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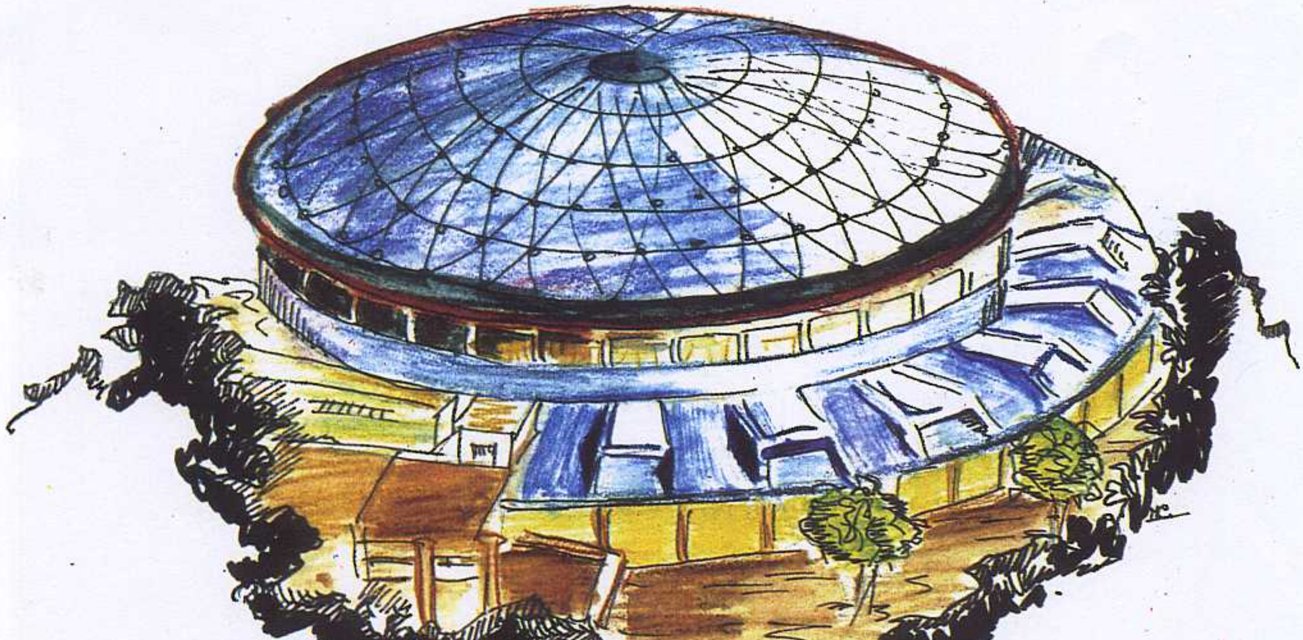

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CORRECTIONS TO $K \to \pi\gamma\gamma$ FROM $K \to 3\pi$

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Abstract

In the framework of chiral perturbation theory we compute the $\Delta I = 3/2$ contribution to $K_L \to \pi^0\gamma\gamma$. As for the $\Delta I = 1/2$ transition, the result turns out to be unambigously predicted and finite. We discuss also the effects on this decay coming from $O(p^4)$ corrections to $K \to 3\pi$. As a by-product we have also a prediction for the CP conserving amplitude $A(K_L \to \pi^0 e^+ e^-)$.

1. Introduction.

Rare meson decays are very useful for our understanding of particle physics [1]. Chiral perturbation theory (CHPT) [2,3] is very suitable for this research; it fulfills this task, satisfying symmetry requirements (the interactions have to be invariant under $SU(3)_L \otimes SU(3)_R$) and having the mesons as the Goldstone bosons associated with spontaneous breaking of the symmetry. Already at tree level all the low energy theorems, PCAC and soft pion properties are recovered. Furthermore, though the theory is non renormalizable, we do require unitarity, which is obtained perturbatively by considering also pion loops. Divergences in the loops are absorbed by corresponding counterterms, which then depend on the renormalization scale μ of the loops. Due to non renormalizability of the theory, new counterterms, which can be determined from experiment [3,4], have to be added order by order to the theory. Actually, the $O(p^4)$ correction to the lowest order strong Lagrangian can be predicted reasonably well by vector meson exchange [5], which also gives an $O(p^4)$ contribution to the $A(K \to 3\pi)$ amplitude, improving the lowest order weak Lagrangian result [6].

