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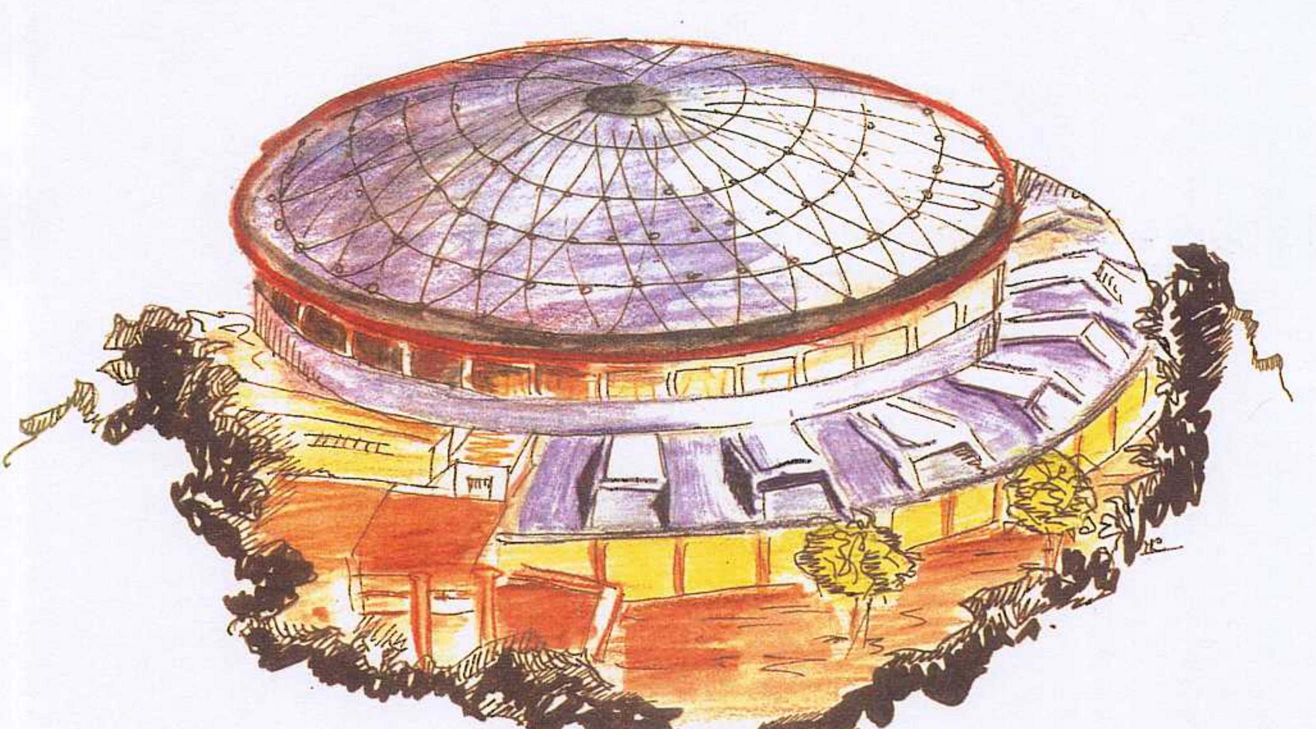

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CP AND CPT VIOLATION IN NEUTRAL KAON DECAYS

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Introduction

The study of CP violation in neutral K decays has been, from the outset, the primary goal of a high-luminosity Φ-factory, like DAFNE. At the beginning, attention was focussed almost exclusively on the measurement of the direct CP-violation parameter, ε'/ε. Due to the very interesting work of Buchanan et al.[1], it has been realized that, in addition, a Φ-factory offers the unique possibility to make a clean test of the CPT symmetry, at the level of precision of 10^{-4} or so, and which is independent from possible conspiracies, still allowed in the presently measured quantities.

The aim of the present article is to provide a self-consistent introduction to the phenomenology of CP and CPT violation in neutral Kaon decays and to give a first illustration of the impact of DAFNE on the issue of the CPT symmetry. More details are provided in the subsequent papers in this Chapter[2,3]. Present calculations of ε'/ε in the Standard Theory are also reviewed here, while CP violation in charged Kaon decays is deferred to a later part of this Report [4].

Of course, there exist in the literature many excellent reviews of the subject, starting from the classical and influential article of Lee and Wu [5]. In the more recent literature, CPT symmetry in Kaon decays has been analyzed by Barmin et al.[6] and, with reference to the experiments at a Φ-factory, in reff.[7,1].

1. Hamiltonian matrix, eigenvalues and mixing

The hamiltonian of the neutral Kaon system, in the particle rest frame, is a complex, 2x2 matrix:

$$H = \begin{bmatrix} h & l \\ m & n \end{bmatrix} = M - \frac{i}{2}\Gamma \qquad (1.1)$$

We are in the basis: up = $|K^0\rangle$, down = $|\bar{K}^0\rangle$; h, l, m, n are complex numbers (so that H depends upon 8 real parameters), and M and Γ hermitian matrices.

The sign of the antihermitian part of H is determined by the requirement that it must lead to exponential *damping* of the wave-function, for $t \to +\infty$. With the sign given in Eq.(1.1), the time-dependent factor of the wave-function is: $\exp(-iEt) = \exp(-iMt - \frac{1}{2}\Gamma t)$, which is correct, provided that Γ has eigenvalues ≥ 0.

We shall also write:

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{22} \end{bmatrix} \quad ; \quad \Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{22} \end{bmatrix}$$

Symmetries:

CP : $|K^0\rangle \to |\bar{K}^0\rangle$, $|\bar{K}^0\rangle \to |K^0\rangle$ (1.2)
T : makes the complex conjugate (1.3)

it follows (τ_i= Pauli matrices):

CP : $M \to \tau_1 M \tau_1$, $\Gamma \to \tau_1 \Gamma \tau_1$, (1.4)
T : $M \to M^*$, $\Gamma \to \Gamma^*$ (1.5)

If we develop both M and Γ in the basis of the Pauli matrices plus the identity,

$X = c_0 \mathbf{1} + c_i \tau_i$; $c_{0,..,3}$ real

we are lead to the following Table, which gives the sign taken by each component under CP, T and CPT transformations.

Tab.1

	c_0,	c_1,	c_2,	c_3
CP:	+	+	-	-
T:	+	+	-	+
CPT:	+	+	+	-

We recover the familiar result that CPT implies equal diagonal elements. Also, CP conservation implies CPT conservation (unlike in 2π decay amplitudes, see below).

There is some freedom to redefine the phases of the states $|K^0\rangle$ and $|\bar{K}^0\rangle$. Strangeness is conserved by strong and e.m. interactions, so we can make the change:

$|n, S\rangle \to e^{-i\alpha S} |n, S\rangle$

i.e.:

$$|K^0\rangle \rightarrow e^{-i\alpha}|K^0\rangle \; ; \; |\bar{K}^0\rangle \rightarrow e^{+i\alpha}|\bar{K}^0\rangle \qquad (1.6)$$

In the new basis, the CP transformations would not be given by (1.2) anymore, and the matrix H would not satisfy the rules given above, even if CP *is* conserved. All this amounts to say that CP is conserved if there exists a change of phase of the form (1.6) such that, in the new basis, M and Γ have vanishing components along τ_2 and τ_3. This happens when M_{12} and Γ_{12} have the same phase and we conclude that the phase-invariant condition for CP conservation is:

$$\arg\left(\frac{\Gamma_{12}}{M_{12}}\right) = 0 \qquad \text{(CP symmetry)} \qquad (1.7)$$

Diagonal elements are not affected by the phase change, so that the condition for CPT symmetry is always:

$$\begin{cases} M_{11} = M_{22} \\ \Gamma_{11} = \Gamma_{22} \end{cases} \qquad \text{(CPT symmetry)} \qquad (1.8)$$

Eigenvalue equation:

$$\det\begin{bmatrix} h-\lambda & l \\ m & n-\lambda \end{bmatrix} = 0 \quad \text{leads to:}$$

$$\lambda_\pm = \frac{1}{2}[h + n \pm \sqrt{(h-n)^2 + 4lm}\ ]; \qquad (1.9)$$

note: $(\lambda_+ - h)(\lambda_- - h) = -lm$.

