



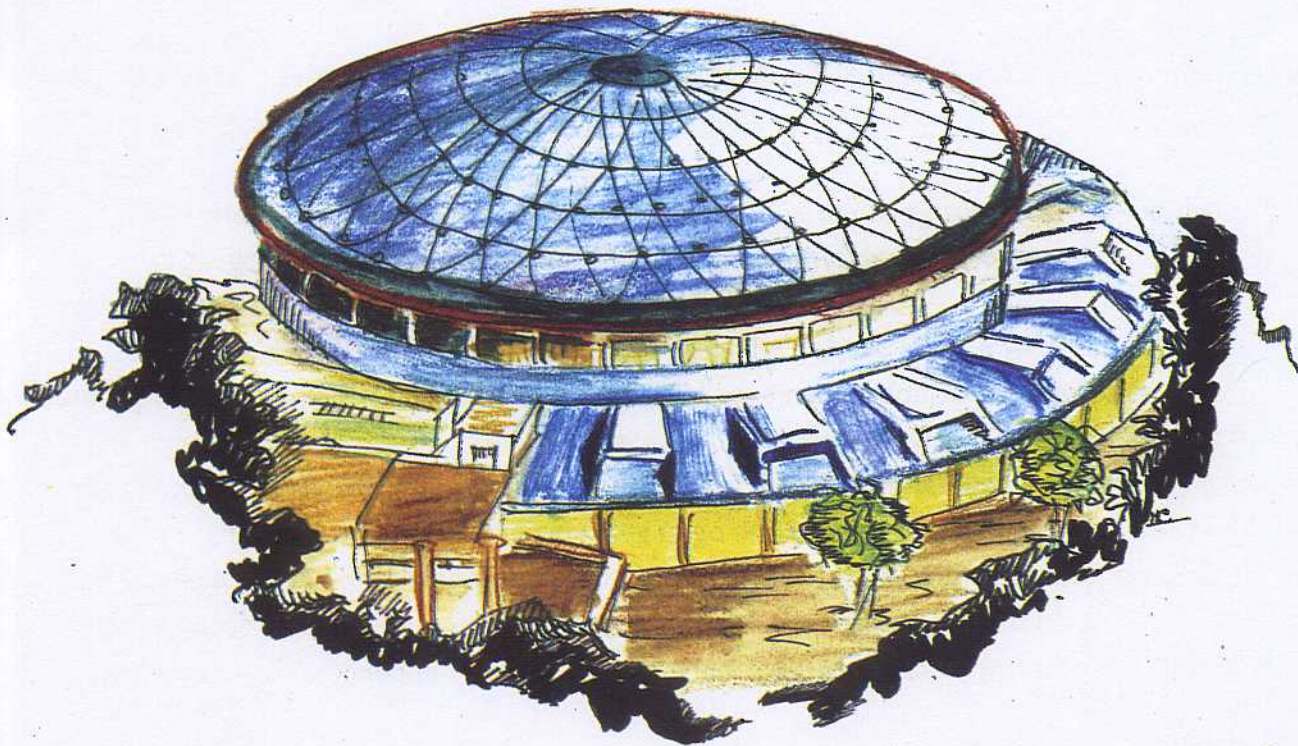
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**CONFINEMENT IN NONCOMPACT NONABELIAN GAUGE THEORIES
ON THE LATTICE**



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**CONFINEMENT IN NONCOMPACT NONABELIAN GAUGE THEORIES
ON THE LATTICE**

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ABSTRACT

We have investigated by Monte Carlo simulation a recently proposed lattice regularization of gauge theories where the gauge fields are noncompact but gauge-invariance is exact. We have found a nonvanishing value of the string tension.

1. – We have investigated by Monte Carlo simulation a lattice regularization of non abelian gauge theories where the gauge fields are noncompact⁽¹⁾, and we have found a nonvanishing value of the string tension.

The present regularization differs from previous noncompact ones^(2,3,4) in that it is exactly gauge-invariant and renormalizable. Gauge-invariance is enforced by means of auxiliary fields which decouple in the continuum limit.