$K_S \to \gamma\gamma$ [7] and $K_L \to \pi^0\gamma\gamma$ [8] play an important role in CHPT; they have no $O(p^2)$ tree level contribution, since the external particles are neutral; for the same reason there are no $O(p^4)$ counterterms. This fact has two implications: 1) the chiral meson loops are finite and so free of the ambiguity of the cut-off; 2) these are the only contribution $O(p^4)$; no dependence on counterterms or other unknown constants.

Thus, CHPT predicts unambiguously at $O(p^4)$ these two decays. While there is a substantial agreement between theory [7,9] and experiments [10] for $K_S \to \gamma\gamma$

$$\Gamma(K_S \to 2\gamma)_{Th} = 1.52 \cdot 10^{-11} eV, \tag{1.1}$$

$$\Gamma(K_S \to 2\gamma) = (1.8 \pm 0.8) \cdot 10^{-11} eV. \tag{1.2}$$

for $K_L \to \pi^0\gamma\gamma$ the situation is still controversial. The theoretical prediction of CHPT at $O(p^4)$ for the $\Delta I = 1/2$ amplitude has been calculated [8], giving the branching ratio

$$\mathrm{Br}(K_L \to \pi^0\gamma\gamma)_{Th} = .68\ 10^{-6}, \tag{1.3}$$

to compare with the experimental values

$$Br(K_L \to \pi^0\gamma\gamma) = \begin{cases} (2.1 \pm 0.6) \cdot 10^{-6} & & \text{NA31} \\ (1.86 \pm 0.6 \pm 0.6) \cdot 10^{-6} & (\sqrt{q^2} > 280\ MeV) & \text{E731} \end{cases} \tag{1.4}$$

($\sqrt{q^2}$ is the two photon invariant mass) at CERN [11] and FNAL [12] respectively, which are somewhat bigger than the prediction. Nevertheless the predicted spectrum, which is dominated at this order by the absorptive part $K_L \to \pi^0\pi^+\pi^- \to \pi^0\gamma\gamma$, looks in good agreement with experiment. Indeed, NA31 experiment seems to exclude a big dispersive contribution at low q^2.

The decay $K_L \to \pi^0 e^+ e^-$ has three kinds of contributions [14,15,13]: direct CP violation, mass CP violation $K_L \to \pi^0 \gamma^* \to \pi^0 e^+ e^-$ and CP conserving $K_L \to \pi^0 \gamma^* \gamma^* \to \pi^0 e^+ e^-$. The CP violating contributions are expected to give a $Br(K_L \to \pi^0 e^+ e^-) \simeq 10^{-11}$; while, since $O(p^4)$ $K_L \to \pi^0 \gamma\gamma$ gives an helicity suppressed CP conserving amplitude, only the $O(p^6)$ contribution might give an appreciable rate to the branching ratio of the CP conserving process, which indeed naive power counting would predict of order 10^{-14} [16,13]; still in this framework $O(p^6)$ vector meson exchange diagrams [13,17,18] might enhance both $Br(K_L \to \pi^0 \gamma\gamma)$ and the CP conserving $Br(K_L \to \pi^0 e^+ e^-)_{CP}$, but it is unlikely to obtain values for these branching ratios of about $2 \cdot 10^{-6}$ and 10^{-11} respectively. Phenomenological models with large vector [19,20] or scalar [21] meson exchange diagrams can obtain these values, but would alter the spectrum $d\Gamma/dq^2$, particularly at low q^2, where they would predict a drastic increase.

In this paper we will first give the $O(p^4)$ $\Delta I = 3/2$ contribution to $K_L \to \pi^0 \gamma\gamma$; then we will discuss some implications of the $O(p^4)$ corrections to the $K \to 3\pi$ amplitude to $K_L \to \pi^0 \gamma\gamma$, $K_L \to \pi^0 e^+ e^-$ and to the CP violating charge asymmetry of $K^\pm \to \pi^\pm \gamma\gamma$.