Eigenvectors:

$$v = \begin{bmatrix} p \\ q \end{bmatrix}$$

$$\left(\frac{q}{p}\right)_\pm = \frac{(\lambda_\pm - h)}{l} = \frac{n-h \pm \sqrt{(h-n)^2+4lm}}{2l} \qquad (1.10)$$

The identification of K_L and K_S states is obtained by going to the symmetric limit of exact CP (and CPT). In this limit, see (1.7) and (1.8):

$$\lambda_\pm = h \pm \sqrt{lm}$$

$$\frac{l}{m} = \frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*} = \frac{M_{12}}{M_{12}^*} = \text{phase factor} =_{DEF} e^{-i2\alpha}$$

so that:

$$v_+ = p_+ \begin{bmatrix} 1 \\ e^{i\alpha} \end{bmatrix} = p_+ e^{i\alpha/2} \begin{bmatrix} e^{-i\alpha/2} \\ e^{i\alpha/2} \end{bmatrix} = \text{CP- even (after phase redefinition)}$$

$$v_- = -q_- \begin{bmatrix} e^{-i\alpha} \\ -1 \end{bmatrix} = -q_- e^{-i\alpha/2} \begin{bmatrix} e^{-i\alpha/2} \\ -e^{i\alpha/2} \end{bmatrix} = \text{CP- odd (after phase redefinition)}$$

Therefore, by continuity, we identify:

$$|K_S\rangle = v_+ \; ; \; |K_L\rangle = v_- \qquad (1.11)$$

Conventionally, the eigenvectors are written in terms of two complex numbers, $\varepsilon_{L,S}$, defined by:

$$|K_S\rangle = v_+ = N_S\,(|K_1\rangle + \varepsilon_S\,|K_2\rangle)\;;\;\; N_S^{-2} = 1 + |\varepsilon_S|^2 \qquad (1.12)$$

$$|K_L\rangle = v_- = N_L\,(|K_2\rangle + \varepsilon_L\,|K_1\rangle)\;;\;\; N_L^{-2} = 1 + |\varepsilon_L|^2 \qquad (1.13)$$

with:

$$|K_{1,2}\rangle = \frac{1}{\sqrt{2}}\,(|K_0\rangle \pm |\bar{K}^0\rangle) \qquad (1.14)$$

The Hamiltonian matrix is determined by 8 real parameters, which we can substitute with the 2 complex eigenvalues and the 2 complex mixing parameters, $\varepsilon_{S,L}$. The relevant formulae are:

$$(m_S+m_L) - \frac{i}{2}\,(\Gamma_S+\Gamma_L) = h + n \qquad (1.15)$$

$$(m_L-m_S) + \frac{i}{2}\,(\Gamma_S-\Gamma_L) = -\sqrt{(h-n)^2 + 4lm} \qquad (1.16)$$

$$\frac{1-\varepsilon_S}{1+\varepsilon_S} = \frac{n-h+\sqrt{(h-n)^2+4lm}}{2l} \qquad (1.17)$$

$$\frac{1+\varepsilon_L}{1-\varepsilon_L} = \frac{n-h+\sqrt{(h-n)^2+4lm}}{2m} \qquad (1.18)$$

In the CPT symmetric limit, h=n and there is only one mixing parameter, $\varepsilon = \varepsilon_S = \varepsilon_L$, determined by:

$$\frac{1+\varepsilon}{1-\varepsilon} = \sqrt{\frac{l}{m}} = \sqrt{\frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}} \qquad \text{(CPT exact)}$$

To keep contact with the CPT-invariant limit, the ε's are conveniently written as:

$$\varepsilon_S = \varepsilon_M + \Delta \qquad (1.19)$$

$$\varepsilon_L = \varepsilon_M - \Delta \qquad (1.20)$$

2. K→ 2π decay amplitudes, I

The strong interaction, S-wave, phase shifts for π-π scattering are defined as:

$$<2\pi, I; \text{out} \mid 2\pi, I; \text{in}> = e^{2i\delta_I} \qquad (2.1)$$

where I denotes the total isospin (I=0, 2) and a c.o.m. energy equal to the Kaon mass is understood. In fact, since $2\pi \not\to 3\pi$ and 4π states are excluded by energy conservation, (2.1) can be rewritten as:

$$\mid 2\pi, I; \text{in}> = e^{2i\delta_I} \mid 2\pi, I; \text{out}> \qquad (2.2)$$

2π decay amplitudes of K^0 and $\bar{K}^0$ are defined according to:

$$A(K^0 \to 2\pi, I) = <2\pi, I; \text{out} \mid H_W \mid K^0> =_{DEF} \sqrt{\frac{3}{2}}(A_I+B_I)\, e^{i\delta_I} \qquad (2.3)$$

$$A(\bar{K}^0 \to 2\pi, I) = <2\pi, I; \text{out} \mid H_W \mid \bar{K}^0> =_{DEF} \sqrt{\frac{3}{2}}(A_I^*-B_I^*)\, e^{i\delta_I} \qquad (2.4)$$

(The factor $\sqrt{2/3}$ is inserted to simplify later formulae, amplitudes are normalized as in ref.[Lusign. et al.]). Symmetry relations are as follows.

$$\text{CP:} \quad <2\pi, I; \text{out} \mid H_W \mid K^0> = <2\pi, I; \text{out} \mid CP^{-1}(H_W)CP \mid \bar{K}^0> \qquad (2.5)$$

$$\text{T:} \quad <2\pi, I; \text{out} \mid H_W \mid K^0>^* = <2\pi, I; \text{in} \mid T^{-1}(H_W)T \mid K^0> =$$

$$= e^{-2i\delta_I} <2\pi, I; \text{out} \mid T^{-1}(H_W)T \mid K^0> \qquad (2.6)$$

Writing, for fixed I:

$$A = A^{(1)} + iA^{(2)}$$

$$B = B^{(1)} + iB^{(2)}$$

the symmetry properties of the various components of the amplitude are those given in the Table.

Tab.2

	$A^{(1)}$	$A^{(2)}$	$B^{(1)}$	$B^{(2)}$
CP:	+	-	-	+
T:	+	-	+	-
CPT:	+	+	-	-

In this case, we may have CP conserved and CPT violated or viceversa. B is exclusively a signal of CPT violation.

3. The Wu-Yang phase convention

It is very important (and helpful!) to keep track of the phase arbitrariness embodied by Eq.(1.6). Frome the formulae above, one has:

$$X \to e^{-Qi\alpha} X \tag{3.1}$$

with:

$$Q = 0;\ -2;\ +2,\quad \text{for } X = h \text{ or } n;\ l;\ m \tag{3.2}$$

$$Q = 1,\qquad \text{for } X = A_I \text{ or } B_I \tag{3.3}$$

We can use the phase arbitrariness to make one of the quantities in (3.2) and (3.3) to be real . The Wu-Yang convention[8] requires:

$$A_0 = \text{real} \geq 0 \tag{3.4}$$

This convention is phenomenologically very useful, as we shall see presently. It corresponds to shift as much T violation as possible away from the (dominant) $\Delta I=1/2$ non leptonic amplitude into the (suppressed) $\Delta I=3/2$ one.

In the usual parametrization of the KM matrix, precisely the opposite happens, namely T-violation appears, predominantly, in $\Delta I=1/2$ amplitude, due to t-quark exchange in penguin diagrams, while the $\Delta I=3/2$ amplitude is predominantly real (except for electroweak penguin effects, which may become important for large values of the t-quark mass, see Sect.5). Of course, amplitudes computed in the latter (KM) convention can be transformed back in the Wu-Yang phase convention by the transformation (3.1) with:

$$\alpha \approx -\frac{\text{Im}A_0}{A_0} \tag{3.5}$$

We adopt the Wu-Yang convention in the following.

