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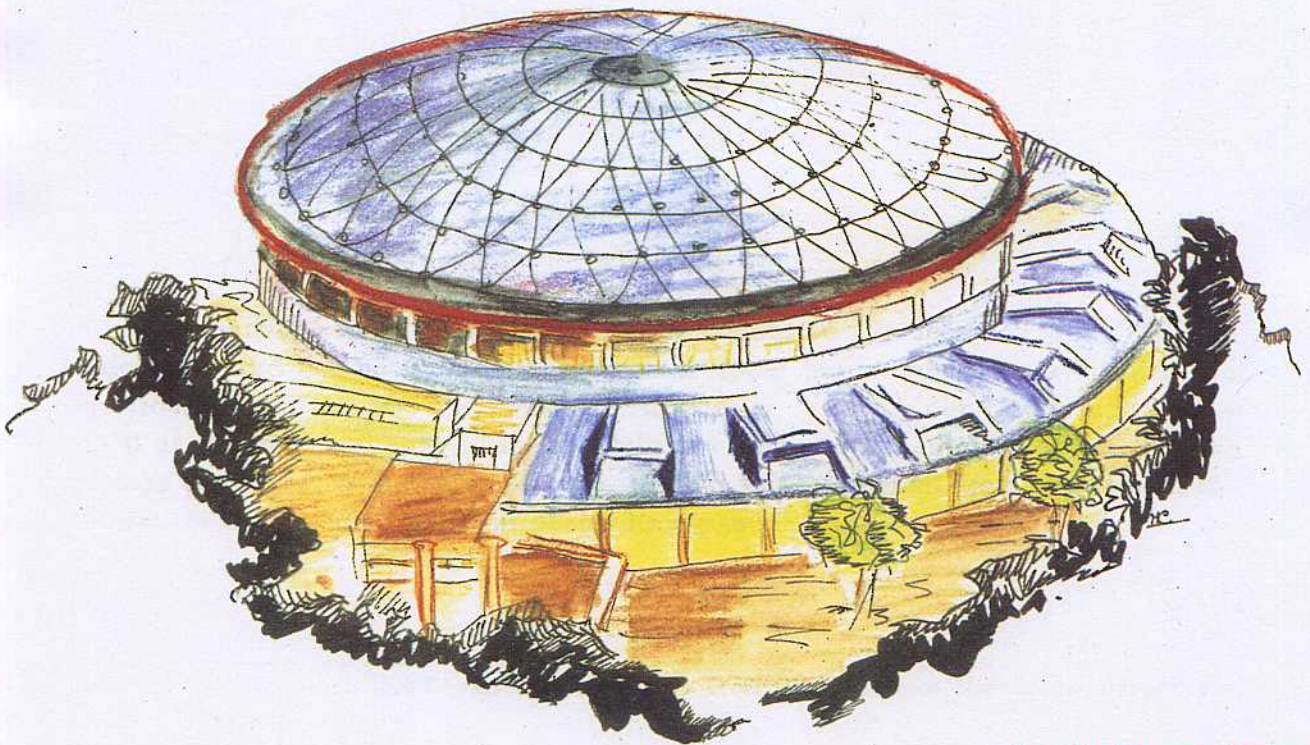
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**INTERMEDIATE VECTOR MESON CONTRIBUTIONS TO $V^0 \rightarrow P^0 P^0 \gamma$
DECAYS**



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INTERMEDIATE VECTOR MESON CONTRIBUTIONS

TO $V^0 \rightarrow P^0 P^0 \gamma$ DECAYS

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Abstract

The contribution of intermediate vector mesons to the decays $V^0 \rightarrow P^0 P^0 \gamma$ is calculated and compared with experimental upper limits, whenever available, and previous estimates. For some decays, like ϕ or $\omega \rightarrow \pi^0 \pi^0 \gamma$ our results update the existing numbers and suggest the feasibility of their experimental detection. For the decays $\phi \rightarrow K^0 \bar{K}^0 \gamma$ and $\phi, \omega \rightarrow \pi^0 \eta \gamma$, we find values different from the ones in the literature and discuss the origin of the discrepancy.

