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**HIGGS AND TOP: MISSING LINKS OF THE STANDARD MODEL**

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## Higgs and Top: Missing Links of the Standard Model

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### General Introduction

In the beginning, the Standard Model (SM) is well delineated between matter and gauge: All matter is made out of spin- half leptons and quarks and all forces are due to spin-one gauge particles. Until the intervention of the gang of (four) Higgs (spin zero-bosons), both the matter fields and the gauge fields are all massless. Spontaneous breakdown mechanism then produces masses for (perhaps all) the fermions and some of the bosons and in the process divides the Higgs sector into two parts: three out of the initial four fields end up as part of gauge and the one remaining as part of the matter fields. Thus, it seems appropriate to label Higgs as the missing link between matter and gauge sectors of the standard model. This explains the first part of the title. On the other hand, the sixth quark called top <sup>(1)</sup> is so far simply missing from the matter family tree and this explains the second part of the title.

In these lectures, we shall concentrate on models which initially have only *fermions* and some symmetries and where other structures including some local symmetries would arise only upon renormalization. Since such ideas are not so common let us spend some time motivating such a framework.

Based on one's experience with perturbation theory, it has become customary to assume that there are more symmetries present at the classical level than at the quantum level, i.e., some symmetries get "broken" by quantum corrections. But there exist non-perturbative models in field theory where this is not true. A realistic example exhibiting

such a phenomenon has been provided by Terazawa et al<sup>(2)</sup> through a classical 4-fermion interaction Lagrange density possessing a global  $SU(3) \times SU(2) \times U(1)$  symmetry for leptons and quarks. They have shown that the standard  $SU(3) \times SU(2) \times U(1)$  model with a spontaneous breakdown results after one-loop renormalization. Their example shows that there appears a local  $SU(3)$  color symmetry whereas at the classical level there had been present only a global color symmetry. In another example<sup>(3)</sup>, starting from a global  $SU(N)$  multiplet of Majorana fermions with a vector 4-fermion coupling, it was found that after renormalization there resulted an effective Lagrange density with local  $SU(N)$  gauge symmetry as well as supersymmetry.

There is, however, an awkward feature in both of the above models regarding the dynamical growth of the vector bosons. The vector bosons are first introduced with a "mass" which is removed after a regularization designed so that the local gauge symmetry emerges. Thus, the scale and gauge invariance are first manifestly broken and then restored. In section 3 we shall exhibit a model<sup>(4)</sup> which is both scale and gauge invariant and which employs a gauge invariant regularization procedure. Hence, it does not suffer from the above deficiency. We shall exhibit this model in some detail and discuss a few of its interesting consequences.

As in our daily lives any thing which increases our mass (or more commonly the weight) leads to many problems, so does the Higgs. But before discussing the Higgs problem in particular, we recall some generalities about masses in subsections (i) and (ii).

#### (i) Massless photons

It is widely known that the Schrödinger equation for an electron has no classical analogue<sup>(5)</sup>. It is less well-known that the Maxwell equations for the  $\mathbf{E}$  and  $\mathbf{B}$  fields (in vacuum) are correct even quantum mechanically including the fact that they are described by spin one matrices.

Proof:

In the vacuum,  $\mathbf{E}$  and  $\mathbf{B}$  are free of divergence and we have

$$(\text{curl } \mathbf{E}) = -(\partial \mathbf{B} / c \partial t)$$

and

$$(\text{curl } \mathbf{B}) = (\partial \mathbf{E} / c \partial t).$$

These two real equations can be combined into one complex equation by introducing the transverse field

$$\mathbf{F} = \mathbf{E} + i\mathbf{B}.$$

Then,

$$i (\partial \mathbf{F} / \partial t) = c \text{curl} \mathbf{F}.$$

This looks almost like the Schrödinger equation, but let us complete the picture by defining the momentum operator  $\mathbf{p}$  with components  $p_j = -i\hbar\partial_j$ , and the spin matrices  $\mathbf{S}_i$  by  $(\mathbf{S}_i)_{jk} = -i\epsilon_{ijk}$ . We find

$$i\hbar(\partial\mathbf{F}_i/\partial t) = c(-i\epsilon_{ijk})p_j\mathbf{F}_k = c(\mathbf{S}\cdot\mathbf{p})_{ik}\mathbf{F}_k$$

