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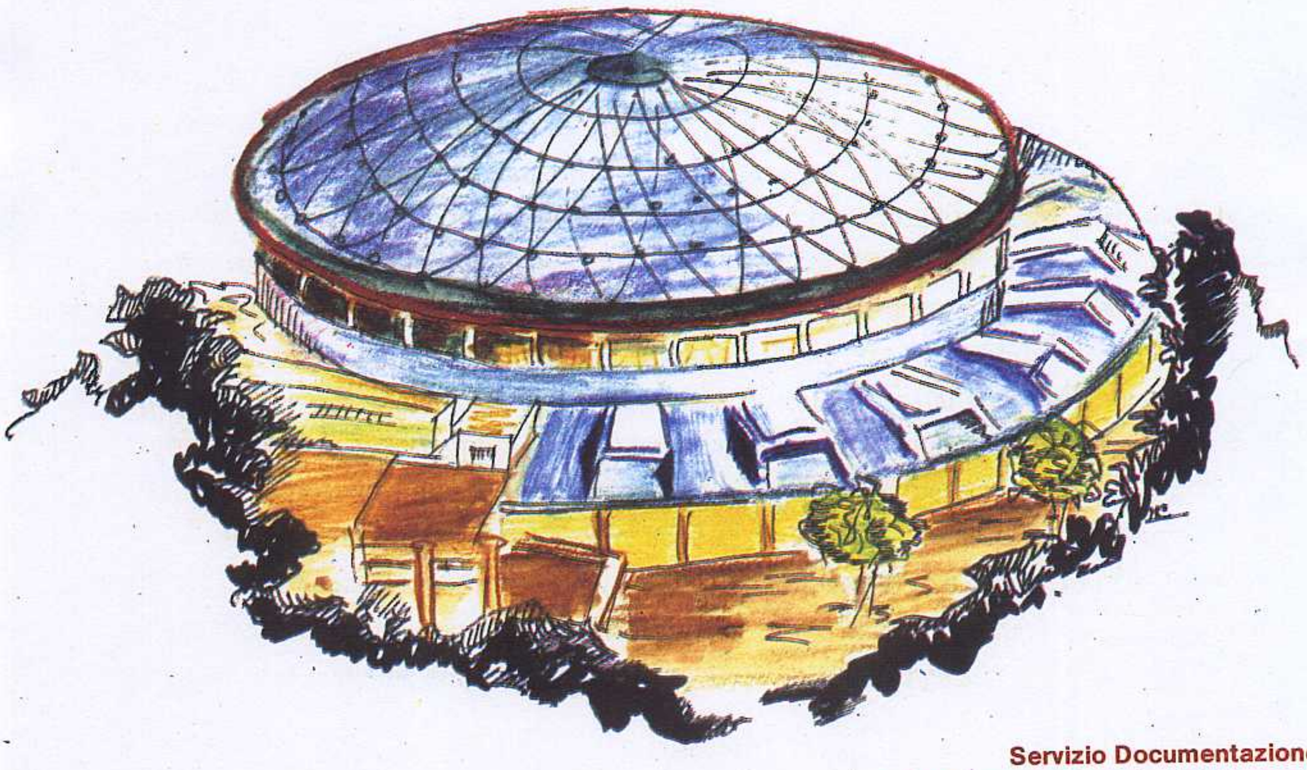
CHIRAL PERTURBATION THEORY PREDICTS PION PION PRODUCTION VIA PHOTON PHOTON FUSION

Contribution to the DAΦNE Physics Handbook



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CHIRAL PERTURBATION THEORY PREDICTS PION PION PRODUCTION VIA PHOTON PHOTON FUSION

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ABSTRACT

We describe the low energy physics of the $\gamma\gamma \rightarrow \pi\pi$ reaction. Then we discuss new high-luminosity experiments that will provide the opportunity to measure the 2-loop effects, within chiral perturbation theory, for the $\pi^0\pi^0$ case.

1. INTRODUCTION

We show in the following how the cross-section for the reaction $\gamma\gamma \rightarrow \pi\pi$ is predicted on the basis of the $SU(3)_L \otimes SU(3)_R$ approximated invariance. The theoretical results are confronted with the experimental data presently available. An important observation is that the production of a pair of charged pions is dominated by the Born amplitude. We also show how the contribution of chiral loops to first order has been included. The corresponding cross-section fits quite well the data from the MARK-II group. The process $\gamma\gamma \rightarrow \pi^0\pi^0$ is of great interest as a test of the chiral loop structure. In this case the cross-section is not yet predicted with enough precision to confront the present data from the Crystal Ball collaboration. The calculation of the two-loop amplitude is under way and its inclusion in the cross-section will give presumably a sizeable effect. Also the opportunities for new measurements are described. With the luminosity planned at DAΦNE there appears the exciting possibility of measuring two-loop effects in the reaction $\gamma\gamma \rightarrow \pi^0\pi^0$. An interesting addition to this discussion concerns the study of the cross-section dependence on small values of the photon invariant masses. This can be achieved by detecting the outgoing electron (positron) at small angles.

2. THE TWO-PHOTON PRODUCTION OF CHARGED PION PAIRS

The leading contribution to the $\gamma\gamma \to \pi^+\pi^-$ cross-section arises from the tree-level Feynman graphs. Choosing the frame

$$\varepsilon_1 \cdot k_1 = \varepsilon_1 \cdot k_2 = \varepsilon_2 \cdot k_1 = \varepsilon_2 \cdot k_2 = 0 \, , \tag{1}$$

the Born amplitude reads[1]

$$A^{(B)}(\gamma\gamma \to \pi^+\pi^-) = 2ie^2 \left[\varepsilon_1 \cdot \varepsilon_2 - \frac{p_+ \cdot \varepsilon_1 \; p_- \cdot \varepsilon_2}{p_+ \cdot k_1} - \frac{p_+ \cdot \varepsilon_2 \; p_- \cdot \varepsilon_1}{p_+ \cdot k_2} \right] . \tag{2}$$