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Radiative decays of low-mass vector-mesons into a single photon and a pair of neutral pseudoscalars have not been detected up-to-now. Only upper limits for three branching ratios of this type of processes have been established (see PDG [1],[2]), namely, $\text{BR}(\omega \rightarrow \pi^0\pi^0\gamma) \leq 4 \cdot 10^{-4}$, $\text{BR}(\phi \rightarrow \pi^0\pi^0\gamma) \leq 10^{-3}$ and $\text{BR}(\phi \rightarrow \pi^0\eta\gamma) < 2.5 \cdot 10^{-3}$. This contrasts with the situation concerning similar radiative decays involving a charged pseudoscalar pair or only one neutral pseudoscalar in the final state. In the first case (particularly, in the observed $\rho^0 \rightarrow \pi^+\pi^-\gamma$ process) one has to deal with a bremsstrahlung photon, thus obtaining much larger branching ratios but reducing the physical interest of such an emission. In the second one, whose best example is the long-ago detected $\omega \rightarrow \pi^0\gamma$ decay, the two-particle phase-space enlarges the rate and the absence of charged particles makes the dynamics more interesting. Indeed, many succesful theoretical ideas such as Vector-Meson-Dominance (VMD), quark model, (anomalous) effective lagrangians... have been originated or tested in this kind of $V \rightarrow P\gamma$ transitions.

With the advent of high-luminosity, low-energy e^+e^- -machines, rare decays of vector mesons with branching ratios even smaller than 10^{-6} will be studied. More precisely, the Frascati Φ -Factory is expected to provide $\sim 10^{10}$ ϕ -decays per year thus allowing for analyses of final states such as $\pi^0\pi^0\gamma$, $\pi^0\eta\gamma$ and, in principle, $K^0\bar{K}^0\gamma$. Their dynamics includes aspects of low-energy hadron physics and, particularly, the effects of well-known vector mesons and of largely unknown scalar mesons. Among the latter, one has the $f_0(975)$ and $a_0(980)$ mesons, whose rather controversial nature could be hopefully clarified [3]. Indeed, their effects will probably manifest themselves in the above ϕ -decays, superimposed to the well understood contributions of vector mesons. The purpose of the present note is to provide a rather exhaustive analysis of these contributions. On the one hand, they are essential for understanding new effects and, on the other, previous studies [4, 5, 6, 7] are based on old data or turn out to be rather incomplete and not entirely free from some contradictions.

All the couplings of our amplitudes can be deduced from the two well-known lagrangians obeying the SU(3)-symmetry dictates

$$L(V\gamma) = -2egf^2 A^\mu \text{tr}(QV_\mu) \tag{1}$$

$$L(VVP) = \frac{G}{\sqrt{2}} \epsilon^{\mu\nu\alpha\beta} \text{tr}(\partial_\mu V_\nu \partial_\alpha V_\beta P)$$

where A^μ is the photon field, Q is the quark charge matrix, V stands for the vector meson nonet and $G = 3\sqrt{2}g^2/(4\pi^2 f)$ is the $\rho^0\omega\pi^0$ coupling constant. The latter played an essential role in conventional VMD results, together with the $V\gamma$ coupling constant, f_V . Both of them have been written in terms of g and $f_\pi \equiv f$ to make contact with modern Chiral and effective lagrangian schemes. From the first eqs.(1) one immediately obtains the $V\gamma$ -couplings

$$f_\rho = f_\omega/3 = -\sqrt{2}f_\phi/3 = \sqrt{2}g = 5.9 \tag{2}$$

in good agreement with the data [1]. With this value of $g = 4.2$ and the pion decay constant $f = 132$ MeV, eqs.(1) satisfactorily describe the set of data for $V \rightarrow P\gamma$. For instance, one predicts $\Gamma(\omega \rightarrow \pi^0\gamma) = 0.76$ MeV quite close to the experimental

value [1] of 0.72 ± 0.05 MeV. Similarly, one can adjust the experimental result [1], $\Gamma(\phi \rightarrow \pi^0 \gamma) = 5.8 \pm 0.6$ assuming that the small contamination of non-strange quarks in the ϕ meson is given by

