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MASS GENERATION IN THE STANDARD MODEL

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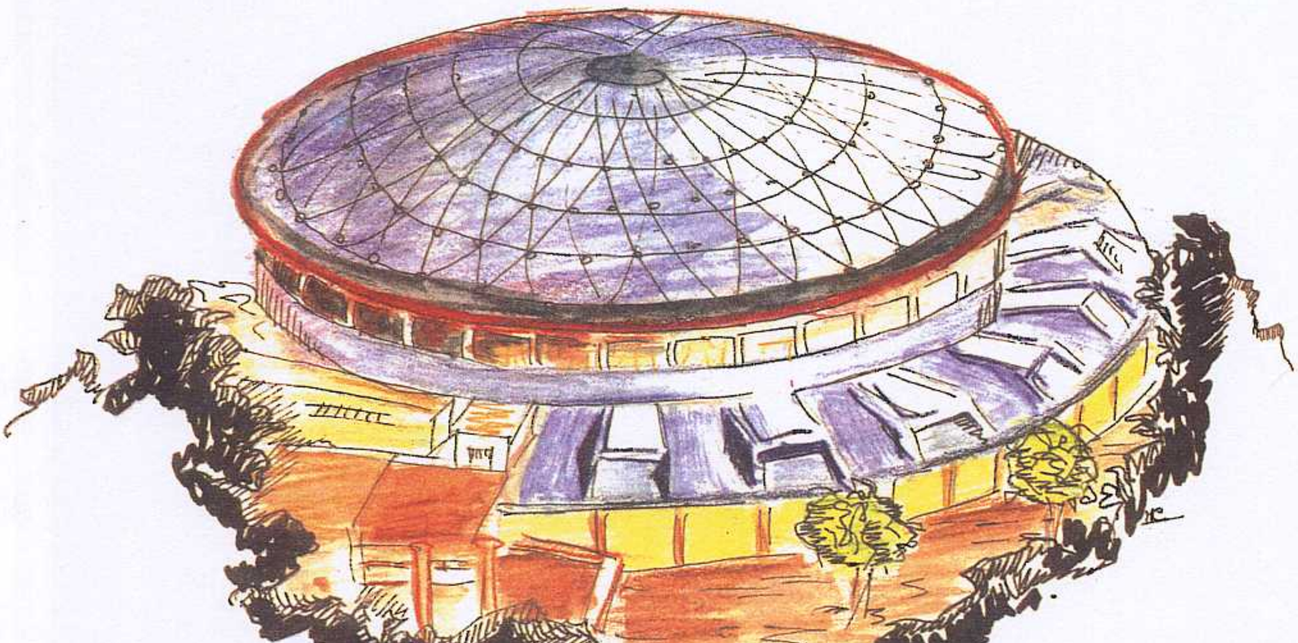

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MASS GENERATION IN THE STANDARD MODEL

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ABSTRACT

A proposal of mass generation in the Standard Model (SM) is presented, based on the idea that the violent fluctuations of quantum gravity at the Planck scale plays a nature regulator for the SM with fundamental length $a_p \sim 10^{-33}$cm, thus gauge invariant four-interactions are necessary due to the "no-go" theorem of Nielsen and Ninomiya, forbidding a sensible formulation of the usual electroweak action on a such "Planck lattice". We present that for certain values of the new four-fermion couplings a spontaneous violation of SM chiral symmetry emerges which (i) avoids the "no-go" theorem, (ii) produces a $\bar{t}t$ condensate model without appearance of the low-energy Higgs boson. The mass generation of other fermions and intermediate gauge bosons, are briefly discussed.

1. Introduction

In this lecture, I shall outline and discuss that the gauge principle realized in the SM prevents from massive fermions and gauge bosons, and the quantum gravity acts as a nature regulator called the "Planck lattice"(PL) which breaks gauge symmetry and thus gauge invariant four-fermion interactions are necessary by the "no-go" theorem of Nielsen and Ninomiya. In this framework of the SM in the PL, gauge principle is maintained and however all fermions as well as intermediate gauge bosons can acquire their masses. The structure of the lecture will thus be as follows. In Section 2 the possibility of quantum gravity may endow space-time with "foam" structure and the usual Higgs mechanism for mass generation in the SM are briefly discussed. Section 3 will provide an brief outline of the "no-go" theorem of Nielsen

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and Ninomiya and the necessity of the gauge invariant four-fermion interactions. The resulting $\bar{t}t$-condensate without the low-energy Higgs boson will be presented in Section 4. The discussions of other fermions and intermediate gauge bosons acquiring their masses are briefly followed in Section 5 and Section 6. And finally I shall close in Section 7 with a few comments and conclusions.

