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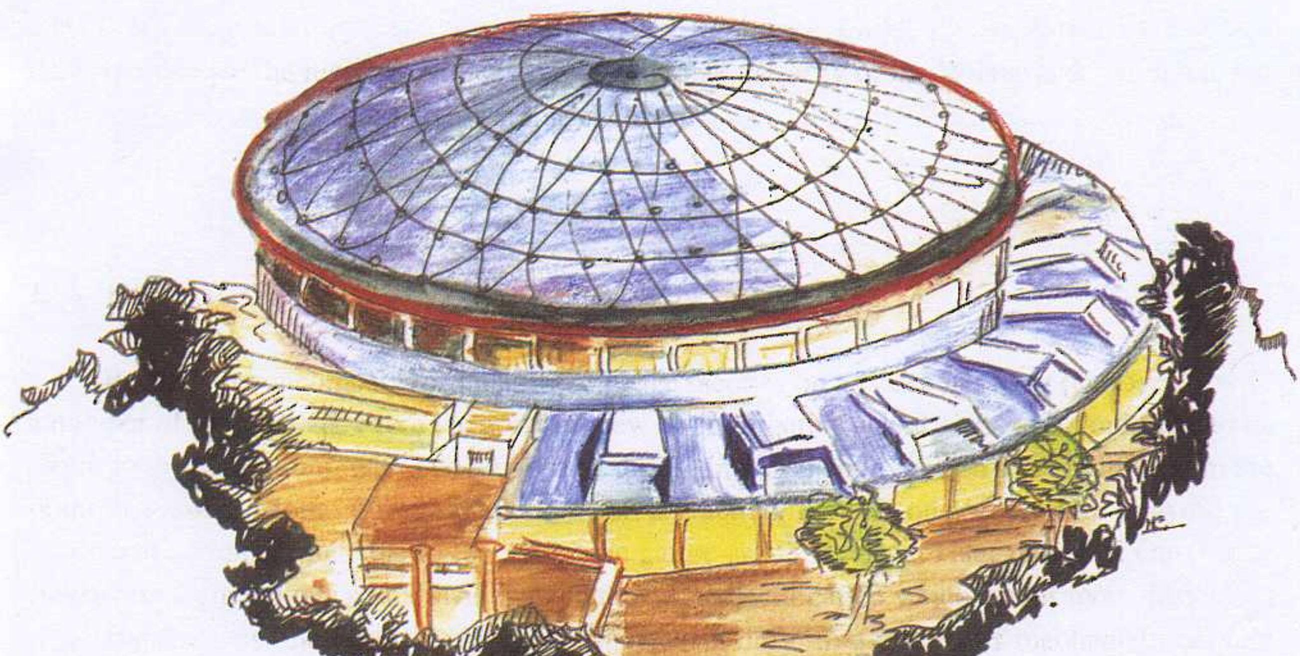

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NON COMPACT GAUGE THEORIES ON A LATTICE: THE CASE OF SCALING DIMENSION 2 FOR THE AUXILIARY FIELD

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ABSTRACT

We investigate a new lattice regularization of gauge theories where gauge–invariance with non compact gauge fields is realized by the help of auxiliary fields. For the gauge group SU(2) there is only one auxiliary field. We construct the complete action for scaling dimension 2 of this field. In this case the regularization ammounts to relax unitarity of the Wilson link variables, but has the same renormalization scale parameter showing the stability of Wilson's fixed point.

1. – INTRODUCTION

Recently a noncompact formulation of gauge theories on a lattice has been proposed[1)], with a number of motivations. From the point of view of perturbative calculations one would like to be closer to the continuum, with Faddev–Popov terms simpler than in Wilson's definition. From the point of view of Monte Carlo calculations, on a practical ground, one would like to avoid the numerical complications due to the use of the gauge group invariant Haar measure. On a more fundamental ground one would like to avoid the decompactification which makes more difficult to understand issues related to confinement. In Wilson definition in fact its mechanism is quite simple at strong coupling where it is strictly related to the compactness of the gauge variables, while in the scaling regime (which should approximate the continuum where the gauge fields are noncompact) it has not yet been fully understood. The extreme possibility that at strong coupling confinement be an artifact of compacteness has been considered also in ref. 1. In order to avoid difficulties with decompactification, some authors[2,3] have defined gauge theories on the lattice by direct discretization of the continuum action. No confinement has been found, but this can be due to the explicit breaking of gauge–invariance by such a procedure. In another attempt[4] the gauge–invariant variables have been used which emerge as solutions of a non renormalizable gauge fixing. Also in this case no confinement has been found. Among the possible reasons for this negative result there is the lack of renormalizability, which seems in fact essential to relate confinement to monopole condensation by the abelian projection method[5]. A possible explanation[3] for the absence of confinement common to all the above results is that, being close to the continuum, all the above regularizations have a scale close to the continuum one. The physical dimension of the lattice is in such a case so small that the quark–quark potential inside it cannot be linear. It is obvious that to clarify this situation one must introduce non compact gauge fields by respecting gauge–invariance and renormalizability.