$$\epsilon = -0.059 \pm 0.004, \quad (3)$$

where the sign comes from observed $\omega - \phi$ interference effects in $e^+ e^- \rightarrow \pi^+ \pi^- \pi^0$ [1] (also compatible with the modulus of eq.(3)). The combined use of both lagrangians (1) leads to the correct values for the axial-anomaly, *i.e.*, $\Gamma(\pi^0 \rightarrow \gamma \gamma) = 7.6$ eV (experiment [1] requires 7.7 ± 0.6 eV), and for the SU(3) rotated processes $\eta, \eta' \rightarrow \gamma \gamma$ [8]. In the last cases one needs to implement the quark content of the η and η' mesons through the phenomenologically preferred [4, 8] η - η' mixing angle $\theta_P = \arcsin(-1/3) = -19.5^\circ$ leading to

$$\begin{aligned} \eta &\sim \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} - s\bar{s}) \\ \eta' &\sim \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d}) + \sqrt{\frac{2}{3}}s\bar{s} \end{aligned} \quad (4)$$

All these results confirm the correctness of the lagrangians (1). Essentially, these have been a part of the traditional successes of VMD combined with simple quark model arguments. From a more modern point of view, eqs.(1) are the main ingredients of the anomalous part of Chiral lagrangians [9, 10] incorporating vector mesons as gauge fields of a "hidden" symmetry [11, 12]. In any case, the lagrangians (1) and the quoted values for their coupling constants can be (and have been) used to predict the intermediate vector-meson contributions to $V^0 \rightarrow P^0 P^0 \gamma$ to which we now turn.

From the kinematical point of view these processes involve the two following amplitudes

$$\begin{aligned} \{a\} &= (\epsilon^* \cdot \epsilon) (q^* \cdot q) - (\epsilon^* \cdot q) (\epsilon \cdot q^*) \\ \{b\} &= -(\epsilon^* \cdot \epsilon) (q^* \cdot P) (q \cdot P) - (\epsilon^* \cdot P) (\epsilon \cdot P) (q^* \cdot q) \\ &\quad + (\epsilon^* \cdot q) (\epsilon \cdot P) (q^* \cdot P) + (\epsilon \cdot q^*) (\epsilon^* \cdot P) (q \cdot P) \end{aligned} \quad (5)$$

where $\epsilon(\epsilon^*)$ are the polarizations of the final photon (initial vector meson V^0), and q ($q^* = q + p + p'$) are the corresponding four-momenta; $P = p + q$ and $P' = p' + q$ are those for the virtual (intermediate) vector mesons (V and V') of the direct and crossed terms. The total amplitude is then found to be

$$A(V^0 \rightarrow P^0 P^0 \gamma) = C_{V^0 P^0 P^0 \gamma} \left(\frac{G^2 e}{g\sqrt{2}} \right) \left\{ \frac{P^2 \{a\} + \{b(P)\}}{M_V^2 - P^2 - iM_V \Gamma_V} + \frac{P'^2 \{a\} + \{b(P')\}}{M_{V'}^2 - P'^2 - iM_{V'} \Gamma_{V'}} \right\} \quad (6)$$

where V^0 is the decaying vector meson. The intermediate ones, V and V' , can be either the ω or the ρ -mesons, with $V = V'$ in $\pi^0 \pi^0 \gamma$ and $V \neq V'$ in $\eta \pi^0 \gamma$ -decays; for $\phi \rightarrow K^0 \bar{K}^0 \gamma$ one obviously has $V = K^{*0}$ and $V' = \bar{K}^{*0}$. The coefficient C is the same for both terms (using SU(3)-symmetric couplings) and changes from process to process according to well-known quark-model or nonet-symmetry rules:

$$1 = C_{\rho^0 \pi^0 \pi^0 \gamma} = 3 C_{\omega \pi^0 \pi^0 \gamma} = \frac{3\sqrt{3}}{\sqrt{2}} C_{\rho^0 \pi^0 \eta \gamma} = \sqrt{\frac{3}{2}} C_{\omega \pi^0 \eta \gamma} = -\frac{3}{\sqrt{2}} C_{\phi K^0 \bar{K}^0 \gamma} \quad (7)$$

and

$$\epsilon = 3 C_{\phi \pi^0 \pi^0 \gamma} = \sqrt{\frac{3}{2}} C_{\phi \pi^0 \eta \gamma}$$

for the ϕ -decays where the Zweig-rule is operative.