Here $\varepsilon_{1,2}$ are the photon polarization vectors, $k_{1,2}$ are the momenta of the incoming photons and p_+ , p_- are the momenta of the outgoing pions. Also the $O(p^4)$ chiral corrections have been computed[2]

$$A^{(1)}(\gamma\gamma \to \pi^+\pi^-) = 2ie^2 \left[a \; \varepsilon_1 \cdot \varepsilon_2 - \frac{p_+ \cdot \varepsilon_1 \; p_- \cdot \varepsilon_2}{p_+ \cdot k_1} - \frac{p_+ \cdot \varepsilon_2 \; p_- \cdot \varepsilon_1}{p_+ \cdot k_2} \right] , \tag{3}$$

where a is given by

$$a = 1 + \frac{2s}{F_\pi^2} (L_9{}^r + L_{10}{}^r) + L(s) \, . \tag{4}$$

The first (second) contribution to a arises from the tree graphs with vertices in the $O(p^2)$ ($O(p^4)$) term of the perturbative expansion of the chiral effective lagrangian $L^{(2)}$ ($L^{(4)}$, see ref. (3)). The last term in eq. (4) represents the contribution of the 1-loop graphs with vertices in $L^{(2)}$. The pion decay constant is denoted by[4] $F_\pi = 93.15$ MeV. For the pion and kaon masses we take the values $m_\pi = 137$ MeV, $m_K = 494$ MeV.

The complex loop-function L(s) has been calculated[2] in chiral perturbation theory including pion and kaon loops to $O(p^4)$

$$L(s) = - \frac{1}{32\pi^2 F_\pi^2} \left[\frac{3}{2} s + (m_\pi \ln Q_\pi)^2 + \frac{1}{2} (m_K \ln Q_K)^2 \right] , \tag{5}$$

where

$$Q_i = \frac{\sqrt{1 - \frac{4 m_i^2}{s}} + 1}{\sqrt{1 - \frac{4 m_i^2}{s}} - 1} \, , \; i = \pi, K \, . \tag{6}$$

The differential cross-section is readily obtained from the amplitude $A^{(1)}(\gamma\gamma \to \pi^+\pi^-)$

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi}{2s} \beta\alpha^2 \left[2|a|^2 - \mathrm{Re}(a) \frac{4 (\beta \sin\theta)^2}{1 - (\beta\cos\theta)^2} + \frac{4 (\beta\sin\theta)^4}{[1 - (\beta\cos\theta)^2]^2} \right] , \tag{7}$$

where θ is the scattering angle and β is the velocity of the pions in the center of mass frame. Integrating over θ yields the total cross-section in the range $|\cos\theta| \leq Z$

$$\sigma(s, |\cos\theta| \leq Z) =$$

$$\frac{\pi}{s}\beta\alpha^2\left[2Z\,|a|^2 + 2(\delta - 2Z)\,\mathrm{Re}(a) + 4Z + 2Z\,\frac{(1-\beta^2)^2}{1-(\beta Z)^2} - (3+\beta^2)\delta\right], \tag{8}$$

with

$$\delta = \frac{1-\beta^2}{\beta}\ln\frac{1+\beta Z}{1-\beta Z}\ . \tag{9}$$

The Born total cross-section corresponds to setting a=1 in $\sigma(s, |\cos\theta| \leq Z)$. Both cross-sections are plotted in Fig. 1, where we integrated over the values of θ in the range determined by Z=0.6.

For the $O(p^4)$ correction (a -1) in eq. (4), we input the value of the chiral lagrangian coefficient predicted by chiral perturbation theory

$$(L_9^r + L_{10}^r) = (1.43 \pm 0.27)\cdot 10^{-3}\ . \tag{10}$$

This value is obtained from the ratio between the measured value of the axial vector coupling constant $F_A = 0.0117 \pm 0.0020$ and the CVC value of the vector coupling constant $F_V = 0.0259 \pm 0.0005$ in the radiative pion decay(4)

$$\frac{F_A}{F_V} = 0.452 \pm 0.086\ . \tag{11}$$

The use of the CVC prediction in place of the measured value $F_V = 0.023^{+0.015}_{-0.013}$ may be motivated by observing that the data on F_A have been analyzed assuming the value of F_V based on the CVC assumption(4). From ref. (3) we find that the ratio in eq. (11) depends on the sum of the two renormalized parameters L_9^r and L_{10}^r as follows:

$$\frac{F_A}{F_V} = 32\pi^2(L_9^r + L_{10}^r)\ . \tag{12}$$

It has been noticed(3) that the sum of the unrenormalized coefficients of the effective lagrangian $(L_9 + L_{10})$ is ultraviolet finite

$$(L_9^r + L_{10}^r) = (L_9 + L_{10})\ . \tag{13}$$

Hence, this sum is renormalization scale independent, even though the individual tree-level parameters are not. The size of the corrections to the lowest order cross-section of the reaction $\gamma\gamma \to \pi^+\pi^-$ does not depend very much on the precise value of this sum within the errors quoted in eq. (10).

