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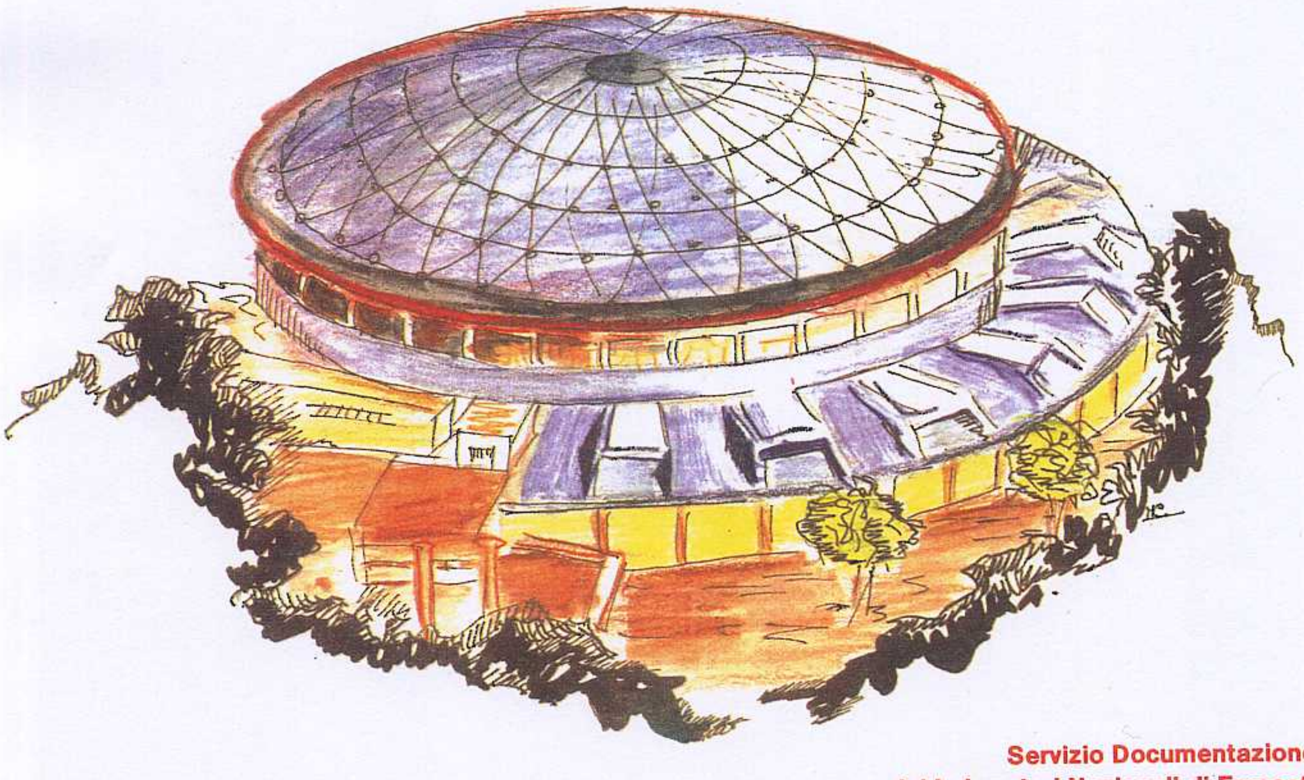

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THE STANDARD MODEL ON A PLANCK LATTICE: EMERGENCE OF THE $\bar{t}t$–CONDENSATE AND DISAPPEARANCE OF THE HIGGS PARTICLE

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ABSTRACT

Motivated by the possibility that the violent fluctuations of quantum gravity at the Planck scale might endow space-time with a lattice structure with lattice constant $a_p \sim 10^{-33}$cm (the Planck lattice), we have reformulated the Standard Model (SM) on such lattice by adding Nambu-Jona Lasinio terms, quadrilinear in the Fermi fields, necessary to avoid the "no-go" theorem of Nielsen and Ninomiya. We find that for certain values of the new Fermi-couplings a spontaneous violation of the SM chiral symmetry emerges which (i) avoids the "no-go"theorem, (ii) produces a kind of $\bar{t}t$ condensate model,(iii) does not exhibit the low-energy scalar particle, that in the continuum $\bar{t}t$-condensate surrogates the Higgs boson.

