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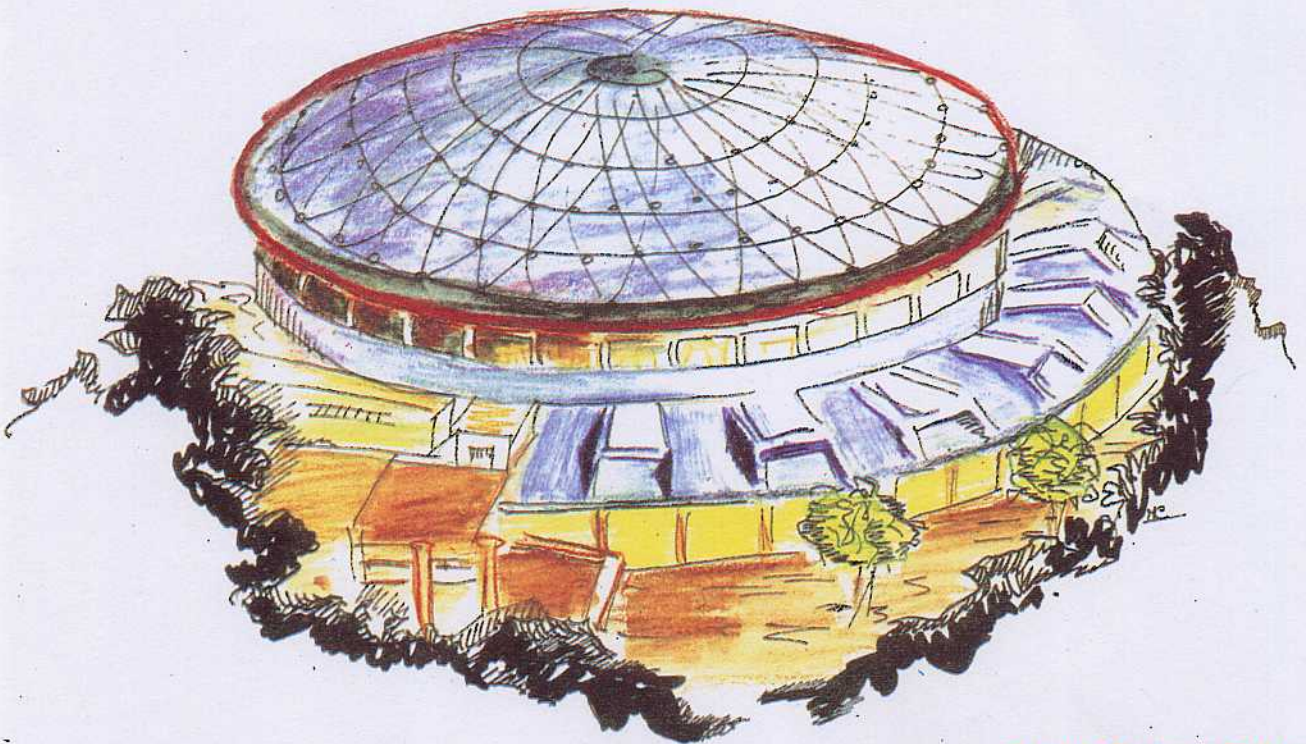
LNF-92/034 (P)

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 $V^0 \rightarrow P^0 P^0 \gamma$  DECAYS**

Contribution to the DAΦNE Theory Study Group



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**CHIRAL PERTURBATION THEORY**  
**AND RADIATIVE  $V^0 \rightarrow P^0 P^0 \gamma$  DECAYS**

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**Abstract**

An attempt to extend Chiral Perturbation Theory to radiative vector-meson decays into two neutral pseudoscalars,  $V^0 \rightarrow P^0 P^0 \gamma$ , is presented. The effects of (chiral) loops are found to be important. Unambiguous predictions are given for  $\phi \rightarrow \pi^0 \pi^0 \gamma$ ,  $\pi^0 \eta \gamma$ ,  $\rho \rightarrow \pi^0 \pi^0 \gamma$  and other processes to be measured in proposed, low-energy  $e^+ e^-$  machines.