4. K→ 2π decay amplitudes, II

From Eqs.(2.3) and (2.4) and the isospin Clebsch-Gordan coefficients, one finds the decay amplitudes in the $\pi^+\pi^-$ and $\pi^0\pi^0$ channels[1]:

$$A(K^0 \to \pi^+\pi^-) = (A_0+B_0)\, e^{i\delta_0} + \frac{1}{\sqrt{2}} (A_2+B_2)\, e^{i\delta_2} \quad (4.1)$$

$$A(K^0 \to \pi^0\pi^0) = (A_0+B_0)\, e^{i\delta_0} - \sqrt{2}\, (A_2+B_2)\, e^{i\delta_2} \quad (4.2)$$

For $\bar{K}^0$ amplitudes, $A \to A^*$, $B \to -B^*$. Also:

$$A(K^+ \to \pi^+\pi^0) = \frac{3}{2} (A_2+B_2)\, e^{i\delta_2} \quad (4.3)$$

We can derive, from these formulae and from the experimental K_S and K^+ decay rates, the values of the CP and CPT-conserving amplitudes:

$$A_0 = 2.7\ 10^{-7}\ \mathrm{GeV} \quad (4.4)$$

$$\omega = \frac{A_2}{A_0} = 0.045 \quad (4.5)$$

as well as:

$$\cos(\delta_2-\delta_0) = 0.52; \qquad |\delta_2-\delta_0| = 59^0 \quad (4.6)$$

The value of the phase is in reasonable agreement with the one found from pion production in π-Nucleon scattering and in K_{e4} decay:

$$\delta_0-\delta_2 = (41.4\pm8.1)^0 \qquad \text{(Devlin \& Dickey[9])}$$

$$\delta_0-\delta_2 = (46.3\ ^{+2.7}_{-4.0})^0 \qquad \text{(Shenk[10])} \quad (4.7)$$

After these preliminaries, we proceed to derive the formulae for the 2π decays of K_L. One defines:

$$\eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}; \quad \eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} \quad (4.8)$$

and finds, from Eqs.(1.13) and (4.1-2) (Wu-Yang convention used throughout, terms of second order in CP/CPT violation and first order multiplied by ω^2 are neglected):

---

[1]The decay rates in $\pi^+\pi^-$ or $\pi^+\pi^0$ are given by: Rate = $\frac{p_{decay}}{8\pi M^2}|A|^2$. For $\pi^0\pi^0$ there is an additional factor of 1/2, for the identical particles.

$$\eta_{+-} = \varepsilon_L + \frac{A(K_2 \to \pi^+\pi^-)}{A(K_1 \to \pi^+\pi^-)} = \varepsilon + \varepsilon' \qquad (4.9)$$

$$\eta_{00} = \varepsilon_L + \frac{A(K_2 \to \pi^0\pi^0)}{A(K_1 \to \pi^0\pi^0)} = \varepsilon - 2\,\varepsilon' \qquad (4.10)$$

with:

$$\varepsilon = \varepsilon_M - (\Delta - \frac{ReB_0}{A_0}) \qquad (4.11)$$

$$\varepsilon' = \frac{ie^{i(\delta_2-\delta_0)}}{\sqrt{2}}\,\omega[\frac{ImA_2}{A_2} - i\,(\frac{ReB_2}{A_2} - \frac{ReB_0}{A_0})] \qquad (4.12)$$

Eqs.(4.9) and (4.10) are formally identical to those of the exact CPT limit, but with a different relation between ε and the mass mixing parameters, and with an additional contribution of the B's to ε'.

In the limit of vanishing Δ and B, one obtains in (4.12) the usual expression for ε', in the notation appropriate to the Wu-Yang convention. As noted before, given the amplitudes computed with a different phase convention, e.g. with the usual KM phases, we obtain the amplitudes in the Wu-Yang frame by making, for any ΔS=-1 amplitude X, the replacement:

$$ImX \to ImX - ReX\,\frac{ImA_0}{A_0}$$

In this way, we obtain from Eq.(4.12) the phase-convention independent expression:

$$\varepsilon' = \frac{ie^{i(\delta_2-\delta_0)}}{\sqrt{2}}\,\omega\,[\frac{ImA_2}{A_2} - \frac{ImA_0}{A_0} - i\,(\frac{ReB_2}{A_2} - \frac{ReB_0}{A_0})] \qquad (4.14)$$

The structure of Eq.(4.14) can be read very simply, with reference to the definition of ε' given in Eq.(4.9). The factor $e^{i(\delta_2-\delta_0)}$ arises from the final state interaction, the dominant final states being I=2 and I=0 for the numerator and denominator of the ratio in Eq.(4.9). The further factor of i arises because ε', being CP violating, must violate T-reversal (i.e. be imaginary, apart from the final state interaction phases) in a CPT conserving theory. CPT violation, indeed, appears as a further, T-conserving imaginary contribution to the square bracket in (4.14).

5. Standard Theory prediction of ε'/ε

The parameter ε' is uniquely related to CP (and CPT) violation in the transition amplitude, see Eq.(4.14). In the usual terminology, $\varepsilon' \neq 0$ characterizes "milli-weak" theories, i.e. theories in which the weak interaction itself has a small but detectable CP (CPT) violating component[11]. The mass mixing parameter, ε, arises as a 2nd order weak effect, in which the CP-odd and CP-even parts combine to give the $\Delta S=\pm 2$ quantities, M_{12} and Γ_{12}.

Another, logically independent, possibility is that the observed CP-violation is the 1st order manifestation of a new interaction with a $\Delta S=\pm 2$ component. In the mass matrix, the new interaction competes with the 2nd order weak contribution. Therefore even a very weak interaction, of strenght 10^{-3} $(G_F M_P)^2 \approx 10^{-13}$ would give rise to the observed CP violation. The latter type of theory is called "superweak"[12], and it has the obvious prediction that no CP-violation is visible in channels where 1st order weak interactions occur, i.e. it predicts $\varepsilon'=0$.

Milliweak theories are in danger [11] to contradict the very tight experimental limits to the electric dipole of the neutron, a T and P-violating, $\Delta S=0$, effect. Unless a special cancellation occurs, we expect any hadron to have an e.d.m. of the order of:

$$\mu_E \approx e\, r_P\, (G_F M_P)\, |\varepsilon| \approx e \cdot 10^{-21} \mathrm{cm} \qquad (5.1)$$

$r_P \approx 10^{-13}$cm is the proton radius, which gives the general dimension, G_F, the Fermi constant, is associated to P-violation while $|\varepsilon|$, in a milliweak theory, characterizes the generic strenght of T-violation. The present experimental upper bound to the neutron e.d.m. is[13,14]:

$$(\mu_E)_{neutron} < 1.2 \cdot 10^{-25}\ e\ \mathrm{cm} \qquad (5.2)$$

much too small to be compatible with (5.1), which therefore calls for a very special cancellation.

The Standard, six flavour, Theory[15] is a milliweak theory in which such a special cancellation does occur[16]. CP-violation (rather, T-violation) arises because different components of the weak charged current have non-vanishing relative phases. However, the one-loop correction to the electromagnetic current of a given quark, e.g. the d-quark, is given by a sum of terms in which each complex entry, corresponding to, say, $d \to c$ (V_{cd}) is multiplied by the amplitude for the inverse process $c \to d$ (V_{cd}^*). To this order, the correction is real and the e.d.m vanishes to one loop, which brings already the estimate (5.1) down to 10^{-24}. A further suppression is due to the fact that any CP-violating effect, in the Standard theory, must involve the light quark mass differences, which brings in powers of m_{quark}/m_W. This is because one could rotate away the CP-violating phase when any two quark of the same charge are degenerate in mass. Finally, as shown by Shabalin[17], the e.d.m. of the quark vanishes also at two-loops, which brings in another factor of 10^{-3}.

In conclusion, current estimates are that the e.d.m. of the neutron in the Standard Theory is predicted at a completely unobservable level[17]:

$$(\mu_E)_{neutron} \approx e.10^{-31}\mathrm{cm} \qquad \text{(Standard Theory)} \tag{5.3}$$

The above discussion underlines the importance of a positive measurement of ε'/ε. The first calculation of ε'/ε in the Standard Theory is due to Gilman and Wise[18]. We summarize here the most recent analyses[19,20].