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The problem of how mass gets generated in the Standard Model (SM) of fundamental interactions, $SU_c(3) \otimes SU_L(2) \otimes U_Y(1)$, is perhaps the most important that is now facing both theoretical and experimental high-energy physics. As well known, the mechanism of mass generation now generally considered, based on the Anderson-Higgs-Kibble mechanism associated to a fundamental local scalar isodoublet Yukawa-coupled to the basic Fermi-fields (quarks and leptons), is also generally believed to be "too ugly" to be really fundamental, leading to the conviction that it must be but the simple surrogate of a deeper, yet to be found and understood, layer of particle interactions. Of particular interest in this direction is the recently proposed $\bar{t}t$-condensate model (developed by Nambu [1] and other research groups [2]) that, reviving the Nambu-Jona Lasinio (NJL) proposal [3] of a four-fermion chiral interaction, appears to point towards a powerful link between the $W^{\pm}$ and Z° masses and those of the yet to be discovered top-quark and Higgs meson, the latter being essentially a $\bar{t}t$-scalar bound state. However it cannot be denied that the addition of a NJL-term to the usual gauge-invariant Lagrangian density gravely lacks a compelling motivation.

In a recent paper [4], inspired by an interesting result, the "no-go" theorem of Nielsen-Ninomiya [5], which stipulates that no simple "lattice transcription" of the SM Lagrangian exists on any type of lattice (or, equivalently, for any type of "regularized" theories), indicating that a consistent chiral gauge lagrangian on a lattice must necessarily include extra interaction, quadrilinear in Fermi fields. Thus we asked whether the physical incompleteness (lack of mass-generation) of the SM, as formulated in continuous space-time, could not be the symptom of a basic lattice structure of space-time: the arena of physical reality. We also recalled that proposals exist [6], based on the violent quantum fluctuations of the metric field at space-time distances of the order of the Planck-length $a_p \sim 10^{-33}$cm, that endow space-time with a "foamlike" structure of grain-size a_p, thus making it equivalent to a 4-dimensional random lattice of lattice constant a_p, which we shall call a Planck lattice. Accepting now, as our main hypothesis, that we do live on the Planck lattice, the "no-go" theorem of Nielsen and Ninomiya tells us that if we wish to construct a consistent SM lagrangian we must go beyond the simple Planck lattice transcription of the continuum Lagrangian, and add to it new terms, whose simplest structure must be quadrilinear in the fundamental fermion fields. This we did in Ref. [4], obtaining the following expression for the Planck lattice SM effective action

$$S_{PL} = S_G + \sum_F (S_D^F + S_{NJL}^{F1} + S_{NJL}^{F2}), \tag{1}$$

where S_G is the usual Wilson gauge-action, S_D the usual Dirac action, which will both be analyzed in a forthcoming article[7], F=l (F=q) denotes its lepton (quark) sector. The new quadrilinear NJL-terms can be written as:

$$S_{NJL}^{F1} = G_1 \sum_x \{\bar{\psi}_L^{Fi}(x) \cdot \psi_R^{Fj}(x) \bar{\psi}_R^{Fj}(x) \cdot \psi_L^{Fi}(x)\} \tag{2}$$

and

$$S_{NJL}^{F2} = \frac{G_2}{2} \sum_{\pm\mu,x} \left[\bar{\psi}_L^{Fi}(x) G_\mu^L(x) U_\mu^c(x) \cdot \psi_R^{Fj}(x+a_\mu) \bar{\psi}_R^{Fj}(x) G_\mu^R(x) U_\mu^c(x) \cdot \psi_L^{Fi}(x+a_\mu)\right], \tag{3}$$