2. The Planck Lattice

The very-small-scale structure of space-time, the arena of physical reality, has recently attracted a great deal attention. the realization of important role that quantum gravity plays in determining such a structure, coupled with the grave difficulties that this theory faces in the usual, perturbative formulation at distance smaller than the Planck length $a_p \sim 10^{-33}$cm (the Planck mass $\Lambda_p \sim 10^{19}$GeV), is the basic motivation of a umber of theoretical proposals tc overcome such difficulties and provide a solution of this fascinating problem. As well known, the very popular superstring theory belongs to this class of proposals, overcoming the problems of the conventional quantum gravity by postulating that the fundamental "constituents" of space-time, the physical events, at distance smaller than a_p exhibit the rather complex structure of a space-time "string" instead of a simple space-time point. In this way at distance smaller than a_p a whole brave new world emerges, described by the rich spectrum of excitations of the string, whose phenomenology, however, seen from our vantage point appears, to say the least, remote.

At the opposite end we may conceive that precisely due to the violent quantum fluctuations that the gravitational field must exhibit at a_p, space-time somehow "end" there. Either by the creation of a "foam" [1], or by some other mechanism which we need not discuss here, one may conceive that as a result the physical space-time gets endowed with a fundamental length, a_p, and thus the basic arena of physical reality becomes a lattice with lattice constant a_p.

It is this possibility that we wish to explore in a fundamental problem of the SM, the problem of fermion and intermediate gauge boson masses. As is well known the gauge-symmetry principle as realized in the electroweak sector $SU_L(2) \otimes U_Y(1)$ demands that, at lagrangian level, all fermions and gauge bosons must be massless. In order to avoid an obvious theoretical disaster and save the gauge principle, the Higgs mechanism [2] had to be grafted upon the beautiful gauge lagrangian, with a completely *ad hoc* negative squared mass in order to secure a spontaneous symmetry breaking mechanism for the generation of fermion masses as well as gauge boson masses. It is for this reason that most people regard the Higgs mechanism at best as a simple, rough approximation to real situation, and deny such field any fundamental reality. And this is perfectly in line with what one learns from superconductivity, where Higgs mechanism was introduced as a simple, phenomenological means to describe the more complicated dynamics of Cooper pairs.

3. The Necessity of Four-fermion Interactions

Suppose now that, following the considerations reported in Section 2, we wish to write the SM without Higgs field on the PL, a profound result obtained more than ten years ago [3] in the form of a "no-go" theorem, tells us that there is no consistent way to transpose straightforwardly on a lattice the lagrangian of the continuum theory. While this is not the place for a detailed discussion of this important result, we limit ourselves to remarking that the origin of the "no-go" theorem is quite simple, stems from the peculiar dispersion relation of fermion on a lattice. According to it, in the long wave-length limit more than one species appear in the spectrum and, for instance of a left-handed Weyl field ψ_L with chiral charge, say $\chi = -1$, there appear four right-handed and four left-handed species. A situation which is clearly at variance with experimental observation.

Wilson [4] has shown how can one modify the lagrangian by adding a simple bilinear term, so as to remove the unwanted "replicas" from the long wave-length regime. However, this can only be done by sacrificing chiral invariance, and "no-go" theorem shows that no bilinear modification can be made to obey the chiral gauge-principle. Thus if we are to go ahead in our program of putting the SM on the PL without sacrificing the chiral gauge-principle, the "no-go" theorem tells us that the simple transposition of the continuum lagrangian must be supplemented by extra-terms that are least quadrilinear in fundamental fermion fields. Familiarity with the original Nambu-Jona-Lasinio model [5] [6] immediately reminds us that such quadrilinear terms can be made to obey the chiral gauge-principle. Thus an $SU_c(3)\otimes SU_L(2)\otimes U_Y(1)$ chiral-gauge invariant lagrangian which evades *in principle* the "no-go" theorem can be written on the PL as [7]

$$S_{PL} = S_G + \sum_F (S_D^F + S_{NJL}^{F1} + S_{NJL}^{F2}), \qquad [3.1]$$

where S_G is the usual Wilson gauge-action, S_D the usual Dirac action, which will both be analyzed in a forthcoming article [8], F=l (F=q) denotes its lepton (quark) sector. The simplest new quadrilinear NJL-terms are as follows [9]:

