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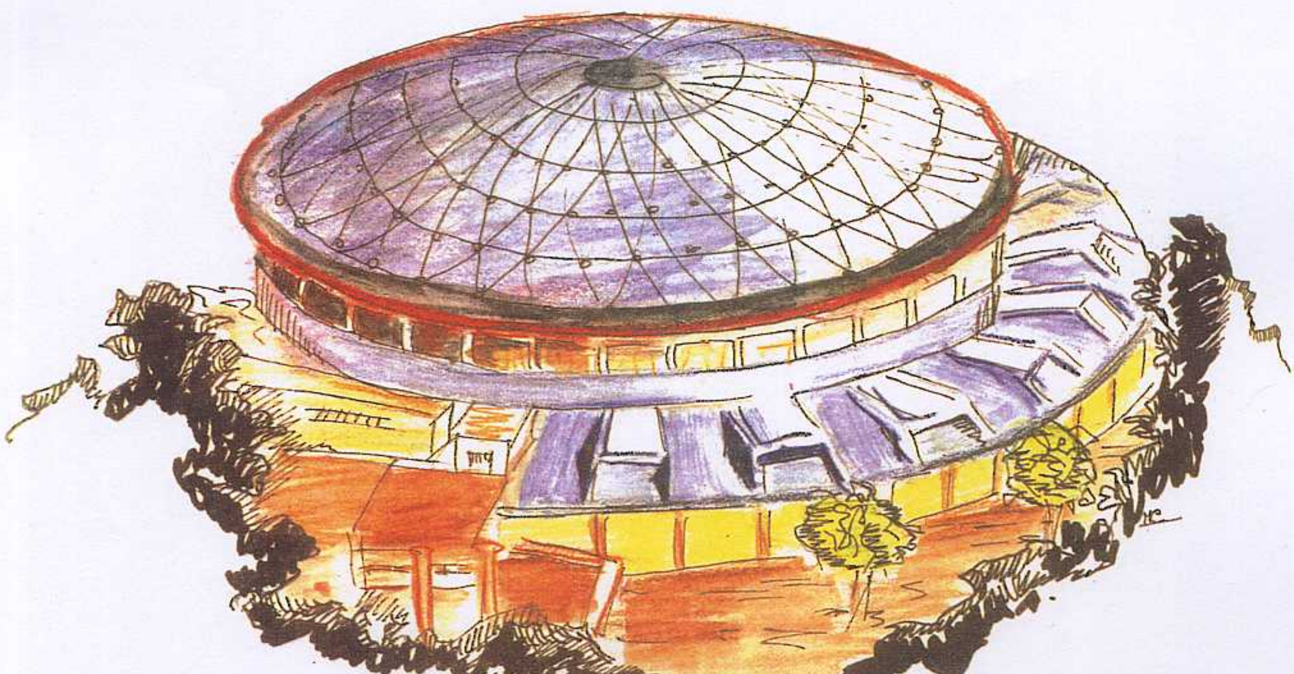

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A theoretical study of the c and b fragmentation function in e^+e^- annihilation

G. Colangelo

INFN, Laboratori Nazionali di Frascati, Italy

P. Nason

INFN, Gruppo Collegato di Parma, Sezione di Milano, Italy

Abstract

We present an analysis of the c and b fragmentation functions which includes in a consistent fashion leading and next-to-leading perturbative contributions, effects due to soft gluon emission and a parametrization of effects of non-perturbative origin. We show that the data on D meson production at intermediate energy constrains the parametrization of the effects of non-perturbative nature. We can therefore make a prediction for the B fragmentation function. Results for the B, D, and D^* fragmentation functions at LEP energy are given, and compared with existing data.

The study of the b fragmentation function at e^+e^- colliders is a subject of considerable experimental and theoretical interest. From an experimental point of view, the knowledge of the QCD production properties of b flavoured hadrons at LEP have considerable impact on b physics studies. While the study of b fragmentation below the Z peak has been limited by the lack of statistics[1,2,3,4], LEP results[5,6] are very promising, and certainly one expects substantial improvements in the future. The knowledge of the heavy quark fragmentation function in e^+e^- annihilation has also considerable importance in the understanding of heavy quark production at high p_T in other contests, like heavy quark hadroproduction[7] and photoproduction.