where the indices i, j denote fermion families; the Dirac indices are denoted by scalar product ".". The gauge link $U_\mu^c(x)$ connects left-and right-handed quark fields in neighbouring points so as to have the $SU_c(3)$ gauge symmetry. The chiral gauge links $G_\mu^L(x)$ ($G_\mu^R(x)$) connect left-handed (right-handed) fermion fields to enforce $SU_L(2) \otimes U_Y(1)$ chiral gauge symmetry. $G_{1,2}$ are two, yet unspecified, Fermi-type $O(a_p^2)$ coupling constants which are assumed universal for both the lepton and quark sectors. Thus action (1) is invariant under chiral gauge symmetries $SU^c(3) \otimes SU_L(2) \otimes U_Y(1)$ and a global $U_L(3) \otimes U_R(3)$ in generation space. In addition it evades, at least in principle, the "no-go theorem". The question now is: does it yield a long wave-length spectrum in agreement with observation (free from the unwanted doublers)?

In order to get a positive answer to this crucial question it is necessary that the quadrilinear terms S^{F1}_{NJL} and S^{F2}_{NJL} develop a dynamical chiral symmetry breaking through the following non-zero vacuum expectation values (V_4 is the 4-dimensional volume)

$$\begin{aligned} m_F^{ij} &= -\frac{G_1}{2V_4}\sum_x \langle \bar\psi_i^F(x)\psi_j^F(x)\rangle; \\ \bar r_F^{ij} &= \frac{G_2}{4V_4}\sum_{\mu,x}\{\langle \bar\psi^F_{Li}(x)U^c_\mu(x)\psi^F_{Rj}(x+a_\mu)\rangle + \text{h.c.}\}. \end{aligned} \tag{4}$$

Notice that the analogous expectation values of the psendoscalar (γ_5) operators can be made to vanish in an appropriate gauge. Indeed, should this happen, one would obtain the following effective lattice action of the Wilson type [8]:

$$\begin{aligned} S^{eff}_{PL} &= S_G + S_D \\ &+\sum_{xF}\left\{\bar\psi^F(x)M_F\psi^F(x) - \frac{1}{2}\sum_\mu\left(\bar\psi^F(x)G^L_\mu(x)\bar r_F G^R_\mu U^c_\mu(x)\psi^F(x+a_\mu) + \text{h.c.}\right)\right\}, \end{aligned} \tag{5}$$

where $M_F = m_F + 4\bar r_F$. Thus through dynamical symmetry breaking the obligatory (in order to get rid of the doublers) Wilson term ($r_F = a_p\bar r_F$) gets produced together with a mass term (m_F) which, according to our action, must necessarily come with it. In this way the evasion of the "no-go theorem" entails an extra bonus: the generation of a fermion mass term. The $SU_L(2)\otimes U_Y(1)$ and $U_L(3)\otimes U_R(3)$ symmetries are clearly broken and the only surriving gauge symmetries $SU_c(3)$ and $U_{em}(1)$[7]. Given the quadrilinear NJL-terms (2) and (3), we construct an effective potential in terms of m_F, $\bar r_F$ [7]. A non-trivial dynamical symmetry breaking may emerge only if the matrices m_F, $\bar r_F$ obey a set of coupled, self-consistent equations obtained from variation of the effective potential. We call these equations "gap equations", that turn out to have the following approximate matrix-form in the weak-isospin space

$$m_F a_p + 4r_F = 2g_1\int_l \frac{m_F a_p + 2r_F\sin^2\frac{l_\mu}{2}}{\text{Den}^F(l)}, \tag{6}$$

$$r_F = -g_2 \int_l (\cos) \frac{m_F a_p + 2r_F \sin^2 \frac{l_\mu}{2}}{\mathrm{Den}^F(l)}, \tag{7}$$

where $g_{1,2} a_p^2 = N_c G_{1,2}$; $r_F = a_p \bar{r}_F$; $\int_l = \int_{-\pi}^{\pi} \frac{d^4 l}{(2\pi)^4}$; $(\cos) = \sum_\mu \cos l_\mu$ and

$$\mathrm{Den}^F(l) = \sin^2 l_\mu + (m_F a_p + 2r_F \sin^2 \frac{l_\mu}{2})^2. \tag{8}$$