Previous calculations with non compact gauge fields have given a vanishing value of the string tension. Such a result, if confirmed, would have related confinement to compactness of the gauge fields, a conclusion also supported by the widespread belief that a gauge-invariant regularization with noncompact gauge fields could not exist on the lattice. Our result avoids such a conclusion, but it would be interesting to go deeper into the previous calculations which are not conclusive because they are incomplete. In all of them with one exception⁽⁴⁾ lattice regularization was done by direct discretization with consequent explicit breaking of gauge-invariance. Such a breaking has indeed been considered responsible for the vanishing of the

string tension. There remains nevertheless the possibility⁽³⁾ that these regularizations, being close to the continuum, might have a scale parameter close to the continuum one. In such a case it is also possible that the physical dimension of the lattice spacing be so small in the scaling window that inside actual lattices there be no linear potential.

In the only exception where gauge-invariance was enforced on the lattice⁽⁴⁾, this was done by using gauge-invariant variables. These variables are the solutions of a non renormalizable gauge-fixing, so that perturbation theory cannot be used to evaluate the scale parameter, and strictly speaking such a parameter could not exist at all. It has not been shown in fact that this regularization has a continuum limit. Since perturbation theory cannot be used for this purpose, one should study some physical quantity (different from the string tension) to determine whether it would scale or not.

In conclusion the present result is not in contradiction with previous investigations, showing the equivalence, as far as confinement is concerned, between compact and noncompact regularizations provided they are both gauge-invariant and renormalizable.

2. - To make the paper reasonably self contained we report the essential features of the regularization we have studied. In the whole paper we will restrict ourselves to the SU(2) gauge group.

The basic element is the covariant derivative

$$D_{\mu} = \left(\frac{1}{a} - W_{\mu} \right) I + \mathcal{A}_{\mu}, \quad \mathcal{A}_{\mu} = T_a \mathcal{A}_{\mu a}, \quad (1)$$

where a is the lattice spacing, T_a are the generators of the gauge group normalized according to

$$\begin{aligned} [T_a, T_b] &= i \epsilon_{abc} T_c \\ \{T_a, T_b\} &= \frac{1}{2} \delta_{ab}, \end{aligned} \quad (2)$$

\mathcal{A}_{μ} is the gauge field and W_{μ} is the auxiliary field. They transform according to

$$\begin{aligned} \mathcal{A}'_{\mu a} &= \mathcal{A}_{\mu a} + (1 - a W_{\mu}) \Delta_{\mu} \vartheta_a - \epsilon_{abc} \mathcal{A}_{\mu b} \left(1 + \frac{a}{2} \Delta_{\mu} \right) \vartheta_c \\ W'_{\mu} &= W_{\mu} + \frac{1}{4} a \mathcal{A}_{\mu a} \Delta_{\mu} \vartheta_a. \end{aligned} \quad (3)$$

In the above formulae ϑ_a are the parameters of the transformation and Δ_{μ} the right derivative

$$\Delta_{\mu} f(x) = \frac{1}{a} [f(x + \mu) - f(x)], \quad (4)$$

μ being the unit vector in the positive μ -direction.

From the transformations of \mathcal{A}_μ and W_μ it follows that

$$D_\mu(x) \rightarrow g(x) D_\mu(x) g^+(x + \mu) \quad (5)$$

for a gauge transformation $g(x) = e^{i T_a \theta_a}$.

There are two invariants

$$\mathcal{L}_P = -\frac{1}{2} \beta \sum_{\mu > \nu} \text{Tr} D_\mu(x) D_\nu(x + \mu) D_\mu^+(x + \nu) D_\nu^+ \quad (6)$$

$$t_\mu I = D_\mu D_\mu^+ - \frac{1}{a^2} I. \quad (7)$$

We will study the gauge field Lagrangian

$$\begin{aligned} \mathcal{L}_G = \mathcal{L}_P + 3\beta \frac{1}{a^2} \sum_{\mu} t_\mu + \frac{1}{2} \beta \sum_{\mu \neq \nu} t_\mu(x) t_\nu(x + \mu) \\ - \frac{1}{16} \beta a^2 \sum_{\mu \neq \nu} (\Delta_\mu t_\nu - \Delta_\nu t_\mu)^2 + \frac{1}{2} \gamma^2 \sum_{\mu} t_\mu^2. \end{aligned} \quad (8)$$

The additional terms with respect to \mathcal{L}_P have been determined⁽⁵⁾ in order to have Euclidean invariance in the continuum limit and a propagator of W_μ constant in momentum space. This ensures its decoupling in the continuum limit where \mathcal{L}_G becomes the Yang-Mills Lagrangian density plus the free Lagrangian of the field t_μ . For $\gamma = \infty$ we get exactly Wilson's formulation.

