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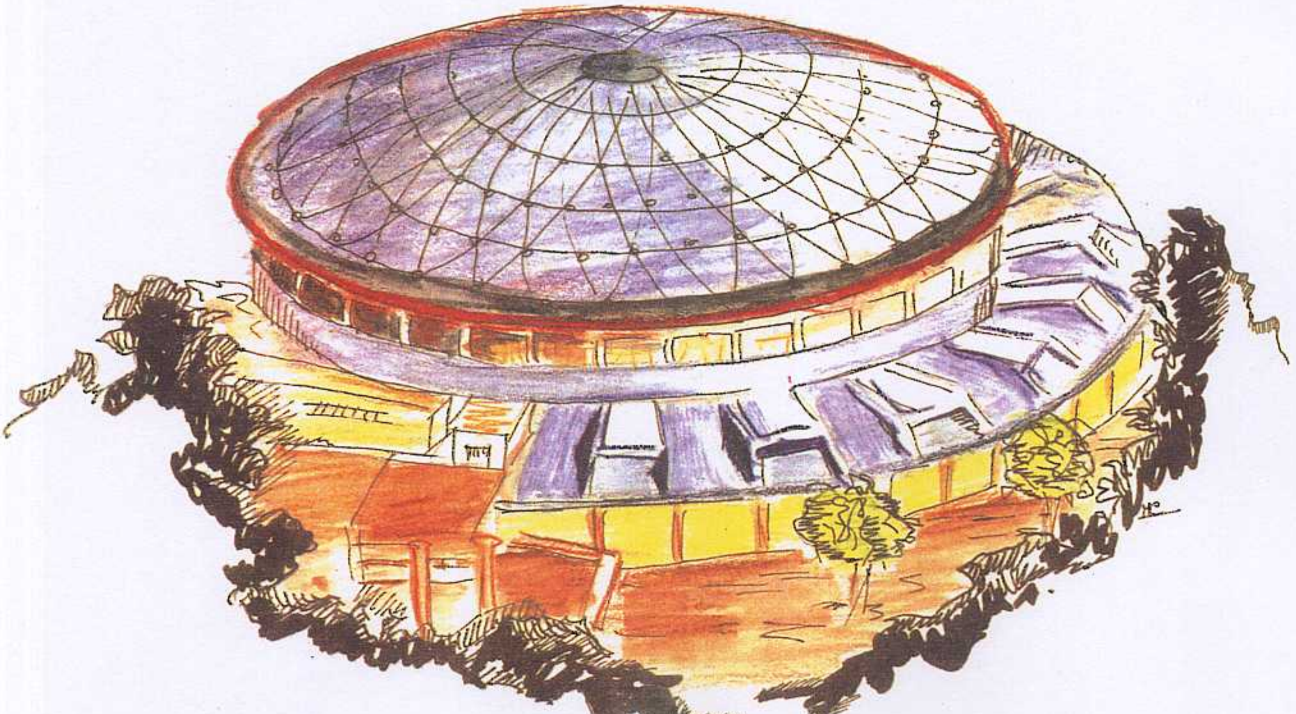
V. Patera, A. Pugliese:

ASYMMETRIES AND TIME EVOLUTION IN THE $\overline{K}^0$ K^0SYSTEM

Contribution to the DAΦNE Physics Handbook



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Asymmetries and time evolution in the $\bar{K^0}K^0$ system

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Abstract

Starting from the time evolution of the C-odd $\bar{K^0}K^0$ system, we study the CP, T and CPT violation by means of the experimentally measurable asymmetries. In particular in $K_L \to 2\pi$ CP violating decays an accuracy of $\simeq 2 \times 10^{-4}$ and 3×10^{-3} on $\Re\left(\frac{\epsilon'}{\epsilon}\right)$ and $\Im\left(\frac{\epsilon'}{\epsilon}\right)$ seems to be achievable. CP, T and CPT violation parameters can be explored in K_S semileptonic decays with an accuracy of the order of 10^{-3}.

1 Introduction

The $\bar{K^0}\ K^0$ state produced in the decay of the ϕ resonance is odd under charge conjugation and is therefore an antisymmetric $K_L\ K_S$ state. Therefore in a ϕ factory it is possible to have in the same detector a K_L and a K_S decay without the need of a regenerator and with relative flux of K_L and K_S perfectly known.

For this reason the ϕ factory is very suitable to study CP violation in K decays and to measure the ratio $\frac{\epsilon\prime}{\epsilon}$[1]. Moreover the coherence of the initial state supplies several independent tests of CPT[2, 3] different from the measurement of $\Im\left(\frac{\epsilon\prime}{\epsilon}\right)$ and the presence of a pure K_S beam allows the determination of the branching ratios of some suppressed K_S decays like the semileptonic ones.

To extract $\frac{\epsilon\prime}{\epsilon}$ from experimental data, the time evolution of the initial $\bar{K^0}K^0$ state can be exploited and time asymmetries in $\pi^+\pi^-, \pi^0\pi^0$ final state can be studied, as suggested in [4]. Otherwise the so called double ratio R of NA31[5] and E731[6] experiments can be measured by tagging the K_S and K_L[7]. Another method is the comparison of branching ratios of decays with different charge configurations[8].

In the following we will refer , for the experimental set-up, to a ϕ-factory similar to the DAFNE project[9]. At that facility a conservative estimate of the achievable luminosity is $L = 5 \times 10^{32}\ cm^{-2}s^{-1}$ with corresponding production of 2.5×10^{10} ϕ per year.

2 Time evolution and the real part of $\frac{\epsilon\prime}{\epsilon}$

Let us consider the following decay chain

$$\phi \rightarrow (\bar{K^0}K^0) = \frac{1}{\sqrt{2}}[K_S K_L - K_L K_S] \rightarrow \pi^+\pi^-, \pi^0\pi^0 \tag{1}$$

The differential rate of events with a $\pi^+\pi^-$pair produced at distance d_c from the interaction vertex and a $\pi^0\pi^0$ pair produced at distance d_n is given by

$$\begin{aligned} N(d_c, d_n) \ = \ & \Gamma_S^{+-}\Gamma_S^{00}\left[\mid \eta^{+-}\mid^2 \exp\left(-\frac{d_c}{d_L} - \frac{d_n}{d_S}\right) + \mid \eta^{00}\mid^2 \exp\left(-\frac{d_n}{d_L} - \frac{d_c}{d_S}\right)\right. \\ & \left. -2\exp\left(-\frac{\Gamma}{\Gamma_S}\frac{d_c + d_n}{d_S}\right) Re\left(\eta^{+-}(\eta^{00})^* \exp(-i\frac{\Delta m}{\Gamma_S}\frac{d_c - d_n}{d_S})\right)\right] \end{aligned} \tag{2}$$

Where

$$\Gamma_S^{ab} = \Gamma\left(K_S \rightarrow \pi^a\pi^b\right) \qquad \Gamma = \frac{1}{2}(\Gamma_S + \Gamma_L) \qquad \Delta m = m_L - m_S$$

$$\begin{aligned} \eta^{+-} &= \frac{A(K_L \rightarrow \pi^+\pi^-)}{A(K_S \rightarrow \pi^+\pi^-)} = \epsilon + \epsilon\prime \\ \eta^{00} &= \frac{A(K_L \rightarrow \pi^0\pi^0)}{A(K_S \rightarrow \pi^0\pi^0)} = \epsilon - 2\epsilon\prime \end{aligned} \tag{3}$$

d_S and d_L are the mean decay paths of the K_S and the K_L. From the decay of a ϕ at rest d_S = 0.58 cm and d_L =340 cm.

