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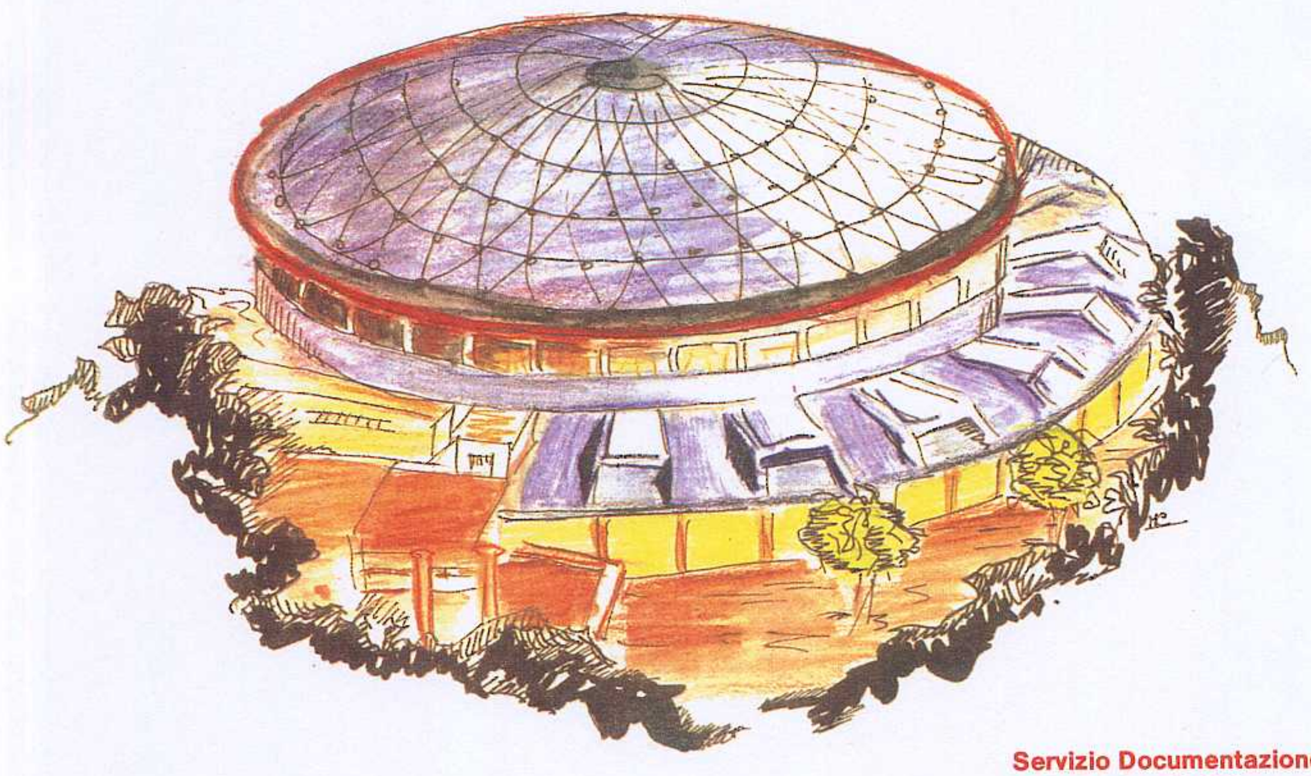

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Angular Distributions for $\phi \to \pi\pi\gamma$ decays at DAΦNE

GILBERTO COLANGELO

Universitá di Tor Vergata, Roma, Italy, and
INFN, Laboratori Nazionali di Frascati, Italy

PAULA J. FRANZINI

*Laboratoire de Physique Théorique ENSLAPP,**
B.P. 110, F-74941 Annecy-Le-Vieux Cedex, France

ABSTRACT

We derive the complete angular distribution for the $\phi \to \pi\pi\gamma$ processes with two pions in a C-even state which are of interest for the study of the $f_0(975)$ at a high luminosity e^+e^- collider such as the DAΦNE ϕ–factory. These are $e^+e^- \to \phi \to f_0\gamma \to \pi\pi\gamma$, the background reaction $e^+e^- \to \phi \to \pi\rho \to \pi\pi\gamma$ and, for charged pions, the process $e^+e^- \to \rho \to \pi\pi\gamma$ and the interference of the f_0 signal with this process.