$$S_{NJL}^{F1} = G_1 \sum_x \{\bar\psi_L^{Fi}(x)\cdot\psi_R^{Fj}(x)\bar\psi_R^{Fj}(x)\cdot\psi_L^{Fi}(x)\} \qquad [3.2]$$

and

$$S_{NJL}^{F2} = \frac{G_2}{2}\sum_{\pm\mu,x}\left[\bar\psi_L^{Fi}(x)G_\mu^L(x)U_\mu^c(x)\cdot\psi_R^{Fj}(x+a_\mu)\bar\psi_R^{Fj}(x)G_\mu^R(x)U_\mu^c(x)\cdot\psi_L^{Fi}(x+a_\mu)\right], \qquad [3.3]$$

where the indices i, j denote fermion families; the Dirac indices are denoted by scalar product "$\cdot$". The gauge link $U_\mu^c(x)$ connects left-and right-handed quark fields in neighboring points so as to have the $SU_c(3)$ gauge symmetry. The chiral gauge links $G_\mu^L(x)$ ($G_\mu^R(x)$) connect left-handed (right-handed) fermion fields to enforce $SU_L(2)\otimes U_Y(1)$ chiral gauge symmetry. $G_{1,2}$ are two, yet unspecified, Fermi-type $O(a_p^2)$ coupling constants which are assumed universal for both the lepton and quark sectors. Thus action [3.1] is invariant under chiral gauge symmetries $SU^c(3)\otimes$

$SU_L(2) \otimes U_Y(1)$ and a global $U_L(3) \otimes U_R(3)$ in generation space. In addition it evades, at least in principle, the "no-go theorem". The question now is: does it yield a long wave-length spectrum in agreement with observation (free from the unwanted doublers)?

4. $\bar{t}t$ Condensate and Composite Particles

In order for the action [3.1] to get rid of unwanted doubler and avoid *in practice* the "no-go" theorem, it is necessary that the quadrilinear terms S^{F1}_{NJL} and S^{F2}_{NJL} develop a dynamical chiral symmetry breaking through the following non-zero vacuum expectation values [7] (V_4 is the 4-dimensional volume)

$$m_F^{ij} = -\frac{G_1}{2V_4}\sum_x \langle \bar{\psi}_i^F(x)\psi_j^F(x)\rangle;$$
$$\bar{r}_F^{ij} = \frac{G_2}{4V_4}\sum_{\mu,x}\{\langle \bar{\psi}^F_{Li}(x)U^c_\mu(x)\psi^F_{Rj}(x+a_\mu)\rangle + \text{h.c.}\}. \qquad [4.1]$$

Indeed, should this happen, one would obtain the following effective lattice action of the Wilson type [4]:

$$S^{eff}_{PL} = S_G + S_D$$
$$+\sum_{xF}\left\{\bar{\psi}^F(x)M_F\psi^F(x) - \frac{1}{2}\sum_\mu\left(\bar{\psi}^F(x)G^L_\mu(x)\bar{r}_F G^R_\mu U^c_\mu(x)\psi^F(x+a_\mu) + \text{h.c.}\right)\right\}, \qquad [4.2]$$

where $M_F = m_F + 4\bar{r}_F$. Thus through dynamical symmetry breaking the obligatory (in order to get rid of the doublers) Wilson term ($r_F = a_p\bar{r}_F$) gets produced together with a mass term (m_F) which, according to our action, must necessarily come with it. In this way the evasion of the "no-go theorem" entails an extra bonus: the generation of a fermion mass term. The $SU_L(2) \otimes U_Y(1)$ and $U_L(3) \otimes U_R(3)$ symmetries are clearly broken and the only surviving gauge symmetries $SU_c(3)$ and $U_{em}(1)$ [9]. Given the quadrilinear NJL-terms [3.2] and [3.3], we construct an effective potential in terms of m_F, $\bar{r}_F$ [8]. A non-trivial dynamical symmetry breaking may emerge only if the matrices m_F, $\bar{r}_F$ obey a set of coupled, self-consistent equations obtained from variation of the effective potential. We call these equations "gap equations", that turn out to have the following approximate matrix-form in the weak-isospin space [9]

$$m_F a_p = 2g_1\int_l \frac{m_F a_p}{\text{Den}^F(l)},$$
$$r_F = g_1\int_l \frac{r_F \sin^2\frac{l_\mu}{2}}{\text{Den}^F(l)}, \qquad [4.3]$$
$$r_F = -g_2\int_l (\cos)\frac{m_F a_p + 2r_F\sin^2\frac{l_\mu}{2}}{\text{Den}^F(l)}$$

where $g_{1,2}a_p^2 = N_c G_{1,2}$; $r_F = a_p\bar{r}_F$; $\int_l = \int_{-\pi}^{\pi}\frac{d^4 l}{(2\pi)^4}$; $(\cos) = \sum_\mu \cos l_\mu$ and

$$\text{Den}^F(l) = \sin^2 l_\mu + (m_F a_p + 2r_F\sin^2\frac{l_\mu}{2})^2. \qquad [4.4]$$