The partition function is

$$Z = \int dW_\mu \int d\mathcal{A}_{\mu a} e^{-\sum_x a^4 \mathcal{L}_G} \quad (9)$$

because the measure for integration over W_μ and $\mathcal{A}_{\mu a}$ is 1. This might be a significant simplification for gauge groups $SU(N)$ with $N > 2$.

Both in the perturbative calculation⁽⁵⁾ and in the present numerical simulation the "polar" representation of the covariant derivative has been used

$$D_\mu = \frac{1}{a} \sqrt{1 + t_\mu} \quad U_\mu \stackrel{\text{def}}{=} \frac{1}{a} \rho_\mu U_\mu \quad (10)$$

where U_μ are link unitary variables. In terms of these variables

$$Z = \int d\rho_\mu \rho_\mu^3 \int dU e^{-\sum_x a^4 \mathcal{L}'_G} \quad (11)$$

where dU is the Haar measure on the $SU(2)$ group and

$$\mathcal{L}'_G = -\frac{1}{2} \sum_{\mu > \nu} \beta \rho_{P\mu\nu} \text{Tr} U_{P\mu\nu} + V(\rho). \quad (12)$$

In the above equation

$$U_{P\mu\nu} = U_\mu(x) U_\nu(x + \mu) U_\mu^\dagger(x + \nu) U_\nu^\dagger$$

$$\rho_{P\mu\nu} = \rho_\mu(x) \rho_\nu(x + \mu) \rho_\mu(x + \nu) \rho_\nu,$$

while $V(\rho)$ can be determined by comparison with Eq. (8). For $\gamma = \infty$, ρ_μ is fixed to one, and this explains how Wilson's formulation is recovered.

The renormalization group parameter of the present noncompact regularization has been determined by a one loop calculation⁵⁾. As a result of this calculation it has been shown that in the scaling regime γ is not an arbitrary parameter, but it must behave at large values of β as

$$\gamma = \gamma_1 \beta + \gamma_2, \quad (13)$$

where γ_1 is arbitrary while γ_2 (independent of β) is calculable but has not been evaluated. In the numerical simulation we will use the above expression of γ not only in the scaling window but for all values of β , neglecting γ_2 . We will comment on this approximation after presentation of the numerical results.

The renormalization scale parameter Λ_{NC} of this regularization has been related⁽⁵⁾ to that of Wilson Λ_W

$$\Lambda_{NC} = \Lambda_W e^{-\frac{12\pi^2}{11} E} \quad (14)$$

where

$$E = 0.2208 \beta \frac{1}{\gamma^2}, \quad (15)$$

so that in the continuum limit ($\beta \rightarrow \infty$) according to Eq. (13) $\Lambda_{NC} \rightarrow \Lambda_W$.

3. - We have performed our numerical simulation on a 12^4 lattice using the polar variables. Since in such a representation the link matrices U_μ enter linearly into the action (12), it is possible to use the standard heat bath method for the integral over the U_μ 's. For the integral over ρ_μ we used instead the Metropolis algorithm.

In Fig. 1 we show the expectation value $\langle \frac{1}{2} \text{Tr} U_P \rangle$ as a function of β for different values of γ_1 . The expectation value $\langle \frac{1}{2} \text{Tr} U_P \rangle$ of Wilson regularization is also reported. We see that Wilson's result is recovered for large γ_1 , while decreasing γ_1 the crossover is shifted to larger values of β .