⋆ *URA 14-36 du CNRS, associée à l'E.N.S. de Lyon, et au L.A.P.P. d'Annecy-le-Vieux*

The Frascati ϕ factory DAΦNE,[1] beginning in 1995, will deliver a luminosity $\mathcal{L}\sim 10^{32}$ cm^{-2} s^{-1}, producing of the order of 5 billion ϕ's in four months of machine-on time. While the primary goal of DAΦNE is the study of CP-violation, an important complementary goal is spectroscopy. One interesting example is the study of the lightest scalar ($I^G\ J^{PC} = 0^+\ 0^{++}$) meson, the $f_0(975)$. If the f_0 is interpreted as a normal two-quark bound state, predictions for many of its measured properties fail: for example its total width (much narrower than expected), its $\gamma\gamma$ couplings, and the ratio $\Gamma(f_0 \rightarrow \pi\pi)/\Gamma(a_0 \rightarrow \pi\pi)$, a_0 being the lightest $1^-\ 0^{++}$ meson. Thus, more exotic possibilities have been proposed, such as four-quark and $K\bar{K}$ bound states (for recent discussions and references, see for example Refs. 2 and 3). One measurable quantity that would help in choosing among these possibilities is the branching ratio BR($\phi \rightarrow f_0\gamma$). Currently, it is only known to be less than 2×10^{-3}. For a four quark state it is expected[2] to be $\mathcal{O}(10^{-4})$, for an $s\bar{s}$ state, $\mathcal{O}(10^{-5})$, and for a diffuse $K\bar{K}$ system, smaller than $\mathcal{O}(10^{-5})$. In any case, DAΦNE will be a good place to study the f_0, with 5000 to 500,000 f_0's expected in four months of full luminosity.

In Refs. 4, 5, and 6, we studied in detail the feasibility of f_0 spectroscopy at a high luminosity e^+e^- collider such as DAΦNE. We found that it should be highly successful, in both the neutral mode $\pi^0\pi^0\gamma$ and the charged mode $\pi^+\pi^-\gamma$. One will be able to determine the sign of the $\phi f_0\gamma$ coupling, currently unknown, thanks to the interference between the f_0 signal and the final state radiation process (FSR), $e^+e^- \rightarrow \rho \rightarrow \pi\pi\gamma$. The BR($\phi \rightarrow f_0\gamma$) will be determinable in each mode to accuracies varying from a fraction of a percent to ten percent, over the expected range of signal and background BR's, and depending on the sign of the interference.

Background is a particular problem in the charged mode. In this case there are two main sources: initial state radiation, $e^+e^- \rightarrow \rho^*\gamma \rightarrow \pi^+\pi^-\gamma$, which can be some two to three orders of magnitude larger than the signal, but contributes incoherently, as it is the only process in which the two pions are in a C–odd state; and FSR, which is a factor of ten smaller than initial state radiation, but does interfere with the signal. The initial state radiation background is already well-known (see, e.g., Refs. 7, 8, 4). It is very sharply peaked for small angles between the photon and the beam, θ_γ. For example, by restricting $|\cos\theta_\gamma| < 0.9$, this background is reduced by a factor of 7; however, the signal will at the same time be reduced somewhat. A knowledge of the full angular distributions of the relevant C–even two–pion processes (the signal $e^+e^- \rightarrow \phi \rightarrow f_0\gamma \rightarrow \pi\pi\gamma$ and the background (VDM) $e^+e^- \rightarrow \phi \rightarrow \pi\rho \rightarrow \pi\pi\gamma$, and, for charged pions, FSR, and the interference between FSR and the signal) is thus essential to choose cuts that improve the signal to background ratio, and is the subject of the present Letter. The angular distributions are of course also generally necessary for any simulations of $e^+e^- \to \phi \to \pi\pi\gamma$, and carry more information than the integrated photon spectra.

One expects the photon distribution due to FSR to be peaked along the pions, that due to $f_0 \to \gamma\pi\pi$ to have a $(1 + \cos^2\theta_\gamma)$ dependence, and the interference contribution some combination of the two. One would hope to enhance the signal to background ratio quoted in Ref. 4 by applying cuts to the collinearity between photon and pions. For this to be done in a realistic way, one should consider a real detector, and use a Monte Carlo that generates events following our distributions, and that takes into account properly the geometry and efficiency of the detector. Such an analysis has been done in Ref. 6. With the two cuts $|\cos\theta_\gamma| < 0.9$ and $|\cos\theta_{\pi\gamma}| < 0.9$ (where $\theta_{\pi\gamma}$ is the angle between the pions and the photon in the dipion rest frame) one retains 80% of the signal and improves the signal to background ratio by a factor of 5 to 6.