A last motivation concerns the possibility that non compact regularizations might have a different scale, providing us with a flexibility to be exploited by choosing the scale more convenient to the problem at hand.

The new regularization, in order to enforce gauge invariance at finite lattice spacing, makes use of auxiliary fields which decouple in the continuum limit. In the case of SU(2), to which we restrict ourselves in the present paper, in addition to the Yang–Mills fields it contains only a vector field. Elimination of this field by a gauge–invariant constraint leads to Wilson's formulation[6]. In this paper we want to discuss the case in which the auxiliary field is retained so that the gauge fields are non compact.

The regularization is such that it is, a priori, possible to assign different scaling dimension to the auxiliary field. We will consider the case of scaling dimension 2. Its kinetic terms are therefore irrelevant, and we will show that they can altogether be eliminated by a proper choice of the coupling constants, so that its propagator be local (constant in momentum space).

In Sect. 2. we will construct all the local gauge–invariant terms which are also invariant under the inversion of any single axis. We will then constrain the coupling constants of such terms by requiring stability (existence of a lower bound for the Euclidean action) and Euclidean invariance in the continuum limit. For completeness we will present the coupling to scalar fields and we will complete the action by introducing the Feynman gauge–fixing throught the BRS procedure.

Having so established the general form of the Lagrangian, in Sect. 3. we shall compare to one loop the renormalization parameter Λ_{NC} of the present non compact regularization to that of Wilson, Λ_W, along the line of Dashen and Gross[7].

It turns out that

$$\ln \frac{\Lambda_W}{\Lambda_{NC}} \propto g^2$$

vanishing in the continuum limit. This shows that Wilson's theory is stable against violations of unitarity of the link variables.

2. – CONSTRUCTION OF THE COMPLETE ACTION

The gauge transformations of the gauge field $\mathcal{A}_\mu$ and of the auxiliary field W_μ are(*)

$$\mathcal{A}'_{\mu a} = \mathcal{A}_{\mu a} + (1 - a\, W_\mu)\, \Delta_\mu \vartheta_a - \varepsilon_{abc}\, \mathcal{A}_{\mu b}\, (1 + \tfrac{a}{2}\Delta_\mu)\, \vartheta_c \tag{2.1}$$

$$W'_\mu = W_\mu + \frac{1}{4}\, a\, \mathcal{A}_{\mu a}\, \Delta_\mu\, \vartheta_a, \tag{2.2}$$

where a is the lattice spacing and

$$\Delta_\mu f(x) = \frac{1}{a}\, [f(x+\mu) - f(x)] \quad , \quad \mu_\nu = \delta_{\mu\nu}\,. \tag{2.3}$$

We will also need the left derivative

$$\Delta_\mu^{(-)} f(x) = \frac{1}{a}\left[f(x) - f(x-\mu)\right]. \tag{2.4}$$

The limit for $a \to 0$ reproduces the continuum gauge transformations for $\mathcal{A}_\mu$, but it is not defined for W_μ depending on a possible rescaling of this field with a. As we will see the scaling dimension 2 allows the rescaling

$$W_\mu = aV_\mu. \tag{2.5}$$

Such a rescaling is also natural from the point of view of the transformations, because if we start with $W_\mu = 0$ only fields of order a will be generated. The continuum limit of the transformations for V_μ is obviously

$$V'_\mu = V_\mu + \frac{1}{4}\, \mathcal{A}_{\mu a}\, \Delta_\mu\, \vartheta_a. \tag{2.6}$$

The basic element of the regularization is the covariant derivative(*)

$$D_\mu = \left(\frac{1}{a} - W_\mu\right) I + i\, \mathcal{A}_\mu \quad , \quad \mathcal{A}_\mu = \mathcal{A}_{\mu a}\, T_a, \tag{2.7}$$

T_a being the generators of the group normalized according to

$$[T_a, T_b] = i\, \varepsilon_{abc}\, T_c$$

$$\{T_a, T_b\} = \frac{1}{2}\, \delta_{ab}. \tag{2.8}$$

Under gauge transformations

$$D_\mu(x) \to g(x)\, D_\mu(x)\, g^\dagger(x+\mu). \tag{2.9}$$

The strength and the Yang–Mills Lagrangian are defined in analogy to the continuum

$$F_{\mu\nu}(x) = \frac{1}{i}[D_\mu(x)\,D_\nu(x+\mu) - D_\nu(x)\,D_\mu(x+\mu)] \to g(x)\,F_{\mu\nu}(x)\,g^\dagger(x+\mu+\nu), \tag{2.10}$$