The matrices  $\mathbf{S}$  obey  $[\mathbf{S}_i, \mathbf{S}_j] = i\epsilon_{ijk}\mathbf{S}_k$ , and  $\mathbf{S}^2 = S(S+1)$  with  $S = 1$ . Thus, the Maxwell equations are the Schrödinger equation for the photon and the electromagnetic  $\mathbf{E}$  and  $\mathbf{B}$  fields are indeed the wave functions of the photon.

QED

How is it that Maxwell could find the correct "wave functions" for the photon long before the advent of quantum mechanics? Two physical circumstances account for this "miracle" (not true for non-relativistic electrons, for example):

- (i) The photon has zero mass and so Planck's constant drops out from the field equations.
- (ii) The photon (being a spin one object) obeys Bose statistics. Many photons can thereby exist in the same quantum state. The electromagnetic field can be "Bose condensed".

It is the photon "condensate" that is ultimately responsible for the power generated at an electricity plant to be transformed for our everyday use in our homes. That so many photons can condense into the same mode is perhaps the best understood form of macroscopic quantum behavior.

It is a sobering thought that while the above is achieved so elegantly and easily via Maxwell equations, it is impossible to obtain it in perturbation theory after the canonical quantization. What is being said is simply that the vacuum average of  $\langle \mathbf{E} \rangle$  and  $\langle \mathbf{B} \rangle$  are zero since they are linear in the creation and annihilation operators. To obtain a non-vanishing classical limit, one has to invoke coherent states for the vacuum. But that is another story and shall not be told here.

Of course, the "real" quantum mechanics does enter into the photon sector once it is coupled to the matter field. The " $\hbar$  physics" begins when ordinary derivatives  $\partial_\mu$  for the matter fields get promoted to covariant derivatives  $D_\mu = \partial_\mu - ieA_\mu/\hbar c$ .

We may ask what happens in the non-abelian case. Since the field-strength tensor

$$\mathbf{F}_{\mu\nu}^a = \partial_\mu \mathbf{A}_\nu^a - \partial_\nu \mathbf{A}_\mu^a - (g/\hbar c) f_{abc} \mathbf{A}_\mu^b \mathbf{A}_\nu^c,$$

itself depends on  $\hbar$  through the non-linear term, here the situation is dramatically different even before the intervention of the gang of Higgs. It is interesting to observe that for the unbroken theory, e.g.,  $SU(3)_{color}$ , only color-singlet states are physical and for operators corresponding to these there is indeed a classical Maxwell type limit. As an example, consider an operator such as

$$\mathbf{X}_{\mu\nu} = \mathbf{Y}_\mu^a \mathbf{Z}_\nu^a.$$

The equation of motion for  $X$  has no covariant derivative and hence a classical (Maxwellian) limit exists.

(ii) Non-analyticity in Mass:

Perturbation theory is in general not reliable for discussion of masses near  $m$  equal to zero. The reason being that generally Green's functions are not analytic near  $m = 0$ .

Proof:

Consider an arbitrary Feynman diagram for a generic Green's function with any number of loop momentum integrations over some mass( $m$ ) in a propagator. If we take enough number of derivatives with respect to  $m$  and then set  $m$  to zero the Feynman integral would diverge signalling non-analyticity there.

QED.

The Gap equation in superconductivity or in its resuscitated version of the Top Model is a case in point. An extreme example of non-analyticity in mass occurs if we consider the antisymmetric part of the photon propagator generated via one-fermion loop in (2+1)-dimensions:

$$\begin{aligned} \mathbf{D}_{\mu\nu}^{(-)}(k) &= e^2 \int [(d^3 p)/(2\pi)^3] \text{Tr}[\gamma_\mu(\not{p} - \not{k} - m + i\epsilon)^{-1} \gamma_\nu(\not{p} - m + i\epsilon)^{-1} - (\mu \leftrightarrow \nu)] \\ &= ie^2 \epsilon_{\mu\nu\lambda} k^\lambda \mathbf{F}(k^2). \end{aligned}$$