From the above amplitudes, the partial widths are obtained performing a numerical integration of

$$\Gamma(V \rightarrow PP'\gamma) = \left(\frac{1}{2}\right) \frac{1}{192\pi^3 M_V} \int dE \int dE_P \sum_{pol} |A(V \rightarrow PP'\gamma)|^2 \quad (8)$$

In eq.(8) the factor (1/2) has to be included only in $\pi^0\pi^0$ decays and the limits of the integration over the photon energy E are 0 and $[M_V^2 - (m_P + m_{P'})^2]/(2M_V)$, while those for the pseudoscalar energies E_P are given by

$$\frac{1}{2} \left[(M_V - E) \left(1 + \frac{m_P^2 - m_{P'}^2}{M_V^2 - 2M_V E} \right) \pm E \sqrt{1 - \frac{2(m_P^2 + m_{P'}^2)}{M_V^2 - 2M_V E} + \left(\frac{m_{P'}^2 - m_P^2}{M_V^2 - 2M_V E} \right)^2} \right]$$

Our results are shown in the two last columns of Table 1. For comparison we also include (first column) the upper limit for the three experimentally studied decay rates [1, 2] and the predictions of other authors [4, 6, 7] who have worked in our same context. Our results are not incompatible with those by Singer [5], who first noticed the simple relation $\Gamma(V^0 \rightarrow \pi^+\pi^-\gamma) = 2\Gamma(V^0 \rightarrow \pi^0\pi^0\gamma)$ for the VMD part of the rate. This relation allows for a comparison of our results with those by Renard [6], quoted (in parenthesis) in the second column of Table 1. The accompanying values are the original ones [6] simply corrected by the present-day data for $\Gamma(\omega \rightarrow \pi^0\gamma)$ and ϵ , and turn out to be in agreement with our predictions. The agreement with ref.[7] is somewhat less satisfactory. Finally, we disagree in the complete list of numerical predictions quoted in ref.[4] even if the initial expressions for the lagrangians are the same (notice that our coupling constant g has been defined as 1/2 of that in ref.[4]).

Concentrating for the moment on ϕ -decays, one first observes that our vector-meson dominated mechanism predicts a completely negligible $\Gamma(\phi \rightarrow K^0\bar{K}^0\gamma)$, contrasting with the (four orders of magnitude larger) prediction from ref.[4]. We have carefully analyzed our calculation in order to understand the origin of the large numerical difference. Since this channel contains exclusively soft photons ($E \lesssim 25$ MeV),

Table 1:

Global contribution of intermediate vector mesons to decay rates (in eV) and branching ratios (last column) for different $V^0 \rightarrow P^0 P^0 \gamma$ transitions as predicted by several authors. Experimental upper limits are also quoted.

Decay rates (in eV)	EXP [1, 2]	Ref. [6]	Ref. [7]	Ref. [4]	This calculation	
					Γ	B.R.
$\Gamma(\rho \rightarrow \pi^0\pi^0\gamma)$	—	(2.5) $1.6 \cdot 10^3$	—	$4.3 \cdot 10^3$	$1.6 \cdot 10^3$	$1.1 \cdot 10^{-5}$
$\Gamma(\rho \rightarrow \pi^0\eta\gamma)$	—	—	—	593	0.061	$4 \cdot 10^{-10}$
$\Gamma(\omega \rightarrow \pi^0\pi^0\gamma)$	$< 3.4 \cdot 10^3$	(350) 227	—	690	235	$2.8 \cdot 10^{-5}$
$\Gamma(\omega \rightarrow \pi^0\eta\gamma)$	—	—	—	53	1.39	$1.6 \cdot 10^{-7}$
$\Gamma(\phi \rightarrow \pi^0\pi^0\gamma)$	$< 4.4 \cdot 10^3$	(250) 54	45	153	51	$1.2 \cdot 10^{-5}$
$\Gamma(\phi \rightarrow \pi^0\eta\gamma)$	$< 11 \cdot 10^3$	—	35	228	23.9	$5.4 \cdot 10^{-6}$
$\Gamma(\phi \rightarrow K^0\bar{K}^0\gamma)$	—	—	—	0.18	$1.2 \cdot 10^{-5}$	$2.7 \cdot 10^{-12}$