2. $\Delta I = 3/2$

At order p^2, the chiral Lagrangian is

$$L = L_{\Delta S=0} + L_{\Delta S=1}. \tag{2.1}$$

One has

$$L_{\Delta S=0} = \frac{1}{4} f^2 Tr D_\mu U D^\mu U^\dagger + \frac{f^2}{2} Tr U^\dagger \mu M + \frac{f^2}{2} Tr U \mu M, \tag{2.2}$$

where

$$\begin{aligned} U &= e^{\frac{2i}{f}\pi_a T_a}, \quad D_\mu U = \partial_\mu U + ieA_\mu [Q, U] \\ M &= diag(m_u, m_d, m_s) \quad Q = diag(2/3, -1/3, -1/3) \\ T_a &= \frac{\lambda_a}{2}, \quad Tr T_a T_b = \frac{1}{2}\delta_{ab}, \quad f \simeq F_\pi = 93.3 MeV; \end{aligned} \tag{2.3}$$

the λ_a are the Gell-Mann matrices and μ is the correct factor to reproduce the right meson masses.

The CP-conserving $\Delta S = 1$ weak Lagrangian consists of two pieces: the octect and 27-plet:

$$L_{\Delta S=1} = \frac{1}{4} f^2 h_8 Tr\lambda_6 D_\mu U D^\mu U^\dagger + \frac{h_{27} f^2}{4} [T_{ij}^{kl} (U D_\mu U^\dagger)_k^i (U D^\mu U^\dagger)_l^j + h.c.] \tag{2.4}$$

where the tensor T is the U=1, ΔS=1, ΔQ=0 element of the 27 with components:

$$T_{13}^{12} = T_{13}^{21} = T_{31}^{21} = T_{31}^{12} = \frac{3}{5}, \quad T_{23}^{22} = T_{32}^{22} = T_{33}^{23} = T_{33}^{32} = -\frac{3}{10}. \tag{2.5}$$

From $K \to \pi\pi$ decays we have at order p^2

$$h_8 = 3.2 \cdot 10^{-7} \qquad h_{27} = -1 \cdot 10^{-8}. \tag{2.6}$$

The general amplitude for $K_L \to \pi^0\gamma\gamma$ is given by

$$M(K_L(p) \to \pi^0(p_3)\gamma(k_1,\varepsilon_1)\gamma(k_2,\varepsilon_2)) = \varepsilon_{1\mu}\varepsilon_{2\nu} M^{\mu\nu}(p_3,k_1,k_2) \tag{2.7}$$

where ε_1, ε_2 are the photon polarizations and $M^{\mu\nu}$, if CP is conserved, is made of two invariant amplitudes:

$$M^{\mu\nu} = \frac{A(y,z)}{M_K^2}(k_2^\mu k_1^\nu - k_1 k_2 g^{\mu\nu})$$

$$+\frac{2B(y,z)}{M_K^4}(-pk_1 pk_2 g^{\mu\nu} - k_1 k_2 p^\mu p^\nu + pk_1 k_2^\mu p^\nu + pk_2 p^\mu k_1^\nu), \tag{2.8}$$

where

$$y = |p(k_1 - k_2)|/M_K^2, \qquad z = (k_1 + k_2)^2/M_K^2$$

Note that $A(y,z)$ and $B(y,z)$ are symmetric for $k_1 \leftrightarrow k_2$, as required by Bose symmetry.

The physical region in the adimensional variables y and z is given by

$$0 \le y \le \frac{1}{2}\lambda^{\frac{1}{2}}(1, r_\pi^2, z) \quad 0 \le z \le (1 - r_\pi)^2 \tag{2.9}$$

where

$$\lambda(1, r_\pi^2, z) = 1 + z^2 + r_\pi^4 - 2(z + z r_\pi^2 + r_\pi^2) \quad \text{and} \quad r_\pi = \frac{m_{\pi^0}}{m_{K^0}}. \tag{2.10}$$

From (2.8), (2.9) and (2.10) we obtain the double differential rate for unpolarized photons:

$$\frac{d^2\Gamma}{dy\ dz} = \frac{m_K}{2^9\pi^3}\left\{ z^2|A+B|^2 + [y^2 - (\frac{(1+r_\pi^2 - z)^2}{4} - r_\pi^2)]^2|B|^2 \right\} \tag{2.11}$$