From a point of view of perturbative QCD, the production of a heavy quark in e^+e^- collision is a well understood problem. Leading logarithmic effects in the heavy quark fragmentation function have been computed a long time ago[8]. Recently, in ref. [9], the next-to-leading effects have been computed. The effects of the soft gluons in the large x limit have also been included[9,10].

Unfortunately, the known heavy quarks are not heavy enough for the purely perturbative description to be sufficient to describe the data well. Our theoretical knowledge of the heavy quark fragmentation function can be summarized as follows:

- the perturbative evolution equation, including the resummation of soft gluon effects, and the next-to-leading corrections. If we have an initial condition for the evolution equation, we can predict the value of the fragmentation function at any scale with next-to-leading accuracy.
- Non perturbative effects must obey a scaling law as a function of the heavy quark mass[8,11,12]. Therefore, if we measure the D meson fragmentation function we should be able to extract the non-perturbative contribution, and scale it in order to obtain the non-perturbative part of the B meson fragmentation function.
- perturbative and non-perturbative effects cannot be fully disentangled. In other words, the size of non perturbative effects will be related to the choice of the perturbative parameters. Therefore, if we change the matching scales for evolution, or value of the mass, or the value of Λ, we will end up attributing a different magnitude to the non-perturbative effects.

Keeping these three points in mind, we have performed an analysis of the fragmentation function for heavy flavoured mesons. We used the low energy data on D production in order to extract the effects of non-perturbative origin. This effects were then scaled to the B mass, allowing us to make a prediction for the B fragmentation function at LEP energy.

In the following sections we give a detailed description of our procedure and results.

1. Description of the method

The formulae for the perturbative contribution to the heavy quark fragmentation function were taken from ref. [9], where they are given in moment space. Their value as a function of x can be obtained by performing an inverse Mellin transformation via a complex integration in moment space. In applying these formulae to phenomenology there are various sources of uncertainty that must be taken into account. First of all, the perturbative formula for the fragmentation function depends upon a matching scale μ_0. The effects of gluons with a transverse momentum larger than μ_0 are resummed by the evolution equation, while gluons with smaller momenta are treated in fixed order of perturbation theory. The scale μ_0 should be taken of the order of the heavy quark mass, in order to avoid large logarithms of μ_0/m in the fixed order expression. Of course, if our calculation was extended to all order in perturbation theory, the result would not depend upon μ_0. Therefore, the uncertainty due to the choice of μ_0 can be seen as an estimate of higher order effects. We will consider a variation of μ_0 between $m/2$ and $2m$ as an estimate of these effects. There is an uncertainty also in the mass of the heavy quark, which can cause an error in our prediction. In all cases, however, we have verified that this uncertainty is much smaller than all the others, and can therefore be neglected. The value of $\Lambda_{\rm QCD}$ is also subject to considerable uncertainty. We have always considered therefore the three possibilities $\Lambda_5 = 100,\ 200$ and 300 MeV, which roughly correspond to the presently allowed experimental range.

We also include a parametrization of non-perturbative effects of the following form

$$D^{\rm NP}(x) = A(1-x)^\alpha x^\beta, \qquad \text{with} \quad \frac{1}{A} = \int (1-x)^\alpha x^\beta dx \tag{1}$$

to be convoluted with the perturbative expression. We prefer this parametrization, with respect to the more commonly used ones, because its Mellin transform has a simple expression, which is easily continued to the complex plane (so that we can

Table I: Fitted values of the non perturbative parameters for D^0 mesons for different values of Λ_5 and μ_0.

		$\mu_0 = m_c/2$	$\mu_0 = m_c$	$\mu_0 = 2m_c$
$\Lambda_5 = 0.1$ GeV	α_D	0.6	1.0	1.3
	β_D	5.4	3.67	4.37
$\Lambda_5 = 0.2$ GeV	α_D	0.61	0.35	0.8
	β_D	13.5	3.05	3.2
$\Lambda_5 = 0.3$ GeV	α_D	****	1.0	0.6
	β_D	****	7.04	2.73

compute the full fragmentation function by a single complex contour integration, with no need of a subsequent convolution in x). We then performed fits to ARGUS[13] data for the D^0 fragmentation function at 10.6 GeV, using $m_c = 1.5$ GeV, $\mu_0 = m_c/2$, m_c, $2m_c$ and $\Lambda_5 = 100$, 200, 300 MeV, using as free variables the parameters α and β. In table I we give the fitted values of α and β. The results of the fits for $\Lambda_5 = 200$ MeV and the different choices of μ_0 are shown in fig. 1. For $\Lambda_5 = 100$ and 300 MeV we obtain fits of similar quality, except that no fit is possible for $\mu_0 = m_c/2$ and $\Lambda_5 = 300$ MeV.