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Strong, electromagnetic and weak interactions of pseudoscalar mesons  $P$  at low energies are known to be well described by effective chiral lagrangians. More precisely, Chiral Perturbation Theory ( $\chi PT$ )[1] offers an accurate description of the whole set of existing data in terms of a systematic expansion in powers of external momenta  $p$  or pseudoscalar masses  $m_P$ . At lowest order (tree-level), the  $\chi PT$  lagrangian contains an ordinary and an anomalous sector, which are of order  $p^2$  and order  $p^4$ , respectively. The non-renormalizability of the theory implies that higher order contributions – generated by the so-called chiral loops– may contain a divergent part which has to be eliminated by appropriated, higher-order counterterms. The finite part of the latter (the so-called low-energy constants) can be fixed at a given energy from experimental data[1] and, hopefully, from  $QCD$  first principles[2]. Somehow in between these two extreme possibilities one can also expect relating the remaining, finite part of the counterterms with the parameters of the well-known meson-resonances. Indeed, the latter do not strictly appear in the  $\chi PT$  lagrangian in spite of providing the more prominent effects in the low-energy region where  $\chi PT$  is confidently operative. The assumption that the finite part of the counterterms is dominated (or saturated) by the contribution of meson-resonances was already advanced by Gasser and Leutwyler[1], successfully developed and tested by Ecker et al.[3] and further confirmed by other authors in both the ordinary[4] and the anomalous[5, 6, 7, 8] sectors. In other words, implementing the strict  $\chi PT$  lagrangian –dealing solely with pseudoscalar and electroweak currents– with the effects of meson-resonances leads to a more complete and realistic scheme with a largely increased predictive power.

With the advent of high-luminosity, low-energy  $e^+e^-$ -machines a large amount of  $\rho$ ,  $\omega$  and  $\phi$  vector-mesons will be produced (one expects some  $10^{10}$   $\phi$ 's per year at Daphne, Frascati [9]) and their decay modes accurately analyzed. Most of these decay modes involve exclusively pseudoscalar-mesons and (at most) one photon. In this sense and at this decay stage,  $\chi PT$  effects and, particularly, chiral loop contributions could be quite relevant. For concreteness, let us consider  $\phi \rightarrow \pi^0\pi^0\gamma$  decay for which a rather low branching ratio should be expected (the available experimental upper limit is[10]  $BR(\phi \rightarrow \pi^0\pi^0\gamma) < 10^{-3}$ ). There is a two-fold reason for that: neutral particles cannot radiate copiously (bremsstrahlung) photons and, moreover, the Zweig rule suppress  $\phi$ -decays into pions. In the  $\chi PT$  context this double suppression is at once avoided through the contributions of charged-kaon loops. If so, the smallness of the  $\phi \rightarrow \pi^0\pi^0\gamma$  branching ratio will no longer hold and the analysis of this and related decays could evidenciate the effects of the (otherwise elusive) chiral loops. Notice, however, that we are pushing  $\chi PT$  somewhat outside its original context which did not allow for the inclusion of  $\phi$  and other resonances. Our purpose is to compute some consequences of this extended version of  $\chi PT$  to allow for future comparison with experimental data.

To this aim the closely related (both are Zweig forbidden)  $\phi \rightarrow \pi^0\pi^0\gamma$  and  $\phi \rightarrow \pi^0\eta\gamma$  decays will be discussed, along with the (Zweig allowed)  $\phi \rightarrow K^0\bar{K}^0\gamma$  decay mode.  $\Phi$ -factories are expected to provide valuable data on these  $K$ -loop dominated processes in a near future. In these cases the relevant range of energies extends from

the lowest limits (where  $\chi PT$  can safely be trusted) up to the  $\phi$ -mass, where the perturbative series could lose its convergence ( Notice, however, that  $\chi PT$  successfully explains  $\eta'(960)$  decays in this energy region). For all these reasons we also consider lower mass  $\rho$  and  $\omega$  decays, such as  $\rho \rightarrow \pi^0\pi^0\gamma$  (proceeding mainly through charged-pion loops) and  $\omega \rightarrow \pi^0\pi^0\gamma$ ,  $\omega \rightarrow \pi^0\eta\gamma$  and  $\rho \rightarrow \pi^0\eta\gamma$  (with only  $K$ -loops in the good  $SU(2)$  limit). An important, common feature of all these  $\phi$ ,  $\rho$  and  $\omega$  radiative decays into uncharged pseudoscalars is that no-counterterms are required. As we shall see, one loop contributions are always finite and well definite thus allowing for a clear cut comparison with future data when available.