We see that for any $g_2 \neq 0$, one gets $r > 0$, which removes the doublers through a Wilson-type mechanism. By using now the gap equation [4.3] for $m_F a_p \neq 0$ and the chain approximation, for the Goldstone modes $\langle \bar{\psi}_i(x)\gamma_5\psi_i(x)\bar{\psi}_i(0)\gamma_5\psi_i(0)\rangle$ and scalar modes $\langle \bar{\psi}_i(x)\psi_i(x)\bar{\psi}_i(0)\psi_i(0)\rangle$, we find the inverse propagators

$$\Gamma^{-1}_{Fp}(q) = B^2(q) I^F_p(q); \quad I^F_p(q) = -\frac{N_c}{2}\int_l \frac{\sum_\mu \left(\cos^2 l_\mu + r_F^2 \sin^2 l_\mu\right)}{\mathrm{den}^F\left(l+\frac{q}{2}\right)\mathrm{den}^F\left(l-\frac{q}{2}\right)}; \qquad [4.5]$$

and

$$\Gamma^{-1}_{Fs}(q) = 2\left(B^2(q) I^F_s(q) + 4M_F^2\right); \; I^F_s(q) = \frac{N_c}{4}\int_l \frac{\sum_\mu (\cos^2 l_\mu)}{\mathrm{den}^F\left(l+\frac{q}{2}\right)\mathrm{den}^F\left(l-\frac{q}{2}\right)}, \qquad [4.6]$$

where $B^2(q) = \sum_\mu \left(\frac{4}{a_p^2}\sin^2\frac{q_\mu a_p}{2}\right)$ and

$$4M_F^2 = 4\int_l \frac{\left[m_F + \frac{2r_F}{a_p}\sin^2\frac{l_\mu}{2}\right]^2}{\mathrm{den}^F\left(l+\frac{q}{2}\right)\mathrm{den}^F\left(l-\frac{q}{2}\right)}. \qquad [4.7]$$

For $q_\mu a_p \ll 1$, we can calculate numerically the position of the pole of the scalar mode, which turns out to be of the order of the Planck mass instead of $4m_F^2$. Indeed one gets

$$4M_F^2 = 4m_F^2 + 0.8r\frac{m_F}{a_p} + 0.9\frac{r^2}{a_p^2}, \qquad [4.8]$$

which for $r = 1$ pushes this pole at the Planck mass, making it thus disappear from the observable, low energy spectrum. Analogously, we find charged Goldstone modes appearing in the flavored channels corresponding to the quantum numbers of the $W^\pm$ bosons.

We shall now analyse the solutions of [4.3]. If we look for "physical" solutions, for which the eigenvalues of the mass matrix are such that $m_F a_p \ll 1$ and the doublers are removed by $r_F \neq 0$, then equation [4.3] for r approximately decouples and becomes

$$1 = -g_2 \int_l (\cos)\frac{2\sin^2\frac{l_\mu}{2}}{\sin^2 l_\mu + 4r^2(\sin^2\frac{l_\mu}{2})^2}. \qquad [4.9]$$

Notice that, assuming a universal coupling constant G_2 in [10], the Wilson parameters for quarks and leptons are different because of the color number N_c. The solution $r = r_F(g_2)$ is reported in fig. 1.