We shall now analyse the solutions of these equations. By an appropriate global transformation belonging to the symmetry group of lagrangian (1) we can diagonalize the m_F and r_F. In addition we can separate eqs.(6) into two "homogeneous" gap equations. Thus we write:

$$m_F a_p = 2g_1 \int_l \frac{m_F a_p}{\mathrm{Den}^F(l)}, \tag{9}$$

$$r_F = g_1 \int_l \frac{r_F \sin^2 \frac{l_\mu}{2}}{\mathrm{Den}^F(l)}, \tag{10}$$

$$r_F = -g_2 \int_l (\cos) \frac{m_F a_p + 2r_F \sin^2 \frac{l_\mu}{2}}{\mathrm{Den}^F(l)}. \tag{11}$$

If we look for "physical" solutions, for which the eigenvalues of the mass matrix are such that $m_F a_p \ll 1, m_F^5 a_p \ll 1$ and the doublers are removed by $r_F \neq 0$, then eq. (11′) approximately decouples and becomes

$$1 = -g_2 \int_l (\cos) \frac{2\sin^2 \frac{l_\mu}{2}}{\sin^2 l_\mu + 4r^2 (\sin^2 \frac{l_\mu}{2})^2}. \tag{12}$$

Notice that, assuming a universal coupling constant G_2 in (1), the Wilson parameters for quarks and leptons are different because of the color number N_c. The solution $r = r_F(g_2)$ is reported in Fig. 1. We see that for any $g_2 \neq 0$, one gets $r > 0$, which removes the doublers through a Wilson-type mechanism. By using now the gap equation (10) for $m_F a_p \neq 0$ and the chain approximation, for the Goldstone modes $\langle \bar{\psi}_i(x)\gamma_5\psi_i(x)\bar{\psi}_i(0)\gamma_5\psi_i(0)\rangle$ and scalar modes $\langle \bar{\psi}_i(x)\psi_i(x)\bar{\psi}_i(0)\psi_i(0)\rangle$, we find the inverse propagators

$$\Gamma_{Fp}^{-1}(q) = B^2(q) I_p^F(q); \qquad I_p^F(q) = -\frac{N_c}{2} \int_l \frac{\sum_\mu (\cos^2 l_\mu + r_F^2 \sin^2 l_\mu)}{\mathrm{den}^F(l + \frac{q}{2})\, \mathrm{den}^F(l - \frac{q}{2})}; \tag{13}$$

$$\Gamma_{Fs}^{-1}(q) = 2(B^2(q) I_s^F(q) + 4M_F^2); \; I_s^F(q) = \frac{N_c}{4} \int_l \frac{\sum_\mu (\cos^2 l_\mu)}{\mathrm{den}^F(l + \frac{q}{2})\, \mathrm{den}^F(l - \frac{q}{2})} \tag{14}$$

where $B^2(q) = \sum_\mu \left(\frac{4}{a_p^2}\sin^2\frac{q_\mu a_p}{2}\right)$ and

$$4M_F^2 = 4\int_l \frac{\left[m_F + \frac{2r_F}{a_p}\sin^2\frac{l_\mu}{2}\right]^2}{\mathrm{den}^F\left(l+\frac{q}{2}\right)\mathrm{den}^F\left(l-\frac{q}{2}\right)}. \tag{15}$$

For $q_\mu a_p \ll 1$, we can calculate numerically the position of the pole of the scalar mode, which turns out to be of the order of the Planck mass instead of $4m_F^2$. Indeed one gets

$$4M_F^2 = 4m_F^2 + 0.8r\frac{m_F}{a_p} + 0.9\frac{r^2}{a_p^2}, \tag{16}$$

which for $r = 1$ pushes this pole at the Planck mass, making it thus disappear from the observable, low energy spectrum. Analogously, we find charged Goldstone modes appearing in the flavoured channels corresponding to the quantum numbers of the $W^\pm$ bosons.