$$\mathcal{L}_{YM} = \frac{1}{8}\,\beta \sum_{\mu\neq\nu} T_r\,F^\dagger_{\mu\nu}\,F_{\mu\nu} = \frac{1}{16}\beta \sum_{\mu\neq\nu} \Big\{(\Delta_\mu \mathcal{A}_{\nu a} - \Delta_\nu \mathcal{A}_{\mu a}) - \Big[\,\frac{1}{2}\varepsilon_{abc}\,\mathcal{A}_{\mu b}\,\mathcal{A}_{\nu c}(x+\mu)$$

$$+ W_\mu\,\mathcal{A}_{\nu a}(x+\mu) + W_\nu(x+\mu)\,\mathcal{A}_{\mu a} - (\mu \leftrightarrow \nu)\,\Big]\Big\}^2$$

$$+\frac{1}{4}\beta \sum_{\mu\neq\nu} \Big\{(\Delta_\mu W_\nu - \Delta_\nu W_\mu) + \Big[\,\frac{1}{4}\vec{\mathcal{A}}_\mu \cdot \vec{\mathcal{A}}_\nu(x+\mu) - W_\mu W_\nu(x+\mu) - (\mu \leftrightarrow \nu)\Big]\Big\}^2 \tag{2.11}$$

In the formal continuum limit the auxiliary field W_μ obviously decouples. To ensure its decoupling also at the quantum level in ref. (1) a potential was introduced in order to give to W_μ a divergent mass for vanishing lattice spacing. This decoupling mechanism is likely to be necessary because the coupling of the vector field W_μ is not protected by a gauge symmetry. In constructing the potential we must require that if W_μ develops a nonvanishing expectation value at the semiclassical level, this should be different from $\frac{1}{a}$, since otherwise D_μ is no longer a (covariant) derivative. Let us in fact notice that the nonhomogeneous gauge transformations (2.1) arise from a spontaneous breaking of homogeneous transformations for the fields $\mathcal{A}_\mu$ and $\hat{W}_\mu = 1 - a\,W_\mu$. The breaking occurs if $<\hat{W}_\mu> \neq 0$, namely $<W_\mu> \neq \frac{1}{a}$.

The only other invariant in the present regularization is t_μ defined by

$$t_\mu I = D_\mu D^\dagger_\mu - \frac{1}{a^2}\,I = \left[\frac{1}{4}\,\vec{\mathcal{A}}^2_\mu + W^2_\mu - \frac{2}{a}\,W_\mu\right] I\,, \tag{2.12}$$

so that the potential must be a function of it. In ref. (1) it was chosen to be

$$\mathcal{L}_c = \beta_c \sum_\mu t^2_\mu. \tag{2.13}$$

If in the continum limit

$$\beta_c \sim (a/\lambda)^{2-\varepsilon}, \qquad \varepsilon > 0,\ \lambda \text{ a parameter with the dimension of a lenght,} \tag{2.14}$$

the mass of W_μ is of the order of $\lambda^{\frac{\varepsilon}{2}-1}\,a^{-\frac{\varepsilon}{2}}$. In the present paper we confine ourselves to the case $\varepsilon = 2$. In this case the mass of W_μ is of the order of a^{-1} and dominates the kinetic terms in the whole Brillouin zone, so that W_μ has scaling(**) dimension 2. Let us mention for future reference that for $\beta_c = \infty$ one gets Wilson's regularization[6].

Finally it is perhaps worth while noticing that the measure for W_μ and $\mathcal{A}_\mu$ in the path integral is 1.

Let us now start our program by defining the operator P_μ representing the inversion of the μ–axis ($x \to i_\mu x$)

$$P_\mu^{-1} D_\nu P_\mu = \begin{cases} D_\mu^\dagger (i_\mu x - \mu) , & \text{for } \mu = \nu \\ D_\nu (i_\mu x) , & \text{for } \mu \neq \nu \end{cases} \tag{2.15}$$

so that

$$P_\mu^{-1} t_\nu (x) P_\mu = t_\nu (i_\mu x - \delta_{\mu,\nu}\mu), \quad \text{for any choice of } \mu \text{ and } \nu.$$

Notice that $\mathcal{L}_{YM}$ is not parity–invariant. Both $\mathcal{L}_{YM}$ and t_μ, however, are invariant under the trasformations(***)

$$\begin{aligned} \mathcal{A}_\mu &\to - \mathcal{A}_\mu \\ W_\mu &\to - W_\mu + \frac{2}{a} \end{aligned} \tag{2.16}$$

which change D_μ into $-D_\mu$.

We select the couplings which are invariant under gauge transformations and under permutations and inversions of the lattice axes, generating the whole cristallographic group.

