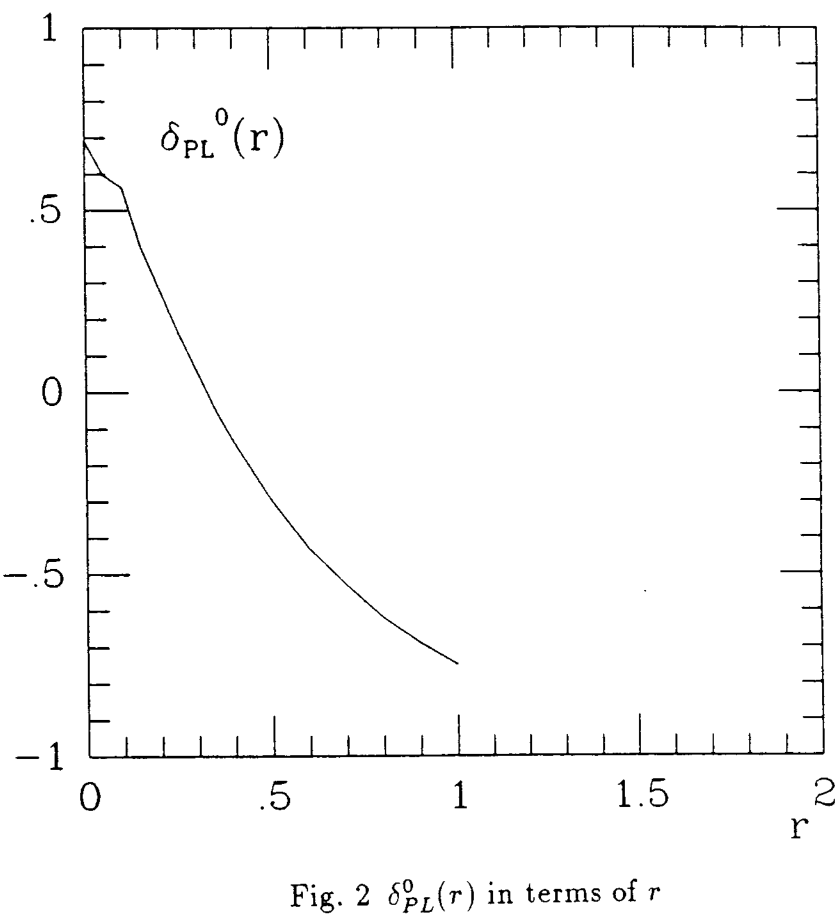

Fig. 2 $\delta^0_{PL}(r)$ in terms of r