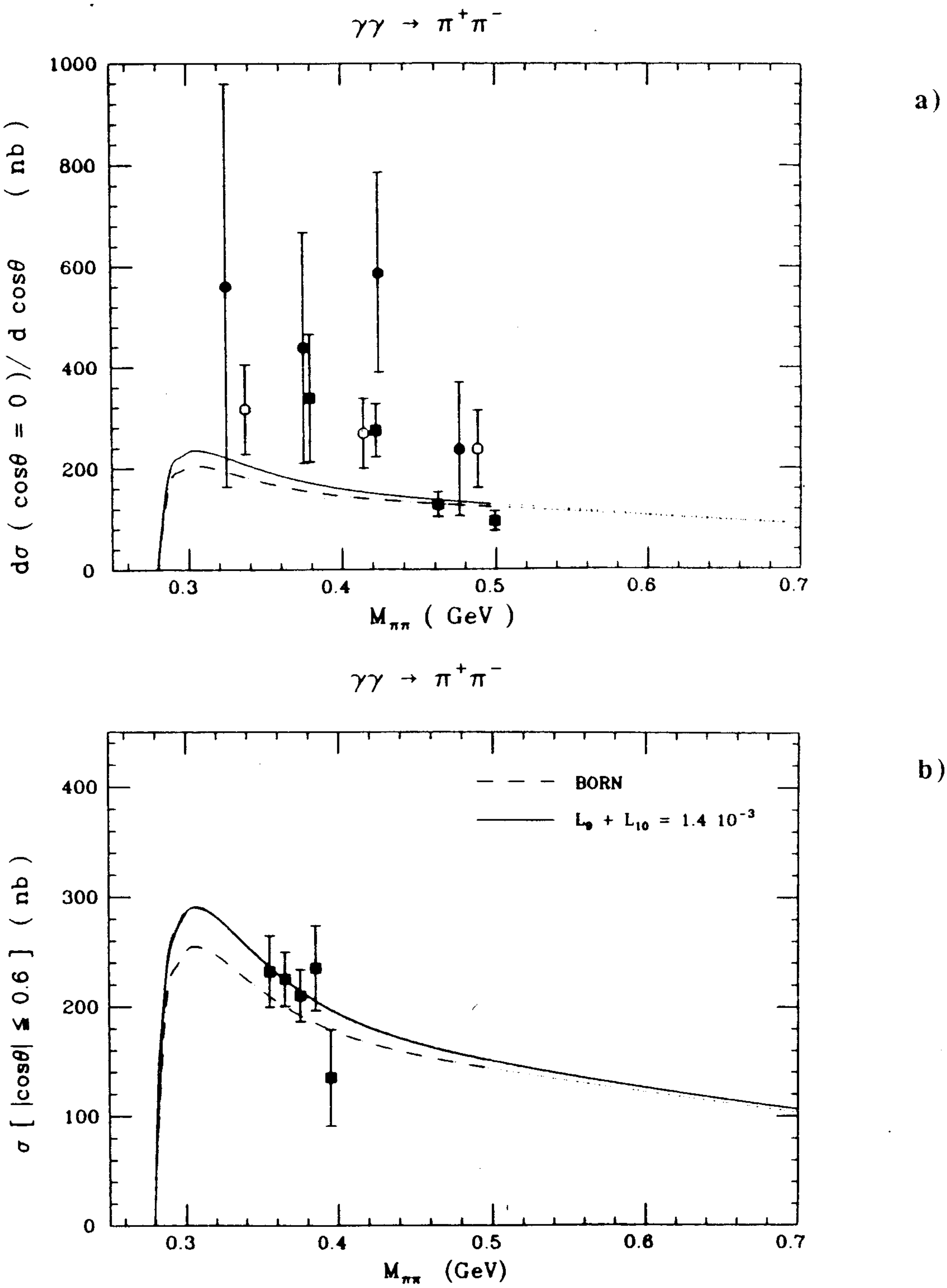

FIG. 1 - a) Present available data for $(d\sigma/d\cos\theta)_{\gamma\gamma\to\pi^+\pi^-}$ at $\theta_{CM} = 90°$ in the $M_{\pi\pi} < 0.5$ GeV region. The experimental points are: DM1[(5)] (○), DM2[(5)] (●), PLUTO[(10)] (■). The theoretical predictions are: Born (dashed line); chiral theory with $(L_9^r + L_{10}^r) = 1.4\cdot10^{-3}$ (full line); b) MARK-II[(6)] total cross section data for $M_{\pi\pi} < 0.5$ GeV. The theoretical curves are the same as in 1a).In both cases the dotted lines indicate the continuation of the chiral predictions above $M_{\pi\pi} = 0.5$ GeV where unitarity effects are important.

This reaction can be studied at DAΦNE by measuring the process $e^+e^- \to e^+e^-\pi^+\pi^-$. This cross-section is related to the $\gamma\gamma \to \pi^+\pi^-$ cross-section as follows:

$$\left(\frac{d\sigma}{ds}\right)_{e^+e^- \to e^+e^- \pi^+\pi^-} = \sigma_{\gamma\gamma \to \pi^+\pi^-}(s)\, L_{\gamma\gamma}(s) \ , \tag{14}$$

where the $\gamma\gamma$ luminosity function reads

$$L_{\gamma\gamma}(s) = -\frac{1}{s}\eta^2 \left[(2+z)^2 \ln z + 2(3+z)(1-z)\right] , \tag{15}$$

with

$$\eta = -\frac{\alpha}{\pi}\ln\left(\frac{E}{m}\right), \qquad z = \frac{s}{4E^2} . \tag{16}$$

Here m denotes the electron mass and E (= 0.51 GeV) is the energy of each lepton beam. An important consequence is borne of the fact that the cross-section for $\gamma\gamma \to \pi^+\pi^-$ is dominated by the tree-level contribution (at e.g. $\sqrt{s}$ = 0.4 GeV, the Born amplitude contributes about 90% of the value of the cross-section, including the $O(p^4)$ correction). Owing to the smallness of the correction, even a measurement of the cross-section with high statistical accuracy yields a value of $(L_9 + L_{10})$ affected by sizeable uncertainties. Assuming the value of $(L_9 + L_{10})$ obtained from only the experimental point at $\sqrt{s}$ = 0.4 GeV, one needs a 0.6% statistical accuracy on the measured cross-section, in order to have a 5% statistical error on this measurement of $(L_9 + L_{10})$. This corresponds to the integrated luminosity $L_{e^+e^-} = 2\cdot 10^{39}$ cm^{-2} . In the hypotesis that the DAΦNE luminosity will have the value $5\cdot 10^{32}$ cm^{-2} s^{-1}, we get a running time of 0.4 year ($4\cdot 10^6$ s). In fact, we can obtain an independent measurement of the $(L_9 + L_{10})$ coefficient (which is a constant in s) for each experimental point. Accordingly, the statistical error on $(L_9 + L_{10})$ can be further reduced. One should bear in mind that the chiral perturbation theory prediction of eq. (10) is affected by a 20% uncertainty. Additional information may be obtained by measuring the shape of the angular distribution and comparing with eq. (7).

The prediction in eq. (8) is based on a perturbative expansion in external momenta and quark masses. In order to establish the region of validity of this result, one needs to calculate the amplitude $A^{(2)}(\gamma\gamma \to \pi^+\pi^-)$ to the next order $O(p^6)$. This information is still lacking. It is generally expected that this result be valid up to a center of mass energy of 0.5 GeV. This low energy bound stems from two criteria. Firstly, there is a general agreement of the data on kaon decays with the predictions of chiral perturbation theory, indicating that the arguments based on chiral symmetry would be reliable up to this bound. Then, the relative magnitude of the contribution of resonance exchange is usually considered as an indicator for the breakdown of the low energy expansion. In the reaction $\gamma\gamma \to \pi^+\pi^-$ the mass of the lowest lying resonance is of the order of 1 GeV. Hence it is quite reasonable that its contribution to the cross section would not be sizeable in the abovementioned low energy region. Empirically, one remarks the smallness of the corrections to the Born cross-section, a fact that confirms the validity of the perturbative expansion of the effective lagrangian in the momenta and quark masses, at least in the low energy region $\sqrt{s} \leq 0.5$ GeV.