The rate of the events inside the detector with a fixed value of the difference $d = d_c - d_n$ is :

$$
\begin{aligned}
N(d) &= \frac{\Gamma_S^{+-}\Gamma_S^{00}}{2\Gamma}\left\{\left[|\eta^{+-}|^2\left(\exp\left(\frac{-d}{d_L}\right) - \exp\left(\frac{-D}{d_L} - \frac{D-d}{d_S}\right)\right) + |\eta^{00}|^2 \exp\left(\frac{-d}{d_S}\right) - \right.\right. \\
&\left. - 2\exp\left(\frac{-\Gamma}{\Gamma_S}\frac{d}{d_S}\right)\Re\left(\eta^{+-}\left(\eta^{00}\right)^* \exp\left(-i\frac{\Delta m}{\Gamma_S}\frac{d}{d_S}\right)\right)\right] \times \theta(d) + \\
&+ \left[|\eta^{00}|^2\left(\exp\left(\frac{d}{d_L}\right) - \exp\left(\frac{-D}{d_L} - \frac{D+d}{d_S}\right)\right) + |\eta^{+-}|^2 \exp\left(\frac{d}{d_S}\right) - \right. \\
&\left.\left. - 2\exp\left(\frac{\Gamma}{\Gamma_S}\frac{d}{d_S}\right)\Re\left(\eta^{00}\left(\eta^{+-}\right)^* \exp\left(i\frac{\Delta m}{\Gamma_S}\frac{d}{d_S}\right)\right)\right] \times \theta(-d)\right\}
\end{aligned}
\tag{4}
$$

where $D \gg d_S$ is the maximum decay path inside the detector. In eq.(4) the terms proportional to $\exp\left(-\frac{D}{d_S}\right)$ have been neglected. In a typical detector at DAFNE, $D \simeq 150\ cm.$ and the K_L acceptance $S_L = 1 - \exp\left(\frac{-D}{d_L}\right)$ is therefore equal to 35%.

If $\epsilon\prime \neq 0$ there is an asymmetry between the events with positive and negative values of d

$$A(d) = \frac{N(|d|) - N(-|d|)}{N(|d|) + N(-|d|)} = A_R(d) \times \Re\left(\frac{\epsilon\prime}{\epsilon}\right) - A_I(d) \times \Im\left(\frac{\epsilon\prime}{\epsilon}\right) \tag{5}$$

The study of the $|d|$ dependence of the asymmetry allows, in principle, the determination of the real and imaginary part of $\left(\frac{\epsilon\prime}{\epsilon}\right)$. The $A_R(d)$ and $A_I(d)$ coefficients, shown in Fig.(1), are defined as :

$$
\begin{aligned}
A_R(d) &= 3\frac{\exp\left(\frac{-|d|}{d_L}\right) - \exp\left(\frac{-|d|}{d_S}\right)}{\exp\left(\frac{-|d|}{d_L}\right) + \exp\left(\frac{-|d|}{d_S}\right) - 2\cos\left(\frac{\Delta m}{\Gamma}\frac{|d|}{d_S}\right)\exp\left(\frac{-\Gamma|d|}{\Gamma_S d_S}\right)} \\
A_I(d) &= 3\frac{2\sin\left(\frac{\Delta m}{\Gamma_S}\frac{|d|}{d_S}\right)\exp\left(\frac{-\Gamma}{\Gamma_S}\frac{|d|}{d_S}\right)}{\exp\left(\frac{-|d|}{d_L}\right) + \exp\left(\frac{-|d|}{d_S}\right) - 2\cos\left(\frac{\Delta m}{\Gamma}\frac{|d|}{d_S}\right)\exp\left(\frac{-\Gamma|d|}{\Gamma_S d_S}\right)}
\end{aligned}
\tag{6}
$$

In eq.(5) terms proportional to $\left(\frac{\epsilon\prime}{\epsilon}\right)^2$ have been neglected .

It can be seen that $A_R(d)$ becomes nearly independent on d , and equal to 3, for $d >> d_S = 0.6\ cm$. On the other hand $A_I(d)$ is strongly dependent of d and vanishes for $d >> d_S$. Therefore a measurement of the asymptotic value of $A(d)$ or of the value of the integrated asymmetry A

$$A = \frac{N(d>0) - N(d<0)}{N(d>0) + N(d<0)} \simeq 3\Re\left(\frac{\epsilon\prime}{\epsilon}\right) \tag{7}$$

allows a clean determination of $\Re\left(\frac{\epsilon\prime}{\epsilon}\right)$.The statistical error on A is given by:

$$\sigma_A = \sqrt{\frac{(1+A)\times(1-A)}{N}} \tag{8}$$

where N is the number of $\phi \to \pi^+\pi^-, \pi^0\pi^0$ events. At the the reference DAFNE luminosity the statistical error on $\Re\left(\frac{\epsilon\prime}{\epsilon}\right)$ is then :

$$\sigma_{\Re\left(\frac{\epsilon\prime}{\epsilon}\right)} \simeq \frac{\sigma_A}{3} \simeq \frac{1}{3\sqrt{N}} \simeq 1.7 \times 10^{-4} \tag{9}$$

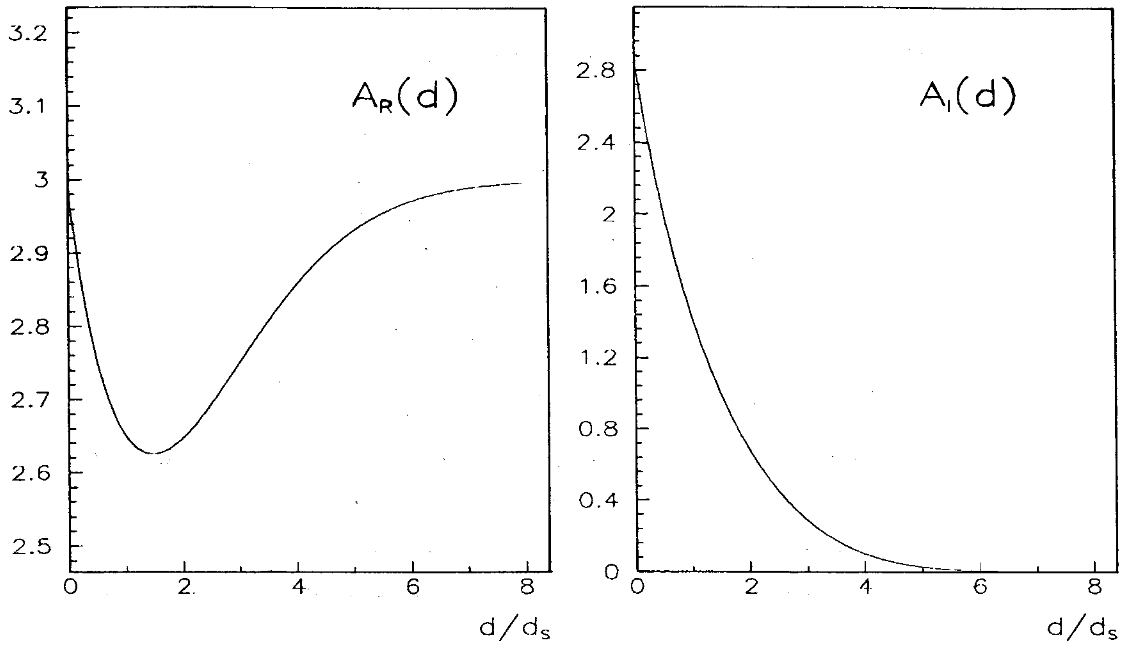

Figure 1: Coefficients of $\Re\left(\frac{\epsilon'}{\epsilon}\right)$ and $\Im\left(\frac{\epsilon'}{\epsilon}\right)$ in the N(d) distributions

3 The $\Im\left(\frac{\epsilon'}{\epsilon}\right)$ and the experimental decay vertex resolution

The integrated asymmetry A allows a precise determination of $\Re\left(\frac{\epsilon'}{\epsilon}\right)$ but gives no information on the imaginary part of $\frac{\epsilon'}{\epsilon}$.

To overcome this problem a further method can be exploited to measure both $\Re\left(\frac{\epsilon'}{\epsilon}\right)$ and $\Im\left(\frac{\epsilon'}{\epsilon}\right)$ from the $K_L K_S \to \pi^0\pi^0, \pi^+\pi^-$ decay time difference: the experimental distribution $N(d)$ can be fitted by the theoretical distribution of eq.(4) and $\Re\left(\frac{\epsilon'}{\epsilon}\right)$ and $\Im\left(\frac{\epsilon'}{\epsilon}\right)$ can be used as free parameters of the fit.