Since K_L and π^0 are neutral there are no tree level $O(p^2)$ and no $O(p^4)$ counterterm contributions to the amplitude. At order $O(p^4)$ the amplitude $B(y,z)$ is zero since there are not enough powers of momenta at this order. The result for the octect amplitude $A^{(8)}(z)$, which at this order depends only on z, is [8]

$$A^{(8)}(z) = \frac{h_8 \alpha m_K^2}{4\pi f^2}\left[(1-\frac{r_\pi^2}{z})f(\frac{z}{r_\pi^2}) - (1-\frac{r_\pi^2}{z}-\frac{1}{z})f(z)\right] \tag{2.12}$$

$$f(x) = 1 + \frac{1}{x}\ln^2\frac{\beta(x)-1}{\beta(x)+1}, \qquad \beta(x) = \sqrt{1-\frac{4}{x}} \tag{2.13}$$

The function $f(x)$ is real for $x \leq 4$ and complex for $x \geq 4$. More explicitly it is written

$$f(x) = \begin{cases} 1 - \frac{4}{x}\arcsin^2\frac{\sqrt{x}}{2} & x \leq 4 \\ 1 + \frac{1}{x}[\ln^2\frac{1-\beta(x)}{1+\beta(x)} - \pi^2 + 2i\pi\ln\frac{1-\beta(x)}{1+\beta(x)}] & x \geq 4 \end{cases} \tag{2.14}$$

In (2.12) the contribution proportional to $f(z)$ comes from the kaon loops and so does not have absorptive part, while the one proportional to $f(z/r_\pi^2)$ comes from pion loops and it has absorptive part, since the pions can be on shell. Correspondingly, the kaon contribution is much less than the one of pions. As for the $A(K_S \to \gamma\gamma)$ this is an excellent test for chiral perturbation theory. In particular the spectrum and the width depend upon the hypothesis of pion loop, and can be considered as a test of CHPT as a quantum field theory. Indeed, the peak in the z spectrum is due to the absorptive part.

It might be interesting to calculate the contribution of the 27, which due to the lacking of the corresponding counterterms, turns out to be finite and unambiguously predicted at $O(p^4)$:

$$A_{1/2}^{(27)}(z) = -\frac{h_{27}\alpha m_K^2}{20\pi f^2}(1-\frac{r_\pi^2}{z})f(\frac{z}{r_\pi^2}) \tag{2.15a}$$

$$A_{3/2}^{(27)}(z) = \frac{h_{27}\alpha m_K^2}{8\pi f^2}\left[\frac{3 - r_\pi^2 - 14r_\pi^4 - (5-14r_\pi^2)z}{(1-r_\pi^2)z}\right] f(\frac{z}{r_\pi^2}) \tag{2.15b}$$

The z spectrum for the y independent amplitudes (2.12, 2.15) is given by

$$\frac{d\Gamma}{dz} = \frac{m_K}{2^{10}\pi^3}[(1+r_\pi)^2 - z]^{\frac{1}{2}}[(1-r_\pi)^2 - z]^{\frac{1}{2}} z^2 |A|^2 \tag{2.16}$$

Due to the finiteness of the $\Delta I = 3/2$ contribution to $K_L \to \pi^0\gamma\gamma$, CHPT predicts, at $O(p^4)$, the $\Delta I = 1/2$ rule for this decay. Indeed, a small negative interference is predicted for the branching ratio:

$$Br(K_L \to \pi^0\gamma\gamma)^{(8)} = 0.68 \cdot 10^{-6}, \quad Br(K_L \to \pi^0\gamma\gamma)^{(8+27)} = 0.61 \cdot 10^{-6}. \tag{2.17}$$

These are dominated by the absorptive contributions:

$$Br(K_L \to \pi^0\gamma\gamma)^{(8)}_{abs} = 0.46 \cdot 10^{-6}, \quad Br(K_L \to \pi^0\gamma\gamma)^{(8+27)}_{abs} = 0.43 \cdot 10^{-6}. \tag{2.18}$$

In Fig.1 the spectrum $d\Gamma/dz$ for the $O(p^4)$ amplitudes (2.12) plus (2.15) is reported with the relative absorptive and dispersive contributions.

3. $O(p^6)$ from $K \to 3\pi$.