In Fig. 1 we compare the predicted cross-section with the available data. The cross-section results for $\gamma\gamma \to \pi^+\pi^-$ obtained by the DM1, DM2 and PLUTO groups are reviewed in ref. (5). Additional experimental results have been gathered by the MARK-II collaboration at SLAC[6]. A general property of the experimental points in Fig. 1 is that the measured cross-section in the low energy region is higher than the Born model prediction. All experiments but the SLAC one obtain a large enhancement of the cross-section at low energy. This could be the result of a systematical error[5] in subtracting (with low statistics) the e^+e^- and $\mu^+\mu^-$ background signals, which are large compared to the $\pi^+\pi^-$ signal in the low-energy region. This interpretation is confirmed by the high statistics MARK-II results, whose uncertainties are dominated by systematic errors. These data exclude a large enhancement at low energy and are consistent with the chiral prediction, including the $O(p^4)$ corrections. In the low energy region there are no results from the TPC[7] and CELLO[8] experiments. In ref. (9) a crossing symmetry is used to relate the $\gamma\gamma \to \pi\pi$ reaction to the Compton scattering formulae. This allows to obtain independent information on the pion polarizabilities (customarily derived from the Compton and Compton-like processes) also from the reaction $\gamma\gamma \to \pi\pi$. A large enhancement effect at low energy would be consistent with an electric pion polarizability much larger than the value $\alpha_{\pi^\pm} = (2.65 \pm 0.50)\cdot 10^{-4}\,\mathrm{fm}^3$ expected, on the basis of the relation valid to $O(p^4)$ in chiral perturbation theory[3]

$$\alpha_{\pi^\pm} = \frac{4\alpha}{m_\pi F_\pi^2}(L_9^r + L_{10}^r)\ . \qquad (17)$$

In order to investigate this possibility, in ref. (9) we fitted all the available data with the cross-section formulae predicted by chiral perturbation theory, including the $O(p^4)$ corrections, in the restricted energy region $\sqrt{s} \leq 0.5$ GeV. The results obtained for the combination $(L_9 + L_{10})$ and for $\alpha_{\pi^\pm}$ are reported in Table I.

TABLE I - a) Values of $(L_9^r + L_{10}^r)$ and $\alpha_{\pi^\pm}$ obtained from the $\gamma\gamma \to \pi^+\pi^-$ data with $\sqrt{s} \leq 0.5$ GeV. (Systematic errors are not included.)

	$(L_9^r + L_{10}^r) \times 10^3$	$\alpha_{\pi^\pm} \times 10^4$ (fm^3)	No. of points
PLUTO[10]	10.3 ± 2.6	19.1 ± 4.8	2
DM1[5]	9.3 ± 2.5	17.2 ± 4.6	3
DM2[5]	14.2 ± 4.0	26.3 ± 7.4	4

TABLE I - b) Values of $(L_9^r + L_{10}^r)$ and $\alpha_{\pi^\pm}$ obtained from the MARK-II data (Fig. 2b) for $\gamma\gamma \to \pi^+\pi^-$, including systematic errors, with $\sqrt{s} \leq 0.5$ GeV.

	$(L_9^r + L_{10}^r) \times 10^3$	$\alpha_{\pi^\pm} \times 10^4$ (fm^3)	No. of points
MARK-II[6]	1.2 ± 0.9	2.2 ± 1.6	5

The differential cross-section values at 90^0 for the DM1 and DM2 experiments have been inferred from the data analysis of ref. (5). Additional problems are associated with the interpretation of the PLUTO results. In the energy range between 0.5 and 0.7 GeV the PLUTO data lie consistently below the Born cross-section and they rise above it only for very low energy values[10]. Since this suggests the effect of modifications of the Born model that we do not consider in the present analysis, the experimental points in this energy range are disregarded. Taking into account only the two lowest energy points (at $\sqrt{s}$ = 0.38 and 0.42 GeV) the corresponding PLUTO result $\alpha_{\pi^\pm} = (19.1 \pm 4.8)\cdot 10^{-4}\ \mathrm{fm}^3$ is consistent with the DM1/DM2 values. The PLUTO estimated systematic uncertainty below 0.5 GeV is ± 20%. As a result, a systematical error $(\pm\ 5.7)\cdot 10^{-4}\ \mathrm{fm}^3$ must be added to the PLUTO value for $\alpha_{\pi^\pm}$. Systematical uncertainties not included in the quoted errors very likely affect all the data of Table Ia. Nevertheless, the DM1 and DM2 data seemingly indicate that the pion polarizability is much larger than the chiral prediction $\alpha_{\pi^\pm} = (2.65 \pm 0.50)\cdot 10^{-4}\ \mathrm{fm}^3$. Conversely, the value for $\alpha_{\pi^\pm}$ deduced from the MARK-II measurements is in good agreement with the chiral prediction (see Fig. 2b), in evident contrast to the DM1/DM2/PLUTO results. However, the systematic errors which have been included in the analysis of the MARK-II data are still too large to provide a useful constraint. This experimental situation clearly needs further investigations before drawing any definitive conclusions about the possibility to interpret the enhancement of the $\gamma\gamma \to \pi^+\pi^-$ cross-section near threshold as evidence for some unpredicted dynamical effect in this energy region, or rather as simply reflecting a systematical error in the identification of pion pairs by missing mass in the DM1/DM2/PLUTO analysis. It is worth noticing that the existence of the basic connection between pion polarizability and radiative pion beta decay expressed by eqs. (12), (17) can be otained, independently from the chiral approach, by simply using soft-pion techniques, PCAC, and current algebra[11,12]. Thus, an evidence for a violation of the predicted values (10) and $\alpha_{\pi^\pm} = (2.65 \pm 0.50)\cdot 10^{-4}\ \mathrm{fm}^3$ could have consequences that are not only connected with chiral perturbation theory.