The calculation of ε'/ε goes through several steps.

i) Determination of the effective weak, non-leptonic Hamiltonian, H_{eff}. This is dominated by short-distance, QCD effects. Different terms in H_{eff} can be classified according to their transformation properties under chiral SU(3)xSU(3)[21]. The dominant term transforms as $(8_L,1_R)$, corresponding to the so-called octet-enhancement, while $\Delta I=3/2$ transitions are produced by a $(27_L,1_R)$ component. The $(8_L,1_R)$ component has a complex coefficent which arises, in the usual KM phase convention, because of t-quark exchange in the so-called penguin diagrams. Electroweak penguin diagrams give rise instead to $(8_L,8_R)$ components, also with a complex coefficient.

ii) The $(8_L,1_R)$ term gives rise to a non-vanishing value of $\mathrm{Im}A_0$, thus giving a first contribution to Eq.(4.14). Contributions to $\mathrm{Im}A_2$ arise from two different sources. The first is due to isospin breaking: the octet component contributes to $\mathrm{Im}A_2$ a term proportional to the quark mass difference, m_d-m_u. Although this difference is small:

$$\frac{(m_d-m_u)}{m_s} \approx \frac{3\mathrm{MeV}}{150\mathrm{MeV}} = 0.02$$

it is partly compensated by the fact that such a term appears in Eq.(4.14) divided by $\mathrm{Re}A_2$ and is therefore enhanced by a factor of ω^{-1} with respect to the previous one, see Eq.(4.5). A second contribution to $\mathrm{Im}A_2$ arises from the $(8_L,8_R)$ component. The small Wilson coefficient with which it appears in H_{eff} is partly compensated by the factor ω^{-1} and also by the fact that chiral symmetry does not require the matrix element of the $(8_L,8_R)$ to vanish for vanishing external momenta, as is the case for both the $(8_L,1_R)$ and $(27_L,1_R)$. The raising of the Wilson coefficient of the $(8_L,8_R)$ term is responsible for the decrease of ε'/ε at the increase of the t-quark mass.

iii) Matrix elements of the effective Hamiltonian are parametrized in terms of the so-called B-factors, scale factors which measure the deviation of the true matrix element from the one computed in the vacuum insertion approximation. At present, systematic calculations of the B-factors have been carried on with lattice QCD, QCD sum-rules and the $1/N_c$ expansion.

The resulting predictions of ε'/ε vs. m_t are shown in Figs.1 and 2, for B-factors computed in the $1/N_c$ expansion or with lattice QCD, respectively. Very good agreement of the two calculations is indicated. ε'/ε is generally expected in the 10^{-3} range, with the possibility of it being very small for $m_t \approx 200$ GeV, due to the electroweak penguin effect mentioned before.

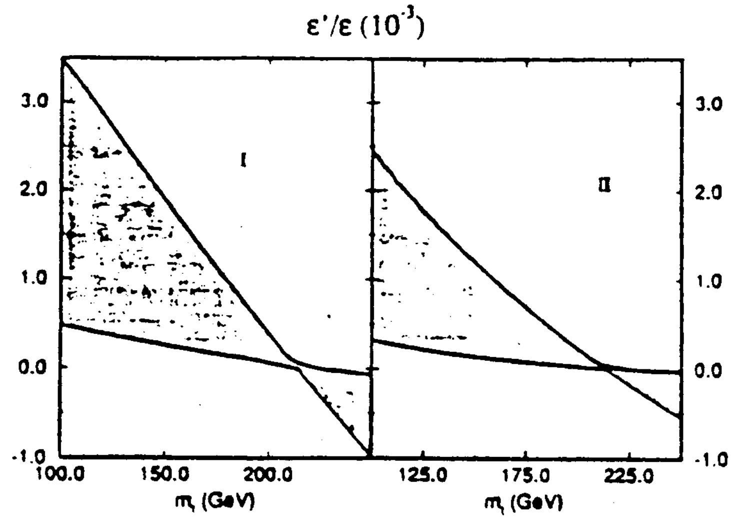

Fig.1. Upper and lower limits of ε'/ε vs. m_t, in the first and second quadrant of the CP-violating phase, δ, second paper of ref.[19]. Parameter ranges used: $0.09 \leq |V_{ub}/V_{cb}| \leq 0.17$, $0.036 \leq |V_{cb}| \leq 0.046$, $0.1\text{GeV} \leq \Lambda_{QCD} \leq 0.3\text{GeV}$, $125 \leq m_s \leq 200\text{MeV}$. Other quantities are taken in the leading 1/N expansion.

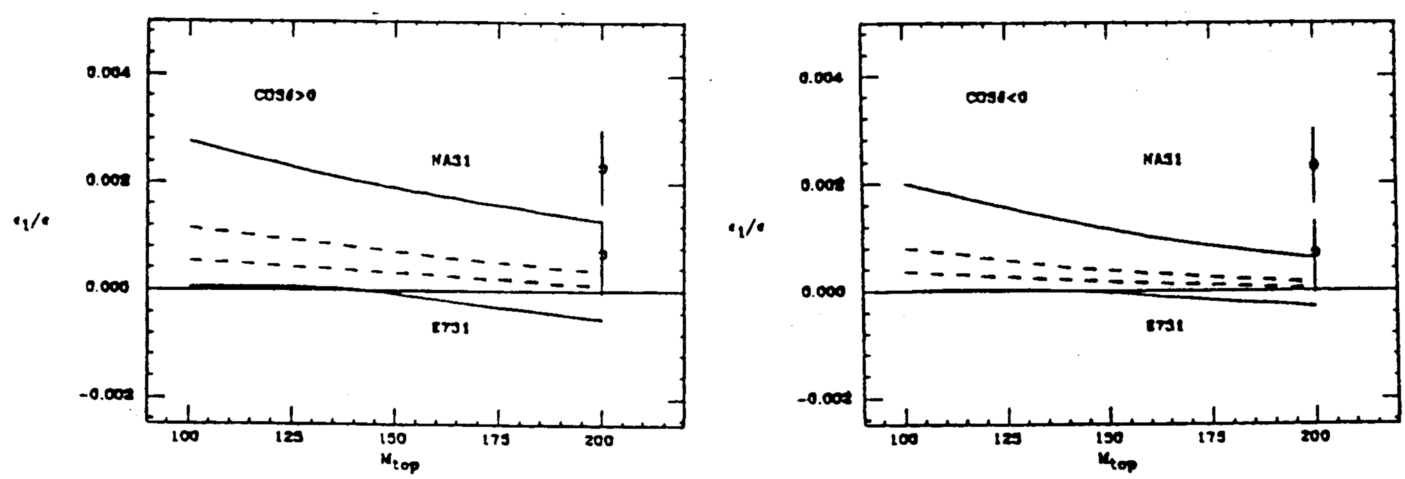

Fig.2. Upper and lower limits of ε'/ε vs. m_t, in the first and second quadrant of the CP-violating phase, δ, ref.[20]. Parameter ranges used: $0.08 \leq |V_{ub}/V_{cb}| \leq 0.14$, $0.042 \leq |V_{cb}| \leq 0.051$, $0.1\text{GeV} \leq \Lambda_{QCD} \leq 0.3\text{GeV}$, $140 \leq m_s \leq 200\text{MeV}$. Other quantities are taken from lattice QCD calculations.

6. Semileptonic amplitudes

We focuse on K_{e3} decays of K^0 and $\bar{K}^0$. On general grounds, there are 4 independent matrix elements, related to the (complex) form factors of the transitions:

$$\begin{cases} K^0 \rightarrow e^+ \nu_e \pi^- \\ \bar{K}^0 \rightarrow e^- \bar{\nu}_e \pi^+ \end{cases} \qquad (\Delta S = \Delta Q) \tag{6.1}$$

$$\begin{cases} K^0 \rightarrow e^- \bar{\nu}_e \pi^+ \\ \bar{K}^0 \rightarrow e^+ \nu_e \pi^- \end{cases} \qquad (\Delta S = - \Delta Q) \tag{6.2}$$

Time-reversal relates each form factor to its complex conjugate, CP relates K^0 to $\bar{K}^0$ form factors. This suggests to parametrize the amplitudes according to:

$$< e^+ \nu_e \pi^- |H_W|K^0> = a + b \tag{6.3}$$

$$< e^- \bar{\nu}_e \pi^+ |H_W|\bar{K}^0> = a^* - b^* \tag{6.4}$$

$$< e^- \bar{\nu}_e \pi^+ |H_W|K^0> = c + d \tag{6.5}$$

$$< e^+ \nu_e \pi^- |H_W|\bar{K}^0> = c^* - d^* \tag{6.6}$$

a and b (c and d) obey the same symmetry properties as the non-leptonic amplitudes A_I and B_I (see Tab. 2), i.e.: b and d are CPT violating, imaginary parts are all T-violating. c and d describe possible violations of the $\Delta S = \Delta Q$ rule.We consider Re a of order unity and keep first order terms in all the other quantities.