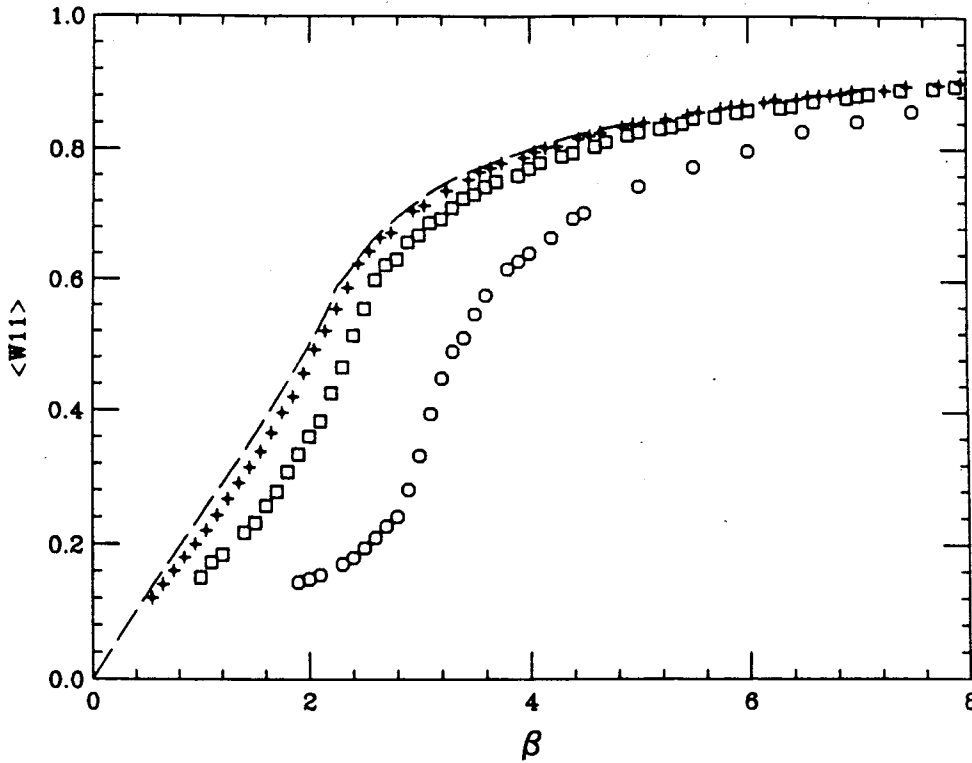


FIG. 1 - The expectation value of $\langle \frac{1}{2} \text{Tr} U_P \rangle$ as a function of β for $\gamma_1 = 5$ (crosses); 2.5 (squares); 1 (circles). The slashed line corresponds to Wilson's model.

In Fig. 2 we report the Creutz ratio

$$\chi_I = - \ln \frac{W_{I-1,I-1} W_{II}}{W_{I-1,I} W_{I,I-1}}, \quad (16)$$

where $W_{I,J}$ is the expectation value of the rectangular Wilson loop $I \times J$.

Due to the existence of the field ρ_μ there are (at least) two possible definitions of the Wilson loop

$$W_C^{(1)} = \langle \prod_{\ell \in C} D_\ell \rangle = \langle \prod_{\ell \in C} \rho_\ell U_\ell \rangle \quad (17)$$

$$W_C^{(2)} = \langle \prod_{\ell \in C} U_\ell \rangle$$

which should coincide in the continuum limit. We have evaluated both of them finding that they indeed coincide within statistical errors in the scaling window for all the values of γ_1 explored. In Fig. 2 we have reported the Creutz ratio obtained from $W_C^{(2)}$.