To obtain these distributions, it is essential to account for the polarization of the decaying resonance, *as produced in an e^+e^- collision*: we must calculate Feynman diagrams starting with e^+e^- and average over e^+e^- spins. The amplitude for $\phi \to \gamma X$ has the general form $A(\phi \to \gamma X) = \epsilon^{*\mu} R_\mu$, which defines R_μ, with $\epsilon^{*\mu}$ the polarization of the ϕ. The complete amplitude for $e^+e^- \to \phi \to \gamma X$ is given by

$$\mathcal{A}(e^+e^- \to \phi \to \gamma X) = \bar{e}\gamma_\mu e R_\mu \frac{1}{D_\phi(s)}, \tag{1}$$

where $1/D_\phi(s)$ is the ϕ propagator. The differential cross-section is given by

$$d\sigma(e^+e^- \to \phi \to \gamma X) = \sum_{\text{spins}} |\mathcal{A}|^2 \mathcal{K}. \tag{2}$$

For a decay to two particles, 1 and 2:

$$\mathcal{K} = \frac{1}{8\pi} \frac{|\vec{p}_1|}{M_\phi^2} \frac{\mathrm{d}\phi_1}{2\pi} \frac{\mathrm{d}\cos\theta_1}{2}. \tag{3}$$

For a decay to three particles, 1, 2 and 3:

$$\mathcal{K} = \frac{1}{64\pi^3} \frac{1}{M_\phi^2} \mathrm{d}E_1 \mathrm{d}E_2 \frac{\mathrm{d}\phi_1}{2\pi} \frac{\mathrm{d}\cos\theta_1}{2} \frac{\mathrm{d}\phi_2}{2\pi}. \tag{4}$$

As usual, $\mathcal{K}$ has an extra factor of $\frac{1}{2}$ if there are two identical particles in the final state. We note, for future convenience, that existing calculations done for ϕ decay with the ϕ unpolarized are easily converted by replacing the polarization vector product $\epsilon^{*\mu}\epsilon^{*\nu}$

with $l_-^\mu l_+^\nu + l_+^\mu l_-^\nu - g^{\mu\nu} l_- \cdot l_+$ and dividing by M_ϕ^2 to get the decay width of a ϕ originating in an e^+e^- collision. Here $l_\pm$ are the e^+, e^- 4–momenta. Likewise in the following p, q are 4–momenta and $p \cdot q$ is the Lorentz invariant scalar product. For $\phi \to f_0\gamma$,

$$R_\mu = eG_s\left(\epsilon_\mu q \cdot q^* - q_\mu \epsilon \cdot q^*\right), \tag{5}$$

where q is the photon momentum, ϵ its polarization, and q^* the ϕ momentum. To then get the $\phi \to f_0\gamma \to \pi^+\pi^-\gamma$ decay amplitude we need only multiply by the coupling $g_s = A(f_0 \to \pi^+\pi^-)$, and the f_0 propagator, $1/(M_{f_0}^2 - iM_{f_0}\Gamma_{f_0} - s')$, where $s' = M_{\pi\pi}^2 = M_\phi^2 - 2M_\phi E_\gamma$. There is no further angular dependence from the isotropic decay of the f_0 to pions. The amplitude must also be damped by the bound quark pair wavefunction (see, for example, Refs. 9, 6).

For FSR,

$$R_\mu = 2\sqrt{2}eg\left(\left(p_{-\mu} - \frac{1}{2}q_\mu^*\right)\frac{\epsilon \cdot p_+}{q \cdot p_+} + \left(p_{+\mu} - \frac{1}{2}q_\mu^*\right)\frac{\epsilon \cdot p_-}{q \cdot p_-} + \epsilon_\mu\right), \tag{6}$$

where $p_\pm$ are the pion momenta, and q^* here is the ρ momentum. The interference between these processes is proportional to $2R_\mu R'_\nu$, using the two R's given in eqs. (5) and (6).