The amplitude for the process $K(p) \to \pi(p_3)\pi(p_1)\pi(p_2)$ is generally expanded in powers of the Dalitz plot variables

$$X = \frac{(s_2 - s_1)}{m_\pi^2}, \quad Y = \frac{s_3 - s_0}{m_\pi^2} \tag{3.1}$$

where $s_i = (p - p_i)^2$ and $s_0 = (s_1 + s_2 + s_3)/3$ and 3 indicates the "odd" charged pion. For the decays $K_L(p) \to \pi^0(p_3)\pi^+(p_1)\pi^-(p_2)$ and $K^+(p) \to \pi^-(p_3)\pi^+(p_1)\pi^+(p_2)$ the isospin decomposition, neglecting the phase shifts, up to quadratic terms, is written as [22,23,4],

$$A(K_L \to \pi^0\pi^+\pi^-) = (\alpha_1 + \alpha_3) - (\beta_1 + \beta_3)Y + (\zeta_1 - 2\zeta_3)(Y^2 + \frac{X^2}{3}) + (\xi_1 - 2\xi_3)(Y^2 - \frac{X^2}{3}) \tag{3.2}$$

$$A(K^+ \to \pi^+\pi^+\pi^-) = (2\alpha_1 - \alpha_3) + (\beta_1 - \frac{1}{2}\beta_3 + \sqrt{3}\gamma_3)Y -$$

$$-2(\zeta_1 + \zeta_3)(Y^2 + \frac{X^2}{3}) - (\xi_1 + \xi_3 - \xi_3')(Y^2 - \frac{X^2}{3}) \tag{3.3}$$

where the subscripts 1 and 3 refer to the $\Delta I = 1/2, 3/2$ transitions. Among the experimental values of α_i and β_i, obtained by a general fit of all $K \to 3\pi$ amplitudes [23,4], and the $O(p^2)$ CHPT theoretical predictions from (2.4) and (2.6), there is a 20-30% disagreement. The inclusion of $O(p^4)$ contributions [4] overcomes this discrepancy, fixing also the quadratic terms, which are vanishing at the lowest order.

The absorptive part of $A(K_L \to \pi^0\gamma\gamma)$ is due to $K_L \to \pi^0\pi^+\pi^- \to \pi^0\gamma\gamma$; thus we can improve the $O(p^4)$ result, using for the absorptive part the physical amplitude $A(K_L \to 3\pi)$, obtaining

$$Br(K_L \to \pi^0\gamma\gamma)^{physical}_{abs} = 0.61 \cdot 10^{-6} \tag{3.4a}$$

to compare with (2.18).

As a way to resum the $O(p^6)$ effects on $A(K_L \to \pi^0\gamma\gamma)$ of the $O(p^4)$ corrections to $A(K \to 3\pi)$, we might decide, now, to change the values of h_8 and h_{27} from (2.6), giving a dispersive contribution $Br(K_L \to \pi^0\gamma\gamma)^{(8+27)}_{disp} = .18 \cdot 10^{-6}$, to those fitting α_1 and α_3, giving

$$Br(K_L \to \pi^0\gamma\gamma)^{physical}_{disp} = 0.25 \cdot 10^{-6} \tag{3.4b}$$

Due to the very small increase, this is not a crucial change. Conclusively we quote the total branching ratio

$$Br(K_L \to \pi^0\gamma\gamma)^{physical}_{tot} = 0.86 \cdot 10^{-6} \tag{3.5}$$

still faraway from the experimental value. In Fig.1, we report the result of the absorptive, dispersive and total spectra obtained with these corrections.

The amplitude (3.3) can be used to extract the absorptive part of the loop amplitude of $K^+ \to \pi^+\gamma\gamma$. For the dispersive part, CHPT is less predictive than for the case of $K_L \to \pi^0\gamma\gamma$, since counterterms are present in the chiral Lagrangian at $O(p^4)$. Nevertheless, the $O(p^4)$ loop amplitude turns out to be finite [16].

Due to CP violation, the counterterm amplitude A_{CT} is in general complex; actually its imaginary part has been estimated for $m_t/m_c \sim 100$ [16]:

$$A_{CT} = \frac{h_8 \alpha m_K^2}{8\pi f^2}\hat{c} \qquad\qquad |Im\hat{c}| \sim 3\cdot 10^{-3} \tag{3.6}$$

where $\hat{c}$ is a scale independent constant. The interference between the absorptive part of the loop amplitude and the counterterm contribution might give a non vanishing value to the charge asymmetry of the process, defined by

$$\frac{\Gamma(K^+ \to \pi^+\gamma\gamma) - \Gamma(K^- \to \pi^-\gamma\gamma)}{\Gamma(K^+ \to \pi^+\gamma\gamma) + \Gamma(K^- \to \pi^-\gamma\gamma)} \tag{3.7}$$