One should introduce analogous amplitudes for muonic decays, but we will leave this understood, in the following, to avoid a too heavy notation.

The following notations are also used[2,3]:

$$y = - \frac{b}{a} \tag{6.7}$$

$$x = \frac{c^* - d^*}{a + b}; \quad \bar{x} = \frac{c^* + d^*}{a - b} \tag{6.8}$$

with:

$$\Delta S = \Delta Q \text{ exact:} \qquad x = \bar{x} = 0, \tag{6.9}$$

$$\text{CPT exact:} \qquad x = \bar{x},\ y = 0, \tag{6.10}$$

$$\text{T exact:} \qquad x, \bar{x}, y = \text{real}, \tag{6.11}$$

$$\text{CP exact:} \qquad x = \bar{x},\ y = \text{imaginary}. \tag{6.12}$$

For convenience, we shall also define:

$$\Delta' = \Delta - \frac{Red}{Rea} - i\frac{Imc}{Rea} \tag{6.13}$$

$$\Gamma^{e}_{L/S} = \Gamma(K_{L/S} \rightarrow e^{+} +..) + \Gamma(K_{L/S} \rightarrow e^{-} +..) + (e \rightarrow \mu) \tag{6.14}$$

with Δ defined in Eqs.(1.19) and (1.20) and $\Gamma^{sl}_{L/S} = \Gamma^{e}_{L/S} + \Gamma^{\mu}_{L/S}$.

The following relations are immediate:

$$A_L = \frac{\Gamma(K_L \rightarrow e^{+}\nu_e\pi^{-}) - \Gamma(K_L \rightarrow e^{-}\bar{\nu}_e\pi^{+})}{\text{sum}} = 2(Re\varepsilon_L + \frac{Reb}{Rea} + \frac{Red}{Rea}) =$$

$$= 2\,(\,Re\varepsilon_M - Re\Delta' + \frac{Reb}{Rea}) \tag{6.15}$$

$$A_S = 2(Re\varepsilon_M + Re\Delta' + \frac{Reb}{Rea}) \tag{6.16}$$

$$\Delta^{e} = \frac{\Gamma^{e}_{S} - \Gamma^{e}_{L}}{\text{sum}} = 2\frac{Rec}{Rea} \tag{6.17}$$

There are in all four semileptonic rates, which can be expressed in terms of the three combinations given above plus the average rate, which determines $(Rea)^2$. In addition, to study the correlated decays of the K_L-K_S pair produced at a Φ-factory, it is convenient to introduce the complex quantities:

$$\eta_{l+} = \frac{<e^{+}\,\nu_e\,\pi^{-}\,|H_W|K_L>}{<e^{+}\,\nu_e\,\pi^{-}\,|H_W|K_S>} = 1 - 2\frac{Rec}{Rea} - 2\Delta' - 2\,i\,\frac{Imd}{Rea} \tag{6.18}$$

$$\eta_{l-} = \frac{<e^{-}\,\bar{\nu}_e\,\pi^{+}\,|H_W|K_L>}{<e^{-}\,\bar{\nu}_e\,\pi^{+}\,|H_W|K_S>} = -\,(1 - 2\frac{Rec}{Rea} + 2\Delta' - 2i\,\frac{Imd}{Rea}) \tag{6.19}$$

In the Standard Theory, CPT and CP are conserved in semileptonic processes and the $\Delta S=\Delta Q$ rule is obeyed to a very good precision[22], with ($g_8 \approx 5$ is the relative strenght of the octet non-leptonic amplitude):

$$|x| \approx g_8 \frac{G_F}{\sqrt{2}} f_\pi^2 \approx 7\; 10^{-7} \tag{6.20}$$

In the current x current picture there is, in fact, little space for the violation of these symmetries, given our very good knowledge of the currents themselves.

Violation of CP, CPT or of the $\Delta S=\Delta Q$ rule could arise from contact interactions of quark and leptons (e.g. in composite models) and one should keep an open mind on the possible presence of anomalies in the semileptonic amplitudes. However, $\Delta S=-\Delta Q$ transitions require hadronic operators transforming as 10 + 27 of flavour SU(3), see e.g. the second paper of ref.[22], so that they can be induced by effective quark and lepton operators of dimension higher than four. A typical example is:

$$\Delta H_{eff} = \frac{4\pi}{\Lambda^6} [\partial_\nu(\bar{e}_L\gamma_\mu \nu_{eL})] (\bar{u}_L\gamma^\nu d_L)(\bar{s}_L\gamma^\mu d_L) \tag{6.21}$$

with Λ the compositness scale, which leads to the (rather generous) estimate:

$$|x| \approx \frac{4\pi}{G_F\Lambda^2} [\frac{1\mathrm{GeV}}{\Lambda}]^4 \approx 10^{-10} [\frac{1\mathrm{TeV}}{\Lambda}]^6 \tag{6.22}$$

The result (6.22) justifies the neglect of $\Delta S=-\Delta Q$ amplitudes, still keeping open the possibility of CPT violation.

For the sake of brevity, the case in which semileptonic amplitudes are assumed to conserve both CPT and the $\Delta S=\Delta Q$ rule will be called **Scheme I** in the following. **Scheme II** will be the case in which CPT is relaxed, still keeping exact the $\Delta S=\Delta Q$ rule. We shall also comment on **Scheme III**, where Eqs.(6.3) to (6.6) are considered in full generality.

7. Comparison of notations

The recent analyses[6,1] of neutral Kaon mixing and decays without assumption of CPT, do not follow a unified set of notations. To facilitate the comparison, we give below the translation table from ours to their notations.

Tab.3

our notations	Barmin et al. ref.[6]	Buchanan et al. ref.[1]
$\varepsilon_{S,L}=\varepsilon_M \pm \Delta$	$\varepsilon_{S,L}=\varepsilon \pm \Delta$	$\varepsilon_{S,L}=\varepsilon_K \pm \delta_K$
$\eta_{+-}=\varepsilon+\varepsilon'$	$\eta_{+-}=\varepsilon_0+\varepsilon'$	$\eta_{+-}=\varepsilon+\varepsilon'$
ε'	ε'	ε'
$\frac{\mathrm{Re}B_0}{A_0}$	a	$\frac{\mathrm{Re}B_0}{A_0}$
a, b; Eqs.(6.3-4)	b=0	a,b
c, d; Eqs.(6.5-6)	c=d=0	c=d=0
	(Scheme I)	**(Scheme II)**

8. Unitarity constraints.

We know little about the real part of the Hamiltonian, the mass matrix M, which is sensitive to virtual particle, high-energy effects (this is, for instance, the case in the Standard Theory, where T violation in M is determined by t-quark exchange). On the other hand, unitarity relates the imaginary part of H, the matrix Γ, to the real decays of the neutral Kaons, about which we have considerably more information:

$$\Gamma_{ab} = \sum_n 2\pi\, \delta(M_K - E_n) \langle a | H_W | n\rangle\langle n | H_W | b\rangle \tag{8.1}$$

In particular, we know that the 2π, I=0 final state is by far the most prominent one in K^0 and $\bar{K}^0$ decays and this simple fact gives interesting restrictions on the parameters $\varepsilon_{L,S}$.