For $\phi \to \pi\rho \to \pi^+\pi^-\gamma$ (VDM),

$$R_\mu = \left(\frac{\epsilon G^2 e}{3g_e\sqrt{2}}\right)\left(\frac{\epsilon_{\mu\alpha\beta\gamma}(q_\alpha + p_\alpha^+)q_\beta^*\epsilon_{\gamma\delta\lambda\sigma}q_\delta(q_\lambda + p_\lambda^+)\epsilon_\sigma}{M_\rho^2 - (q+p^+)^2 - iM_\rho\Gamma_\rho}\right) + (p^+ \leftrightarrow p^-) \tag{7}$$

where $G = (3\sqrt{2}g_e^2)/(4\pi^2 f_\pi)$.

The most compact way to display our results is to borrow the notation of Creutz and Einhorn.[7] We initially did our calculations without knowing of this reference, and in later checking agreement with the results given in ref. 7, found a factor of 1/2 and a minus sign in their expression that we disagreed with. We have subsequently confirmed with the authors[10] that these differences* were previously unnoticed typographical errors in their published results. We define the dilepton and dipion scalar invariants $s = (l_- + l_+) \cdot (l_- + l_+)$ and $t = (p_+ + p_-) \cdot (p_+ + p_-)$; the pion velocity in the dipion rest frame, $\beta_\pi = \sqrt{1 - \xi/(1-x)}$; and the angles: ϕ, between the $e^+e^-\gamma$ plane and the $\pi^+\pi^-\gamma$ plane in either the lab frame or the dipion rest frame; θ_γ, between photon and

* The differences consist of an extra 1/2 in the second line of eq. (11) in their paper, and an opposite sign on the $\cos\gamma\cos\theta_{\pi\gamma}$ term in the same line. Other apparent differences in form come from different conventions (our $\cos_{\pi\gamma}$ is defined with the opposite sign from theirs) and are of no consequence.

beam in the lab frame; and $\theta_{\pi\gamma}$, between the pions and the photon *in the dipion rest frame*. Here we use $x = 2E_\gamma/\sqrt{s}$ and $\xi = 4m_\pi^2/s$. The pion energy is related to $\theta_{\pi\gamma}$ by

$$y = 1 - \frac{x}{2}\left(1 + \cos\theta_{\pi\gamma}\sqrt{1 - \frac{\xi}{1-x}}\right). \tag{8}$$

where $y = 2E_\pi/s$. E_γ and E_π are the energies of the photon and one of the pions in the lab frame. Our sign convention for $\cos\phi$ is specified by

$$(l_- - l_+)\cdot(p_- - p_+) = -\beta_\pi\sqrt{st}\,(\gamma\cos\theta_{\pi\gamma}\cos\theta_\gamma + \sin\theta_{\pi\gamma}\sin\theta_\gamma\cos\phi). \tag{9}$$

The most general form[7] for the C–even $\pi^+\pi^-\gamma$ vertex may be written in terms of three factors H_1, H_2 and H_3 (we give the explicit forms for these factors in the cases we are considering in eqs. (12)–(16)):

$$\begin{aligned} R_\mu = C\Bigg(&\left[\epsilon\cdot q^*\frac{q\cdot(p_- - p_+)}{q\cdot q^*} - \epsilon\cdot(p_- - p_+)\right]\left[H_1\left(q^*_\mu - \frac{sq_\mu}{q\cdot q^*}\right)\right. \\ &\left. + H_2\left((p_- - p_+)_\mu - \frac{q\cdot(p_- - p_+)}{q\cdot q^*}q_\mu\right)\right] + H_3\left[\epsilon_\mu - \frac{\epsilon\cdot q^*}{q\cdot q^*}q_\mu\right]\Bigg). \end{aligned} \tag{10}$$

C here denotes the overall multiplicative factor in the front of each R_μ in eqs. (5)–(7). We then get the general expression