It must be stressed that this procedure is very sensitive to the experimental resolution on the measurement of d. The information contained in the shape of the $N(d)$ distribution can be easily washed out, in particular in the region of interest for the determination of $\Im\left(\frac{\epsilon'}{\epsilon}\right)$, where $d \simeq d_S$. In fact only in this range of d values $A_I(d) \neq 0$ and the strongly variating behaviour of $A_I(d)$ can be smeared out by a bad vertex reconstruction.

This effect is shown in fig.2 where the theoretical distribution is compared to a simulated experimental distribution with a gaussian error on the d measurement equal to 5 mm.

To take into account the effect of the finite experimental resolution we assume that the corresponding error distribution $g_\sigma(x)$ is well known and we define an experimental decay time difference distribution :

$$f_\sigma^{exp}(y) = \int_{-\infty}^{\infty} f(d)\, g_\sigma(y-d)\, d(d) \qquad (10)$$

y is the actual measured $d_c - d_n$ distance. The $\Re\left(\frac{\epsilon'}{\epsilon}\right)$ and $\Im\left(\frac{\epsilon'}{\epsilon}\right)$ parameter can now be extracted by fitting the data with this corrected distribution.

The statistical accuracy that can be achieved in the determination of a parameter p

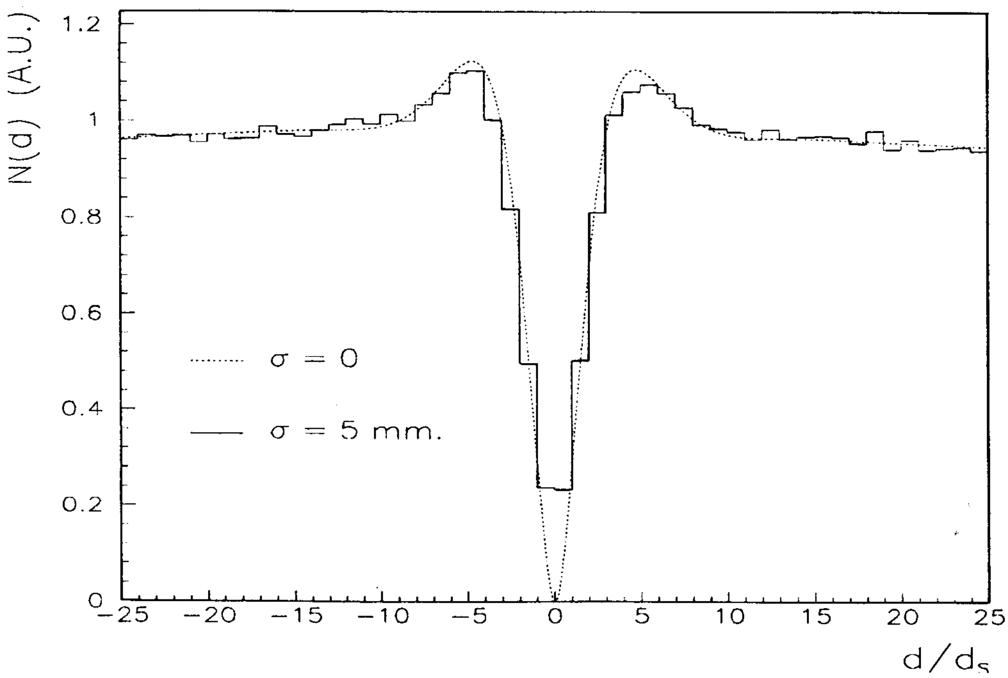

Figure 2: Comparison between the theorical N(d) distribution and that obtained with a $\sigma = 5mm$ vertex resolution

from a N events sample distributed with a function $f(x,p)$ is given by :

$$\sigma_p = \frac{1}{\sqrt{N}} \left(\int \frac{1}{f} \left[\frac{df(x;p)}{dp} \right]^2 dx \right)^{-\frac{1}{2}} \tag{11}$$

In this way we can study the statistical accuracy achievable on $\Im\left(\frac{\epsilon'}{\epsilon}\right)$ with respect to different experimental resolution. To take into account eventual non-gaussian tails in the vertex reconstruction we define a very general experimental error distribution made of a sum of n = 1,2,3 gaussians , each with increasing σ, $\sigma_k = k\sigma$ and decreasing weight $= \frac{1}{k}$:

$$g_\sigma^n(x) = \frac{1}{\sigma\sqrt{2\pi}} \left(\sum_{k=1}^{n} \frac{1}{k} \right)^{-1} \sum_{k=1}^{n} \frac{1}{k^2} \exp\left(\frac{-x^2}{2\sigma^2 k^2} \right) \tag{12}$$

Resolution functions with a given σ and different index n have nearly the same FWHM, approximately equal to 2.36 times σ . On the other hand the RMS, and also the "tails", of the distributions grow with the index n. In this way we parametrized the width and the "tails" of the experimental resolution function by the σ and the index n of distribution $g_\sigma^n(x)$.

For a function with n = 1 (gaussian error) the "corrected" distribution of eq (10) is given by :

$$\begin{aligned} f_\sigma^{exp}(y) \quad = \quad & \frac{C_1^2}{\sigma\sqrt{2\pi}} \left\{ \exp\left(-y + \frac{\sigma^2}{2} \right) \int_{-y}^{\infty} \exp\left[-\frac{1}{2} \left(\frac{t}{\sigma} + \sigma \right)^2 \right] dt \; - \right. \\ & \left. \exp\left(\beta y + \frac{\sigma^2 \beta^2}{2} \right) \int_{-\infty}^{-y} \exp\left[-\frac{1}{2} \left(\frac{t}{\sigma} - \beta\sigma \right)^2 \right] dt \right\} + \end{aligned}$$

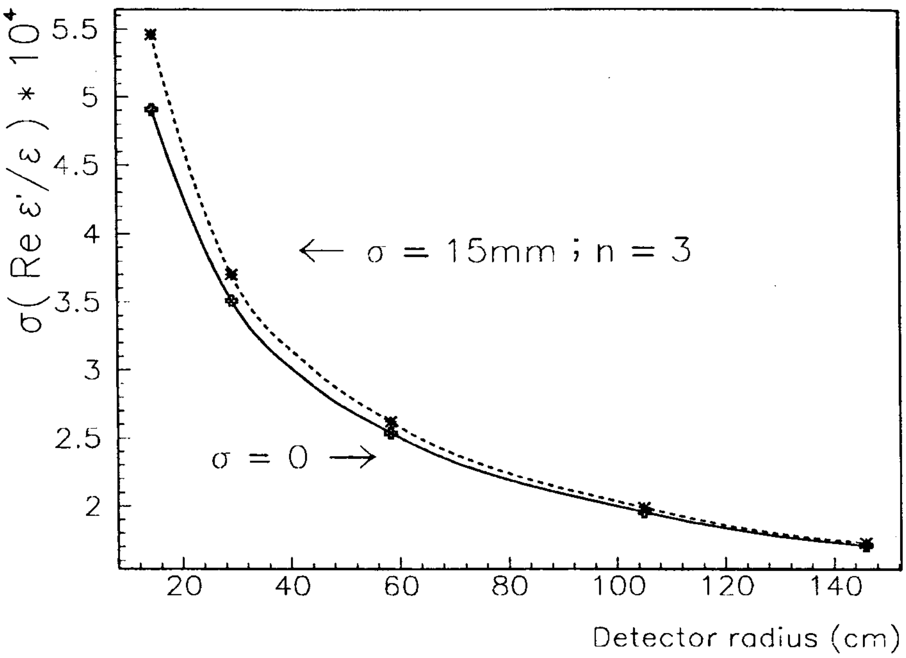

Figure 3: Statistical accuracy achievable on $\Re\left(\frac{\epsilon'}{\epsilon}\right)$ with respect to different detector sizes