3. LEARNING FROM $\gamma\gamma \to \pi^0\pi^0$

In order to get further insight in the problem discussed above, we now bring into focus the low energy $\gamma\gamma \to \pi^0\pi^0$ production. This process involves only neutral particles. This entails that the tree-level diagrams contribute to the amplitude only with terms of sixth order, or higher, in the external momenta and quark masses. Summarizing, the perturbative expansion of the amplitude yields the following result. The $O(p^2)$ amplitude vanishes. The only contribution to the $O(p^4)$ amplitude comes from the 1-loop diagrams. Since there are no counterterms available to this order, these diagrams have cancelling divergences and the 1-loop amplitude is finite. In this respect, the production of neutral pions in photon photon collisions is very different from the charged pion case, where the $O(p^4)$ contribution to the amplitude is only a small correction to the leading order. Furthermore, the actual value of this correction depends on a free parameter, i.e. the sum of the tree-level coefficients $(L_9 + L_{10})$ in the chiral lagrangian, a quantity which must be taken from the measurements in radiative pion beta decay. Conversely, the $O(p^4)$ correction to the current algebra value (i.e. zero) of the $\gamma\gamma \to \pi^0\pi^0$ amplitude does not depend on the free value of the low-energy constants in the effective lagrangian and is completely specified in terms of the known quantities m_π, F_π and the proton electric charge.

For real photons, the transition amplitude can be expressed in terms of two gauge-invariant functions A(s,t,u), B(s,t,u)

$$\mathcal{A}(\gamma\gamma \to \pi^0\pi^0) = A(s,t,u)\left[-\frac{s}{2}\varepsilon_1\cdot\varepsilon_2 + \varepsilon_1\cdot k_2\,\varepsilon_2\cdot k_1\right]$$

$$+ B(s,t,u)\left[\varepsilon_1\cdot\varepsilon_2(t-u)^2+2(t-u)(\varepsilon_1\cdot\Delta\,\varepsilon_2\cdot k_1-\varepsilon_1\cdot k_2\,\varepsilon_2\cdot\Delta)-2s\varepsilon_1\cdot\Delta\,\varepsilon_2\cdot\Delta\right], \quad (18)$$

where $\Delta = p_1-p_2$ is the difference of the 4-momenta of the pions in the final state. Since B(s,t,u) has the dimensions of an inverse mass to the fourth power, whereas A(s,t,u) has the dimensions of an inverse mass squared, only the function A receives a contribution at the $O(p^4)$ level (which is proportional to $\frac{1}{F_\pi^2}$), while B starts only at the next order. By squaring and averaging over the photon helicities, assuming the cut $|\cos\theta| \le Z$ for the scttering angle, the total cross-section for unpolarized photons is readily obtained from eq. (18) in terms of the functions A, B

$$\sigma(s, |\cos\theta| \le Z) =$$

$$\frac{1}{256\pi}\int_{t_+}^{t_-} dt\left[\;|A + 2(4m_\pi^2 - s)B|^2 + \frac{64}{s^2}(m_\pi^4 - tu)^2\,|B|^2\;\right], \quad (19)$$

where

$$t_\pm = -\frac{1}{2}s\left[\,1 - \frac{2m_\pi^2}{s} \pm Z\sqrt{1 - \frac{4m_\pi^2}{s}}\,\right]. \quad (20)$$

In deriving the cross-section we have assumed the state normalization such that

$$\langle p'|p\rangle = 2p_0(2\pi)^3\,\delta^3(\vec{p}\,' - \vec{p}), \quad (21)$$

as well as the following relation between the S-matrix and the Lorentz invariant amplitude $\mathcal{A}$

$$\langle\pi\pi|S|\gamma\gamma\rangle = (2\pi)^4\,\delta^4(q_1 + q_2 - p_1 - p_2)\,\mathcal{A}\,. \quad (22)$$

The 1-loop amplitude is the sum of the diagrams with pion and kaon loops. The contribution of these diagrams to eq. (18) reads[2,13]

$$A^{(1)} = A^{(\pi\text{-loops})} + A^{(K\text{-loops})} , \tag{23}$$

$$A^{(\pi\text{-loops})} = ie^2 \frac{1}{4\pi^2 F_\pi^2} \left[1 - \frac{m_\pi^2}{s} \right] \left[1 + \frac{1}{s} (m_\pi \ln Q_\pi)^2 \right] , \tag{24}$$

$$A^{(K\text{-loops})} = ie^2 \frac{1}{16\pi^2 F_\pi^2} \left[1 + \frac{1}{s} (m_K \ln Q_K)^2 \right] , \tag{25}$$

$$B^{(1)} = B^{(\pi\text{-loops})} = B^{(K\text{-loops})} = 0 . \tag{26}$$

The expressions of Q_π and Q_K are given in eq. (6). The kaon-loop correction is very small compared to the pion-loop contribution. Yet, for the sake of completeness, we keep this contribution both in the analytic expression of the cross section and in the numerical calculations.

The exchange of vector mesons in the t-channel produces a contribution of order $O(p^6)$ to the scattering amplitude. This has been calculated in refs. (14,15). However the calculations of refs. (14) include terms contributing to orders higher than $O(p^6)$. To be rigorous, such terms should be dropped, in order that the result be consistent with chiral symmetry to $O(p^6)$. Nonetheless the numerical results obtained for the cross-section in refs. (14,15) are in agreement, as should be expected.

The unambiguous inclusion of vector meson resonances in the effective low-energy chiral lagrangian was obtained taking into account the QCD asymptotic structure[16]. The effective coupling relevant for the decays of a vector field V into a photon and a pseudoscalar is produced by the lagrangian[17]

$$L_3 = h_V \, \varepsilon_{\mu\nu\rho\sigma} \, \mathrm{Tr}\, V^\mu \{ u^\nu, f_+^{\rho\sigma} \} , \tag{27}$$

where we use the same notation as ref. (17) and define

$$u_\mu = iuD_\mu Uu \;, \quad f_+^{\mu\nu} = uF_L^{\mu\nu}u + uF_R^{\mu\nu}u , \tag{28}$$

with the covariant derivative $D_\mu U$ and the corresponding nonabelian field strengths as in ref. (3). Exchanging a field V^μ between two couplings (27) yields the $O(p^6)$ terms in the chiral effective lagrangian