The lowest order term ( $p^2$ ) of the  $\chi PT$  lagrangian is

$$L_2 = \frac{1}{8} f^2 \text{tr} \left( D_\mu \Sigma D^\mu \Sigma^\dagger + \chi \Sigma^\dagger + \chi^\dagger \Sigma \right) \quad (1)$$

where  $f = 132$  MeV is the  $\pi$  decay constant and the covariant derivative  $D_\mu \Sigma = \partial_\mu \Sigma + ie A_\mu [Q, \Sigma]$  contains the photon field  $A_\mu$  and the quark charge matrix  $Q = \text{diag} (2/3, -1/3, -1/3)$ . The pseudoscalar nonet  $P$  is the usual  $SU(3)$  matrix appearing in  $\Sigma = \exp(2iP/f)$  with masses given in terms of the quark mass matrix  $\mathcal{M} = \text{diag} (m_u, m_d, m_s)$  contained in the last  $\chi$ -terms of eq.(1). Vector-mesons can be incorporated along the lines of the work by Bando et al.[11] or other (for our purposes) equivalent contexts [12]. The whole (ideally mixed) nonet appears in the usual  $SU(3)$  matrix  $V$  whose diagonal elements are  $(\rho^0 + \omega)/\sqrt{2}$ ,  $(-\rho^0 + \omega)/\sqrt{2}$  and  $\phi$ . Their  $SU(3)$ -symmetric couplings to  $P$ -pairs and to the photon are given by the conventional lagrangians

$$L_{VPP} = ig \text{tr}(V_\mu P \partial^\mu P - V_\mu \partial^\mu P P) \quad (2)$$

$$L_{V\gamma} = -2egf^2 A^\mu \text{tr}(QV_\mu) \quad (3)$$

with  $\sqrt{2}gf = M_V$ , the  $V$ -meson mass, or  $g \simeq 4.2$ . With these values, eqs.(2) and (3) reproduce the  $P$ -photon vertices of eq.(1) and their vector-meson dominated (VMD) form-factors.

Amplitudes for the decay processes  $V^0 \rightarrow P^0 P^0 \gamma$  can now be deduced from the above lagrangians. There is no tree-level contribution and at the one-loop level one requires computing the set of diagrams shown in Fig.1. This leads to the following finite amplitudes  $A(V^0 \rightarrow P^0 P^0 \gamma)_P$ , where the subscript  $P = K$  or  $\pi$  indicates that charged-kaons or pions circulate along the loop and  $q^*(\epsilon^*)$  and  $q(\epsilon)$  stand for the initial vector-meson and photon four-momenta (polarizations), respectively,

$$\begin{aligned} A(\rho^0 \rightarrow \pi^0 \pi^0 \gamma)_K &= A(\omega \rightarrow \pi^0 \pi^0 \gamma)_K = \frac{-1}{\sqrt{2}} A(\phi \rightarrow \pi^0 \pi^0 \gamma)_K = \frac{-1}{\sqrt{2}} A(\phi \rightarrow K^0 \bar{K}^0 \gamma)_K \\ &= \frac{-eg}{4\sqrt{2}\pi^2 f^2} (q^{*2} - 2 q^* q) \{a\} \bar{J}_K \end{aligned} \quad (4)$$

$$A(\rho^0 \rightarrow \pi^0 \pi^0 \gamma)_\pi = \frac{-\sqrt{2}eg}{\pi^2 f^2} (q^{*2} - 2 q^* q - m_\pi^2) \{a\} \bar{J}_\pi \quad (5)$$

$$\begin{aligned} A(\rho^0 \rightarrow \pi^0 \eta \gamma)_K &= A(\omega \rightarrow \pi^0 \eta \gamma)_K = \frac{-1}{\sqrt{2}} A(\phi \rightarrow \pi^0 \eta \gamma)_K = \frac{-2}{3} A(\phi \rightarrow \pi^0 \eta_8 \gamma)_K \\ &= \frac{-eg}{6\sqrt{3}\pi^2 f^2} (3q^{*2} - 6 q^* q - 4m_K^2) \{a\} \bar{J}_K \end{aligned} \quad (6)$$