$$\frac{C_2^2}{\sigma\sqrt{2\pi}}\left\{\exp\left(y+\frac{\sigma^2}{2}\right)\int_{-\infty}^{-y}\exp\left[-\frac{1}{2}\left(\frac{t}{\sigma}-\sigma\right)^2\right]dt+\right.$$
$$\left.\exp\left(-\beta y+\frac{\sigma^2\beta^2}{2}\right)\int_{-y}^{\infty}\exp\left[-\frac{1}{2}\left(\frac{t}{\sigma}+\beta\sigma\right)^2\right]dt\right\}-$$
$$\frac{2C_1C_2}{\sigma\sqrt{2\pi}}\left\{\exp\left(-y\frac{1+\beta}{2}\right)\int_{-y}^{\infty}\exp\left(-\frac{t^2}{2\sigma^2}-t\frac{1+\beta}{2}\right)\cos\left[\chi\left(t+y\right)+\Delta\phi\right]+\right.$$
$$\left.\exp\left(y\frac{1+\beta}{2}\right)\int_{-\infty}^{-y}\exp\left(-\frac{t^2}{2\sigma^2}+t\frac{1+\beta}{2}\right)\cos\left[\chi\left(t+y\right)+\Delta\phi\right]\right\} \qquad (13)$$

$t = d - y$, y and σ are expressed in d_S units. We defined the following quantities:

$$C_1^2 = 1 + 2\Re\left(\frac{\epsilon'}{\epsilon}\right) + \left(\Re\left(\frac{\epsilon'}{\epsilon}\right)\right)^2 \qquad C_2^2 = 1 - 4\Re\left(\frac{\epsilon'}{\epsilon}\right) + 4\left(\Re\left(\frac{\epsilon'}{\epsilon}\right)\right)^2 \qquad (14)$$

$$\beta = \frac{d_S}{d_L} \qquad \chi = \frac{\Delta m}{\Gamma_S} \qquad \Delta\phi = 3\times\Im\left(\frac{\epsilon'}{\epsilon}\right) \qquad (15)$$

The statistical accuracy on the $\Re\left(\frac{\epsilon'}{\epsilon}\right)$ shown in fig. 3 refers to different apparatus sizes and was obtained via eq.(11) using a $\sigma = 0$ (cross) and a $\sigma = 15$ mm , n = 3 (star) experimental resolution. Since the value of this parameter is determined by the difference in height of the two flat wings of the d distribution, the vertex resolution does not affect very much this mesurement. The statistical error on $\Re\left(\frac{\epsilon'}{\epsilon}\right)$ is then dominated by the number of events that can be collected.

On the other hand the accuracy on $\Im\left(\frac{\epsilon'}{\epsilon}\right)$ is critically affected by the resolution, as shown in fig. 4. The obtained $\sigma = 0$ limit is in agreement with earlier results[10]. We assume that the uncertainty on the d determination comes mainly from the neutral vertex reconstruction . A realistic simulation of a $K^0 \rightarrow \pi^0\pi^0$ vertex reconstruction can be

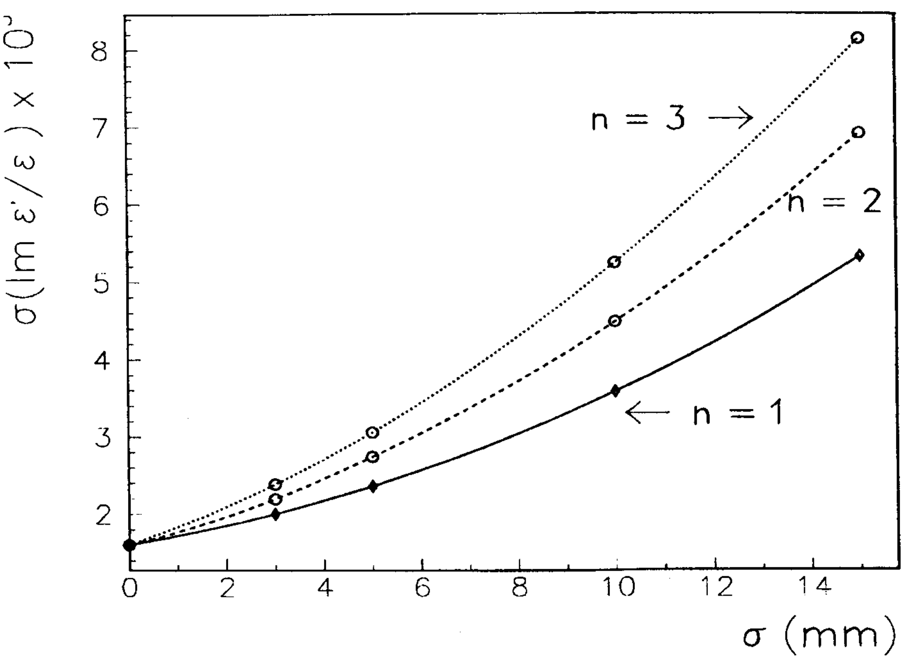

Figure 4: Statistical accuracy achievable on $\Im\left(\frac{\epsilon'}{\epsilon}\right)$ with respect to different vertex resolution

found in [11]. The author obtained a simulated vertex resolution function that is well approximated by a $g_\sigma^n(x)$ with n=2 and "nominal" $\sigma = .7$ cm. The estimated accuracy achievable by this analysis method with such a realistic detector:

$$\sigma_{\Re\left(\frac{\epsilon'}{\epsilon}\right)} = 1.8 \times 10^{-4} \quad ; \quad \sigma_{\Im\left(\frac{\epsilon'}{\epsilon}\right)} = 3.4 \times 10^{-3} \tag{16}$$

These numbers are to be compared with the present experimental situation :

$\Re\left(\frac{\epsilon'}{\epsilon}\right) = (2.3 \pm 0.7) \times 10^{-3}$	$\Im\left(\frac{\epsilon'}{\epsilon}\right) = (-1.2 \pm 17) \times 10^{-3}$	$NA31$[5]
$\Re\left(\frac{\epsilon'}{\epsilon}\right) = (0.6 \pm 0.7) \times 10^{-3}$	$\Im\left(\frac{\epsilon'}{\epsilon}\right) = (+3.1 \pm 9.3) \times 10^{-3}$	$E731$[6]

4 C-even background

Till now we assumed that the 4π events can be generated only by the process in eq.(1). However some years ago it has been pointed out that radiative decay of the ϕ can generate a C-even $\bar{K^0}K^0$ state [12] :

$$\phi \to (f_0 \div a_0) + \gamma = \left(\bar{K^0}K^0\right)_{C=+} + \gamma \to \frac{1}{\sqrt{2}}(K_S K_S - K_L K_L) + \gamma \tag{17}$$

If the branching ratio of process in eq.(17) is not negligible the CP conserving decay of its $K_S K_S$ component can generate a large number of symmetric 4π events that washes out the asymmetry. Moreover due to the small energy of the radiative photon ($E_\gamma < 21MeV$) it will be difficult to identify the process in eq.(17) by kinematical reconstruction or by direct detection of the photon.

The estimated rate of this C-even process is strongly model dependent and the predicted branching ratios range from 10^{-5} to 10^{-9}. A complete review of the subject has been done in[13], and, according to this reference, the smaller estimates for the B.R. seem to be the more likely. In this case the C-even background does not affect the previous discussion. To be more quantitative with a B.R. $\simeq 10^{-7}$ the ratio of C- even and C-odd 4π events is of the order of 10^{-4} and therefore completely negligible.