Disregarding Wess-Zumino type contributions (irrelevant to the present discussion), the amplitude for $K^+ \to \pi^+\gamma\gamma$ has the form given in eq.(2.8). One has:

$$|\Gamma(K^+ \to \pi^+\gamma\gamma) - \Gamma(K^- \to \pi^-\gamma\gamma)| = \frac{m_K^3}{2^7\pi^3}\frac{h_8\alpha}{8\pi f^2}\times$$

$$|Im\hat{c}| \int_{4r_\pi^2}^{(1-r_\pi)^2} z^2 dz \int_0^{\frac{1}{2}\lambda^{1/2}(1,r_\pi^2,z)} dy\ |(A_{abs}(y,z) + B_{abs}(z))|, \tag{3.8}$$

where $r_\pi = m_{\pi^+}/m_{K^+}$ and $\lambda(1, r_\pi^2, z)$ is defined in (2.10).

The use of the amplitude (3.3) with the constants fitted to the experimental data in ref. [4], does not alter significantly the value of the charge asymmetry (3.8) which results equal to $6\cdot 10^{-23}$ MeV, i.e. of the same order of magnitude of the value obtained at $O(p^4)$ in CHPT by the authors of ref.[16]: $4\cdot 10^{-23}$ MeV.

4. The CP conserving decay $K_L \to \pi^0 e^+e^-$.

The CP conserving two-photon exchange decay $K_L \to \pi^0 e^+e^-$ has the absorptive part coming from the two photon discontinuity. Due to the tensor structure shown in (2.8) the part of the $K_L \to \pi^0\gamma\gamma$ amplitude containing $A(y,z)$ gives a contribution to the two photon discontinuity in $A(K_L \to \pi^0 e^+e^-)_{CP}$ which is suppressed by a factor m_e/m_K. This suppression is not present in the contribution coming from the term containing $B(y,z)$ in (2.8) [16]. Dispersion relations for $A(K_L \to \pi^0 e^+e^-)_{CP}$ using the $O(p^4)$ amplitude (2.12), give rise to an m_e/m_K suppression also for the dispersive part of $A(K_L \to \pi^0 e^+e^-)_{CP}$ [16]. Thus the main contribution to this process in CHPT is expected to come from the B-type $O(p^6)$ contribution of $K_L \to \pi^0\gamma\gamma$ to the photon discontinuity amplitude. The dispersive part should not change the size of $\Gamma(K_L \to \pi^0 e^+ e^-)_{CP}$.

While the $O(p^6)$ contributions coming from VMD have been studied by other authors [15,19,20,13] (with different conclusions), here we will be concerned with the value of $B(y,z)$ obtained by including the $O(p^4)$ corrections to the $A(K \to 3\pi)$ amplitudes.

Indeed, the computation of the absorptive part of $A(K_L \to \pi^0 \gamma\gamma)$ from (3.2) will give rise to a model independent $B(y,z)_{abs}$ in (2.8) coming from the terms proportional to X^2 (actually depending only on z),

$$B(z)_{abs} = \frac{\alpha}{3r_\pi^4}(\zeta_1 - 2\zeta_3 - \xi_1 + 2\xi_3)\left[\beta - \frac{5}{3}\beta^3 - \frac{1}{2}(1-\beta^2)^2 \ln\frac{1+\beta}{1-\beta}\right] \qquad (4.1)$$

which gives

$$Br(K_L \to \pi^0 e^+ e^-)_{CP}^{B_{abs}} \simeq 10^{-14} \qquad (4.2)$$

It is interesting, though not conclusive, to give an estimate $B(y,z)_{disp}$ by extrapolating X^2 to off-shell pions. This shows to be equivalent to the use, for the $O(p^4)$ corrections of CHPT to $A(K \to 3\pi)$, of the operators suggested by the authors of ref.[25].

$$B_{tot}(z) = \frac{4}{3\pi}\frac{\alpha}{r_\pi^4}(\zeta_1 - 2\zeta_3 - \xi_1 + 2\xi_3)\times$$

$$\left[\frac{1}{6}\ln\frac{m_\pi^2}{\Lambda^2} + \frac{5}{9} - \frac{13}{12}\beta^2 + \frac{1}{4}(\frac{5}{3}\beta^2 - 1)\beta\ \ln(\frac{\beta+1}{\beta-1}) + \frac{1}{16}(1-\beta^2)^2\ln^2(\frac{\beta+1}{\beta-1})\right] \qquad (4.3)$$