an analytic expression for the amplitude in this low-E limit can be obtained. One has

$$A(\phi \rightarrow K^0 \bar{K}^0 \gamma) \simeq \frac{eG^2}{3g(M_{K^*}^2 - m_{K^0}^2 - iM_{K^*}\Gamma_{K^*})} \left[(p \cdot p' - m_{K^0}^2) \{a\} + \right. \\ \left. \epsilon^* \cdot (p - p') [(\epsilon \cdot p)(q \cdot p') - (\epsilon \cdot p')(q \cdot p)] \right] \quad (9)$$

where p, p' are the pseudoscalar four-momenta, and

$$\Sigma |A(\phi \rightarrow K^0 \bar{K}^0 \gamma)|^2 = \frac{e^2 G^4}{9g^2} \frac{(p \cdot p' - m_{K^0}^2)}{(M_{K^*}^2 - m_{K^0}^2)^2 + M_{K^*}^2 \Gamma_{K^*}^2} \cdot \\ \left[2(p \cdot p')(q^* \cdot q)^2 - (p \cdot p' + m_{K^0}^2) 4(q \cdot p)(q \cdot p') \right] \quad (10)$$

accidentally containing the small numerical factor $(p \cdot p' - m_{K^0}^2) = (M_\phi^2 - 4m_{K^0}^2)/2$ [1]. In other words, the VMD contribution to the $\phi \rightarrow K^0 \bar{K}^0 \gamma$ decay is predicted to be exceptionally suppressed not only by the obviously scarce available phase-space but also due to an almost complete destructive interference in the amplitude (with the opposite sign, the interference term would enlarge the width by almost 2 orders of magnitude). If $\phi \rightarrow K^0 \bar{K}^0 \gamma$ is detected this would signal decay mechanisms [13, 14, 15] different from ours, such as $\phi \rightarrow f_0, a_0 \gamma \rightarrow K^0 \bar{K}^0 \gamma$ or other final-state interactions in $\phi \rightarrow K^+ K^- \gamma \rightarrow K^0 \bar{K}^0 \gamma$. On the contrary, our mechanism predicts sizable contributions to $\phi \rightarrow \pi^0 \pi^0 \gamma$ and $\eta \pi^0 \gamma$ decays. The corresponding photonic spectra can be found in Figs.1 and 2, where the squared modulus and interference terms are separately shown. The latter contribute to enhance the peak at high E in the $\phi \rightarrow \pi^0 \pi^0 \gamma$ spectrum. Roughly one-half of this decay contains a photon with an energy E in the narrow range $400 \text{ MeV} \lesssim E \lesssim 470 \text{ MeV}$. Alternative mechanisms as the above mentioned are expected to produce mainly low-energy photons ($E \lesssim 100 \text{ MeV}$), thus minimizing the interferences and allowing for separated analyses, particularly in $\phi \rightarrow \pi^0 \pi^0 \gamma$.

In the ρ - ω region the BR for the $\pi^0 \eta \gamma$ final states proceeding through VMD is predicted to be clearly smaller than $BR(\rho, \omega \rightarrow \pi^0 \pi^0 \gamma)$. Also, the latter decays are expected to have a photonic spectra peaked at the higher values of the energy. This has been plotted in Fig.3 for $\omega \rightarrow \pi^0 \pi^0 \gamma$; for $\rho \rightarrow \pi^0 \pi^0 \gamma$ one can essentially obtain the same spectrum by simply rescaling the curves. In this ρ - ω region, well below the mass of the established scalar mesons $f_0(975)$ and $a_0(980)$, the contributions from the latter are expected to be rather suppressed. Final-state interactions could lead to a large value for $BR(\rho \rightarrow \pi^0 \pi^0 \gamma)$ through the chain $\rho^0 \rightarrow \pi^+ \pi^- \gamma \rightarrow \pi^0 \pi^0 \gamma$, but very unlikely for $BR(\omega \rightarrow \pi^0 \pi^0 \gamma)$. Thus one can expect that our VMD contributions should provide a substantial part of the latter ω -decay.