It must be stressed that the real part of $\frac{\epsilon'}{\epsilon}$ can be measured also if the B.R. of eq.(17) would be unrealistically large, exploiting the different topological features of the C-even and of the C-odd state[14]. Chosen a distance a, with $d_S \ll a \ll d_L$, let us consider the events with $0 < d_c < a < d_n < D$ (A class events) or with $0 < d_n < a < d_c < D$ (B class events); from a C-odd $\bar{K^0}K^0$ pair one has

$$N_A^- = \frac{\Gamma_S^{+-}\Gamma_S^{00}}{\Gamma_S\Gamma_L}\left[|\eta^{+-}|^2 S_1 + |\eta^{00}|^2 S_2 + Re\left(\eta^{+-}\left(\eta^{00}\right)^*\right) S_3\right] \tag{18}$$

$$N_B^- = \frac{\Gamma_S^{+-}\Gamma_S^{00}}{\Gamma_S\Gamma_L}\left[|\eta^{+-}|^2 S_2 + |\eta^{00}|^1 S_1 + Re\left(\eta^{+-}\left(\eta^{00}\right)^*\right) S_3\right] \tag{19}$$

and from a C-even $\bar{K^0}K^0$ pair :

$$N_A^+ = N_B^+ \simeq \frac{\Gamma_S^{+-}\Gamma_S^{00}}{\Gamma_S\Gamma_L} \mid \epsilon \mid^2 B \tag{20}$$

The S_i and B coefficients are defined in [14]. Their approximate expressions are :

$$\begin{aligned}
S_1 &\simeq \exp(\frac{-a}{d_S})\left[1-\exp\left(\frac{-a}{d_L}\right)\right] \qquad (21)\\
S_2 &= \left[1-\exp\left(\frac{-a}{d_S}\right)\right]\left[\exp(\frac{-a}{d_L})-\exp(\frac{-D}{d_L})\right]\\
S_3 &\simeq -2\frac{\Gamma_S\Gamma_L}{\Gamma^2+\Delta m^2}\exp\left(\frac{-a}{2d_S}\right)\cos\left(\frac{\Delta m}{\Gamma_S}\frac{a}{d_S}\right)\\
B &\simeq \frac{\Gamma_L}{\Gamma_S|\epsilon|^2}\exp\left(\frac{-a}{d_S}\right)\left[1-\exp\left(\frac{-a}{d_S}\right)\right]
\end{aligned} \tag{22}$$

For $d_S << a << d_L$, S_1 and S_3 are negligible, and S_2 is almost equal to the total acceptance of the detector. The B coefficient is suppressed by the exponential factor $\exp\left(\frac{-a}{d_S}\right)$ and, in spite of the large $|\epsilon|^{-2}$ factor, $\frac{B}{S_1} \ll 1$ (i.e. $a = 10\ d_S \simeq 6cm$ $\frac{B}{S_1} \simeq 4.3 \times 10^{-2}$).

The asymmetry between A and B events is

$$\bar{A} = \frac{N_B - N_A}{N_B + N_A} = \frac{|\eta^{+-}|^2 - |\eta^{00}|^2}{|\eta^{+-}|^2 + |\eta^{00}|^2 + \frac{2\alpha|\epsilon|^2 B}{S_2}} \simeq \frac{3Re\frac{\epsilon'}{\epsilon}}{1+\alpha\frac{B}{S_1}} \tag{23}$$

Where α is the ratio of C-even $\bar{K^0}K^0$ present in the initial state:

$$\alpha = \frac{BR\left(\phi \to \left(\bar{K^0}K^0\right)_{c=+1} + \gamma\right)}{BR\left(\phi \to \left(\bar{K^0}K^0\right)_{c=-1}\right)} \tag{24}$$

Eq.(23) shows that for any value of α compatible with measured ϕ branching ratios the C-even background cannot affect the determination of $\Re\left(\frac{\epsilon\prime}{\epsilon}\right)$. In the C region (both decay paths less then a) on the contrary the ratio between the background and the signal is of the order of $10^{4}\alpha$. Since $\Im\left(\frac{\epsilon\prime}{\epsilon}\right)$ is mostly related with the number of the events in the C region, this background must be carefully taken into account.

5 C-even background effect on $Im\left(\frac{\epsilon\prime}{\epsilon}\right)$

The background contribution is symmetric in d and neglecting the K_LK_L component (negligible in this region) we obtain :

$$N^{+}(d) = \alpha \frac{\Gamma_S^{+-}\Gamma_S^{00}}{2\Gamma_S} \exp\left(\frac{-\mid d \mid}{d_S}\right) \tag{25}$$

This distribution overlaps the signal just in the interference zone , $d \simeq d_S$, worsening the resolution on $\Im\left(\frac{\epsilon\prime}{\epsilon}\right)$.

However the signal ($K_LK_S \to \pi^0\pi^0, \pi^+\pi^-$) and the background ($K_SK_S \to \pi^0\pi^0, \pi^+\pi^-$) a have different spatial behaviour: $A_I(d)$ is an odd function of d and the $N^{+}(d)$ in eq.(25) is even. This different behaviour is of help to disentangle the signal contribution from the background.

To study the C-even background effect we assume that a detector implemented at DAFNE has an efficiency of detection of a 21 MeV γ equal to 80 %, according to the results of detailed detector simulation [15]. To introduce the C-even background in our fitting procedure we add the background $N^{+}(d)$ to the initial theoretical $N(d)$ distribution of the signal in the definition of the $f_\sigma(y)$ distribution. Then we compute the accuracy on $\Im\left(\frac{\epsilon\prime}{\epsilon}\right)$ for different $BR(\phi \to f_0 + \gamma)$. Reasonable experimental resolution function $g_\sigma^n(x)$ with n=2 and different values of the of nominal σ was used. The accuracy on $\Im\left(\frac{\epsilon\prime}{\epsilon}\right)$ is shown in the table below compared with that obtained without the background. As it can be seen for realistic vertex resolution the worsening is around the 5% of no-background value.

$\sigma\,(mm)$	0	3	5	10	15
$B_\phi \to f_0 + \gamma = 5\ 10^{-5}$	$2.2\ 10^{-3}$	$2.5\ 10^{-3}$	$3.0\ 10^{-3}$	$4.6\ 10^{-3}$	$7.2\ 10^{-3}$
$B_\phi \to f_0 + \gamma = 0$	$1.7\ 10^{-3}$	$2.2\ 10^{-3}$	$2.8\ 10^{-3}$	$4.5\ 10^{-3}$	$6.9\ 10^{-3}$

6 C-even background measurement

As discussed in[16] the C-even background can be measured detecting the low energy γ associated. Otherwise the event distribution inside the detector can be used to single out the K_LK_L component of the C-even $\bar{K}^0K^0$ state.

If $\mid f_i >$ is one final state produced in a K_L allowed decay :

$$\mid f_i >= \left(e^{\pm}\nu\pi^{\mp}\right)\ ,\ \left(\mu^{\pm}\nu\pi^{\mp}\right)\ ,\ \left(\pi^+\pi^-\pi^0\right) \tag{26}$$

then $\sum_i BR(K_L \to f_i) = 0.78$[17], $\sum_i BR(K_S \to f_i) \simeq 1.1 \times 10^{-3}$ [1]; the total number of

[1] On theoretical ground the K_S semileptonic branching ratio is $\frac{\Gamma_L}{\Gamma_S}BR(K_L \to l + x)$ and that of $K_S \to 3\pi$ is estimated to be $\simeq 10^{-7}$[18]

$| f_i >, | f_j >$ final states with both decay paths larger than a is

$$\sum_{ij} N_D(f_i, f_j) = N_\phi \times BR\left(\phi \to K^0 \bar{K^0}\right) \left\{0.61 \times \frac{\alpha}{2}\left[\exp\left(\frac{-a}{d_L}\right) - \exp\left(\frac{-D}{d_L}\right)\right]^2 + 8.6\ 10^{-4} \exp\left(\frac{-a}{d_S}\right)\left[\exp\left(\frac{-a}{d_L}\right) - \exp\left(\frac{-D}{d_L}\right)\right]\right\} \quad (27)$$

For $a = 20\ d_S$

$$\sum_{ij} N_D(f_i, f_j) = N_\phi \times \left\{3.2\ 10^{-2} \times BR\left(\phi \to \left(\bar{K^0} K^0\right)_{C=+}\right) + 2\ 10^{-13}\right\} \quad (28)$$

With the project luminosity of DAFNE and for a $BR\left(\phi \to \left(\bar{K^0} K^0\right)_{C=+}\right) \simeq 5 \times 10^{-8}$, about 40 of such events in a year are produced. The corresponding contribution from the odd state is $\simeq 10^{-2}$ per year.