From fig. 1, we see that the difference in the quality of the fit due to the choice of scale is quite small. Observe that this does not mean that the scale dependence is small. In fact, for each choice of the scale, the non-perturbative parameters are adjusted in order to fit the data. Therefore, the fact that for some scale choices a fit is not possible, should be viewed as a limitation of the non-perturbative model, which cannot be adjusted in order to compensate for the scale variation. From table I, we notice that for $\Lambda_5 = 200$ MeV and $\mu_0 = m_c/2$ the parameter β_D is very large, so that the expression in formula (1) is strongly peaked around one. We can therefore, by choosing appropriately small values for μ_0, fit the data with no need for a non-perturbative contribution.

The ARGUS data is uncorrected for initial state radiation. Strictly speaking we should include such corrections. In ref. [4], it is shown that at a center of mass energy of 34 GeV they are negligible, and they should therefore be even smaller at 10.6 GeV,

Table II: Values of the non-perturbative parameters to adopt for B mesons for different values of Λ_5 and μ_0.

		$\mu_0 = m_b/2$	$\mu_0 = m_b$	$\mu_0 = 2m_b$
$\Lambda_5 = 0.1$ GeV	α_B	0.88	2.04	2.66
	β_B	28.45	29.84	36.06
$\Lambda_5 = 0.2$ GeV	α_B	0.72	0.595	1.64
	β_B	54.54	18.67	25.69
$\Lambda_5 = 0.3$ GeV	α_B	****	1.46	1.23
	β_B	****	37.76	21.54

since initial state radiation becomes stronger as the energy grows.

It was pointed out long ago[8,11,12] that non-perturbative effects should scale linearly in the mass of the heavy quark. This fact can be easily shown to be a consequence of the infinite quark mass limit, and it is the only sound theoretical information we have about the non-perturbative part of the fragmentation function. We have implemented this scaling law in the following way. We compute the average value of x and the position of the peak given only by the non-perturbative term, eq. (1), for D mesons

$$\langle x_D^{\rm NP}\rangle = \int D_D^{\rm NP}(x) x dx = \frac{\beta_D + 1}{\alpha_D + \beta_D + 2} \qquad (2)$$

$$\frac{d}{dx} D_D^{\rm NP}(x) = 0 \quad \text{at} \quad x = \hat{x}_D^{\rm NP} = \frac{\beta_D}{\alpha_D + \beta_D} \qquad (3)$$

where α_D and β_D are taken from table I. We then compute α_B and β_B in such a way that

$$\begin{aligned} 1 - \langle x_B^{\rm NP}\rangle &= \frac{m_c}{m_b}(1 - \langle x_D^{\rm NP}\rangle) \\ 1 - \hat{x}_B^{\rm NP} &= \frac{m_c}{m_b}(1 - \hat{x}_D^{\rm NP}) \end{aligned} \qquad (4)$$

We obtain therefore the values of α_B and β_B given in table II. These values are then used to make the predictions for the B meson fragmentation function, shown in figs. 2, 3 and 4, together with the LEP data from ref. [5]. As an estimate of the uncertainty associated with the scaling law, we can simply consider extreme values of the ratio of m_c/m_b in eq. (4). Changing the c quark mass between 1.2 and 1.8 GeV induces a change in the average value of x for the B fragmentation function of $\pm 3\%$. We also observe a considerable dependence of the prediction upon the scale choice. This dependence is due to the fact that the perturbative part of the fragmentation function does not scale with the mass of the heavy quark in the same way as the non-perturbative part. It therefore represents a theoretical uncertainty which we cannot eliminate. Thus, while a better determination of Λ_5 could help to choose between figs. 2, 3 and 4, we have no way to decide which of the curves in each figure should be chosen.