We start from Eqs.(1.16) to (1.18), which are easily solved to obtain h-n, l and m in terms of the physical parameters. Separating real and imaginary parts, one finds six relations:

$$2\mathrm{Re}(M_{12}) = -(m_L - m_S) \tag{8.2}$$

$$2\mathrm{Im}(M_{12}) = -(\Gamma_S - \Gamma_L)\left[\mathrm{Re}\left(\frac{\varepsilon_S+\varepsilon_L}{2}\right) + \tan\phi_{SW}\, \mathrm{Im}\left(\frac{\varepsilon_S+\varepsilon_L}{2}\right)\right] \tag{8.3}$$

$$2\mathrm{Re}(\Gamma_{12}) = (\Gamma_S - \Gamma_L) \tag{8.4}$$

$$\mathrm{Im}(\Gamma_{12}) = -(\Gamma_S - \Gamma_L)\left[\tan\phi_{SW}\, \mathrm{Re}\left(\frac{\varepsilon_S+\varepsilon_L}{2}\right) - \mathrm{Im}\left(\frac{\varepsilon_S+\varepsilon_L}{2}\right)\right] \tag{8.5}$$

$$M_{11} - M_{22} = -(\Gamma_S - \Gamma_L)\left[\tan\phi_{SW}\, \mathrm{Re}\left(\frac{\varepsilon_S-\varepsilon_L}{2}\right) - \mathrm{Im}\left(\frac{\varepsilon_S-\varepsilon_L}{2}\right)\right] \tag{8.6}$$

$$\frac{1}{2}(\Gamma_{11} - \Gamma_{22}) = (\Gamma_S - \Gamma_L)\left[\mathrm{Re}\left(\frac{\varepsilon_S-\varepsilon_L}{2}\right) + \tan\phi_{SW}\, \mathrm{Im}\left(\frac{\varepsilon_S-\varepsilon_L}{2}\right)\right] \tag{8.7}$$

We have introduced the "superweak phase", ϕ_{SW}, defined by:

$$\tan\phi_{SW} =_{DEF} \frac{2(m_L - m_S)}{\Gamma_S - \Gamma_L} = 0.9565 \pm 0.0051$$
$$\phi_{SW} = (43.73 \pm 0.15)^0 \tag{8.8}$$

and will denote by v and w the complex numbers:

$$\begin{cases} v = \dfrac{1}{\sqrt{1+\tan^2\phi_{SW}}}(1 + i\tan\phi_{SW}) \\ w = \dfrac{1}{\sqrt{1+\tan^2\phi_{SW}}}(-\tan\phi_{SW} + i) \end{cases} \tag{8.9}$$

Eqs.(8.3) and (8.5) can be seen as specifying the components of $(\varepsilon_S+\varepsilon_L)$ along v and w, regarded as mutually orthogonal vectors in the complex plane, and similarly Eqs.(8.6) and (8.7), for the CPT violating parameter, $(\varepsilon_S - \varepsilon_L)$.

In the first case, we use the fact that the dominant 2π, I=0 amplitude is exactly real, in the Wu-Yang convention. Correspondingly, $Im(\Gamma_{12})$ receives contribution from 2π with I=2, 3π and semileptonic decay modes. In general, the scale of these contributions is suppressed, with respect to the r.h.s. of Eq.(8.5), by a factor of Γ_{K^+}/Γ_S or Γ_L/Γ_S. Thus, to be competitive with the r.h.s., CP violation on the l.h.s. of Eq.(8.5) should be of order unity, which is not the case (rather, as we have indicated in Sect.5, ε' effects in 2π decays are much more suppressed).

More in detail, one may classify the contribution of the most prominent intermediate states as follows (first order terms only are retained).

2π:

$$Im(\Gamma_{12})_{2\pi} = \frac{3}{2} Im\, [(A_0{}^2-B_0{}^2)^* + (A_2{}^2-B_2{}^2)^*] \text{ x (phase space)} =$$

$$= - [\frac{4}{3}\, B(K^+ \to \pi^+\pi^0) \frac{\Gamma_{K^+}}{\Gamma_S} \frac{Im A_2}{Re A_2}]\, \Gamma_S \approx - 2.04\ 10^{-3} \frac{Im A_2}{Re A_2}\, \Gamma_S$$

3π

We approximate 3π decay amplitudes with their value at the center of the Dalitz plot and consider only $\Delta I=1/2$ contribution to the CP-conserving transition. Defining CP-violating parameters[2] according to[4]:

$$\varepsilon_{+-0}' = \frac{A(K_1 \to \pi^+\pi^-\pi^0;\, I=1)}{A(K_2 \to \pi^+\pi^-\pi^0;\, I=1)}\, ;\ \varepsilon_{000}' = \frac{A(K_1 \to 3\pi^0;\, I=1)}{A(K_2 \to 3\pi^0;\, I=1)}$$

one finds:

$$Im(\Gamma_{12})_{3\pi} = \Gamma(K_2 \to \pi^+\pi^-\pi^0)\, Im\, \varepsilon_{+-0}' + \Gamma(K_2 \to 3\pi^0)\, Im\, \varepsilon_{000}' =$$

$$\approx 2.14\ 10^{-3} (Im\, \varepsilon_{+-0}' + 1.74\, Im\, \varepsilon_{000}')\, \Gamma_S$$

semileptonic

$$Im(\Gamma_{12})_{sl} = Im\, (x + \bar{x}\,) \Gamma_L^{sl} \approx 2.26\ 10^{-3}\ Im\, (\frac{x+\bar{x}}{2})\, \Gamma_S$$

---

[2] final state interaction phases cancel in the ratios below, unlike in the case of ε'. Therefore ε_{+-0}' and ε_{000}', being CP-violating, are expected to violate T, i.e. be pure imaginary, if CPT is conserved. CPT violation is introduced by a non-vanishing real part.

(we have averaged the electron and muon contributions). The semileptonic contribution is suppressed, since it requires $\Delta Q=-\Delta S$.

Neglecting completely the l.h.s of (8.5), we conclude that $(\varepsilon_S+\varepsilon_L)$ is orthogonal to w:

$$\mathrm{Arg}(\varepsilon_S+\varepsilon_L) \approx \phi_{SW} \tag{8.10}$$

The component along v is determined by the short-distance sensitive quantity, $\mathrm{Im}(M_{12})$:

$$\frac{\varepsilon_S+\varepsilon_L}{2} = \varepsilon_M \approx \frac{-\mathrm{Im}(M_{12})}{m_L-m_S}\,\frac{\tan\phi_{SW}\,(1+i\tan\phi_{SW})}{(1+\tan^2\phi_{SW})} = \frac{-\,i\,\mathrm{Im}(M_{12})}{m_L-m_S-\frac{i}{2}(\Gamma_L-\Gamma_S)} \tag{8.11}$$

In addition, from the result (4.7), we find:

$$\mathrm{Arg}(ie^{i(\delta_2-\delta_0)}) = (43.7\pm4)^0 \approx \phi_{SW} \tag{8.12}$$

it follows from Eq.(4.12) that ε' is approximately parallel to ε_M, except for CPT-violating effects.

To make a similar analysis for the CPT-violating quantity, $\Delta=(\varepsilon_S-\varepsilon_L)/2$, we first extract the 2π contribution to the r.h.s. of Eq.(8.7). Explicitly (ρ_{+-}, ρ_{00} are the 2π phase-space factors; $\rho_{+-}\approx2\rho_{00}$ for exact isospin symmetry):

$$(\Gamma_{11}-\Gamma_{22})_{2\pi} = \rho_{+-}\,(|A(K^0\to\pi^+\pi^-)|^2 - |A(\bar K^0\to\pi^+\pi^-)|^2) +$$

$$+\,\rho_{00}\,(|A(K^0\to\pi^0\pi^0)|^2 - |A(\bar K^0\to\pi^0\pi^0)|^2) =$$

$$= \rho_{+-}\,\frac{3}{2}\,[4A_0^2\,\frac{ReB_0}{ReA_0} + 4A_2^2\,\frac{Re(A_2{}^*B_2)}{ReA_2}] =$$

$$= 2\,\Gamma(K_S\to2\pi)\,[\,\frac{ReB_0}{A_0} + \omega^2\,\frac{Re(A_2{}^*B_2)}{A_2}] \tag{8.13}$$

With this result , Eq.(8.7) becomes:

$$\frac{1}{2}(\Gamma_{11}-\Gamma_{22})_{res}+\delta\Gamma\,\frac{ReB_0}{A_0} = (\Gamma_S-\Gamma_L)[Re(\Delta-\frac{ReB_0}{A_0})+\tan\phi_{SW}\,Im(\Delta-\frac{ReB_0}{A_0})] \tag{8.14}$$

The suffix res indicates the sum over intermediate states different from 2π, I=0, and:

$$\delta\Gamma = \Gamma(K_S\to2\pi)-\Gamma_S+\Gamma_L \approx \Gamma_L - \Gamma_S^{sl} \approx \Gamma_L - \Gamma_L^{sl} \approx 0.59\ 10^{-3}\Gamma_S$$

(assuming the semileptonic rates af K_S and K_L to be approximately equal).