We also require the couplings to be minimal in the sense that every independent term be at most quartic in D_μ and contain at least a local or a nearest–neighbour coupling. This leads to the following terms:

$$\mathcal{L}_P = -\frac{1}{4} \beta \sum_{\mu \neq \nu} \mathrm{Tr} \left(D_\mu (x) D_\nu (x + \mu) D_\mu^\dagger (x + \nu) D_\nu^\dagger (x)\right) \tag{2.17}$$

$$\mathcal{L}_m = -\frac{\beta_m}{a^2} \sum_\mu t_\mu$$

$$\mathcal{L}_c = \beta_c \sum_\mu t_\mu^2$$

$$\mathcal{L}_1 = \beta_1 \sum_\mu t_\mu (x)\, t_\mu (x + \mu)$$

$$\mathcal{L}_2 = \beta_2 \sum_{\mu \neq \nu} t_\mu (x)\, t_\mu (x + \nu)$$

$$\mathcal{L}_3 = \beta_3 \sum_{\mu \neq \nu} t_\mu (x) \left(t_\nu (x) + 2\, t_\nu (x + \mu) + t_\nu (x + \mu - \nu)\right)$$

$$\mathcal{L}_4 = \beta_4 \sum_{\mu \neq \nu \neq \rho} t_\mu (x) \left(t_\nu (x + \rho) + t_\nu (x + \rho + \mu) + t_\nu (x + \rho - \nu) + t_\nu (x + \rho + \mu - \nu)\right)$$

$$\mathcal{L}_5 = \beta_5 \sum_{\mu\neq\nu} t_\mu(x) \big[t_\nu(x) + t_\nu(x+\mu-\nu) + t_\nu(x-\mu+\nu) + t_\nu(x+2\mu-2\nu) -$$

$$2(t_\nu(x+2\mu-\nu) + t_\nu(x+\mu+\nu) + t_\nu(x+\nu) + t_\nu(x+2\mu) -$$

$$t_\nu(x+\mu) - t_\nu(x+2\mu-\nu))\big]. \tag{2.18}$$

If W_μ has dimension 2, $\mathcal{L}_1$ and $\mathcal{L}_2$ are equivalent to $\mathcal{L}_C$ and $\mathcal{L}_4$ and $\mathcal{L}_5$ to $\mathcal{L}_3$ up to irrelevant terms. We will retain all these terms, however, in order to be able to get for W_μ a propagator strictly local, i.e. proportional to a Kronecker–δ of the lattice site.

Summing all the above terms we have the gauge Lagrangian

$$\mathcal{L}_G = \mathcal{L}_P + \mathcal{L}_m + \mathcal{L}_c + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + 6\beta \frac{1}{a^4}, \tag{2.19}$$

where the last term has been determined in such a way that for $\mathcal{A}_\mu = W_\mu = 0$, $\mathcal{L}_G = 0$.

In order to discuss stability and Euclidean invariance we need the classical potential

$$V = \frac{1}{16}\beta \sum_{\mu\neq\nu} [\, \vec{\mathcal{A}}_\mu^2\, \vec{\mathcal{A}}_\nu^2 - (\vec{\mathcal{A}}_\mu \cdot \vec{\mathcal{A}}_\nu)^2] + \alpha_1 \sum_{\mu\neq\nu} t_\mu t_\nu$$

$$+ \alpha_2 \sum_\mu t_\mu^2 - (\beta_m + 3\beta)\frac{1}{a^2} \sum_\mu t_\mu, \tag{2.20}$$

where

$$\alpha_1 = 4(\beta_3 + 2\beta_4) - \frac{1}{2}\beta$$

$$\alpha_2 = \beta_1 + 3\beta_2 + \beta_c. \tag{2.21}$$

Stability requires V to be bounded from below, which yields

$$3\alpha_1 + \alpha_2 > 0, \qquad \beta_m \text{ arbitrary}$$

$$3\alpha_1 + \alpha_2 = 0, \qquad \beta_m + 3\beta \leq 0. \tag{2.22}$$

The minimum of V is obtained for:

$$W_\mu(x) = \frac{1}{a}\left\{1 - \sigma_\mu(x)\left[1 + \frac{\beta_m + 3\beta}{2(3\alpha_1 + \alpha_2)}\right]^{1/2}\right\}, \qquad \sigma_\mu(x) = \pm 1, \tag{2.23}$$

so that it is degenerate at each site. Such a degeneracy is a consequence of the symmetry (2.16), which is not broken at the quantum level, due to the absence (or irrelevance) of the kinetic terms for W_μ and, as we will see, has to be accounted for also in perturbation theory.