We point out that we could have chosen to use the D^+ data, or the D^* data for our analysis, or a combination of them. Our choice of the D^0 was therefore dictated by the fact that errors are smaller than for the $D^\pm$, and by the fact that the D is more similar to a B than a D^*. Nevertheless, we did perform the same operation with the D^* data. The results for the non-perturbative parameters for the D^* are given in table III, and the quality of the fit is shown in fig. 5 for $\Lambda_5 = 200\,\text{MeV}$. We don't present the corresponding prediction for the B, since we didn't find any dramatic difference in the result. A further problem is due to the fact that the flavour composition of b flavoured mesons at LEP is unknown. Our prediction should therefore strictly apply to B mesons, not including B_s. However, we do not see any theoretical reasons why the inclusion of B_s should produce a relevant uncertainty in our prediction, even under the pessimistic assumption that the B_s rate is a third of the total B mesons rate.

As one can see, figs. 2, 3 and 4 are all compatible with LEP data, expecially if one remembers that there should be a further smearing in x of the order of 3%, due to the uncertainties in the scaling law. Nevertheless, it appears that in all cases the theoretical curves are slightly harder than the data. Results from OPAL are however more compatible with our prediction[14].

On the basis of the non-perturbative parameters given in table I and III, we also give a prediction for the D and D^* fragmentation functions at LEP. The results are reported in fig. 6 and 7.

Data on D production is also available from the CLEO experiment[15]. We have repeated the same analysis with the CLEO data. We find only minor differences in

Table III: Fitted values of the non perturbative parameters for D^* mesons for different values of Λ_5 and μ_0.

		$\mu_0 = m_c/2$	$\mu_0 = m_c$	$\mu_0 = 2m_c$
$\Lambda_5 = 0.1$ GeV	α_{D^*}	0.6	0.6	1.0
	β_{D^*}	5.4	5.4	5.0
$\Lambda_5 = 0.2$ GeV	α_{D^*}	0.01	0.4	0.7
	β_{D^*}	23.2	4.6	4.1
$\Lambda_5 = 0.3$ GeV	α_{D^*}	****	0.2	0.5
	β_{D^*}	****	5.8	5.0

the prediction for the B and D fragmentation functions at LEP.

2. Conclusions

We have studied the fragmentation function in e^+e^- annihilation for D and B mesons in a unified contest, including all the theoretical knowledge which is available at present. We find that the available data is fairly consistent with the theoretical description of this phenomenon.

We point out that the parametrization of non perturbative effects that we obtained from e^+e^- annihilation data should also be applicable in all the process in which the heavy quark are produced at large p_T, like heavy quark production in hadronic or ep collisions.

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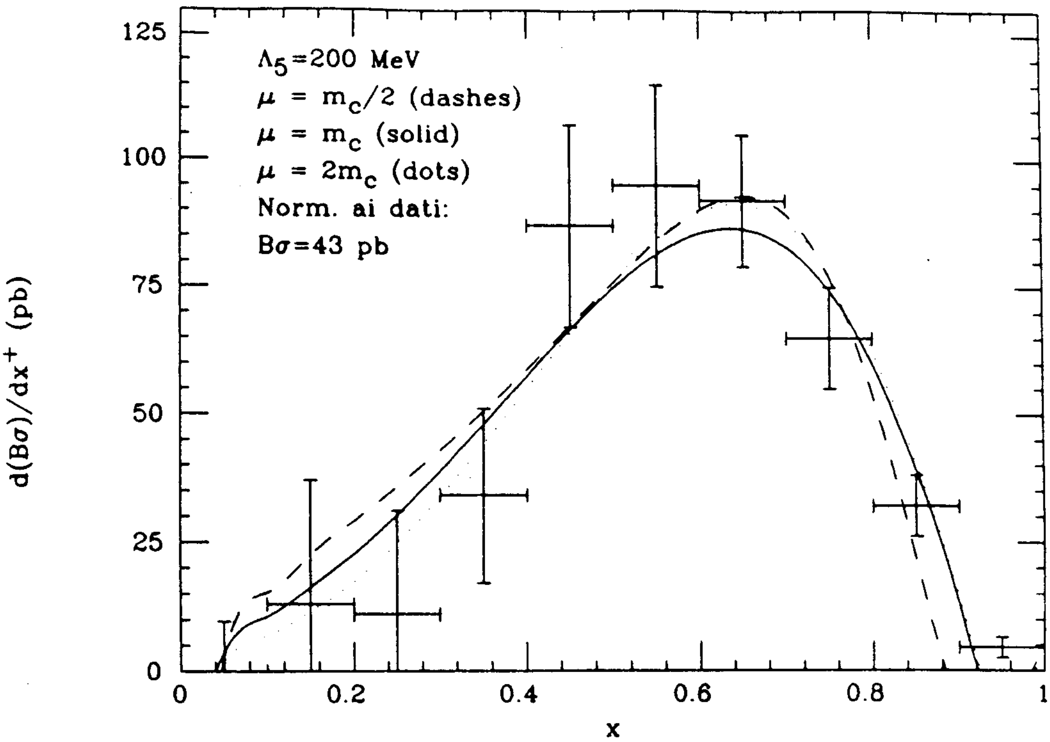