In summary, the well understood contributions of intermediate vector mesons in $V^0 \rightarrow P^0 P^0 \gamma$ decays have been discussed. One predicts $BR(\phi \rightarrow \pi^0 \pi^0 \gamma) = 12 \cdot 10^{-6}$ and $BR(\phi \rightarrow \pi^0 \eta \gamma) = 5.4 \cdot 10^{-6}$, and a characteristic photonic spectrum (peaked at higher energies) in the first decay. Similarly, an exceptionally small contribution is predicted (and its physical origin understood) for the branching ratio $BR(\phi \rightarrow K^0 \bar{K}^0 \gamma)$, namely, $\sim 2.7 \cdot 10^{-12}$. Finally, intermediate vector meson exchange could be the dominant mechanism in $\omega \rightarrow \pi^0 \pi^0 \gamma$ decays with $BR(\omega \rightarrow \pi^0 \pi^0 \gamma) = 2.8 \cdot 10^{-5}$ and a characteristic spectrum peaked at higher photonic energies.

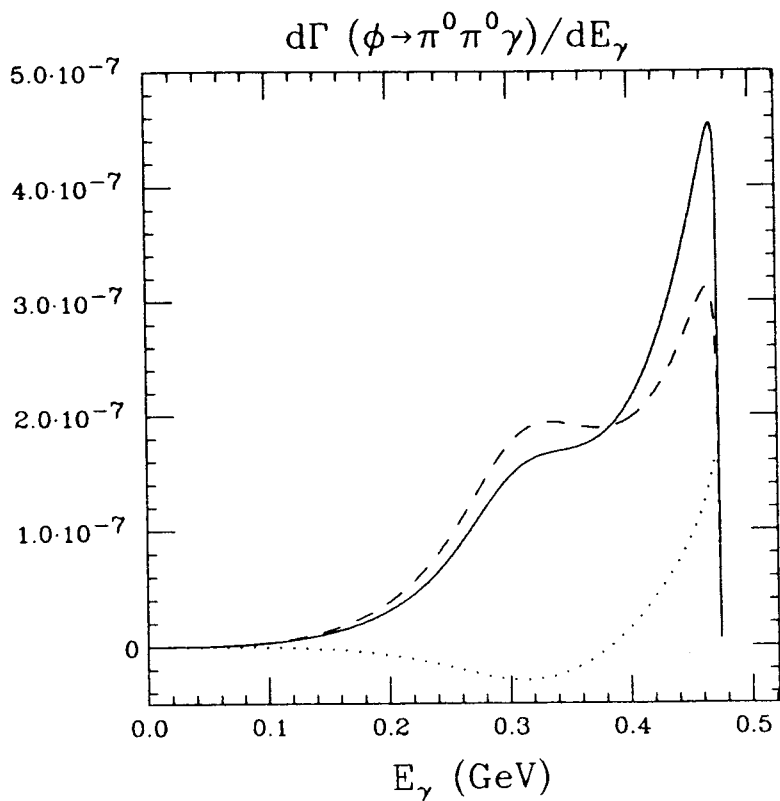


Fig.1 Photonic spectrum generated by intermediate vector-mesons in $\phi \rightarrow \pi^0 \pi^0 \gamma$ (solid line). Dashed and dotted lines correspond to twice the contribution of a single diagram and their interference, respectively.