We fix β_m by the normalization condition

$$W_\mu(x) = 0 \quad \text{for} \quad \sigma_\mu(x) = 1 \,, \tag{2.24}$$

which yields

$$\beta_m = -3\beta \,. \tag{2.25}$$

Let us now come to the constraint of Euclidean invariance in the continuum limit. This constraint in principle concerns the functional generator defined by the Feynman functional integral. It does not apply to the Lagrangian since the lattice regularization is not Euclidean invariant. However, if our theory is asymptotically free, as it is expected to be, its short distance behaviour agrees with perturbation theory. This implies Euclidean invariance for the action in the formal continuum limit. As already noted in this limit the relevant field is $V_\mu = \frac{1}{a} W_\mu$. The potential is in fact Euclidean invariant with respect to the field V_μ shifted according to

$$V_\mu - \frac{1}{8} \vec{\mathcal{A}}_\mu^2 - \frac{1}{2} a^2 V_\mu^2 = -\frac{1}{2} t_\mu \tag{2.26}$$

for

$$\alpha_1 = 0 \,. \tag{2.27}$$

It is easy to check that the kinetic terms of W_μ disappear by taking

$$\begin{aligned} &\beta_1 = \beta_4 = \beta_5 = 0 \\ &\beta_2 = \frac{1}{4} \beta, \\ &\beta_3 = \frac{1}{8} \beta \\ &\beta_c \text{ arbitrary.} \end{aligned} \tag{2.28}$$

Indeed after the above substitutions

$$\mathcal{L}_G = \mathcal{L}_{YM} - \frac{1}{16} \beta a^2 \sum_{\mu \neq \nu} (\Delta_\mu t_\nu - \Delta_\nu t_\mu)^2 + \frac{1}{2} \gamma^2 \sum_\mu t_\mu^2 \tag{2.29}$$

where

$$\gamma^2 = 2 \left(\beta_c + \frac{3}{4} \beta\right) . \tag{2.30}$$

The stability condition (2.22) tells now that $\gamma^2 > 0$.

The above shows that $\mathcal{L}_G$ differs from a combination of $\mathcal{L}_{YM}$ and $\mathcal{L}_c$ only by irrelevant terms.

Noticing that $\mathcal{L}_{YM}$ given by eq. (2.11) can be written in terms of $\mathcal{L}_P$ as

$$\mathcal{L}_{YM} = \mathcal{L}_P + 6\beta \frac{1}{a^4} + \mathcal{L}_m + \frac{1}{2} \beta \sum_{\mu \neq \nu} t_\mu (x)\ t_\nu (x + \mu), \tag{2.31}$$

we see that quadratic terms in t_μ which are not strictly local cancel out in $\mathcal{L}_G$.

Let us finally come to the coupling with scalars. It is simple to see that the only terms invariant under the symmetry (2.16), gauge–and parity–invariant, hermitean and of degree not greater than 4 are

$$\mathcal{L}_S = | \mathcal{D}_\mu \varphi |^2 + m^2 \varphi^+ \varphi + \lambda (\varphi^+ \varphi)^2 \tag{2.32}$$

where

$$\mathcal{D}_\mu \varphi = \frac{1}{2}\Big[D_\mu (x)\ \varphi\ (x + \mu) - D^+_\mu\ (x - \mu)\ \varphi\ (x - \mu) \Big] . \tag{2.33}$$

To complete the Lagrangian we adopt to the lattice Feynman gauge–fixing. We define lattice BRS transformations

$$s\, D_\mu (x) = i \left[c\,(x)\, D_\mu (x) - D_\mu (x)\, c\,(x + \mu) \right] \tag{2.34}$$

$$s\, c\,(x) = i\ c^2 (x)$$

$$s\, \bar{c}\,(x) = i\, b\,(x)$$

$$s\, b\,(x) = 0, \tag{2.35}$$

where

$$c = c_a T_a$$

$$\bar{c} = \bar{c}_a T_a \tag{2.36}$$

are the Faddeev–Popov gosts and

$$b = b_a T_a \tag{2.37}$$

is the Lagrange multiplier.

The gauge–fixing term can now be written

$$\mathcal{L}_{gf} = i\, s\, T_r \left[\bar{c}\, (2 \Delta^{(-)}_\mu D_\mu - b) \right] . \tag{2.38}$$

The Lagrangian we have found is in a form not suitable for perturbative calculations. The difficulty stems from the degeneracy of the classical potential at each site. The field W_μ, due to its scaling dimension, i.e. to the irrelevance (or total absence) of its kinetic terms, will perform large fluctuations between the 2 minima. We can define a field

$$W'_\mu = W_\mu - \frac{1}{a}\left[1 - \sigma_\mu\right] \qquad \sigma_\mu = \pm 1, \tag{2.39}$$

which has small fluctuations, but this will introduce in the action the spin–field σ_μ. Before being able to perform a loop–expansion we must then sum over $\sigma_\mu(x)$ explicitely, and this will introduce non polynomial interactions. At this price we can circumvent the difficulty by introducing as independent variables the Wilson link variables and t_μ, in terms of which, as shown below, the degeneracy disappears.