Fig. 1: Fits to the D fragmentation function at 10.6 GeV, measured at ARGUS.

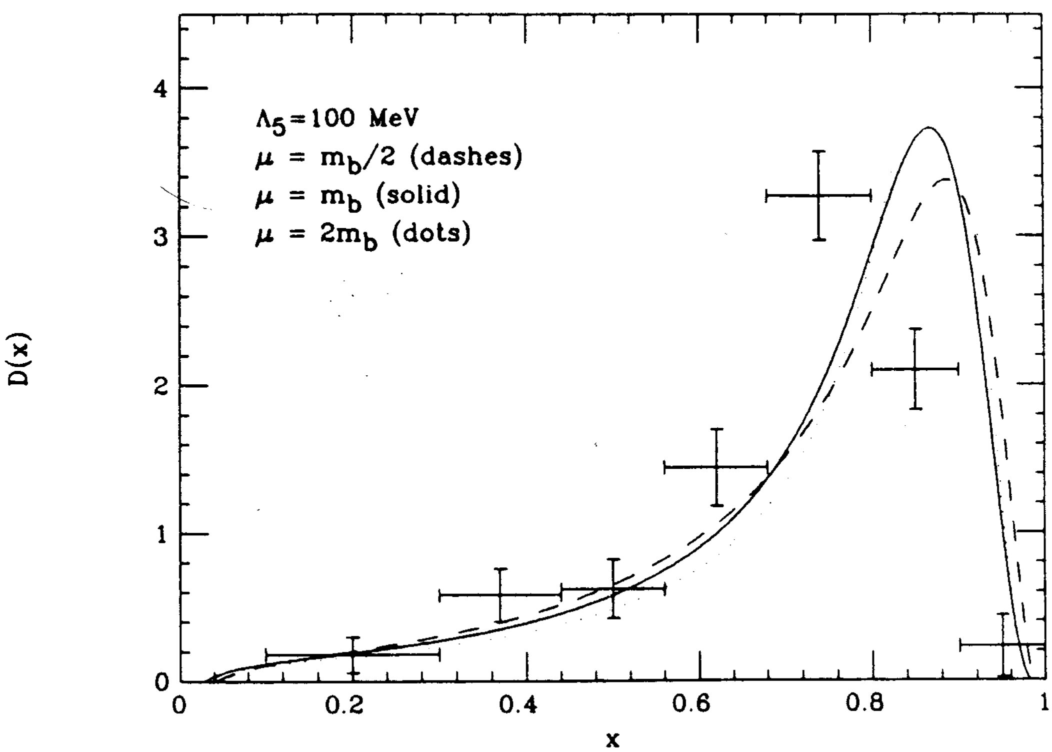

Fig. 2: Prediction for the B fragmentation function at LEP, for $\Lambda_5 = 100$ MeV.