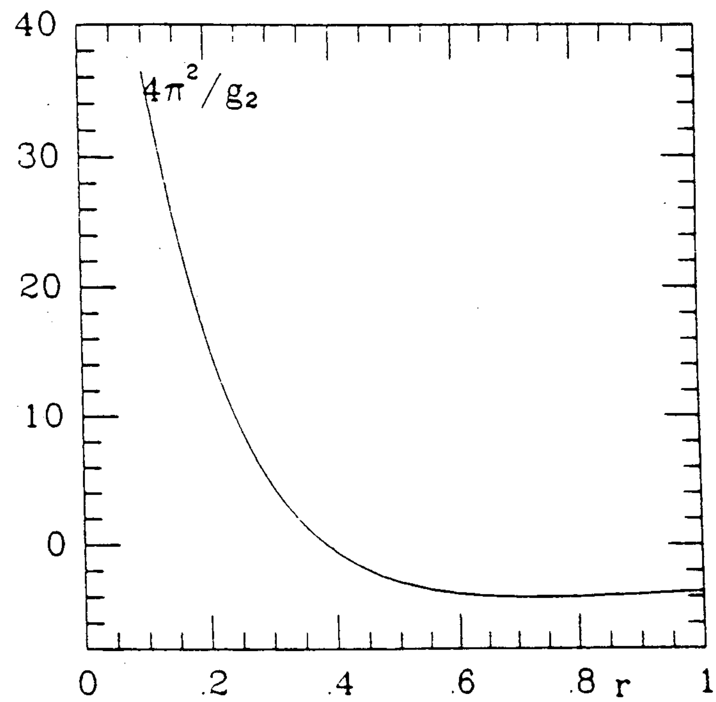

Fig. 1 $\frac{4\pi^2}{g_2}$ as a function of r

In order to understand the implications of obtaining physical solutions $m_F a_p \ll 1$, we use the gap-equations [4.3] to draw the phase diagram (fig. 2), where the phases $(m < 0)$ are separated from the phase $(m > 0)$ by critical lines, on which $m = 0$.

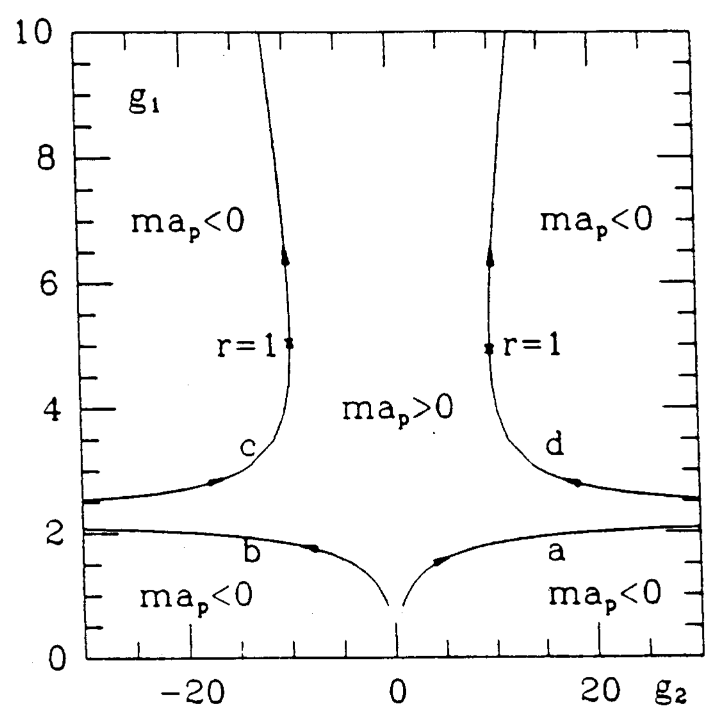

Fig. 2 phase diagram in terms of g_1 and $g_2, a, b(r = 0 \rightarrow 0.385)$ and $c, d(r = 0.385 \rightarrow 1 \rightarrow \infty)$.

This phase diagram is in agreement with the continuum NJL-model that possesses only the coupling constant G_1. The requirement of obtaining physical solutions for quarks $N_c = 3$ with $\bar{r}_q a_p = r_q \sim O(1)$ and $m_F a_p \ll 1$ can clearly be met by a coupling constant $g_2 \gg 1$, in this case one has $r \rightarrow 0.385$. On the other hand the coupling g_1 is required to lie in the vicinity of the critical value g_c^2. This holds for quarks, for it is easy to see that for leptons ($N_c = 1$) equation [4.3] does not allow for massive solutions. Thus leptons at this stage remain massless. In order to have $\bar{t}t$ condensate and get rid of doublers, we chose the mass matrix, which only has one non-zero eigenvalue, and $r = rI$ (I is identity matrix) for every fermions, which are the solution to gap equation [4.3].

5. Fermion Masses without Goldstone Bosons

In this section we wish to outline our strategy to proceed further in our computation of fermion masses, by considering the contributions arising from the photon γ, $W^{\pm}$, and Z° interaction that we have so far neglected. The problem we must now solve is the evaluation of the self-energy function of the fermions with the SM action and supplemented by the interactions [3.2] and [3.3] on the Planck lattice. In order to render this problem manageable we shall work in the "rainbow" approximation of the Dyson equation, which neglects all vertex and gauge propagator corrections. Thus on the Planck lattice we must solve the Dyson-Schwinger equation which we give in diagrammatic form fig. 3.