$$\mathbf{F}(0) = im \int [(d^3 p)/(2\pi)^3] / [p^2 - m^2 + i\epsilon]^2,$$

which is equal to  $(\frac{1}{8\pi})$  and independent of the mass  $m$ . On the other hand, if we had initially set  $m = 0$ , we would have found

$$\mathbf{F}(0) = 0.$$

The above example is of physical interest in the quantum Hall effect and is at the origin of the extreme accuracy of the Hall steps independent of the electron mass even in a messy environment such as the surface of a field effect transistor.

The lesson to be learnt from these analyses is that a theory - especially a gauge theory - with a mass parameter  $m$  near zero may not be "close" to a theory with  $m$  equal to zero.

After these cautionary (and hopefully, not too alarming remarks) let us return to the subject proper which concerns a discussion of a model which at the classical level is scale-invariant with fermions as the only fields. Dynamics for the gauge and Higgs sector are "grown" from the non-linear interactions between the fermions.

## 1. Composite Higgs / Top Model

### (i) Higgs: Elementary or Composite ?

If one employs scalar mesons to do the symmetry breaking, it is relevant to ask whether such scalars are "elementary" or "composite", since the lore is that elementary scalars ruin asymptotic freedom. ( See some discussion to the contrary in Section 3 regarding elementary scalars and asymptotic freedom). But the elementary vs. composite question is quite subtle as Eguchi<sup>(6)</sup> has illustrated in the renormalizable  $\sigma$ - model. The essential point is that what is elementary for one "classical" lagrangian may appear as composite with respect to another starting lagrangian after renormalization. Two bare lagrangians one with an "elementary" scalar field with its own degree of freedom and the other with a scalar introduced as an external (constraint) field, after renormalization become equivalent. The renormalization group analysis for the two systems would be the same except for the boundary conditions which would be different in the two cases.

In the next section, we shall be using similar techniques for the gauge fields where it leads to very interesting structures.

### (ii) Top Model

In the top model<sup>(6,7)</sup>, which is modelled after the BCS theory of superconductivity, the idea is to start with massless gauge and fermi fields as elementary and introduce a 4-fermion interaction term. One further assumes that the Fermi coupling constant is beyond a critical value such that there is a non-vanishing vacuum expectation value for  $\langle \bar{\Psi}\Psi \rangle$ . Such a condensate leads to masses for the fermions and gauge bosons. Here, as in the Nambu-Jona Lasinio model, one initially finds the relationship between the scalar ( read Higgs ) and the fermion ( read the Top quark ) to be  $m_{Higgs} = 2 m_{Top}$ . After renormalization, the factor 2 may be reduced to 1.1. However, it appears that the top mass remains rather high.

It is not unlikely that the difficulty with the top model is due to the violation of scale invariance arising from the 4-fermion interaction term and the treatment of the vector fields as elementary (in the sense of Eguchi). These are avoided in the model considered below.

## 2. Purely Fermionic Scale Invariant Interactions

As stated above, inspired by the pioneering work of Nambu and Jona-Lasinio on generating spin-zero bosons from 4 -Fermion interactions, several attempts have been made to utilize similar mechanisms to generate gauge bosons as well( both for Abelian and non-Abelian cases). But as noted earlier, an unattractive feature of this approach is that the initial couplings are non-scale-invariant and non-gauge-invariant and subsequently compensating non-gauge-invariant renormalizations have to be performed to render the final result gauge invariant.