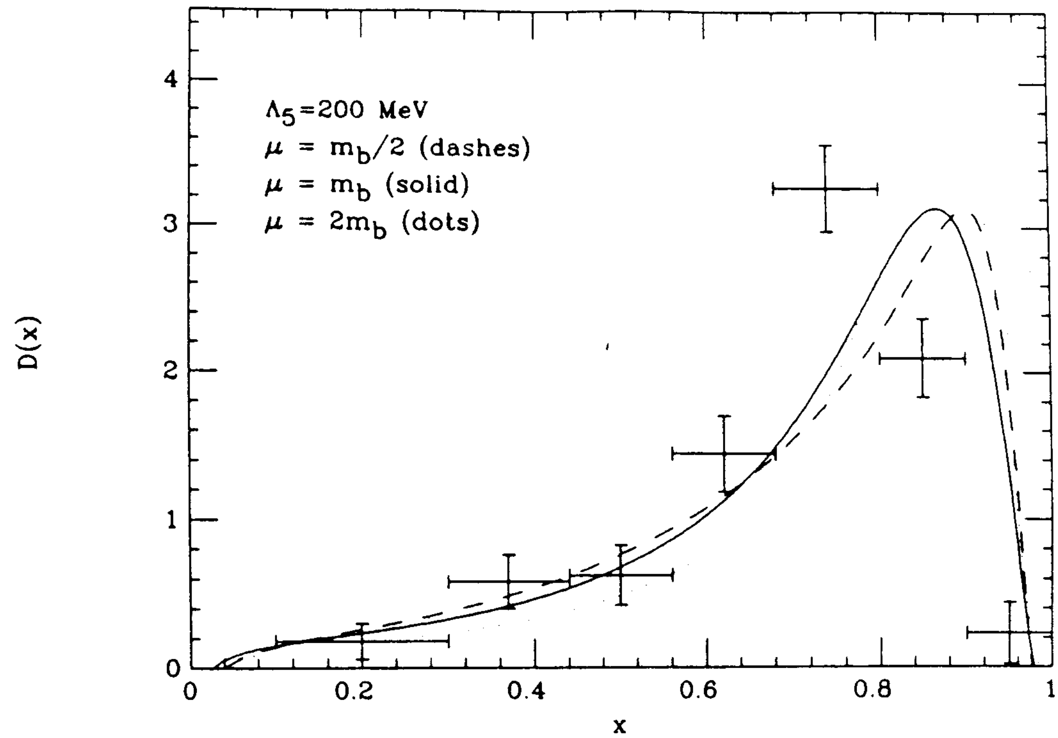

Fig. 3: Prediction for the B fragmentation function at LEP, for $\Lambda_5 = 200$ MeV.

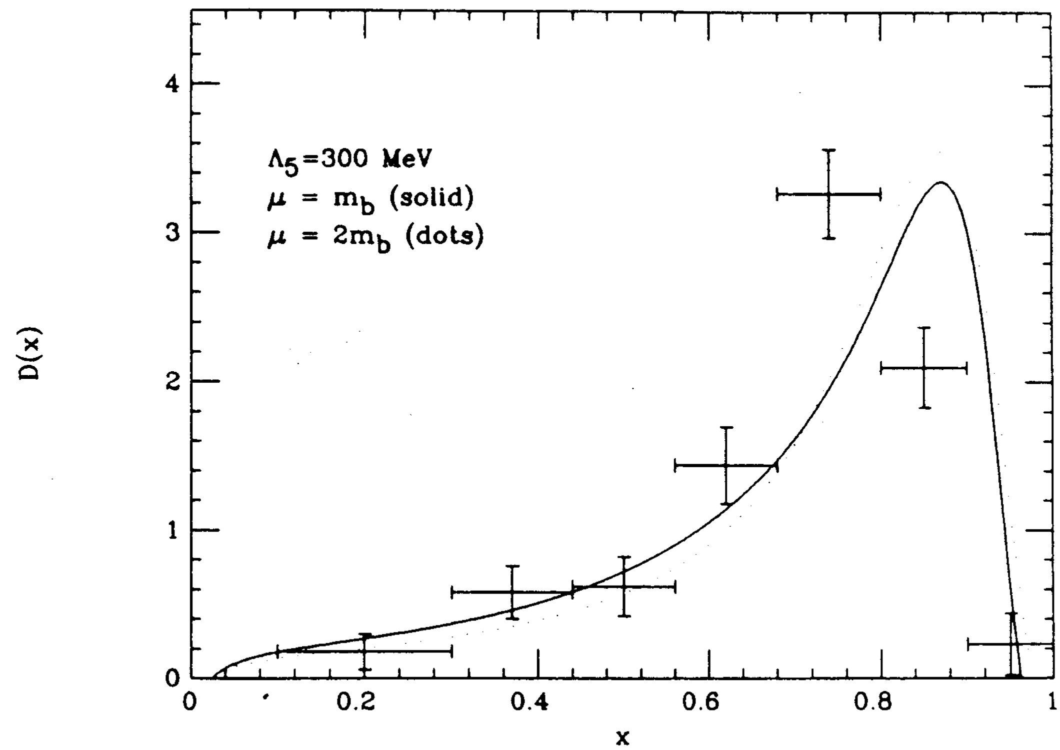

Fig. 4: Prediction for the B fragmentation function at LEP, for $\Lambda_5 = 300$ MeV.