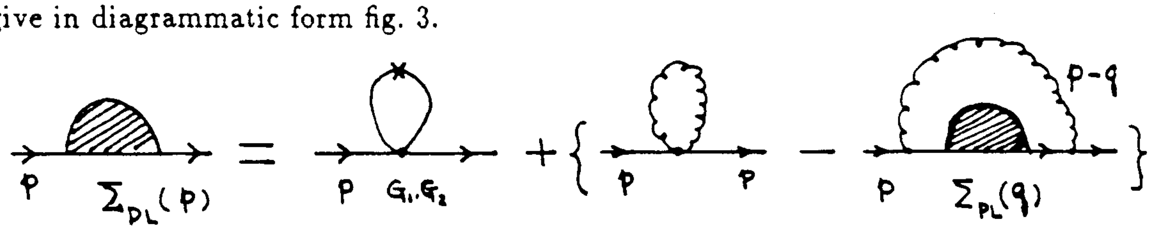

Fig. 3 The diagrammatic form of Dyson-Schwinger equation

The first diagram in the left-hand side represents the result of the first stage calculation of ref. [9], while the second is a contact interaction typical of gauge theories on a lattice. Please note that in the Dyson equation we have not included colored gluon exchanges for they are related to color confinement, producing an infinite mass-shift for quarks and all colored states. We shall return on this point in a future publication; for, as a matter of fact, in this paper our sole intention is to study the structure of the solutions of equation (fig. 3) on a Planck lattice and compare it with the rather large amount of extant works on the continuum version of the Dyson equation. A final remark on our approximation: the smallness of the coupling constants associated with electroweak exchanges makes it quite plausible.

Let's now turn to equation (fig. 3), where, for illustration purposes, we keep only the exchange of a massive vector gauge-field of mass m_v coupled to a vector current. We can write it as:

$$\Sigma_{PL}(p) = W_r(p) + \bar{g}^2 \int_{-\frac{\pi}{a_p}}^{\frac{\pi}{a_p}} \frac{d^4q}{(2\pi)^4} \frac{1}{S(p-q)^2 + m^2}$$
$$\left(\delta_{\mu\nu} - \xi \frac{S_\mu(p-q)S_\nu(p-q)}{S(p-q)^2 + m_v^2}\right) \mathrm{tr}\left[V^{(2)}_{\mu\nu}(p,p) - V^{(1)}_\mu(p,q)\frac{1}{\gamma_\rho D_\rho(q) + \Sigma_{PL}(q)}V^{(1)}_\nu(p,q)\right], \quad [5.1]$$

where $S_\mu(l) = \frac{2}{a_p}\sin\frac{l_\mu a_p}{2}$, $D_\mu(l) = \frac{1}{a_p}\sin(l_\mu a_p)$ and $\bar{g}^2 = e^2(\frac{g^2(N^2-1)}{2N})$ for $U_{em}(1)(SU(N))$ gauge group. The $W_r(p)$ is the self-energy of fermion generated through the Nambu-Jona Lasinio mechanism [5]. As discussed in ref. [9]for one-loop contribution, $W_r(p) = M_t + \frac{2r}{a_p}\sum_\mu \sin^2\frac{p_\mu a_p}{2}$; M_t is non-vanishing for the top-quark only and, however r is different from zero for all fermions. The vertices [10] are $(k_\mu = \frac{(p_\mu+q_\mu)a_p}{2})$

$$V^{(1)}_\mu(p,q) = (\gamma_\mu \cos k_\mu + r\sin k_\mu); \quad V^{(2)}_{\mu\nu}(p,q) = a_p(-\gamma_\mu \sin k_\mu + r\cos k_\mu)\,\delta_{\mu\nu}. \quad [5.2]$$

In this paper, the only property of $W_r(p)$; that we need specify, is its being $O(a_p)$ for $pa_p \ll 1$.

We limit our attention to fermions other than the top quark and to values of the external momentum p such that $pa_p \ll 1$, and in this kinematical region the self energy function $\Sigma_{PL}(p)$ should not and does not differ from its continuum limit version $\Sigma_c(p)$. One of the main novelties of [5.1] is the non trivial interplay between the continuum-limit region, i.e., for momenta $(qa_p \ll 1)$, and the truly discrete region, which is probed for momenta $qa_p \simeq 1$. In order to study such interplay it is important to introduce a "dividing scale" ϵ, such that $pa_p \ll \epsilon \ll \pi$. Separating the integration region in [5.1] into two regions $[0,\epsilon]^4$ and $[\epsilon,\pi]^4$, we may write our integral equations as