We point out that the divergence of this $O(p^6)$ loop amplitude has consequences on both endpoints of the spectrum $d\Gamma(K_L \to \pi^0\gamma\gamma)/dz$. Contrarily to the $O(p^4)$ (2.13), the (4.3) is non-vanishing for $z \to 0$; thus the spectrum starts with a non zero value (see fig.2). For $4r_\pi^2 \le z \le (1-r_\pi)^2$, while the dispersive part of (4.3) keeps almost constant, the dispersive $O(p^4)$ (2.12) goes almost to zero consistently with the finiteness of this contribution. We emphasize that there are other dispersive contributions at $O(p^6)$, but the divergent ones keep these properties; now to show the size of these effects, we add the contribution of dispersive part of (4.3) to (3.5) obtaining different predictions, reported below, according to different values of Λ. Also the corresponding two photon-discontinuity contributions to $K_L \to \pi^0 e^+ e^-$ are reported:

Λ	$Br(K_L \to \pi^0\gamma\gamma)$	$Br(K_L \to \pi^0 e^+ e^-)_{CP}$
770 MeV	$2.05 \cdot 10^{-6}$	$0.8 \cdot 10^{-12}$
1000 MeV	$2.3 \cdot 10^{-6}$	$1. \cdot 10^{-12}$

In Fig.2 is reported the spectrum for $\Lambda = 770$ MeV.

5. Conclusions

We have shown that the 27 contribution of $O(p^4)$ to $K_L \to \pi^0\gamma\gamma$, is suppressed by $\Delta I = \frac{1}{2}$ rule, while absorptive contribution from $O(p^4)$ $K \to 3\pi$ increase by about 30% the width keeping inaltered the normalized spectrum $\Gamma^{-1} d\Gamma/dz$. While these contributions keep $Br(K_L \to \pi^0 e^+ e^-)_{CP} \leq 10^{-14}$, the dispersive contributions from quadratic terms in $K \to 3\pi$ might bring $Br(K_L \to \pi^0 e^+ e^-)_{CP} \sim 10^{-12}$, which is still less than the CP violating part. $O(p^6)$ dispersive contributions to $K_L \to \pi^0\gamma\gamma$, though not fully analyzed in this paper, might give relevant contribution for $z \geq 4r_\pi^2$.

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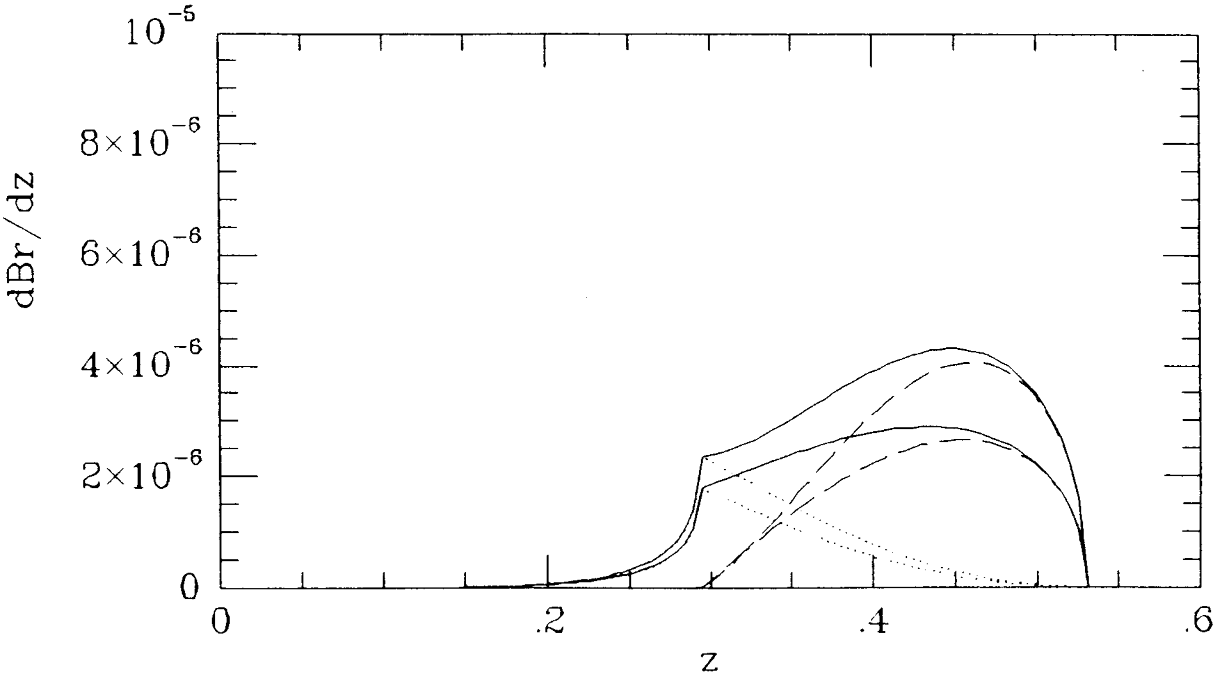