3. – THE RENORMALIZATION SCALE PARAMETER

We will follow the procedure of Dashen and Gross[7] which is based on a generalization of the background field method to the lattice. We will replace in $\mathcal{L}_P$ (but not in the gauge fixing) the covariant derivative D_μ by the background derivative

$$\hat{D}_\mu = D_\mu U^o_\mu, \tag{3.1}$$

where

$$U^o_\mu = e^{i a \alpha^o_\mu}, \tag{3.2}$$

and we will redefine the gauge–fixing term according to

$$\mathcal{L}_{gf} = i\, s\, T_r\left\{\bar{c}\,(x)\, \frac{1}{a}\left[\hat{D}_\mu\,(x)\, U^+_\mu(x) - U^+_\mu(x-\mu)\;\; \hat{D}_\mu(x-\mu)\right] - \frac{1}{2}\, b\,(x)\right\}. \tag{3.3}$$

The BRS transformations are now

$$\begin{aligned} s\,\hat{D}_\mu\,(x) &= i\left[c\,(x)\,\hat{D}_\mu\,(x) - \hat{D}_\mu\,(x)\,c\,(x+\mu)\right] \\ s\,U^o &= 0, \end{aligned} \tag{3.4}$$

the other transformations remaining unchanged.

Willing to compare our regularization to that of Wilson, it is convenient to abandon the original cartesian parametrization (2.7) of D_μ, introducing as independent variables the polar field t_μ and the unitary link variable

$$U_\mu = e^{i a \alpha_\mu} \tag{3.5}$$

through the equation

$$D_\mu = \left(1 + a^2 t_\mu\right)^{\frac{1}{2}} \frac{1}{a} U_\mu . \tag{3.6}$$

Such a parametrization makes it clear how unitarity of Wilson link variables has been relaxed and how for $\beta_c = \infty$ it is recoverd, since in such a case $t_\mu = 0$.

Notice that with this choice of variables we have circumvented the difficulty related to the vacuum degeneracy. In terms of these variables $\mathcal{L}_G$ has the form

$$\mathcal{L}_G = \sum_{\square} \mathcal{L}^W_{\square} + \frac{1}{2} \gamma^2 \sum_{\mu} t^2_\mu + \frac{1}{2} \sum_{\mu \neq \nu} T_{\mu\nu} \mathcal{L}^W_{\mu\nu} + \text{polynomials in } t_\mu \tag{3.7}$$

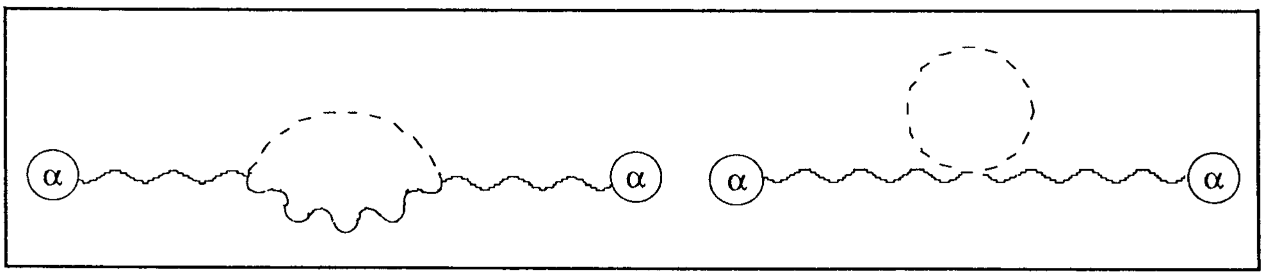
where

$$\mathcal{L}^W_{\mu\nu} = -\frac{1}{2} \beta a^{-4} T_r \left[U_\mu(x)\, U_\nu(x + \mu)\, U^+_\mu (x + \nu)\, U^+_\nu - 1 \right], \tag{3.8}$$

$$\mathcal{L}^W_{\square} = \frac{1}{2} \sum_{\mu \neq \nu} \mathcal{L}^W_{\mu\nu} , \tag{3.9}$$

$$T_{\mu\nu} = \left\{ [1 + a^2 t_\mu (x)]\, [1 + a^2 t_\nu (x + \mu)]\, [1 + a^2 t_\mu (x + \nu)]\, [1 + a^2 t_\nu (x)] \right\}^{1/2} - 1 . \tag{3.10}$$

$\mathcal{L}_G$ has been thus decomposed into the Wilson Lagrangian $\mathcal{L}^W_{\square}$ plus a free term for t_μ plus an interaction term. Such a decomposition makes easy to evaluate the additional Feynman diagrams with respect to those of Wilson which contribute to the effective action to one loop and second order in α^o_μ (see the Figure).



a b

FIG. 1 – Wavy lines refer to quantum gauge fields, circles to background gauge fields and broken lines to the W_μ–field.