7 Semileptonic K_S decays and CP violation

The semileptonic decays of K_S are interesting to test the $\bar{K^0} K^0$ mass matrix and decay amplitudes. Assuming CPT and $\Delta S = \Delta Q$ rule, one has

$$\begin{aligned} \Gamma\left(K_S \to l^+ + x\right) = \frac{1}{2}\frac{|1+\epsilon|^2}{1+|\epsilon|^2}\Gamma\left(K^0 \to l^+ + x\right) &= \Gamma\left(K_L \to l^+ + x\right) \\ \Gamma\left(K_S \to l^- + x\right) = \frac{1}{2}\frac{|1-\epsilon|^2}{1+|\epsilon|^2}\Gamma\left(\bar{K^0} \to l^- + x\right) &= \Gamma\left(K_L \to l^- + x\right) \end{aligned} \quad (29)$$

therefore

$$BR\left(K_S \to l + x\right) = \frac{\Gamma_L}{\Gamma_S} BR\left(K_L \to l + x\right) \quad (30)$$

and the charge asymmetry in K_S decay, δ_S, must be equal to the K_L one, δ_L.

Without imposing CPT symmetry the $\bar{K^0} K^0$ mixing can be parametrized introducing two different ϵ parameters in the following way:

$$\begin{aligned} K_S &= \frac{1}{\sqrt{2\left(1+|\epsilon_S|^2\right)}}\left[\left(1+\epsilon_S\right) K^0 + \left(1-\epsilon_S\right) \bar{K^0}\right] \\ K_L &= \frac{1}{\sqrt{2\left(1+|\epsilon_L|^2\right)}}\left[\left(1+\epsilon_L\right) K^0 - \left(1-\epsilon_L\right) \bar{K^0}\right] \end{aligned} \quad (31)$$

If we consider the following amplitudes :

$$\begin{aligned} M_{11} - \frac{i}{2}\Gamma_{11} &= < K^0 | H | K^0 > \\ M_{12} - \frac{i}{2}\Gamma_{12} &= < K^0 | H | \bar{K^0} > \\ M_{22} - \frac{i}{2}\Gamma_{22} &= < \bar{K^0} | H | \bar{K^0} > \end{aligned} \quad (32)$$

the ϵ parameters can be written:

$$\epsilon_S = \frac{-i\Im(M_{12}) - \frac{1}{2}\Im(\Gamma_{12}) - \frac{1}{2}\left[M_{11} - M_{22} - \frac{i}{2}(\Gamma_{11} - \Gamma_{22})\right]}{m_L - m_S + i\frac{\Gamma_S - \Gamma_L}{2}}$$
$$\epsilon_L = \frac{-i\Im(M_{12}) - \frac{1}{2}\Im(\Gamma_{12}) + \frac{1}{2}\left[M_{11} - M_{22} - \frac{i}{2}(\Gamma_{11} - \Gamma_{22})\right]}{m_L - m_S + i\frac{\Gamma_S - \Gamma_L}{2}} \tag{33}$$

The CPT symmetry would require $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$, recovering the relation $\epsilon_S = \epsilon_L = \epsilon$.

The semileptonic decay amplitude can be parametrized as :

$$A\left(K^0 \to l^+ + x\right) = a(1 - y)$$
$$A\left(\bar{K^0} \to l^- + x\right) = a^*(1 + y^*) \tag{34}$$

here a is the CPT conserving amplitude and y is the CPT violation in the amplitude. Thus the charge asymmetries in semileptonic decays can be written as :

$$\delta_L \simeq 2\Re(\epsilon_L) - 2\Re(y)$$
$$\delta_S \simeq 2\Re(\epsilon_S) - 2\Re(y)$$
$$\delta_L - \delta_S \simeq 2\Re(\epsilon_L - \epsilon_S) \tag{35}$$

At a ϕ factory K_S semileptonic branching ratio can be obtained selecting the events where is observed a decay in one of the previous definited states f_i with path larger than a together a semileptonic decay with path shorter than a.

The number of such events is

$$L_S = N_\phi \times BR\left(\phi \to \bar{K^0}K^0\right)\left[\exp\left(\frac{-a}{d_L}\right) - \exp\left(\frac{-D}{d_L}\right)\right] \times$$
$$\sum_i BR(K_L \to f_i)\left\{ BR(K_S \to l + x) + \right.$$
$$\left. \alpha \times BR(K_L \to l + x)\left[1 - \exp\left(\frac{-a}{d_L}\right)\right]\right\} \tag{36}$$

In eq.(36) the exchanged process $(K_L \to l, K_S \to f_i)$ has been neglected, its probability being less than

$$\exp\left(\frac{-a}{d_S}\right) \times \left[\frac{1 - \exp\left(\frac{-a}{d_L}\right)}{\exp\left(\frac{-a}{d_L}\right) - \exp\left(\frac{-D}{d_L}\right)}\right] = 2.7\ 10^{-6} \tag{37}$$

and only the $K_L K_L$ component of the C-even background has been considered, since the contribution of the $K_S K_S$ component is suppressed by the factor

$$\frac{\left(\frac{\Gamma_L}{\Gamma_S}\right)^2 \exp\left(\frac{-a}{d_S}\right)}{\left[exp\left(\frac{-a}{d_L}\right) - exp\left(\frac{-D}{d_L}\right)\right]\left[1 - exp\left(\frac{-a}{d_L}\right)\right]} \simeq 10^{-8} \tag{38}$$

Inserting the experimental values of BR[17] and fixing $a = 10d_S$ we obtain :

$$L_S = N_\phi \times 9.1 \times 10^{-2} BR\left(K_S \rightarrow l + x\right) \times (1 + 10\alpha) \tag{39}$$

For the expected values of α the C-even background does not affect sensibly the determination of the $BR\left(K_S \rightarrow l + x\right)$.

The charge asymmetry of these events is related to δ_S by

$$\delta_S^{exp} = \frac{L_S^+ - L_S^-}{L_S^+ + L_S^-} = \frac{W_-}{W_+} = \frac{\delta_S + 10 \times \alpha\ \delta_L}{1 + 10 \times \alpha} \tag{40}$$

Where we have defined :

$$\begin{aligned} W_- &= BR\left(K_S \rightarrow l^+ + x\right) - BR\left(K_S \rightarrow l^- + x\right) + \\ &\alpha\left(1 - \exp\frac{-a}{d_L}\right)\left[BR\left(K_L \rightarrow l^+ + x\right) - BR\left(K_L \rightarrow l^- + x\right)\right] \quad (41) \\ W_+ &= BR\left(K_S \rightarrow l^+ + x\right) + BR\left(K_S \rightarrow l^- + x\right) + \\ &\alpha\left(1 - \exp\frac{-a}{d_L}\right)\left[BR\left(K_L \rightarrow l^+ + x\right) + BR\left(K_L \rightarrow l^- + x\right)\right] \quad (42) \end{aligned}$$

for the expected values of α, δ_S^{exp} is about equal to δ_S, moreover

$$\delta_S^{exp} - \delta_L = \frac{\delta_S - \delta_L}{1 + 10 \times \alpha} \tag{43}$$

Eq.(43) shows that the C-even background cannot simulate CPT violation.

We assume conservatively that in the experimental determination of K_S charge asymmetry only the electrons can be used. In fact the main semileptonic decays of neutral kaons are $K \rightarrow e^\pm + \pi^\mp + \nu$ or $K \rightarrow \mu^\pm + \pi^\mp + \nu$ and could be difficult to distinguish $\mu^+\pi^-$ and $\mu^-\pi^+$ final states. With the project luminosity of DAFNE ($2.5 \times 10^{10}\phi$ per year), the statistical error on δ_S is of about $.94 \times 10^{-3}$.