Fig.1. The full lines are total contributions. The lower curve is the $O(p^4)$ 8 + 27 contribution, the upper curve is obtained by adding all $O(p^4)$ $K \to 3\pi$ contributions in the absorptive part and taking h_8 and h_{27} from α_1 and α_3 in the dispersive part. The dashed and dotted lines are the absorptive and dispersive contributions respectively.

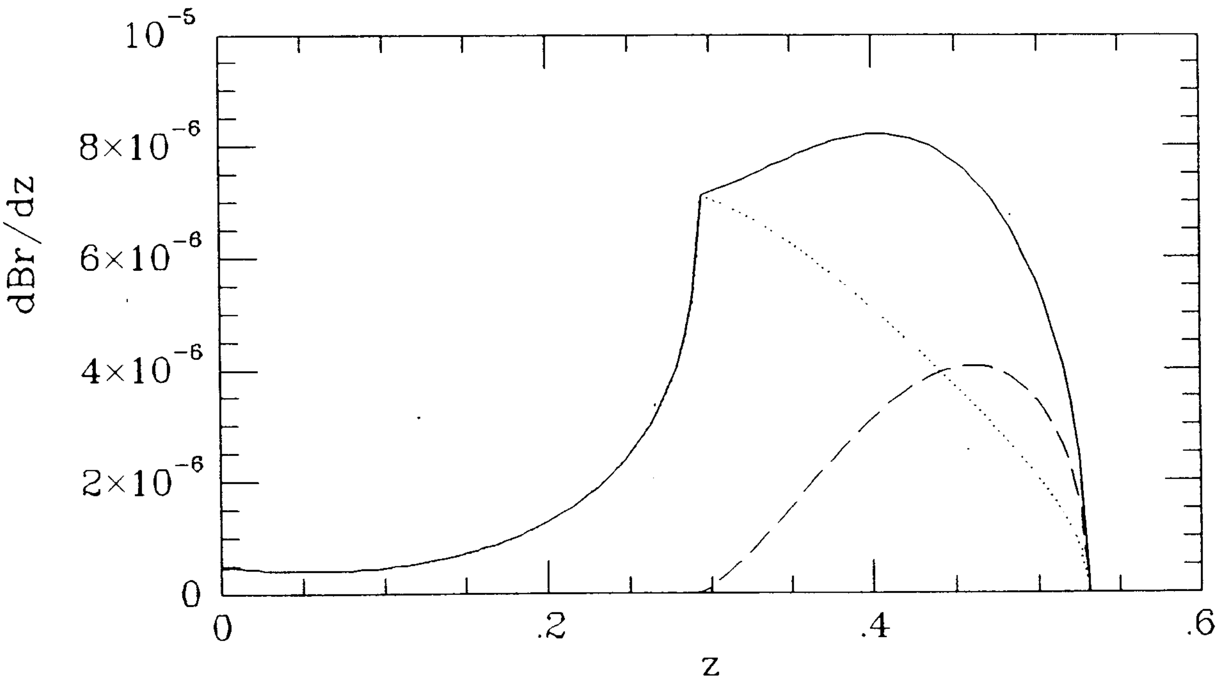

Fig.2. $dBr(K_L \to \pi^0\gamma\gamma)/dz$ obtained by adding for the absorptive part all $O(p^4)$ $K \to 3\pi$ contributions, for the dispersive part h_8 and h_{27} taken from α_1 and α_3 respectively and also B_{disp} in (4.3) is added.