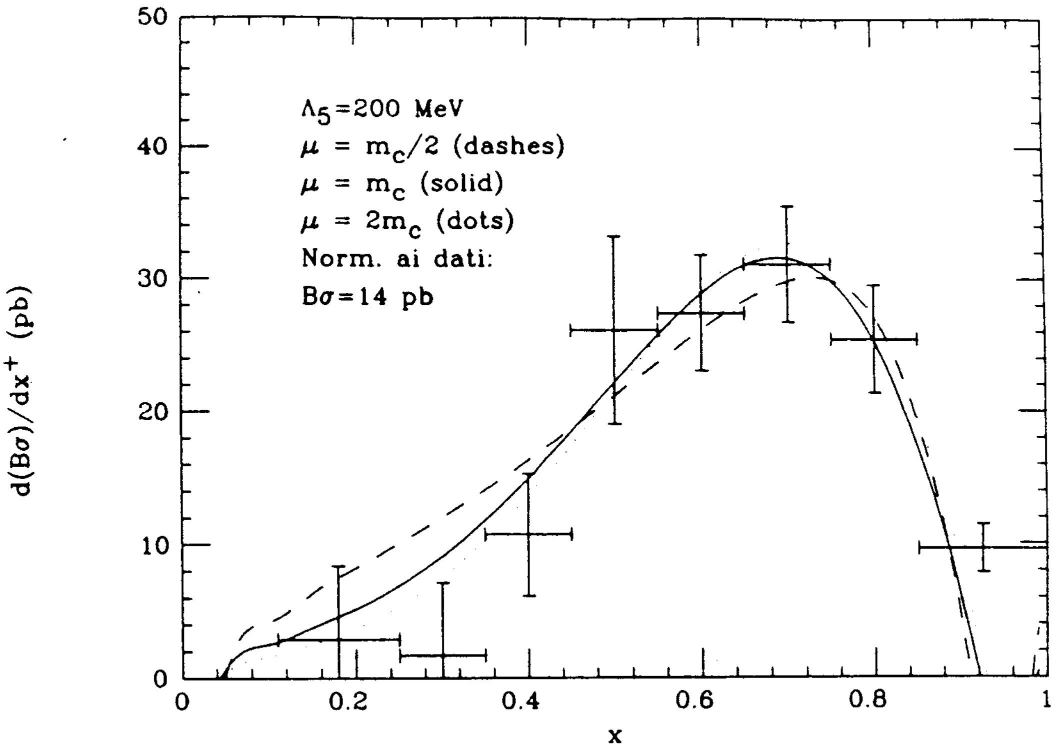

Fig. 5: Fits to the D^* fragmentation function at 10.6 GeV, measured at ARGUS.