$$\mathcal{L}_V^{(6)} = \frac{h_V^2}{M_V^2} \, \mathrm{Tr} \left[\{ u_\lambda , f_{+\mu\nu} \} (\{ u^\lambda, f_+^{\mu\nu} \} + 2 \{ u^\mu, f_+^{\nu\lambda} \}) \right] , \tag{29}$$

where $M_V = M_\rho$ is the common mass of the octet and singlet vector mesons. This gives rise to the $O(p^6)$ scattering amplitude of the form (18), with the form factors

$$A^{(V)} = - i \, C (3s - 8m_\pi^2) , \tag{30}$$

$$B^{(V)} = - i \frac{1}{2} C \ . \tag{31}$$

Here we denote

$$C = \frac{320}{9} \pi \alpha \left[\frac{h_V}{M_V F_\pi} \right]^2 \tag{32}$$

The value of h_V can be fixed using the data for the decays $\rho^+ \to \pi^+\gamma$ and $\omega^0 \to \pi^0\gamma$. These decays have the rates with the smallest experimental errors. From the expression (27), one gets directly[17]

$$| h_V | = \frac{3}{4} F_\pi \sqrt{\frac{12 M_V^3}{\alpha} \frac{\Gamma(\rho^+ \to \pi^+\gamma)}{(M_V^2 - m_\pi^2)^3}} \ . \tag{33}$$

Following ref. (17), one can make for the ω the nonet assumption

$$V_\mu = \frac{1}{\sqrt{2}} \sum_{i=1}^{8} \lambda_i \ V_\mu^{\ i} + \frac{1}{\sqrt{3}} V_\mu^{\ 0} \ , \tag{34}$$

yielding the formula for h_V

$$| h_V | = \frac{1}{4} F_\pi \sqrt{\frac{12 M_V^3}{\alpha} \frac{\Gamma(\omega^0 \to \pi^0\gamma)}{(M_V^2 - m_\pi^2)^3}} \ . \tag{35}$$

Taking the measured masses and widths from ref. (4), we get from eq. (33)

$$| h_V | = (3.62 \pm 0.24) \cdot 10^{-2} \ , \tag{36}$$

which can be taken as the resulting value of $| h_V |$. In addition, the ω^0 decay yields the value obtained from eq. (37)

$$| h_V | = (3.93 \pm 0.14) \cdot 10^{-2} \ . \tag{37}$$

The consistency (within the quoted experimental errors) of the two numbers obtained above provides evidence in favour of the nonet assumption. hence, making the nonet assumption as in ref. (17), one may take the weighted average of these two numbers

$$| h_V | = (3.73 {}^{+0.034}_{-0.035}) \cdot 10^{-2} \ . \tag{38}$$

However, we note that the agreement between the two numbers in eqs. (36) and (37) is not great. Thus, in the numerical evaluation of the cross-sections in the present work, we prefer to stick to the value of the coupling constant obtained in eq. (36) from the ρ^+ decay.

The total cross-section in the range $|\cos \theta | \leq Z$ for unpolarized photons, including the contribution of vector resonance exchange, reads

$$\sigma(s, |\cos\theta| \le Z) = \sigma^{(1)}(s, |\cos\theta| \le Z) + \sigma^{(V)}(s, |\cos\theta| \le Z) + \sigma^{(1\text{-}V)}(s, |\cos\theta| \le Z) \ , \tag{39}$$

where the first two terms denote, respectively, the 1-loop and the vector meson cross-sections and the last contribution represents the interference effect. Plugging eqs. (23-26) and eqs. (30,31) into eq. (19) yields the results

$$\sigma^{(1)}(s, |\cos\theta| \le Z) = \alpha^2 \frac{1}{128\pi^3 F_\pi^4} \sigma_0 \Bigg\{ [f(s)]^2 + 2\left(1 - \frac{m_\pi^2}{s}\right) f(s) \left[1 + \frac{m_\pi^2}{s} g(s)\right] + \left(1 - \frac{m_\pi^2}{s}\right)^2 \left[1 + 2\frac{m_\pi^2}{s} g(s) + \frac{m_\pi^4}{s^2}\left(g(s) + 2\pi^2\right)^2\right]\Bigg\} \ , \tag{40}$$

$$f(s) = \frac{1}{4}\left[1 - \frac{4m_K^2}{s}\left(\arcsin\sqrt{\frac{s}{4m_K^2}}\right)^2\right] \ , \tag{41}$$

$$g(s) = \ln^2 |Q_\pi| - \pi^2 \ , \tag{42}$$

$$|Q_\pi| = \left(1 + \sqrt{1 - \frac{4m_\pi^2}{s}}\right)\left(1 - \sqrt{1 - \frac{4m_\pi^2}{s}}\right)^{-1} \ , \tag{43}$$

$$\sigma_0 = \frac{1}{2} Z s \sqrt{1 - \frac{4m_\pi^2}{s}} \ ; \tag{44}$$

$$\sigma^{(V)}(s, |\cos\theta| \le Z) = \frac{M_V^4}{16\pi s^2} C^2 \left[\sigma_0\left(I_A + \frac{I_B}{b^2 - \sigma_0^2}\right) + I_C \ln\frac{b^2 - \sigma_0^2}{b^2 + \sigma_0^2}\right] \ , \tag{45}$$

$$I_A = \frac{5}{3} b^2 - \frac{1}{3}\sigma_0^2 + \frac{1}{2} s^2 \left(3 - 4\frac{m_\pi^2}{s}\right) \ , \tag{46}$$

$$I_B = \frac{1}{3}\left(4b^4 - \sigma_0^4\right) + \frac{1}{2} s^2 \left(1 + 4\frac{m_\pi^2}{s}\right)\left(2b^2 - \sigma_0^2\right) - b s^3 \left(1 - 2\frac{m_\pi^2}{s}\right) + s^4\left(\frac{5}{16} - \frac{3}{2}\frac{m_\pi^2}{s} + 2\frac{m_\pi^4}{s^2}\right) \ , \tag{47}$$

$$I_C = \frac{3}{2} b^3 + \frac{1}{4} bs^2 \left(5 + 4 \frac{m_\pi{}^2}{s} \right) - \frac{1}{2} s^3 \left(1 - 2 \frac{m_\pi{}^2}{s} \right)$$

$$- \frac{s^4}{b} \left(\frac{5}{32} - \frac{3}{4} \frac{m_\pi{}^2}{s} + \frac{m_\pi{}^4}{s^2} \right) , \tag{48}$$

$$b = M_V{}^2 + \frac{1}{2} s - m_\pi{}^2 , \tag{49}$$

$$\sigma^{(1-V)}(s, |\cos\theta| \leq Z) = - \frac{M_V{}^2}{32\pi^2 F_\pi{}^2} \alpha C \left[2\sigma_0 + M_V{}^2 \ln \frac{b^2-\sigma_0{}^2}{b^2+\sigma_0{}^2} \right]$$

$$\times \left[f(s) + \left(1 - \frac{m_\pi{}^2}{s} \right) \left(1 + \frac{m_\pi{}^2}{s} g(s) \right) \right] . \tag{50}$$

Here the value of the constant C is obtained from eqs. (32), (36). The numerical evaluation of the total cross-section leads to Figs. 2,3.