The term proportional to $\delta\Gamma$ can be safely neglected[3]. Proceeding as before, we write the most important contributions to the l.h.s. of Eq.(8.14) as follows.

2π, I=2:

$$\frac{1}{2}(\Gamma_{11}-\Gamma_{22})_{2\pi,I=2} = B(K_S\to 2\pi)\ \omega^2 \frac{Re(A_2{}^*B_2)}{A_2}\Gamma_S \approx 2.02\ 10^{-3}\frac{Re(A_2{}^*B_2)}{A_2}\Gamma_S$$

3 π

$$\frac{1}{2}(\Gamma_{11}-\Gamma_{22})_{3\pi} = \Gamma(K_2\to\pi^+\pi^-\pi^0)\ Re\ \varepsilon_{+-0}' + \Gamma(K_2\to 3\pi^0)\ Re\ \varepsilon_{000}' =$$

$$\approx 2.14\ 10^{-3}(\ Re\ \ \varepsilon_{+-0}' + 1.74\ Re\ \ \varepsilon_{000}')\ \Gamma_S$$

semileptonic

$$\frac{1}{2}(\Gamma_{11}-\Gamma_{22})_{sl} = 2\frac{Reb}{Rea}\Gamma_L^{sl} \approx 2.26\ 10^{-3}\frac{Reb}{Rea}\ \ \Gamma_S$$

(electron and muon contributions averaged).

It is difficult to say anything more precise about the first term in the l.h.s. of (8.14), except that it should be very small, for the same reasons which justified the neglect of $Im\Gamma_{12}$. DAFNE can improve substantially on the present limits to the above CPT-violating quantities and therefore lead to improved bounds to the unitarity sum (see also ref.[2]), .

If we take the l.h.s. to vanish, Eq.(8.14) leads to the elegant result that the complex number $\Delta-\frac{ReB_0}{A_0}$ is parallel to w, i.e. it is orthogonal to ε:

$$Arg(\Delta-\frac{ReB_0}{A_0}) = \phi_{SW} \pm 90^0$$

so that:

$$\Delta-\frac{ReB_0}{A_0} \approx -\frac{\frac{ReB_0}{A_0}(m_L-m_S)+\frac{1}{2}(M_{11}-M_{22})}{m_L-m_S-\frac{i}{2}(\Gamma_L-\Gamma_S)} \qquad (8.15)$$

or, equivalently:

---

3 even if we want to leave open the possibility that $Re(\Delta-\frac{ReB_0}{A_0})$ is considerably smaller than its individual components[1], we consider it very unlikely a cancellation by three orders of magnitude.

$$\Delta \approx -\frac{1}{2}\;\frac{(M_{11}-M_{22})-i\,\frac{\mathrm{Re}B_0}{A_0}(\Gamma_S-\Gamma_L)}{m_L-m_S-\frac{i}{2}(\Gamma_L-\Gamma_S)} \approx -\frac{1}{2}\;\frac{(M_{11}-M_{22})-\frac{i}{2}(\Gamma_{11}-\Gamma_{22})2\pi}{m_L-m_S-\frac{i}{2}(\Gamma_L-\Gamma_S)}$$

The situation is illustrated in Fig.3 (without paying attention to the relative proportions).

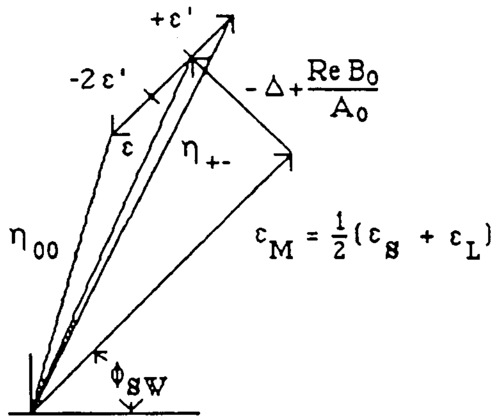

Fig.3. Schematic representation in the complex plane of the relations between ε_M, ε, η_{+-} and η_{00}. ε' is drawn parallel to ε, but can be tilted in presence of CPT violation, Eqs.(4.12) and (8.12).

The phases of ε and Δ are sometime discussed, in the literature, in connection with the Bell-Steinberger relation[23]:

$$[-i(m_S-m_L)+\frac{1}{2}\,(\Gamma_S+\Gamma_L)]\,\langle K_S|K_L\rangle = \sum_f A^*(K_S\to f)\,A(K_L\to f) \qquad (8.16)$$

which can be derived directly from the conservation of probability. By substituting Eqs.(1.12) and (1.13) in the r.h.s., it is immediate to see that the real and imaginary parts of the Bell-Steinberger relation coincide with Eqs.(8.7) and (8.5), respectively.

It could not have been differently. The unitarity condition Eq.(8.1) is all we can say about probability conservation. This equation involves a total of 4 real quantities: $\mathrm{Re}\Gamma_{12}$, $\mathrm{Im}\Gamma_{12}$, Γ_{11} and Γ_{22}. $\mathrm{Re}\Gamma_{12}$ and the average $\Gamma_{11}+\Gamma_{22}$ are related to the CP and CPT conserving total widths, Γ_S and Γ_L; the CP-violating (CPT-conserving) $\mathrm{Im}\Gamma_{12}$ determines the phase of ε_M, while the CPT and CP violating difference, $\Gamma_{11}-\Gamma_{22}$, fixes the phase of the combination Δ-$\mathrm{Re}B_0/A_0$. There can be no other general restrictions.

9. Comparison with present data

We can analyse the data at different levels, according to whether CPT symmetry and the $\Delta S=\Delta Q$ rule are kept exact or released in the semileptonic transitions. Exact CPT and the $\Delta S=\Delta Q$ rule are asssumed by Bermin et al.[6], adopting what we have called Scheme I, while Buchanan et al.[1] adopt Scheme II. We illustrate in detail, in this Section, the results in the scheme I, and will comment, in the next Section, on the impact of DAFNE on the complex of Kaon parameters in Schemes II and III.

Data

$$\begin{cases} |\eta_{+-}| = (2.268\pm0.023)\ 10^{-4} \text{ ref.[14]} \\ \dfrac{|\eta_{00}|}{|\eta_{+-}|} = 0.9935\pm0.0032 \quad \text{ref.[14]} \end{cases} \tag{9.1}$$

$$\begin{cases} \mathrm{Arg}(\eta_{+-}) = \Phi_{+-} = (46.0\pm1.2)^0 \text{ ref.[14]} \\ \Phi_{+-}-\Phi_{00} = (-0.1\pm2.0)^0 \text{ (weighted av. from reff.[24,25])} \end{cases} \tag{9.2}$$

$$A_L = (3.27\pm0.12)\ 10^{-3} \text{ ref.[14]} \tag{9.3}$$

$$\mathrm{Re}\,\frac{\varepsilon'}{\varepsilon} = \begin{cases} (23\pm7)\ 10^{-4} \text{ NA31 [26]} \\ (6\pm7)\ 10^{-4} \text{ E731 [27]} \end{cases} \tag{9.4}$$

Analysis

The smallness of ε'/ε implies that ε is very close to η_{+-} and η_{00}:

$$\varepsilon = \frac{2\eta_{+-}+\eta_{00}}{3} = 2.263\pm0.023\ 10^{-3} \tag{9.5}$$

$$\mathrm{Arg}(\varepsilon) = \Phi_{+-} + \frac{1}{3}(\Phi_{00}-\Phi_{+-}) = (46.0\pm1.4)^0 \tag{9.6}$$