Using the esperimental value [17] $\delta_L = (3.27 \pm 0.12) \times 10^{-3}$ the CPT conservation in Kaon mass matrix ($\delta_S = \delta_L$) can be tested within more than $3 \times \sigma$

$$\frac{\sigma_{\Re(\epsilon_L - \epsilon_S)}}{\Re\left(\epsilon_L\right)} \simeq 0.29 \tag{44}$$

A limit on $\Re\left(y\right)$ (CPT violating amplitude) can be put comparing δ_L with η^{+-} ; allowing CPT violation in $K_L \rightarrow 2\pi$ decay amplitudes and neglecting $\epsilon\prime$ one has [3]:

$$\frac{\delta_L}{2} - \Re\left(\eta^{+-}\right) \simeq -\Re\left(y\right) - \frac{\Re\left(B_0\right)}{\Re\left(A_0\right)} = (0.6 \pm 0.7) \times 10^{-4} \tag{45}$$

where A_0 and B_0 are the CPT conserving and violating $K^0 \rightarrow (2\pi)_{I=0}$ decay amplitutes. On the other hand the CPT violating contribution to η^{+-} is 90^o out of phase with respect to the CPT conserving one[3], comparing the experimental phase of η^{+-} [17] with the superweak phase $\phi_{SW} = arctan\left(\frac{2\Delta m}{\Delta\Gamma}\right)$ one gets

$$\left| \frac{\epsilon_L - \epsilon_S}{2} + \frac{\Re\left(B_0\right)}{\Re\left(A_0\right)} \right| = (0.9 \pm 0.6) \times 10^{-4} \tag{46}$$

Eqs. (45) and (46) show as it is possible to obtain constraints on CPT violating parameters comparing the amplitudes of different weak decays. A very detailed analysis of the constraints that can be obtained in this way is done in [19].

8 Direct tests of T and CPT symmetries

The dilepton events allow direct test of T and CPT symmetries[2, 3]. Long time ago Kabir[20] has shown that T violation implies different probabilities for $K^0 \to \bar{K}^0$ and $\bar{K}^0 \to K^0$ transitions, while CPT requires equal probabilities for $K^0 \to K^0$ and $\bar{K}^0 \to \bar{K}^0$ transitions. Then a T violating asymmetry

$$A_T = \frac{P\left(\bar{K}^0 \to K^0\right) - P\left(K^0 \to \bar{K}^0\right)}{P\left(\bar{K}^0 \to K^0\right) + P\left(K^0 \to \bar{K}^0\right)} \tag{47}$$

and a CPT violating asymmetry

$$A_{CPT} = \frac{P\left(K^0 \to K^0\right) - P\left(\bar{K}^0 \to \bar{K}^0\right)}{P\left(\bar{K}^0 \to \bar{K}^0\right) + P\left(K^0 \to K^0\right)} \tag{48}$$

can be defined.

Both these tests can be done at a ϕ factory, where the initial state is an antisymmetric $K^0\bar{K}^0$ state, if the $\Delta Q = \Delta S$ rule holds. [2]

If at a time t a neutral kaon decays into a positive lepton, the other neutral kaon is at the same time a $\bar{K}^0$ and the sign of the lepton emitted in a subsequent semileptonic decay signals if the $\bar{K}^0$ has changed or conserved its own flavor. Therefore at the $\phi - factory$ the charge asymmetry in equal sign dilepton pairs is equal to A_T

$$A_T = \frac{L^{++} - L^{--}}{L^{++} + L^{--}} = 2\Re\left(\epsilon_L + \epsilon_S\right) - 4\Re\left(y\right) = \delta_L + \delta_S \tag{49}$$

On the other hand time asymmetry in opposite sign dilepton pairs signals CPT violation; if L^{+-} (L^{-+}) is the number of dilepton pairs where the positive lepton is emitted before (after) the negative one, then

$$A_{CPT} = \frac{L^{-+} - L^{+-}}{L^{+-} + L^{-+}} \simeq 2\Re\left(\epsilon_L - \epsilon_S\right) \tag{50}$$

Furthermore, the study of time dependence of opposite sign dilepton events allows the determination of $\Im\left(\epsilon_L - \epsilon_S\right)$.

We can proceed as in section 2 : d_+ and d_- are the decay paths for $K \to l^+ + x$ or $K \to l^- + x$ respectively, and L(d) is the number of opposite sign dilepton events with $d_+ - d_- = d$. The asymmetry between positive and negative values of d is

$$\begin{aligned}
A^L(d) &= \frac{L(|d|) - L(-|d|)}{L(|d|) + L(-|d|)} \\
&= A^L_R(d) \times \Re\left(\epsilon_L - \epsilon_S\right) + A^L_I(d) \times \Im\left(\epsilon_L - \epsilon_S\right) \\
A^L_R(d) &= 2\frac{\exp\left(\frac{-|d|}{d_L}\right) - \exp\left(\frac{-|d|}{d_S}\right)}{\exp\left(\frac{-|d|}{d_L}\right) + \exp\left(\frac{-|d|}{d_S}\right) + 2\exp\left(\frac{-\Gamma}{\Gamma_S}\frac{|d|}{d_S}\right)\cos\left(\frac{\Delta m}{\Gamma_S}\frac{|d|}{d_S}\right)} \\
A^L_I(d) &= 2\frac{\exp\left(\frac{-\Gamma}{\Gamma_S}\frac{|d|}{d_S}\right)\sin\left(\frac{\Delta m}{\Gamma_S}\frac{|d|}{d_S}\right)}{\exp\left(\frac{-|d|}{d_L}\right) + \exp\left(\frac{-|d|}{d_S}\right) + 2\exp\left(\frac{-\Gamma}{\Gamma_S}\frac{|d|}{d_S}\right)\cos\left(\frac{\Delta m}{\Gamma_S}\frac{|d|}{d_S}\right)}
\end{aligned} \tag{51}$$

[2]The effect of the $\Delta S = -\Delta Q$ transitions on CPT and T violating asymmetries will be discussed in appendix.

Even before discussing the sensitivity that can be reached at DAFNE, we investigate the effect of a C-even background on these T and CPT violating asymmetries; in fact, if the initial $\bar{K^0}K^0$ state is C-even, the correlation of the flavor of the two neutral kaons at later times is lost.

Then without assuming CPT one has

$$\frac{K^0\bar{K^0}+\bar{K^0}K^0}{\sqrt{2}}=\frac{K_SK_S-K_LK_L+(\epsilon_L-\epsilon_S)(K_LK_S+K_SK_L)}{\sqrt{2}}. \tag{52}$$

The charge asymmetry in equal sign dilepton pairs is essentially equal to $2\delta_L$ (the main contribution to dilepton pairs comes from the K_LK_L component), therefore,if we call αr the fraction of C-even dilepton pairs, where

$$r=\frac{1}{2}\frac{BR(K_L\to l+x)\,S_L}{BR(K_S\to l+x)}\simeq 10^2 \tag{53}$$

The T violating asymmetry becomes

$$A_T^{exp}=\frac{L^{++}-L^{--}}{L^{++}+L^{--}}=\frac{A_T+\alpha r2\delta_L}{1+\alpha r}=2\delta_L-\frac{\delta_L-\delta_S}{1+\alpha r} \tag{54}$$

As far as the CPT violating asymmetry is concerned, the interference of the two terms in eq.(52) gives rise to a time asymmetry in opposite sign dilepton pairs which vanishes if $\epsilon_L=\epsilon_S$ and is moreover suppressed by the factor Γ_L/Γ_S. Therefore, at leading order in Γ_L/Γ_S

$$A_{CPT}^{exp}=\frac{L^{-+}-L^{+-}}{L^{+-}+L^{-+}}=\frac{A_{CPT}}{1+\alpha r}. \tag{55}$$

The equations (54) and (55) show that the C-even background cannot simulate CPT violation, but can only induce a relative uncertainty less than 10^{-3} on the value of asymmetries.