For their evaluation it is sufficient to expand $T_{\mu\nu}$ to second order in t_μ

$$T_{\mu\nu} \simeq a^2 (t_\mu + t_\nu) + \frac{1}{2} a^3 (\Delta_\mu t_\nu + \Delta_\nu t_\mu) + a^4 \; t_\mu t_\nu$$

$$+ \frac{a^5}{2} (t_\mu \Delta_\mu t_\nu + t_\nu \Delta_\nu t_\mu) - \frac{a^6}{8} (\Delta_\mu t_\nu - \Delta_\nu t_\mu)^2 \tag{3.11}$$

and to retain in $\mathcal{L}^W_{\mu\nu}$ only the quadratic terms

$$\mathcal{L}^W_{\mu\nu} \simeq \frac{1}{16} \beta \left[\vec{G}_{\mu\nu} (\alpha) + \vec{G}_{\mu\nu} (\alpha^o) \right]^2 . \tag{3.12}$$

Finally the part of $\mathcal{L}_G$ relevant to the present calculation is

$$\mathcal{L}_G \simeq \mathcal{L}_o + \mathcal{L}_{int} \tag{3.13}$$

where, taking into account that $\beta = \frac{4}{g^2}$

$$\mathcal{L}_o = \frac{1}{g^2} \frac{1}{4} \sum_{\mu \neq \nu} G^2_{\mu\nu} (\alpha^o) + \frac{1}{2} \gamma^2 \sum_\mu t^2_\mu \tag{3.14}$$

$$\mathcal{L}_{int} = \frac{1}{g^2} \frac{1}{4} \sum_{\mu \neq \nu} T_{\mu\nu} \left[\vec{G}_{\mu\nu} (\alpha) + \vec{G}_{\mu\nu} (\alpha^o) \right]^2 . \tag{3.15}$$

The propagators of t_μ and α_μ are

$$< t_\mu (x) \, t_\nu (y) > = \frac{1}{\gamma^2 a^4} \delta_{\mu\nu} \delta_{xy}$$

$$< \alpha_{\mu a} (x) \; \alpha_{\nu b} (y) > = \frac{g^2}{a^4} K (x - y) \, \delta_{\mu\nu} \, \delta_{ab} , \tag{3.16}$$

where

$$K (x - y) = \frac{1}{L^4} \sum_n K_n \, e^{i \frac{2\pi}{N} n \cdot x} \quad , \qquad L = aN \tag{3.17}$$

$$K_n = \frac{a^6}{2} \left[\sum_\mu (1 - \cos \frac{2\pi}{N} n_\mu) \right]^{-1} , \tag{3.18}$$

L being the lattice edge.

The contribution of diagram a is

$$\frac{1}{g^2\gamma^2}\,(I_1+I_2)\sum_x a^4\,\frac{1}{4}\sum_{\mu\neq\nu} G^2_{\mu\nu}(\alpha^o) \tag{3.19}$$

where

$$I_1=\frac{1}{N^4}\sum_{n_\mu}\frac{\left(1-\cos\frac{2\pi}{N}\,n_1\right)\left(1+\cos\frac{2\pi}{N}n_2\right)}{\sum_\mu 1-\cos\frac{2\pi}{N}\,n_\mu}$$

$$I_2=\frac{1}{N^4}\sum_{n_\mu}\frac{\sin^2\frac{2\pi}{N}\,n_1}{\sum_\mu 1-\cos\frac{2\pi}{N}\,n_\mu}\,. \tag{3.20}$$

Their numerical value is

$$I_1+I_2=0.38323. \tag{3.21}$$

The contribution of diagram b is

$$\frac{1}{2g^2\gamma^2}\sum_x a^4\,\frac{1}{4}\sum_{\mu\neq\nu} G^2_{\mu\nu}(\alpha^o)\,. \tag{3.22}$$

The sum of the two contributions is

$$E\sum_x a^4\,\frac{1}{4}\sum_{\mu\neq\nu} G^2_{\mu\nu}(\alpha^o)\,, \tag{3.23}$$

where

$$E=\frac{1}{g^2\gamma^2}\left(\frac{1}{2}+I_1+I_2\right). \tag{3.24}$$

This is an additional finite term in the background field effective action with the present regularization. It corresponds to a correction to the Wilson renormalization scale parameter. Following Dashen and Gross we find that this correction is