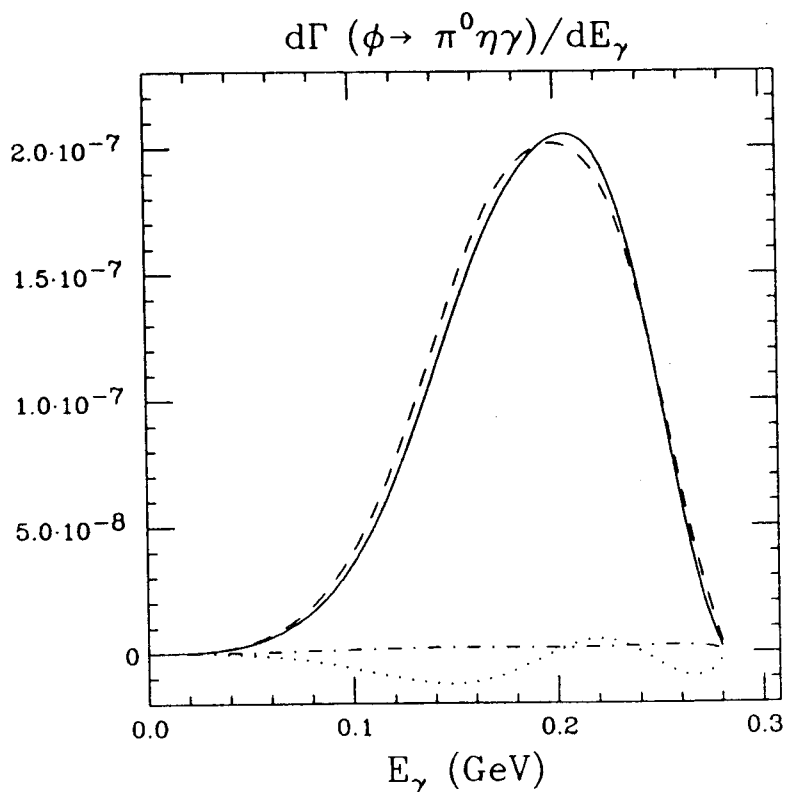


Fig.2 Photonic spectrum in $\phi \rightarrow \pi^0 \eta \gamma$ (solid line). Dashed and dot-dashed lines are the contributions of each diagram and the dotted line corresponds to their interference.

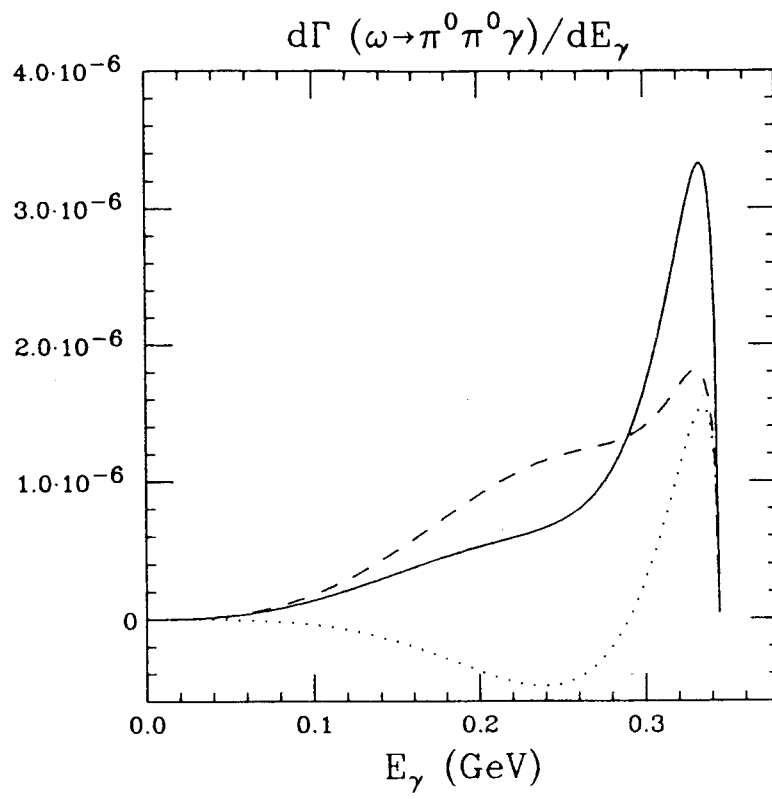


Fig.3 Photonic spectrum in $\omega \rightarrow \pi^0\pi^0\gamma$ with conventions as in Fig.1.

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