In order to understand the implications of obtaining physical solutions $m_F a_p \ll 1$, we use the gap-equations (11) and (11′) to draw the phase diagram (Fig.2), where the phases ($m < 0$) are separated from the phase ($m > 0$) by critical lines, on which $m = 0$. This phase diagram is in agreement with the continuum NJL-model that possesses only the coupling constant G_1. The requirement of obtaining physical solutions for quarks $N_c = 3$ with $\bar{r}_q a_p = r_q \sim O(1)$ and $m_F a_p \ll 1$ can clearly be met by a coupling constant $g_2 \gg 1$, in this case one has $r \to 0.385$. On the other hand the coupling g_1 is required to lie in the vicinity of the critical value g_c^2. This holds for quarks, for it is easy to see that for leptons ($N_c = 1$) equation (10) does not allow for massive solutions. Thus leptons at this stage remain massless.

Promising though it may appear, our solution does not seem yet to avoid a phenomenological disaster, for quarks get equally massive and, furthermore, 36 Goldstone modes appear. The phenomenologically appealing $\bar{t}t$-condensate model, on the other hand, postulates that of 6 quarks only one, the top-quark t, acquires through the NJL-mechanism a large mass, that is comparable with the $W^\pm$ and

Z° masses. Is there any way, by a particular fine-tuning of G_1, to see the emergence of this model from our approach? We shall now show that this does indeed happen. The gap-equation (6) is only true in the lowest approximation; if we go further (by iteration) to calculate the effective potential (the ground state energy), including the contributions from the Goldstone-bosons and the composite scalars to the effective potential (Fig.3), going through the same variational procedure that produces the "mean field" gap-equations, instead of (10) one obtains a new gap-equation:

$$m_F a_p = \frac{2g_1}{(1 + N_q C_0(m_F a_p, r))} \int_l \frac{m_F a_p}{\mathrm{Den}^F(l)} \qquad (17),$$

where $C_0(m_F a_p, r)$ is a finite positive function[7]. The crucial difference between the new and the old gap-equation is its non-trivial dependence on the number of quarks N_q that acquire masses through the NJL-mechanism. The phase diagram and the critical ($ma_p = 0$) lines will now depend strongly on N_q. And in view of the discrete nature of this variable, there will be a discrete and finite number of possible solutions, characterized by N_q. The $\bar{t}t$ condensate model is thus the solution for which $N_q = 1$.

In conclusion, we have shown that on the Planck lattice, whose "raison dêtre" may well reside in the violent quantum fluctuations of the metric field at the Planck scale a_p, a consistent SM requires the addition of extra terms, quadrilinear in the Dirac fields, whose coupling constants can be determined to induce the emergence of the $\bar{t}t$-condensate model with $m_t a_p \ll 1$ and $0 < r \leq 1$: m_t gives rise to the scale of electroweak breakdown and $r \neq 0$ endows the composite scalar and the mirror fermions with masses at the Planck scale, making them disappear from the low-energy spectrum.

We should also stress that, differently from the continuum $\bar{t}t$-condensate model, its Planck lattice version disposes in a nice way of the scalar composite, thus leading to the disappearance from the spectrum of a particle that would resemble the Higgs boson. The implication of this conclusion for the present and future phenomenology is too obvious to need further comments.

At the stage where we now leave our work, and where we have completely neglected the interaction of the fermions with the gauge-fields, the top-quark is the only fermion that has acquired a mass. However the emergence of non-zero Wilson parameters r for all fermions implies a violation of chiral symmetry even in the sector of the fermions that have remained massless. We hope to be able to report on the consequences of this important fact soon.