The superweak phase being:

$$\begin{cases} \tan\phi_{SW} = \dfrac{2(m_L-m_S)}{\Gamma_S-\Gamma_L} = 0.9565\pm0.0051 \\ \phi_{SW} = (43.73\pm0.15)^0 \end{cases}$$

there is, at face value, a slight indication (at 1.5 σ) for a CPT-violating difference between ε and ε_M:

$$\mathrm{Arg}(\varepsilon) - \phi_{SW} = \mathrm{Arg}(\varepsilon) - \mathrm{Arg}(\varepsilon_M) = (2.3\pm1.4)^0$$

Since Δ-ReB_0/A_0 is at right angle with respect to ε_M, see Fig.3, the above result translates into:

$$|- \Delta + \frac{ReB_0}{A_0}| \approx |\varepsilon|[Arg(\varepsilon) - Arg(\varepsilon_M)] = (0.9\pm0.6)\ 10^{-4} \qquad (9.7)$$

In **Scheme I**, the K_L lepton asymmetry, Eq.(9.3), allows already a separate determination of Δ and ReB_0/A_0. From (9.3), (9.5-6) and (6.7) we get:

$$\frac{ReB_0}{A_0} = (-0.6\pm0.7)10^{-4} \qquad (9.8)$$

and from (8.9), (8.15) and (9.7):

$$\Delta = (-0.6\pm0.7)10^{-4} - (0.9\pm0.6)\ 10^{-4}\ w = (-1.2\pm1.1)10^{-4} - i\ (0.7\pm0.4)10^{-4} \qquad (9.9)$$

Using Eq.(8.6), we find a limit to the CPT-violating mass difference[4], M_{11}-M_{22}:

$$\frac{M_{11}-M_{22}}{m_K} = 1.48.10^{-14}\ (\frac{M_{11}-M_{22}}{\Gamma_S-\Gamma_L}) = (3.0\pm2.4)\ 10^{-18} \qquad (9.10)$$

A last possible CPT test is given by the phase of ε'/ε. As seen from Eq.(4.12), the phase of ε'/ε is made of two components:

$$Arg\frac{\varepsilon'}{\varepsilon} = \phi' + \phi_{CPT/CP}$$

$$\phi' = (\frac{\pi}{2} + \delta_2 - \delta_0 - Arg\varepsilon) = (0\pm4)^0$$

$$\phi_{CPT/CP} = (\frac{ReB_2}{A_2} - \frac{ReB_0}{A_0})\ (\frac{ImA_2}{A_2})^{-1} \qquad (9.11)$$

where we have used the result (8.12).

A precise measurement of the real and imaginary parts of ε'/ε allows, in principle, a determination of the CPT violating phase, $\phi_{CPT/CP}$. The smallness of $Re\varepsilon'/\varepsilon$, (Eq.9.4), implies $ImA_2/A_2 \approx 10^{-4}$, so that we can obtain anyway an interesting bound to the CPT violating part of ε'/ε. In formulae, from Eqs.(4.9), (4.10) and (4.12), one finds:

$$Im\frac{\varepsilon'}{\varepsilon} = Im\ (\frac{\eta_{+-}-\eta_{00}}{2\eta_{+-}+\eta_{00}}) \approx \frac{1}{3}(\Phi_{+-}-\Phi_{00}) =$$

$$= \frac{\omega}{\sqrt{2}}[\cos\phi'(\frac{ReB_2}{A_2} - \frac{ReB_0}{A_0}) + \sin\phi'\ (\frac{ImA_2}{A_2})] \approx \frac{\omega}{\sqrt{2}}\cos\phi'(\frac{ReB_2}{A_2} - \frac{ReB_0}{A_0}) \qquad (9.12)$$

---

[4] For comparison, the results quoted by Carosi et al., ref[23], are: $Arg(\varepsilon) = (47.0\pm2.0)^0$; $(M_{11}-M_{22})/m_K < 5\ 10^{-18}$ (95% c.l.).

so that:

$$\left(\frac{ReB_2}{A_2} - \frac{ReB_0}{A_0}\right) = 4.1\ 10^{-4}\ (-0.1\pm2.0)^0 = (0\pm8)10^{-4} \qquad (9.13)$$

Note that the error on the strong interaction phase, δ_2-δ_0, drops out in first order, because $\phi' \approx 0$.

It will be difficult to make this into a much more precise test. DAFNE can produce, anyway, a considerable improvement on the determination of $Im\frac{\varepsilon'}{\varepsilon}$ and of the strong interaction phase.

10. The impact of DAFNE on CPT violating parameters

In Schemes II and III, the presence of the new parameters b, c and d, in the semileptonic sector complicates considerably the matter, with respect to the one found in the previous Section. In this situation, eq.(9.9) still holds for ImΔ:

$$Im\Delta = (-\ 0.7\pm0.4)10^{-4} \qquad (10.1)$$

but the present data give a bound only to the combination $|\Delta - \frac{ReB_0}{A_0}|$, which raises the possibility that the smallness of the r.h.s. of (9.7) may be due to (fortuitous?) cancellations of larger effects.

As pointed out in ref.[1], the observation of correlated K_L-K_S decays at DAFNE will permit to disentangle the individual CPT violating parameters, in Scheme II. In the last istance, this is made possible by the fact that Φ-decay provides a beam of tagged K_S. With the reference integrated luminosity of the present Report (2.5 10^{32} cm^{-2} sec^{-1} times 10^7 sec) one estimates a yearly production of 1.7 10^9 K_S, which corresponds to a statistical error on the asymmetry A_S, Eq.(6.16):

$$\Delta A_S \approx 7\ 10^{-4} \qquad (10.2)$$

As seen from Eqs.(6.12) to (6.16), the observation of the three possible semileptonic asymmetries allows a separate determination of the three parameters: Rec/Rea,Reb/Rea and $Re\Delta'$. The same conclusion is reached starting from the measurement of $|\eta_{I}\pm|$ and of the CP-conserving asymmetry, Δ^{sl} .

In **Scheme II**, $Re\Delta' = Re\Delta$, $Im\Delta$ is given by Eq.(10.1), and we succeed in disentangling Δ from ReB_0/ReA_0, as anticipated. The observation of correlated semileptonic and hadronic decays allows, in addition, to determine the phases of $\eta_{l}\pm$, which, in Scheme II, gives a further check of the imaginary part of Δ.

In **Scheme III**, the phases of $\eta_{l}\pm$ give *Im*c and *Im*d, but the determination of *Re*Δ' is no more sufficient to fix Δ, if we allow a non vanishing value of *Re*d.

The same conclusion is reached by parameter counting. A total of 24 real parameters have been introduced: 8 for the Hamiltonian, 8 for $\pi\pi$ decay and 8 for semileptonic decay amplitudes. After dropping 1 inessential phase and using the 2 unitarit· conditions, we reduce to a total of 21 real parameters. On the other side, there are 16 ..servable, with DAFNE, namely: 2 masses and 2 widths, 2 decay rates in 2π, η_{+-} and η_{00}, 4 semileptonic rates and 2 phases of $\eta_{l}\pm$. The 4 imaginary parts, *Im*a, *Im*b, *Im*B_0 and *Im*B_2 remain indetermined, as well as one combination among *Re*Δ, *Re*B_0, *Re*d.

A conspiracy between CPT-violating parameters in $\Delta S=\Delta Q$ and $\Delta S=-\Delta Q$ amplitudes may seem unlikely. It remains that, in this situation, a separation of Δ from the other parameters requires further experimental input. One possibility is given by experiments like CP-LEAR, where a K^0 (or $\bar{K}^0$) can be tagged on the basis of strangeness, rather than semileptonic decay. In this case, we have access to the further experimental quantity:

$$R(t) = \frac{|\langle e^- +..|H_W|K^0(t)\rangle|^2}{|\langle e^+ +..|H_W|K^0(t)\rangle|^2}$$

which is sensitive to *Re*d, for small times.

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