The number of equal sign electron pairs and that of opposite sign expected at DAFNE is $N_\phi\times .9\times 10^{-5}\simeq 2.3\times 10^5$, therefore the T and CPT violating asymmetries can be measured with a statistical error of about 2.1×10^{-3}.

From equations (53) and (54) we have two independent determinations of the CPT violating parameter $\Re(\epsilon_L-\epsilon_S)$; the combined error is $\sigma_{\Re(\epsilon_L-\epsilon_S)}\simeq 7.5\times 10^{-4}$ somewhat larger than that quoted in eq.(44).

The determination of $\Im(\epsilon_L-\epsilon_S)$ and $\Re(\epsilon_L-\epsilon_S)$ from time asymmetry of opposite sign dilepton events is quite similar to that of the real and imaginary part of $\frac{\epsilon'}{\epsilon}$ from $\pi^0\pi^0,\pi^+\pi^-$ events; then we estimate :

$$\sigma_{\Im(\epsilon_L-\epsilon_S)}\simeq\sigma_{\Im\left(\frac{\epsilon'}{\epsilon}\right)}\frac{\sigma_{\Re(\epsilon_L-\epsilon_S)}}{\sigma_{\Re\left(\frac{\epsilon'}{\epsilon}\right)}}\simeq 1.2\times 10^{-2} \tag{56}$$

In conclusion the asymmetries in leptonic events allow the determination of $\epsilon_L-\epsilon_S$; inserting these values in eqs.(35,45,46), also limits on CPT violating amplitudes y and B_0 can be obtained and almost each CPT violating parameter can be constrained up to an accuracy of $10^{-3}\div 10^{-4}$.

9 Conclusions

The ϕ factory is a very powerful tool to study CP violation and for testing CPT in the neutral kaons system.

The direct CP violation in K^0 decay amplitudes can be measured with higher sensitivity for both $\Re\left(\frac{\epsilon'}{\epsilon}\right)$ and $\Im\left(\frac{\epsilon'}{\epsilon}\right)$. Even if the observation of CP violating $K_S \to 3\pi^0$ decay is at the limit of DAFNE capabilities, the detection of charge asymmetry in semileptonic K_S decay would be the first observation of CP violation in K_S system.

For the first time T and CPT violating asymmetries could be measured and their values, together with those of CP violating parameters in K_L and K_S decays, allow to disentangle the different symmetry breaking effects.

10 Appendix

We discuss now the effect of $\Delta S = -\Delta Q$ transitions semileptonic decays of neutral kaons.

We define:

$$x = \frac{A(\bar{K^0} \to l^+ + x)}{A(K^0 \to l^+ + x)} \qquad \bar{x^*} = \frac{A(K^0 \to l^- + x)}{(\bar{K^0} \to l^- + x)} \tag{57}$$

The difference $(x - \bar{x})$ is CPT violating, while the imaginary part of x, in phase convention where $\Im(A_0) = 0$ is CP violating.

The present experimental limits on x [17] are of about 2×10^{-2} for both real and imaginary part; the theoretical predictions [21] give x essentially real, and $|x|$ of the order $10^{-6} \div 10^{-7}$.

The introdution of the $\Delta S = -\Delta Q$ transitions modifies the semileptonic decay rates of K_S and K_L in the following way:

$$\begin{aligned}
\Gamma\left(K_S \to l^+ + x\right) \quad + \quad \Gamma\left(K_S \to l^- + x\right) &= \frac{1}{2}\left\{\Gamma\left(K^0 \to l^+ + x\right) + \right. \\
&\left. \Gamma\left(\bar{K^0} \to l^- + x\right)\right\}\left[1 + \Re\left(x + \bar{x}\right)\right] \\
\Gamma\left(K_L \to l^+ + x\right) \quad + \quad \Gamma\left(K_L \to l^- + x\right) &= \frac{1}{2}\left\{\Gamma\left(K^0 \to l^+ + x\right) + \right. \\
&\left. \Gamma\left(\bar{K^0} \to l^- + x\right)\right\}\left[1 - \Re\left(x + \bar{x}\right)\right]
\end{aligned} \tag{58}$$

It follows that

$$2\Re(x + \bar{x}) = \frac{\Gamma_S BR(K_S \to l + x)}{\Gamma_L BR(K_L \to l + x)} - 1 \tag{59}$$

With the project luminosity of DAFNE the error on $\Re(x + \bar{x})$ is $\sigma_{\Re(x+\bar{x})} \simeq 4.1 \times 10^{-3}$, an order of magnitude less than the present bound. It should be noted that the main contribution to $\sigma_{\Re(x+\bar{x})}$ comes from the present experimental error on K_S and K_L lifetimes (if $\sigma_{\frac{\Gamma_S}{\gamma_L}} = 0$ one would get $\sigma_{\Re(x+\bar{x})} \simeq 4 \times 10^{-4}$).

The charge asymmetries of K_L and K_S are given by:

$$\begin{aligned}
\delta_L &= 2\Re\left(\epsilon_L\right) - 2\Re(y) - \Re\left(x - \bar{x^*}\right) + 2\Re\left(\epsilon_L\left(x + \bar{x^*}\right)\right) \\
\delta_S &= 2\Re\left(\epsilon_S\right) - 2\Re\left(y\right) + \Re\left(x - \bar{x^*}\right) - 2\Re\left(\epsilon_S\left(x + \bar{x^*}\right)\right)
\end{aligned} \tag{60}$$

If CPT holds the charge asymmetries are modified only by the higher order term $4\Re(\epsilon)\Re(x)$, and $\Delta S = -\Delta Q$ transitions can simulate CPT violation only to this order. With the present experimental limit on x, the CPT conserving part of the difference $\delta_L - \delta_S$ is bounded by:

$$\frac{|\delta_L - \delta_S|}{|\delta_L + \delta_S|} < 4 \times 10^{-2} \tag{61}$$

and this is much smaller than the sensitivity on this parameter achievable at DAFNE.

The expressions of the asymmetries of dilepton pairs are, if the initial state is C-odd:

$$A_T = \frac{L^{++} - L^{--}}{L^{++} + L^{--}} = 2\Re(\epsilon_L + \epsilon_S) - 4\Re(y) = \delta_L + \delta_S \tag{62}$$

and

$$\begin{aligned} A_{CPT} &= \frac{L^{-+} - L^{+-}}{L^{-+} + L^{+-}} = 2\Re\left(\epsilon_L - \epsilon_S\right) - 2\Re\left(x - \bar{x}^*\right) + \\ & 2\Re\left[\left(\epsilon_L + \epsilon_S\right)\left(x + \bar{x}^*\right)\right] + \frac{4}{S_L}\left[\Im\left(\epsilon_L - \epsilon_S\right) - \Im\left(x + \bar{x}\right)\right]\frac{\Delta m \Gamma_L}{\Gamma^2 + \Delta m^2} \end{aligned} \tag{63}$$

Eq.(62) shows that charge asymmetry in equal sign dilepton pairs is still a measure of T violatin in $\bar{K}^0 K^0$ system. On the contrary, the effects of CPT violation and $\Delta S = -\Delta Q$ transitions cannot be disantangled in time asymmetry in opposite sign dilepton pairs, or more generally in their time evolution. Therefore to test CPT the value of x must be supplied by other experiment. Conversely the knowledge of the CPT violating parameters allows the determination of x from this asymmetry.

If CPT holds the time asymmetry can be written as :

$$\bar{A}_{CPT} = 8\Re(\epsilon)\Re(x) - 8\frac{\Im(x)}{S_L}\frac{\Delta m \Gamma_L}{\Gamma^2 + \Delta m^2} \tag{64}$$

and inserting the experimental limits on x :

$$|\bar{A}_{CPT}| < 1.1 \times 10^{-3} \tag{65}$$

A value of A_{CPT} larger than 10^{-3} signals an actual CPT violation or in kaon mass matrix or in $\Delta S = -\Delta Q$ transition amplitudes.

11 Aknowledgents

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