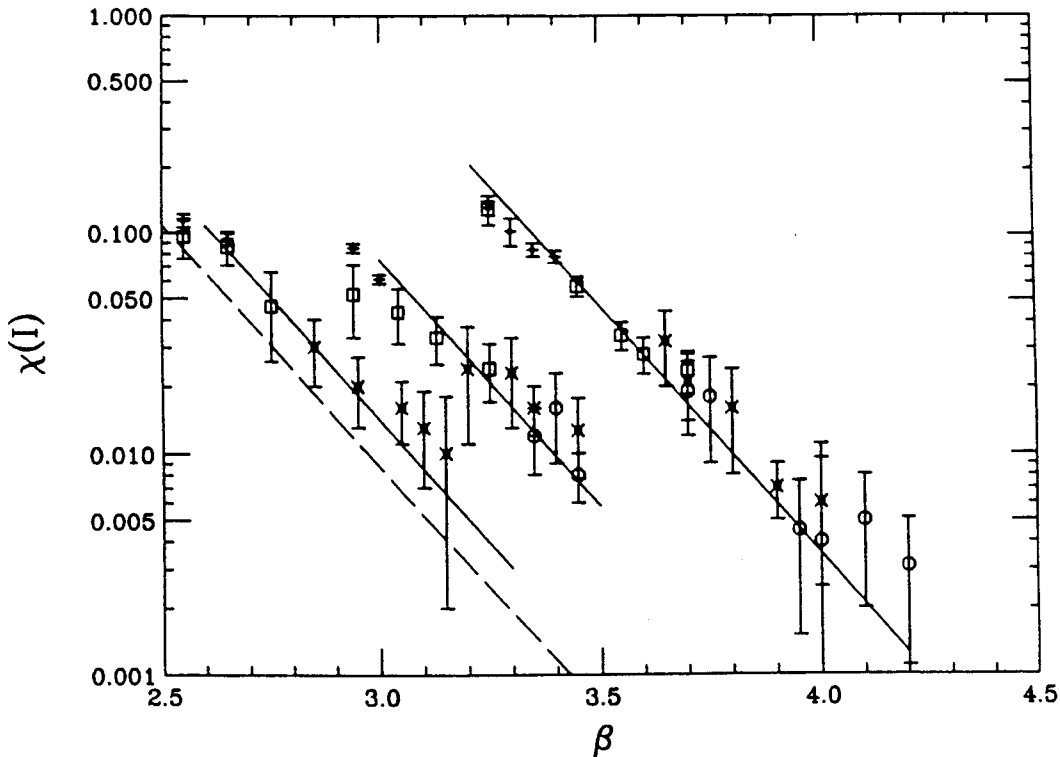


FIG. 2 – The value χ_I as a function of β for decreasing values of γ_1 from left to right ($\gamma_1=5;2,5;1$). The fit to Wilson's values is given by the slashed line. Full circles, squares, crosses and empty circles refer to $I = 3,4,5,6$ respectively.

We calculated the mean values of χ_I using for each value of β and γ_1 30 configurations of the gauge field. These configurations are separated by 50 Monte Carlo sweeps through the lattice. For each link we choosed the new value of ρ_μ by performing 3 Metropolis iterations, while, as already mentioned, we used the heat bath method for U_μ .

In Fig. 2 for each value of γ_1 only points lying in a band around the fit are reported for clarity. The points to be discarded have been chosen by inspection. Always for clarity we have

not reported the points corresponding to Wilson's regularization, but only their fit (broken line). Writing the Creutz ratio in the form

$$\chi = \frac{\sigma}{\Lambda_w^2} \left(\frac{\Lambda_w}{\Lambda_{NC}} \right)^2 \left(\frac{6\pi^2}{11} \beta \right)^{\frac{102}{121}} \exp \left(- \frac{6\pi^2}{11} \beta \right), \quad (18)$$

from the fits we determine the ratio (Λ_w/Λ_{NC}) which is reported in the Table along with the theoretical value given by Eq. (14). Although this theoretical value is increasing with decreasing γ_1 , it does so at a very much lower rate than the corresponding value provided by the Monte Carlo simulation. This discrepancy can be due either to our approximation of neglecting γ_2 in Eq. (13), or to large corrections to the one-loop formula which would not be surprising in view of the value of $g \sim 1$ in the scaling window. A similar situation occurs in the mixed fundamental-adjoint model⁽⁶⁾.

TABLE – The Monte Carlo and the theoretical values of the ratio (Λ_w/Λ_{NC}) for different values of γ_1 . Numerical errors are purely subjective estimates. The uncertainty in the theoretical values is due to the range of β in the scaling window.

γ_1	5	2.5	1
$(\Lambda_w/\Lambda_{NC})_{MC}$	1.3 ± 0.2	3.0 ± 0.2	8.5 ± 0.3
$(\Lambda_w/\Lambda_{NC})_{th}$	1.03 ± 0.05	1.13 ± 0.05	2.0 ± 0.2

In conclusion we think that we have the same kind of evidence for confinement as in Wilson model. Such a result puts the two regularizations on the same footing and eliminates the common assumption that confinement be related to compactness of the gauge fields.

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