$$\begin{aligned} |\mathcal{A}|^2 = C^2 s^2\Bigg(&\frac{1}{2}|H_1|^2\beta_\pi^2 t\sin^2\theta_\gamma\sin^2\theta_{\pi\gamma} + \frac{1}{2}|H_2|^2\beta_\pi^4 t\sin^2\theta_{\pi\gamma} \\ &\times\left(\cos^2\theta_{\pi\gamma} + \frac{t}{s}\sin^2\theta_{\pi\gamma} - (\cos\theta_\gamma\cos\theta_{\pi\gamma} - \sqrt{\frac{t}{s}}\sin\theta_\gamma\sin\theta_{\pi\gamma}\cos\phi)^2\right) \\ &+ |H_3|^2(1+\cos^2\theta_\gamma)/(2s) \\ &+ \mathrm{Re}(H_1H_2^*)\beta_\pi^3 t\sin\theta_\gamma\sin^2\theta_{\pi\gamma}\left(\sqrt{\frac{t}{s}}\sin\theta_{\pi\gamma}\cos\theta_\gamma\cos\phi + \sin\theta_\gamma\cos\theta_{\pi\gamma}\right) \\ &+ \mathrm{Re}(H_1H_3^*)\beta_\pi\sqrt{\frac{t}{s}}\sin\theta_\gamma\cos\theta_\gamma\sin\theta_{\pi\gamma}\cos\phi + \mathrm{Re}(H_2H_3^*)\beta_\pi^2\sqrt{\frac{t}{s}}\sin\theta_{\pi\gamma} \\ &\times\left(\sin\theta_\gamma\cos\theta_\gamma\cos\theta_{\pi\gamma}\cos\phi + \sqrt{\frac{t}{s}}\sin\theta_{\pi\gamma}(1-\sin^2\theta_\gamma\cos^2\phi)\right)\Bigg). \end{aligned} \tag{11}$$

Eq. (11) can be seen to be symmetric under $\pi^+ \leftrightarrow \pi^-$ exchange (change of the sign of $\cos\theta_{\pi\gamma}$) if we note that H_1 is antisymmetric while H_2 and H_3 are symmetric.

For the processes we are considering, we have:

$$f_0: \qquad H_1 = 0, \quad H_2 = 0, \quad H_3 = q\cdot q^* = sx/2, \tag{12}$$

i.e.. one term only, with a $(1+\cos^2\theta_\gamma)$ dependence and

$$\mathrm{FSR}: \qquad H_1 = 0, \quad H_2 = \frac{x}{2s(x+y-1)(1-y)}, \quad H_3 = 1, \tag{13}$$

three terms with a complicated angular dependence. The only clear feature is that H_2 has the expected collinear divergence regulated by the mass of the pions.

For VDM we have

$$H_i = \frac{h_i}{D} + \frac{h_i'}{D(\pi^+ \leftrightarrow \pi^-)}, \qquad D = M_\rho^2 - iM_\rho\Gamma_\rho - (M_\phi^2 + m_\pi^2 - 2M_\phi E_\pi) \tag{14}$$

with

$$h_1 = -\frac{sx}{8}, \quad h_2 = \frac{sx}{8}, \quad h_3 = \frac{s^2}{8}(2x^2 + x\xi - 2x - 2y^2 + 2y). \tag{15}$$

$$h_1' = -h_1, \quad h_2' = h_2, \quad h_3' = h_3(\pi^+ \leftrightarrow \pi^-). \tag{16}$$

The h_i and h_i' terms come from the direct and crossed diagrams, and the separate contributions from these diagrams and their interference may be easily written in terms of the h_i. For example, using $H_i = h_i/|D|$ we get the result for the direct diagram alone, and replacing $|H_i|^2$ by $2h_ih_i'\text{Re}\left(\frac{1}{DD'}\right)$ and $\text{Re}\left(H_iH_j^*\right)$ by $(h_ih_j' + h_i'h_j)\text{Re}\left(\frac{1}{DD'}\right)$ we get the result for their interference.

For the f_0-FSR interference term, each H_iH_j factor in eq. (11) must be replaced by $H_i(f_0)H_j(\text{FSR}) + H_i(\text{FSR})H_j(f_0)$ and the $\mathcal{C}^2$ factor by $\mathcal{C}(f_0)\mathcal{C}(FSR)$. The f_0-VDM interference term can also be calculated this way but is small in comparison.

In conclusion, combining all these equations then gives the full angular dependence for the two–π C–even $\phi_{e^+e^-} \to \pi^+\pi^-\gamma$ processes. $\phi_{e^+e^-} \to \pi^0\pi^0\gamma$ is identical except for the factor of 1/2 in $\mathcal{K}$, and the exclusion of those processes not present in the neutral channel (final and initial state radiation). Three-dimensional graphs of these distributions can be found in Refs. 4, 5 and 6. The angle ϕ may be trivially integrated over by replacing $\cos^2\phi$ by $\frac{1}{2}$, and $\cos\phi$ by zero (after first expanding the squared expression in the second line of eq. (11)). We have checked that integrating these results over $\frac{d\cos\theta_\gamma}{2}$ yields the previously found results[11] derived without starting from e^+e^-.

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