All the above amplitudes are gauge-invariant through the factor

$$\{a\} = (\epsilon^* \epsilon)(q^* q) - (\epsilon^* q)(\epsilon q^*) \quad (7)$$

and in eq.(6) we have used an  $\eta$ - $\eta'$  mixing angle  $\theta_P = \arcsin(-1/3) \simeq -19.5^\circ$  in agreement with recent phenomenological estimates. The Feynman integral  $\bar{J}_P$  is purely real and given by

$$\begin{aligned} J_P &= m_P^2 \bar{J}_P = \int_0^1 dx \int_0^{1-x} dy \frac{xy}{1 + \alpha xy - \beta y(1-y)} \\ &= \frac{1}{2\alpha} + \frac{2}{\alpha^2} \left\{ \left( \arcsin \frac{\sqrt{\beta - \alpha}}{2} \right)^2 - \left( \arcsin \frac{\sqrt{\beta}}{2} \right)^2 \right\} + \\ &\quad \frac{\beta}{\alpha^2} \left\{ \sqrt{\frac{4}{\beta - \alpha} - 1} \arctan \left( \frac{4}{\beta - \alpha} - 1 \right)^{-1/2} - \sqrt{\frac{4}{\beta} - 1} \arctan \left( \frac{4}{\beta} - 1 \right)^{-1/2} \right\} \end{aligned} \quad (8)$$

for  $\beta \equiv q^{*2}/m_P^2 \leq 4$  and  $\beta - \alpha \leq 4$ , with  $\alpha \equiv 2 q^* q/m_P^2$ . Otherwise,  $J_P$  becomes complex and can be deduced from (8) making the substitutions

$$\begin{aligned} \arcsin \frac{\xi}{2} &\rightarrow \frac{\pi}{2} + \frac{i}{2} \ln \frac{1 + \sqrt{1 - 4/\xi^2}}{1 - \sqrt{1 - 4/\xi^2}} \\ \sqrt{\frac{4}{\xi^2} - 1} \arctan \left( \frac{4}{\xi^2} - 1 \right)^{-1/2} &\rightarrow \frac{1}{2} \sqrt{1 - \frac{4}{\xi^2}} \left\{ \ln \frac{1 + \sqrt{1 - 4/\xi^2}}{1 - \sqrt{1 - 4/\xi^2}} - i\pi \right\} \end{aligned} \quad (9)$$

whenever  $\xi \geq 2$ , with  $\xi$  standing for  $\sqrt{\beta}$  and/or  $\sqrt{\beta - \alpha}$ .

Two checks of our above results have been performed. The first consists in recovering the amplitude for  $\eta \rightarrow \pi^0 \gamma \gamma$  deduced in ref.[13] using crossing and our amplitudes (6) for  $\rho$ ,  $\omega$ ,  $\phi \rightarrow \pi^0 \eta \gamma$  followed by the  $V\gamma$  conversion in eq.(3). The second comes from comparing our results with those by Lucio and Pestieau [14] for the  $\phi \rightarrow K^0 \bar{K}^0 \gamma$  amplitude which these authors assume to be dominated by the scalar mesons  $f_0(975)$

or  $a_0(980)$  in the  $K\bar{K}$  channel. Although some dynamical aspects of the two models are different they coincide in the factors  $\{a\}$ , eq.(7), and  $J_K$ , eqs.(8) and (9).

Partial widths for the various  $V^0 \rightarrow P^0 P^0 \gamma$  decays can be deduced from eqs.(4) to (6) giving the corresponding amplitudes at the one-loop and order- $p^4$  level. For  $\phi$ -decays one finds

$$\begin{aligned} \Gamma(\phi \rightarrow \pi^0 \eta \gamma)_K &= 131 \text{ eV}, & \Gamma(\phi \rightarrow \pi^0 \pi^0 \gamma)_K &= 224 \text{ eV} \\ \Gamma(\phi \rightarrow K^0 \bar{K}^0 \gamma)_K &= 0.033 \text{ eV} \end{aligned} \quad (10)$$

where the subscript  $K$  reminds us that only charged-kaon loops are operative. By contrast, these  $K$ -loops give a contribution to  $\rho^0 \rightarrow \pi^0 \pi^0 \gamma$  which is  $10^3$  times smaller than that due to charged-pion loops, namely,

$$\Gamma(\rho^0 \rightarrow \pi^0 \pi^0 \gamma)_\pi = 1.42 \cdot 10^3 \text{ eV} \quad (11)$$

Finally, one also gets

$$\begin{aligned} \Gamma(\rho^0 \rightarrow \pi^0 \eta \gamma)_K &= 0.006 \text{ eV} \\ \Gamma(\omega \rightarrow \pi^0 \eta \gamma)_K &= 0.013 \text{ eV}, & \Gamma(\omega \rightarrow \pi^0 \pi^0 \gamma)_K &= 1.8 \text{ eV} \end{aligned} \quad (12)$$

with  $\pi$ -loop contributions vanishing in the good isospin limit.