$$\Lambda_{NC}=\Lambda_W\,e^{-\frac{12\pi^2}{11}E}\,. \tag{3.25}$$

This is not our final result since the scale parameter is defined in the continuum region, so that we must determine the behaviour of E for vanishing lattice spacing. Its dependence on g seems to give rise to a divergence which would imply absence of scaling and therefore a surprising inconsistency of our regularization. This is however not the case since also the partameter γ depends on a. There is a simple argument to show that the dependence is such that E vanishes proportionally to g^2 in the extreme scaling limit. In order to evaluate radiative corrections to the coupling g and γ in perturbation theory it is convenient to rescale t_μ and α_μ according to

$$t_\mu \to \frac{1}{\gamma} t_\mu$$

$$\alpha_\mu \to g\, \alpha_\mu , \tag{3.26}$$

so getting

$$\mathcal{L}_o = \frac{1}{4} \sum_{\mu \neq \nu} G^2_{\mu\nu} (\alpha^o) + \frac{1}{2} \sum_\mu t^2_\mu$$

$$\mathcal{L}_{int} = \frac{1}{4} \sum_{\mu \neq \nu} T_{\mu\nu} \left(\frac{1}{\gamma} t_\mu \right) \left[\vec{G}_{\mu\nu}(\alpha) + \vec{G}_{\mu\nu} (\alpha^o) \right]^2 \tag{3.27}$$

Now γ appears as an inverse coupling constant in $\mathcal{L}_{int}$. In order to compute the radiative corrections to this coupling constant we have to consider the possible divergences appearing in the two–point functions of t_μ and α_μ and of course the radiative corrections to $\mathcal{L}_{int}$. Owing to the fact that the field t_μ has no renormalizable coupling no logarithmic divergences will appear in its two–point function, and the only diagrams contributing to logarithmic divergent radiative corrections to $\mathcal{L}_{int}$ contain a single $\mathcal{L}_{int}$ vertex and only α_μ–lines. It follows that the divergent corrections to $\mathcal{L}_{int}$ coincide will those to $\mathcal{L}_{\square}^{W}$. They are fairly well known[8] since they do not depend on the regularization.

It turns out that

$$\left(\frac{\gamma_o}{\gamma} \right)^2 = \left(\frac{g}{g_o} \right)^4 \quad \text{up to higher orders,} \tag{3.28}$$

g_o and γ_o being the bare constants, since this ratio is the renormalization constant of $G^2_{\mu\nu}$. So we can write to one loop order

$$\gamma = \gamma_1 \frac{1}{g^2} + \gamma_2 , \tag{3.29}$$

where γ_1 is a free parameter while γ_2 can be determined by taking into account the higher order terms in eq. (3.28). In conclusion

$$E = \left(\frac{1}{2} + I_1 + I_2 \right) \frac{1}{g^2 (\gamma_1/g^2 + \gamma_2)^2} . \tag{3.30}$$

4. – CONCLUSIONS

Let us confront our results to our motivations. From the point of view of perturbation theory, there is no simplification with respect to Wilson regularization owing to the local degeneracy of the vacuum at the semiclassical level.

In the framework of nonperturbative calculations the present regularization avoids the use of the Haar measure and makes it possible to investigate confinement with non compact gauge fields. Actually it has been shown[9)] by a Monte Carlo calculation that there exists a nonvanishing string tension compatible with the scaling determined by Eqs. (3.25) and (3.30).

The renormalization scale parameter turns out to be the same as Wilson's. This is easy to understand if we remember that Wilson regularization is recovered by putting since the very beginning $\beta_c = \infty$, i.e. $\gamma^2 = \infty$ for which $t_\mu = 0$. Here γ^2 grows proportionally to β^2 in the scaling regime. So unitarity of the link variables, rather that being imposed ab initio, is reached gently because of the stability of Wilson fixed point. The hope to introduce a different scale relies therefore on a different choice of ε in eq. (2.14).

In conclusion we have shown that it is possible to construct a consistent regularization with non compact gauge fields. It is then very well possible that there exist others with different properties.

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(*) The following formulae are slightly different from the corresponding ones in refs. 1, 2 due to a different normalization of the generators of SU(2).

(**) We remind that the scaling dimension $[\phi]$ of a field ϕ whose propagator behaves as $p^{-\sigma}$ for large p in the continuum limit, is $[\phi] = \frac{4-\sigma}{2}$.

(***) It is perhaps worth while noticing that this symmetry is nearly local. Both $\mathcal{L}_{YM}$ and t_μ remain in fact unchanged if, for fixed $\hat{\mu}$, we perform the transformation (2.16) on all the sites in the hyperplane orthogonal to $\hat{\mu}$. For instance using this symmetry one can show that

$$< W_\mu (x) > = \frac{1}{a}$$

$$< W_\mu (x) W_\mu (y) > = \frac{1}{a^2}$$

if x and y do not belong to the same hyperplane orthogonal $\hat{\mu}$. This nearly local character of the transformation, however, is broken both by gauge fixing and by interaction with matter fields (see below).