$$\Sigma_c(p) = \frac{\lambda}{4\pi^2}\int_{\Lambda=\epsilon\Lambda_p} d^4q \frac{1}{(p-q)^2 + m_v^2}\frac{\Sigma_c(q)}{q^2+m^2} + \delta_{PL}(r,\epsilon), \quad [5.3]$$

where $\lambda = \frac{3g^2}{4\pi^2}$, and the continuum-limit integral equation, following the common practice, has been linearized, i.e., in the denominator one sets $\Sigma_c(q) \simeq m$, the"physical" mass of the fermion, and the Landau gauge $\xi = 1$, in which the self-energy function for $m_v = 0$ contains a mass renormalization only, is chosen for simplicity. As for $\delta_{PL}(r,\epsilon)$, the contribution to the integral equation from the discrete lattice region, we may write it as $(l_\mu = q_\mu a_p)$:

$$\delta_{PL}(r,\epsilon) \simeq \frac{\lambda r}{a_p}\int_{[\epsilon,\pi]^4} \frac{d^4l}{12\pi^2(4\sin^2(\frac{l_\mu}{2}))}\frac{1}{}\left[\frac{1}{2} - \frac{\frac{1}{r}G_{PL}(l)(-\cos^2(\frac{l_\mu}{2}) + r^2\sin^2(\frac{l_\mu}{2})) + \sin^2(l_\mu)}{G_{PL}(l)^2 + \sin^2(l_\mu)}\right]$$
$$-\lambda\int_{[\epsilon,\pi]^4}\frac{d^4l}{4\pi^2}\frac{\Sigma_c(l)}{(4\sin^2(\frac{l_\mu}{2}))}\left[\frac{-\cos^2(\frac{l_\mu}{2}) + r^2\sin^2(\frac{l_\mu}{2})}{G_{PL}(l)^2 + \sin^2(l_\mu)}\right], \quad [5.4]$$

where we have decomposed $\Sigma_{PL}(l)$ as $a_p\Sigma_{PL}(l) = a_p\Sigma_c(l) + G_{PL}(l)$, and $G_{PL}(l) = W_r(l)$ is non zero only in the discrete lattice region, i.e. for $l \in [\epsilon,\pi]^4$. The dependence on the external momentum pa_p is omitted in $\delta_{PL}(\epsilon, r)$ because of $pa_p \ll l \in [\epsilon,\pi]^4$.

Note that (i) there is a perfect cancellation of gauge-dependent term $a_p^{-1}\xi$ between the contact and the "rainbow" diagrams, that is guaranteed by Ward's identities; (ii) $\delta_{PL}(\epsilon, r)$ cannot vanish if r, as in our case, does not vanish. Thus, on the Planck lattice, the self energy integral equation [5.3] acquires an inhomogeneous term even for $M_t = 0$. This implies the very important consequence that [5.3] admits only massive solutions provided $\lambda > 0$ [11]. The most appealing aspect of this result is that on the Planck lattice mass gets generated for all fermions, but the top-quark, without the appearance of Goldstone bosons: the reason for this fact, extremely important phenomenologically, is the connection of the inhomogeneous term $\delta_{PL}(\epsilon, r)$ with a non zero r-value that, as emphasized, already breaks the chiral symmetry of the four-fermion interaction.

At a moment we are not able to show the cancellation of a_p^{-1} divergence by intrinsic requirement of hidden symmetries in the theory, this linear divergence can be cancelled by simply adding counterterm. We can now write the finite part of dP_L(r,‰) is

$$\delta^f_{PL}(r, \epsilon) = -\int_{[\epsilon,\pi]^4} \frac{d^4 l}{4\pi^2} \frac{\Sigma_c(l)}{(4\sin^2(\frac{l_\mu}{2}) + m_v^2)} \left[\frac{-\cos^2(\frac{l_\mu}{2}) + r^2 \sin^2(\frac{l_\mu}{2})}{G_{PL}(l)^2 + \sin^2(l_\mu)}\right]. \qquad [5.5]$$