Are there other non-negligible contributions to the above processes to be considered? In a recent paper[16], the authors have studied the effects of intermediate vector-mesons in the above decays. These can be considered the conventional VMD-contributions and, as such, they are known to be there and have been computed by several authors[15, 16] along the last twenty years. Alternatively, from the more modern  $\chi PT$  point of view (implemented with the assumption of resonance saturation of counterterms) these contributions should be associated with the finite part of the counterterms required to cancel the divergences arising at higher orders (see refs.[13],[7] and [8]). In any case, the whole dynamics is contained in the  $SU(3)$ -symmetric  $VVP$  lagrangian

$$L_{VVP} = \frac{1}{\sqrt{2}} G \epsilon^{\mu\nu\alpha\beta} \text{tr}(\partial_\mu V_\nu \partial_\alpha V_\beta P), \quad (13)$$

with the  $\rho^0 \omega \pi^0$  coupling constant  $G = 3\sqrt{2}g^2/4\pi^2 f \simeq 14.4 \text{ GeV}^{-1}$  deduced from [10],  $\Gamma(\omega \rightarrow \pi^0 \gamma) = 0.72 \text{ MeV}$ , and the  $V\gamma$  lagrangian quoted in eq.(3). The corresponding VMD amplitudes have the form

$$A(V^0 \rightarrow P^0 P^0 \gamma)_{VMD} = C_{V^0 P^0 P^0 \gamma} \left( \frac{G^2 e}{g\sqrt{2}} \right) \left\{ \frac{P^2 \{a\} + \{b(P)\}}{M_V^2 - P^2 - iM_V \Gamma_V} + \text{crossed} \right\} \quad (14)$$

where  $V^0$  ( $V$ ) is the decaying (intermediate) vector meson with four-momentum  $q^*$  ( $P = q + p$ ),  $\{a\}$  has been given in eq.(7) and

$$\begin{aligned} \{b(P)\} \equiv & - (\epsilon^* \cdot \epsilon) (q^* \cdot P) (q \cdot P) - (\epsilon^* \cdot P) (\epsilon \cdot P) (q^* \cdot q) \\ & + (\epsilon^* \cdot q) (\epsilon \cdot P) (q^* \cdot P) + (\epsilon \cdot q^*) (\epsilon^* \cdot P) (q \cdot P) \end{aligned} \quad (15)$$

The coefficients  $C$  are fixed by  $SU(3)$  to

$$1 = C_{\rho^0 \pi^0 \pi^0 \gamma} = 3 C_{\omega \pi^0 \pi^0 \gamma} = \frac{3\sqrt{3}}{\sqrt{2}} C_{\rho^0 \pi^0 \eta \gamma} = \sqrt{\frac{3}{2}} C_{\omega \pi^0 \eta \gamma} = \frac{-3}{\sqrt{2}} C_{\phi K^0 K^0 \gamma} \quad (16)$$

for the more relevant (Zweig allowed) decays. The remaining ones have been discussed in [16] where further details will be found.

The relative weight of the two contributions so far discussed –the finite chiral loops of eqs.(4) to (6) versus the VMD amplitudes (14)– depends crucially on the decay mode. Let us first discuss  $\rho^0 \rightarrow \pi^0 \pi^0 \gamma$  whose VMD contribution is given by eqs.(14) and (15) with the  $\omega$  mass and width in both propagators. One easily obtains[16]

$$\Gamma(\rho^0 \rightarrow \pi^0 \pi^0 \gamma)_{VMD} = 1.62 \cdot 10^3 eV \quad (17)$$

which is of the same order of magnitude as the pion-loop contribution quoted in eq.(11). The global  $\rho^0 \rightarrow \pi^0 \pi^0 \gamma$  decay width is therefore given by the sum of the two amplitudes leading separately to eqs.(11) and (17). One obtains

$$\begin{aligned} \Gamma(\rho^0 \rightarrow \pi^0 \pi^0 \gamma) &= 3.88 \cdot 10^3 eV \\ BR(\rho^0 \rightarrow \pi^0 \pi^0 \gamma) &= 26 \cdot 10^{-6} \end{aligned} \quad (18)$$

and the photonic spectrum shown (solid line) in Fig.2 clearly peaked at higher energies  $E_\gamma$ . The separated contributions from pion-loops and from VMD, as well as their interference, are also shown in Fig.2 (dashed, dotdashed and dotted lines, respectively).