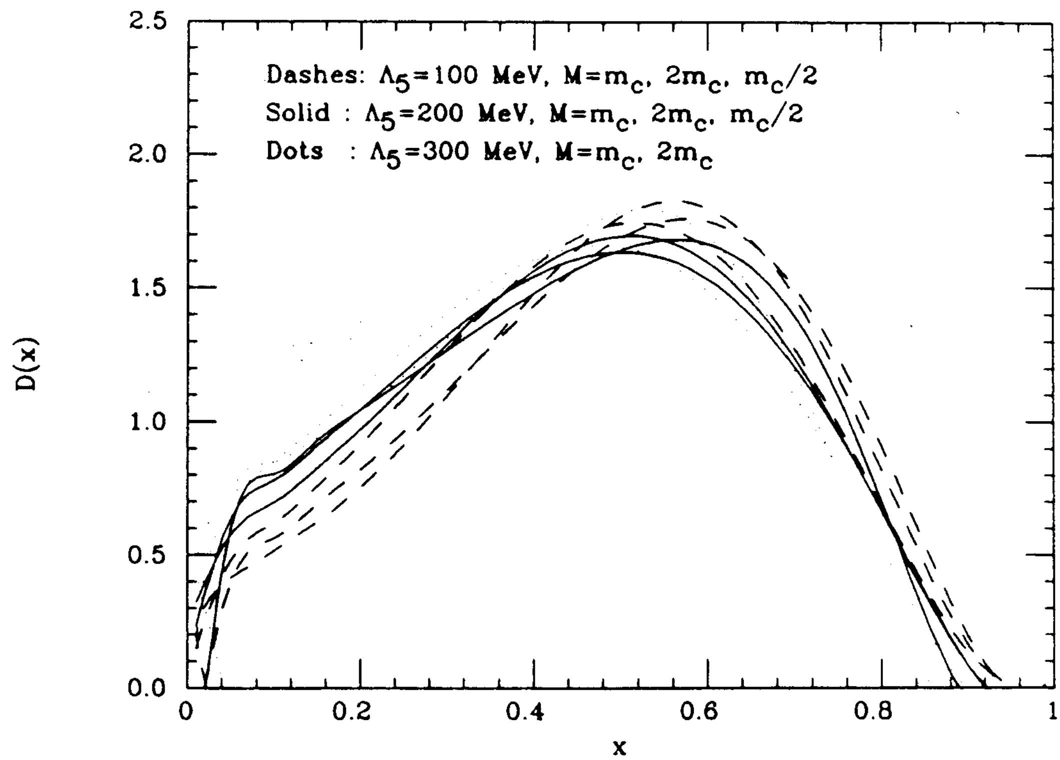

Fig. 6: Prediction for the D fragmentation function at LEP.

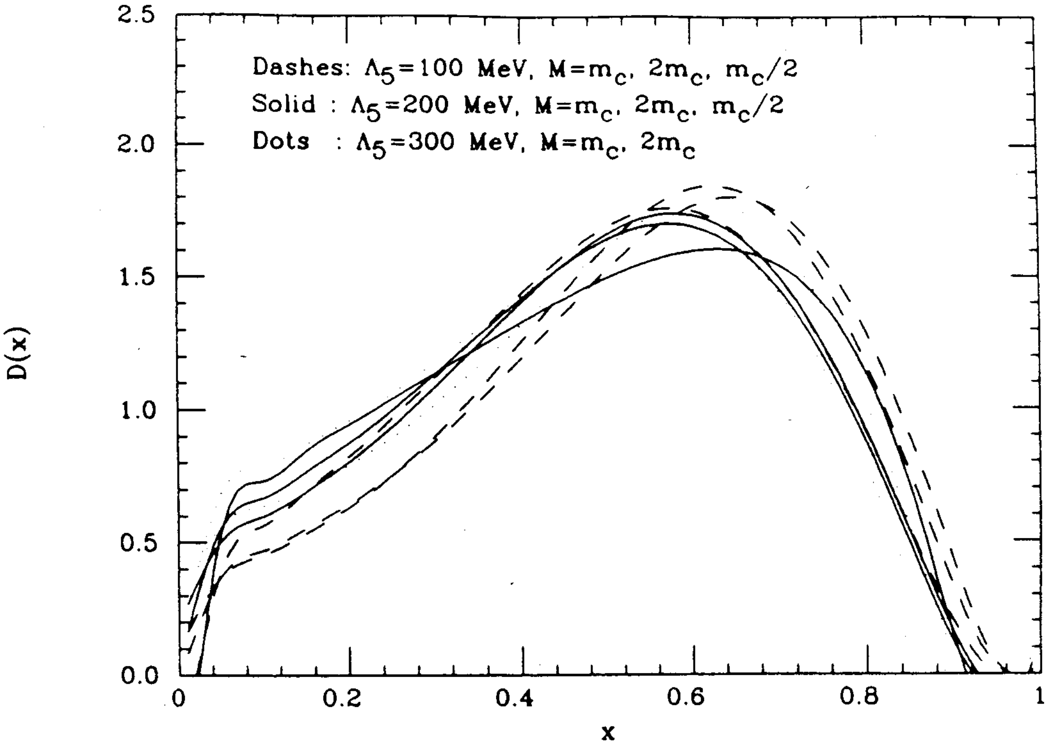

Fig. 7: Prediction for the D^* fragmentation function at LEP.