Let's now address the important question of the ϵ-independence of our result. The introduction of this "dividing scale" in [5.3] is, apart from the requirement $pa_p \ll \epsilon \ll \pi$, rather arbitrary thus no dependence on ϵ should appear in our final results. In order for such independence to occur, as it must occur, it is clear that the ϵ-dependence from the continuum integral in eqn. [5.3], that is clearly logarithmic, must be compensated by an analogous logarithmic term arising in the calculation of $\delta_{PL}(r, \epsilon)$. Thus segregating the $\ell n\, \epsilon$-term in $\delta^f_{PL}(r, \epsilon)$, we may write

$$\delta^f_{PL}(r, \epsilon) = \lambda \delta^1_{PL}(r)\, \ell n\, \epsilon + \lambda \delta^0_{PL}(r), \qquad [5.6]$$

where $\delta^{\circ}_{PL}(r)$ and $\delta^0_{PL}(r)$ is independent of ϵ, [5.3] becomes

$$\Sigma_c(p) = \frac{\lambda}{4\pi^2}\int_{\Lambda_p} d^4 q \frac{1}{(p-q)^2 + m_v^2} \frac{\Sigma_c(q)}{q^2 + m^2} + \lambda \delta^{\circ}_{PL}(r). \qquad [5.7]$$

This is an inhomogeneous equation admitting a finite solution for small λ. We will report this result soon [12].

6. Neutrino and Intermediate Gauge Boson Masses

The D.S. equation we adopted in above contains only vector-like coupling vertices and massless gauge bosons. We are going to focus on the D.S. equation involving V-A couplings and massive gauge bosons. Neutrinos, which belong to a special category in fermion family, have only weak interaction with massive intermediate gauge bosons. In the lattice with spacing a_p, we not only have left-handed neutrinos but also right-handed neutrinos since again there exists a Wilson term which is essentially an interacting term between left- and right- handed species. It is expected that they would acquire small masses by weak interaction with heavy intermediate gauge bosons. Since weak interactions change fermion flavours, the fermion mass matrix we will calculate should not be diagonal in the basis of weak interaction eigenvectors. Thus we expect that the different eigenvalues (fermion masses) would be obtained if the fermion mass matrix is diagonalized and the transfer matrices would be related to the Cabibbo-Kobayashi-Masicawa matrix.

Let us turn to intermediate gauge boson masses. The photon and gluon (without considering their self-interaction) are massless since a vector-like gauge symmetry can be maintained in an appropriate regularization (via a cutoff a_p). This is guaranteed by the so-called Elitzur's theorem [13], which states: a spontaneoús breaking of local symmetry for a symmetrical gauge theory (with cutoff and vector-like gauge coupling) without gauge fixing is impossible. On the other hand, the intermediate gauge bosons are endowed with masses because they couple to fermions in a chiral fashion and chiral-gauge symmetry cannot be maintained in any regularization. We have been trying to prove that it is impossible to maintain gauge symmetry in a regularized gauge theory where the gauge coupling is in chiral fashion (parity violating) [14]. Thus, instead of adding local gauge-variant counterterms to lagrangian for maintaining gauge symmetry order by order, as one usually does for perturbative QED and QCD, we should keep the local gauge-variant terms from the calculation of the two-point function, which turn out to be mass terms of gauge bosons. The gauge boson masses obtained in this way are related to the sum of all fermion masses [15], which are finite once the spontaneous symmetry breakdown happens and fine tuning is performed (as described in section 4). In this way, we will see the counterterm (divergence) structure of a gauge theory is not affected by the occurrence of spontaneous symmetry breakdown. In another words, we do not need new counterterms after fermions and intermediate boson become massive. It is then expected to be renormalizable and all effects from the short wave-length regime ($\Lambda = \epsilon\Lambda_p$) would be renormalized away so as to have meaningful results defined on the physical mass-shell in the long wave-length regime. In addition, the triangle-anomalies from the calculations of the three-point function are cancelled naturally by the fermion contents of the theory, thus conservation of matter source current and Ward identities are preserved and the theory is unitary. This is also important for renormalizability.

7. Conclusions and Remarks

In conclusion, we have shown that on the Planck lattice, whose "raison dêtre" may well reside in the violent quantum fluctuations of the metric field at the Planck scale a_p, a consistent SM requires the addition of extra terms, quadrilinear in the Dirac fields, whose coupling constants can be determined to induce the emergence of the $\bar{t}t$-condensate model with $m_t a_p \ll 1$ and $0 < r \leq 1$: m_t gives rise to the scale of electroweak breakdown and $r \neq 0$ endows the composite scalar and the mirror fermions with masses at the Planck scale, making them disappear from the low-energy spectrum. We should also stress that, differently from the continuum $\bar{t}t$-condensate model, its Planck lattice version disposes in a nice way of the scalar composite, thus leading to the disappearance from the spectrum of a particle that would resemble the Higgs boson. The implication of this conclusion for the present and future phenomenology is too obvious to need further comments. In this frame-work, we are proceeding the calculations of fermion and gauge boson masses, and other problems we mentioned in this paper.

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