The situation changes quite clearly when turning to the other decay modes. Our predictions for the VMD contributions to  $\rho, \omega \rightarrow \pi^0 \eta \gamma$  and  $\omega \rightarrow \pi^0 \pi^0 \gamma$  are[16]

$$\begin{aligned} \Gamma(\rho^0 \rightarrow \pi^0 \eta \gamma)_{VMD} &= 0.061 eV \simeq \Gamma(\rho^0 \rightarrow \pi^0 \eta \gamma) \\ \Gamma(\omega \rightarrow \pi^0 \eta \gamma)_{VMD} &= 1.39 eV \simeq \Gamma(\omega \rightarrow \pi^0 \eta \gamma) \\ \Gamma(\omega \rightarrow \pi^0 \pi^0 \gamma)_{VMD} &= 235 eV \simeq \Gamma(\omega \rightarrow \pi^0 \pi^0 \gamma) \end{aligned} \quad (19)$$

and have been identified with the corresponding global rates because the previously evaluated kaon-loop contributions (12) are one or two orders of magnitude smaller. The physical reason for this suppression is that the usually dominant pion-loops



are isospin-forbidden in these decays. More accurate estimates and the shape of the photonic spectra seem unnecessary due to the smallness of the corresponding branching ratios (only the third one,  $BR(\omega \rightarrow \pi^0\pi^0\gamma) \simeq 28 \cdot 10^{-6}$ , could reasonably allow for detection) and also to the fact that these decay modes are dominated by the well-understood (see [16]) but less-interesting VMD contribution.

By contrast the latter VMD-contribution is expected to be much smaller in  $\phi \rightarrow \pi^0\eta\gamma$  and  $\pi^0\pi^0\gamma$  decays due to the Zweig rule. Indeed, one easily finds[16]

$$\Gamma(\phi \rightarrow \pi^0\eta\gamma)_{VMD} = 23.9 \text{ eV} , \quad \Gamma(\phi \rightarrow \pi^0\pi^0\gamma)_{VMD} = 51 \text{ eV} \quad (20)$$

well below the kaon-loop contributions quoted in eq.(10). Proceeding as before and adding the corresponding amplitudes with the appropriate phases leads to

$$\begin{aligned} \Gamma(\phi \rightarrow \pi^0\eta\gamma) &= 151 \text{ eV} & \Gamma(\phi \rightarrow \pi^0\pi^0\gamma) &= 281 \text{ eV} \\ BR(\phi \rightarrow \pi^0\eta\gamma) &= 34 \cdot 10^{-6} & BR(\phi \rightarrow \pi^0\pi^0\gamma) &= 64 \cdot 10^{-6} \end{aligned} \quad (21)$$

and the photonic spectra shown in Fig.3 and 4. The Zweig allowed kaon-loops are seen to dominate both spectra and decay rates and the predicted branching ratios are large enough to allow for detection and analyses in future  $\phi$ -factories. For completeness we have also computed  $\Gamma(\phi \rightarrow K^0\bar{K}^0\gamma) \simeq \Gamma(\phi \rightarrow K^0\bar{K}^0\gamma)_K \simeq 0.033 \text{ eV}$ , with  $BR(\phi \rightarrow K^0\bar{K}^0\gamma) \simeq 7.6 \cdot 10^{-9}$ , again dominated by kaon-loops (due to the smallness of the VMD contribution discussed in ref.[16]) but far from near-future detection capabilities.

In summary, some vector meson decays into two neutral pseudoscalars and a photon could receive important contributions from chiral loops if Chiral Perturbation Theory ( $\chi PT$ ) is extended in the plausible and well defined way proposed in this note. Some consequences of this extension –the relevance of pion-loops in  $\rho \rightarrow \pi^0\pi^0\gamma$  and the dominance of kaon-loops in  $\phi \rightarrow \pi^0\eta\gamma, \pi^0\pi^0\gamma$ – have been unambiguously predicted thus allowing for future comparison with data. If the latter turn out to confirm our predictions the domain of applicability of  $\chi PT$  and their relevance would be considerably increased. In case of disagreement it is not  $\chi PT$  itself but, rather, our mechanisms to introduce extra resonances which should be suspicious.