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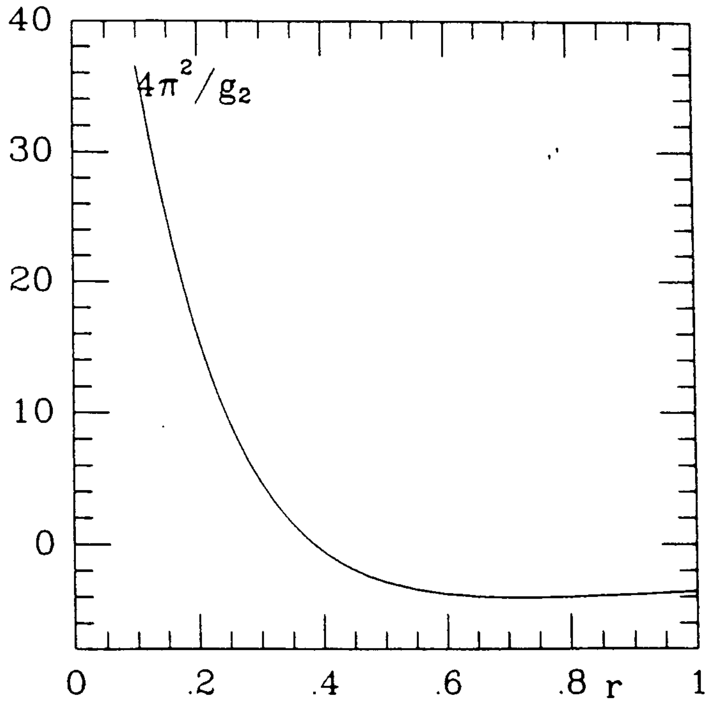

FIG. 1 – $\frac{4\pi^2}{g_2}$ as a function of r.

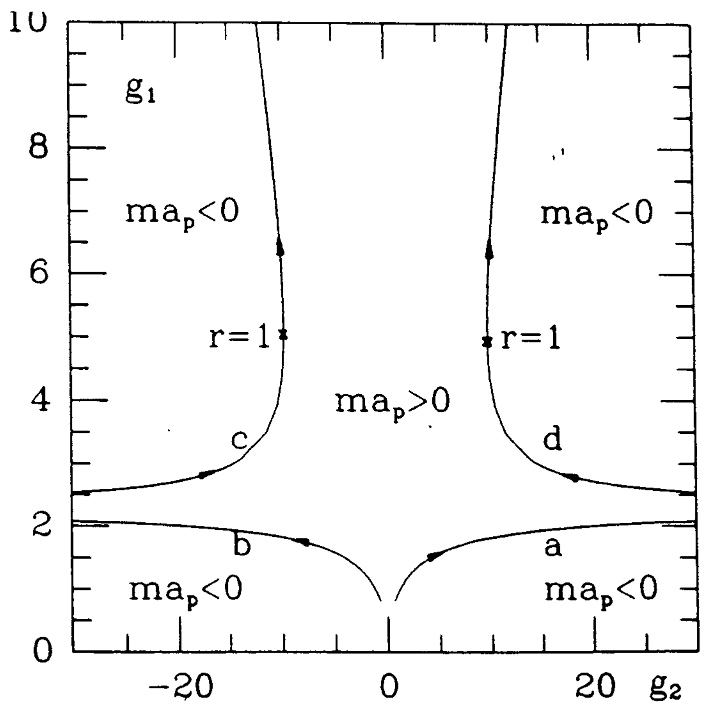

FIG. 2 – Phase diagram in terms of g_1 and g_2, a, b ($r = 0 \rightarrow 0.385$) and c,d ($r = 0.385 \rightarrow 1 \rightarrow \infty$).

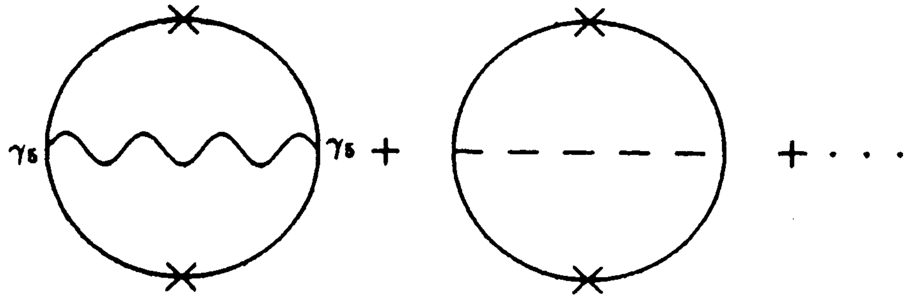

FIG. 3 – The contributions of Goldstone-bosons (wavy line) and composite scalars (dashed line) to the effective potential (ground state energy).