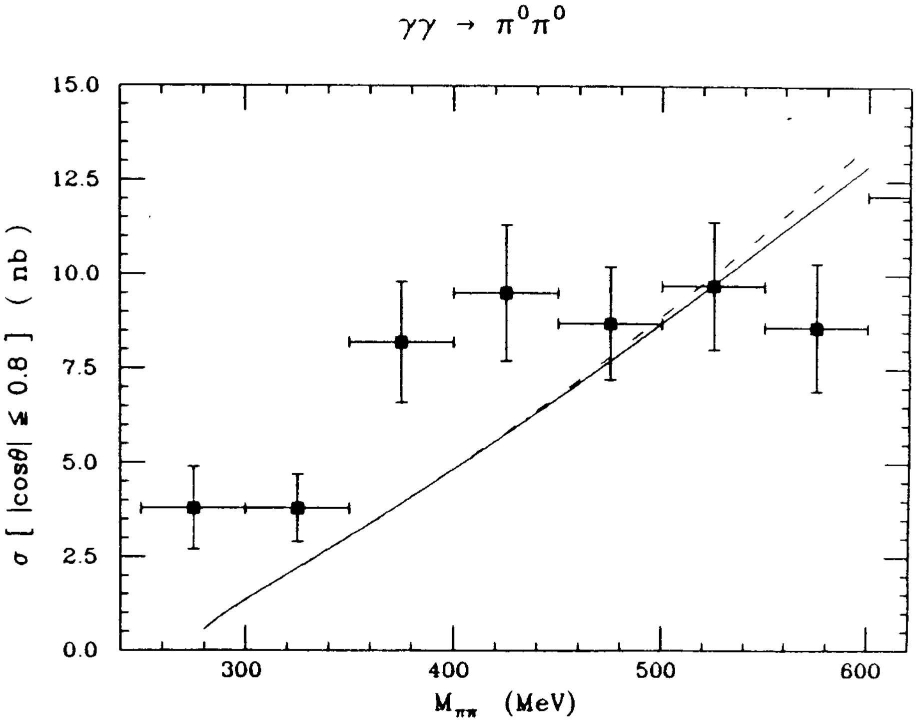

FIG. 2 - The 1-loop contribution[2] to $\sigma(\gamma\gamma \to \pi^0\pi^0)$ including: i) both π and K loops (full line); ii) π loops only (dot-dashed line). The data are taken from the CB experiment[18].

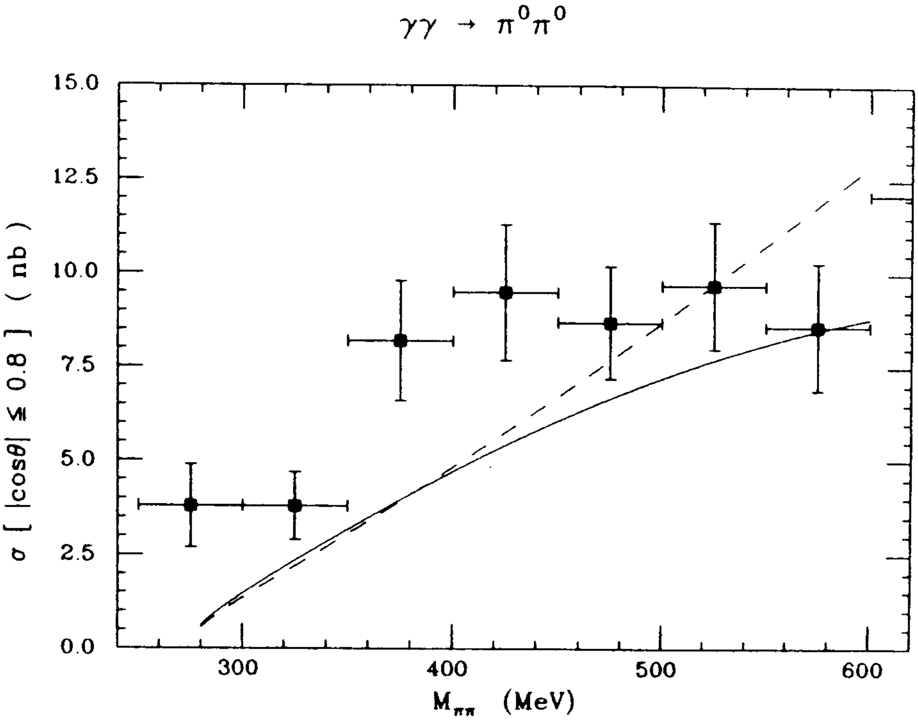

FIG. 3 - The cross-section for $\gamma\gamma \to \pi^0\pi^0$ including: i) both the 1-loop diagrams and the $O(p^6)$ contribution due to the vector-meson resonance exchange in the t-channel (full line); ii) the 1-loop contribution only (dashed line).

In the threshold region ($\sqrt{s} \leq 0.5$ GeV), the Crystal Ball experiment[18] yields results for the total cross-section that are consistently higher than the 1-loop chiral perturbation theory prediction. This puzzling discrepancy could be resolved, within chiral perturbation theory, by calculating the 2-loop effects, which turn out to give a sizeable contribution to the cross-section in the threshold region[19]. This may improve the agreement with the Crystal Ball data, but this still needs to be confirmed. Notice that the contribution of the vector meson resonances in Fig. 3 is rather small in the threshold region and could be completely neglected for $\sqrt{s} \leq 0.4$ GeV.