### Acknowledgements

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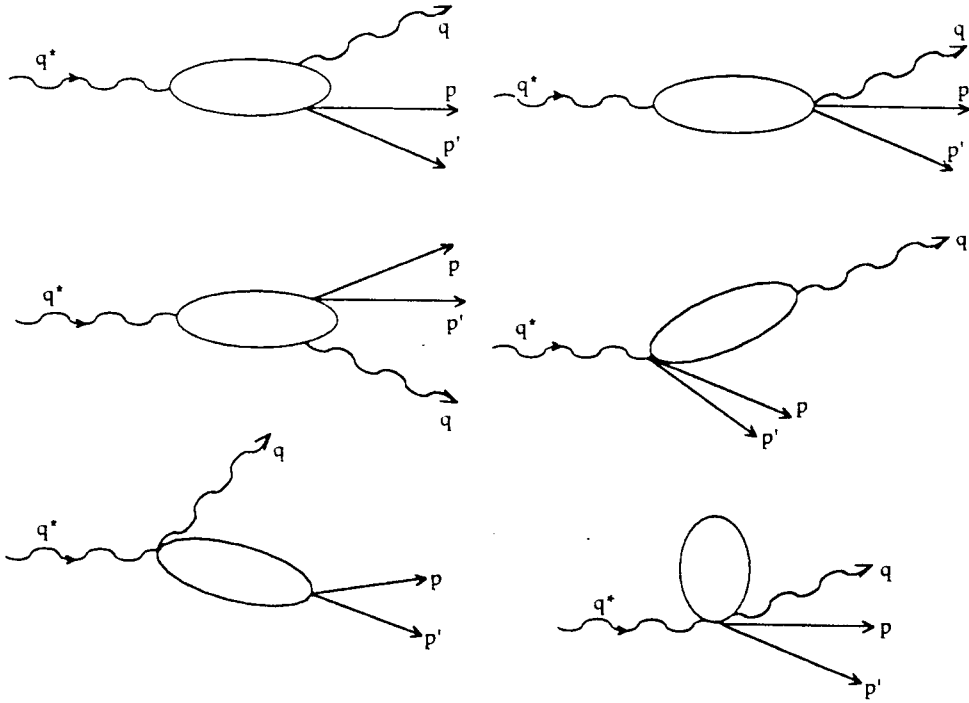


Fig.1 One loop diagrams for  $V^0 \rightarrow P^0 P^0 \gamma$  decays.

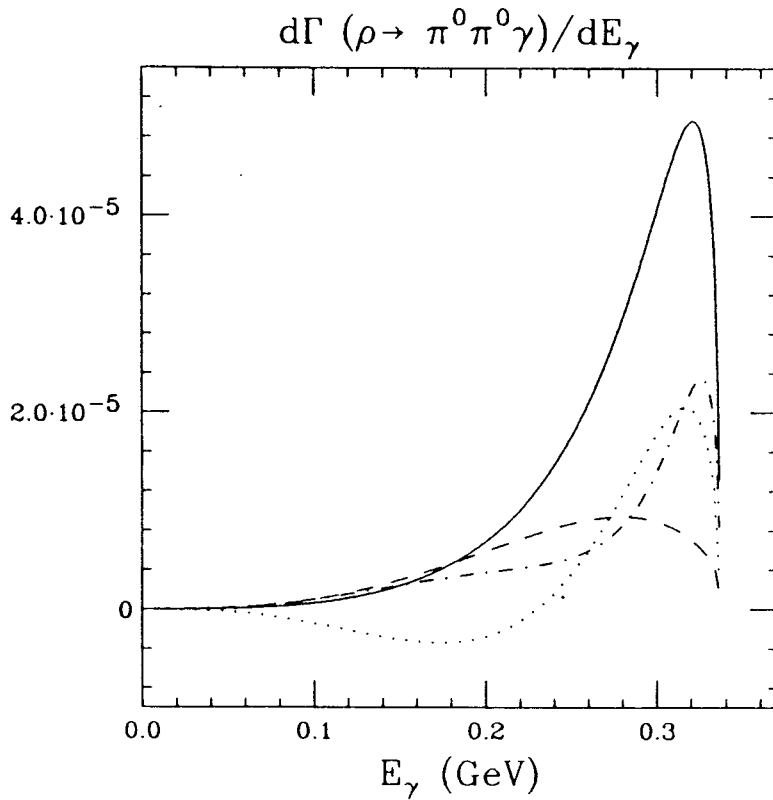
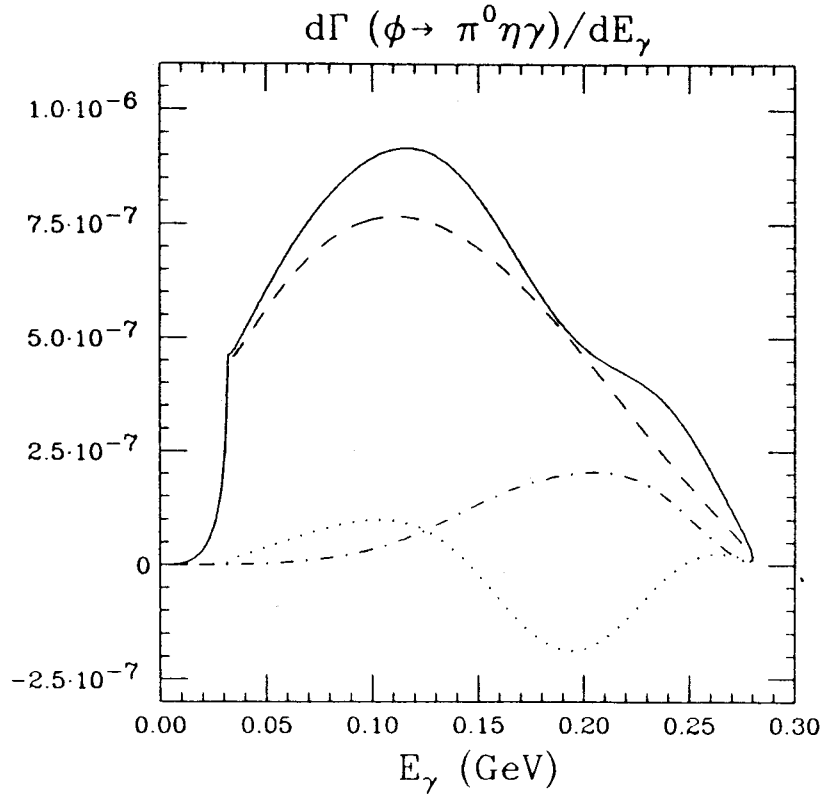
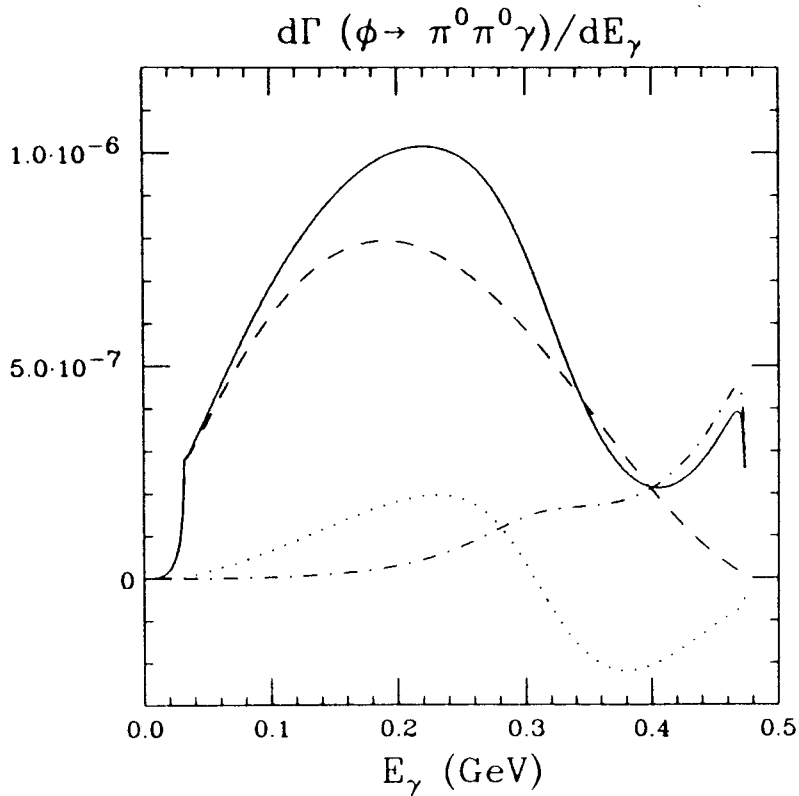


Fig.2 Photonic spectrum in  $\rho^0 \rightarrow \pi^0 \pi^0 \gamma$  (solid line). Dashed line corresponds to the contribution of pion-loops, dotdashed line is the VMD contribution, and dotted line is their interference.



**Fig.3** Photonic spectrum in  $\phi \rightarrow \pi^0 \eta \gamma$  (solid line). Dashed line corresponds to the contribution of kaon loops, dotdashed line is the VMD contribution, and dotted line is their interference.



**Fig.4** Photonic spectrum in  $\phi \rightarrow \pi^0 \pi^0 \gamma$  with conventions as in Fig.3