Recent results support the conjecture that the bulk of the $\gamma\gamma \to \pi^0\pi^0$ cross-section can be related to the phase-shifts for the $\pi^+\pi^- \to \pi^0\pi^0$ process[20,21]. These analyses yield an estimated $\sigma(\gamma\gamma \to \pi^0\pi^0)$ in good agreement with the Crystal Ball measurements. In the paper by Im, as the result of an expansion in the number of π fields, a cross-section is produced which agrees qualitatively with the cross-section given in the paper by Morgan and Pennington. This agreement is not surprising indeed, since in both papers the authors use similar techniques where the result is dominated by the right-hand cut. The two procedures may still differ by the left-hand cut, which however is only a small contribution. So, there exists three results (including the preliminary ones obtained at the 2-loop level[19]) which predict a sizeable enhancement of the cross-section in the range from threshold up to 450 MeV, with respect to the lowest order prediction of chiral perturbation theory.

The estimated cross-sections of refs. (20,21) depend on the choice of the phase-shift parametrization. Unfortunately this dependence cannot be fully explored, given the large errors affecting the Crystal Ball data. For instance, changing the cross-section estimated by Morgan and Pennington by one standard deviation of the Crystal Ball points corresponds to a change of 0.15 in the pion scattering length, i.e. 75% of the central value a_0 = 020 (see Fig. 3 in ref. (20)). This is obtained neglecting other sources of potential uncertainty, in addition to the value of a_0, in the Morgan-Pennington estimate, such as the position of the Adler zero. Thus, it is urgent to measure the cross-section with higher precision. This will provide independent information to be compared with the input of the pion phase-shifts and/or appreciate the contribution of higher order effects in chiral perturbation theory.

The reaction $\gamma\gamma \to \pi^0\pi^0$ can be studied with great detail in the threshold region with high luminosity e^+e^- machines, such as DAΦNE. The number of π^0 pairs $N_{\pi^0\pi^0}$ produced in the range $s \pm \Delta s$ can be obtained from the integrated luminosity $L_{e^+e^-}$

$$N_{\pi^0\pi^0} = \langle \sigma_{\gamma\gamma\to\pi^+\pi^-}(s)\, L_{\gamma\gamma}(s) \rangle_{\Delta s}\, L_{e^+e^-}\, \Delta s \ . \qquad (51)$$

Here we are averaging over the Δs range. Assuming a ±5 MeV bin in the $\pi^0\,\pi^0$ center of mass energy, it is straightforward to see that the integrated luminosity $L_{e^+e^-} = 5\cdot 10^{39}$ cm^{-2} is sufficient, in order to obtain a 1-2% statistical accuracy on the measurement of the generalized polarizability of ref. (9), with an improvement of almost one order of magnitude with respect to the CB experiment. If the DAΦNE luminosity is set to be $5\cdot 10^{32}$ cm^{-2} s^{-1}, then the time of running is 1 year (10^7 s). In fact, with the quoted integrated luminosity, almost 10^4 events should be collected (see the second ref. (14)). A statistical accuracy of a few % is very small with respect to possible systematical error of the order of a few tens %. For the production of charged pion pairs, having a systematical uncertainty of this order of magnitude would prevent to measure the coefficient ($L_9^r + L_{10}^r$) of section 2 with the needed accuracy. Hence a very careful control of the sources of systematical errors is absolutely necessary, in order to exploit the DAΦNE opportunity to test the chiral symmetry predictions for the $\gamma\gamma$ physics at low energy. The best way to achieve this goal is by tagging the lepton beams. At the DAΦNE energies this should not reduce too much the $\gamma\gamma$ luminosity[22].

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REFERENCES

1) S.Brodsky et al., Phys. Rev. **D4**, 1532 (1971).
2) J.Bijnens and F.Cornet, Nucl. Phys. **B296**, 557 (1988).
3) J.Gasser and H.Leutwyler, Nucl. Phys. **B250**, 465 (1985); J.F.Donoghue and B.R.Holstein, Phys. Rev. **D40**, 2378 (1989).
4) Review of Particle Properties (G.P.Yost et al.), Phys. Lett. **B239**, 1 (1990).
5) D.Morgan and M.R.Pennington, Phys. Lett. **192B**, 207 (1987).
6) MARK-II Collaboration (J.Boyer et al.), Phys. Rev. **D42**, 1350 (1990).
7) TPC/$\gamma\gamma$ Collaboration (H.Aihara et al.), Phys. Rev. Lett. **57**, 404 (1986).
8) CELLO Collaboration (H.J.Behrend et al.), 25th International Conference on High Energy Physics, Singapore 1990.
9) D.Babusci et al., Proceedings of the Workshop on Physics and Detectors for DAΦNE, Frascati, April 9-12 1991, p. 383; D.Babusci et al., Phys. Lett. **277B**,158 (1992).
10) PLUTO Collaboration (C.Berger et al.), Z. Phys. **C26**, 199 (1984).
11) V.A.Petrun'kin, Sov.J.Part.Nucl. **12**, 278 (1981); A.I.L'vov, Lebedev preprint 344 (1987); J.L.Friar, Los Alamos preprint LA-UR-88-2845 (1988).
12) M.V.Terentev, Sov. J. Nucl. Phys. **16**, 87 (1972).
13) J.F.Donoghue, B.R.Holstein, and Y.C.Lin, Phys. Rev. **D37**, 2423 (1988).
14) P.Ko, Phys. Rev. **D41**, 1531 (1990); S.Bellucci and D.Babusci, Proceedings of the Workshop on Physics and Detectors for DAΦNE, Frascati, April 9-12 1991, p. 351.
15) J.Bijnens, S.Dawson and G.Valencia, Phys. Rev. **D44**, 3555 (1991).
16) G.Ecker et al., Phys. Lett. **B223**, 425 (1989).
17) G.Ecker, A.Pich and E.de Rafael, Phys. Lett. **B237**, 481 (1990).
18) Crystal Ball Collaboration (H. Marsiske et al.), Phys. Rev. **D14**, 3324 (1990).
19) S.Bellucci and J.Gasser, in preparation.
20) D.Morgan and M.R.Pennington, Phys. Lett. **B272**, 134 (1991).
21) C.J.C.Im, preprint SLAC-PUB-5627 (September 1991).
22) A.Courau, Proceedings of the Workshop on Physics and Detectors for DAΦNE, Frascati, April 9-12 1991, p. 373.