We shall discuss in the following another approach<sup>(4)</sup> which is scale invariant from the outset and employs a gauge invariant renormalization procedure. Consider three families of Dirac fermions for the standard group  $SU(3) \times SU(2) \times U(1)$  where  $\Psi_{\mathbf{k}}$  refers to leptons and  $Q_{\mathbf{k}}$  to the quarks. The index  $k$  refers to the families. As usual we let the left-handed fermions transform as isotopic doublets. Explicitly,

$$\Psi_{kL} = \begin{pmatrix} \nu_k \\ E_k \end{pmatrix}_L \quad (2.1)$$

and

$$Q_{kL} = \begin{pmatrix} U_k \\ D_k \end{pmatrix}_L \quad (2.2),$$

while the right handed fermions (excluding neutrinos)  $E_{kR}$ ,  $U_{kR}$  and  $D_{kR}$  transform as  $SU(2)$  singlets. We impose that the model be locally gauge invariant and so introduce the gauge fields  $T_{\mu}^a$  for  $SU(3)$ ,  $V_{\mu}$  for  $SU(2)$  and  $S_{\mu}$  for  $U(1)$ . However, this does not require introduction of kinetic energies for the gauge fields and hence we do not. Thus, at this classical level, we have only fermion fields which are dynamical since only they possess kinetic energies. Now just as for a scalar field  $\Phi$  the only scale-invariant interaction is of the form  $(\Phi \cdot \Phi)^2$ , for fermion fields  $\Psi$ , the corresponding scale invariant form of interaction is  $(\bar{\Psi}\Psi)^{2/3}$ . Our Lagrange density which satisfies scale and gauge invariance is essentially uniquely prescribed therefore to be

$$\mathbf{L}_1 = \mathbf{L}_{lL} + \mathbf{L}_{lR} + \mathbf{L}_{qL} + \mathbf{L}_{uR} + \mathbf{L}_{dR} + \mathbf{L}_{int} \quad (2.3)$$

where

$$\mathbf{L}_{lL} = \sum_{k=1}^3 i\bar{\Psi}_{kL} [\not{\partial} - i(\tau/2) \cdot \mathcal{V} - i(\mathbf{Y}_{lL}/2)\not{\mathcal{S}}] \Psi_{kL}, \quad (2.4a)$$

$$\mathbf{L}_{lR} = \sum_{k=1}^3 i\bar{E}_{kR} [\not{\partial} - i(\mathbf{Y}_{lR}/2)\not{\mathcal{S}}] E_{kR}, \quad (2.4b)$$

$$\mathbf{L}_{qL} = \sum_{k=1}^{3^*} i\bar{Q}_{kL} [\not{\partial} - i(\tau/2) \cdot \mathcal{V} - i(\mathbf{Y}_{qL}/2)\not{\mathcal{S}} - i(\lambda_a/2)\not{\mathcal{T}}^a] Q_{kL}, \quad (2.4c)$$



$$\mathbf{L}_{uR} = \sum_{k=1}^3 i\bar{U}_{kR} [\not{\partial} - i(Y_{uR}/2)\not{\beta} - i(\lambda_a/2)\not{T}^a] U_{kR}, \quad (2.4d)$$

$$\mathbf{L}_{dR} = \sum_{k=1}^3 i\bar{D}_{kR} [\not{\partial} - i(Y_{dR}/2)\not{\beta} - i(\lambda_a/2)\not{T}^a] D_{kR}, \quad (2.4e)$$

and

$$\mathbf{L}_{\text{int}} = \left[ \sum_{k=1}^3 (f_{kL} \bar{\Psi}_{kL} E_{kR} - f_{ku} \bar{Q}_{kR}^G U_{kL}^c + f_{kd} \bar{Q}_{kL} D_{kR}) \right] \times (\text{H.c.}) \Big]^{2/3}. \quad (2.4f)$$

In Eq.(2.4)  $\bar{Q}_k^G$  is the  $G$ -conjugate field of  $Q_k$ , while  $U_k^c$  is the charge-conjugate field of  $U_k$ . Where ever quarks are involved, a summation on color indices is understood. The above choice is the most economical symmetry-breaking scheme which is scale-invariant.

The rather awkward non-polynomial interaction in Eq.(2.4f) can be converted to a more standard polynomial form upon the introduction of a scalar,  $SU(2)$ -doublet constraint field  $\chi$ . We then have

$$\begin{aligned} \mathbf{L}_{\text{int}} = & \sum_{k=1}^3 \left[ -g_{kl} (\bar{\Psi}_{kL} E_{kR} \chi + \chi^\dagger \bar{E}_{kR} \Psi_{kL}) \right. \\ & - g_{ku} (\bar{Q}_{kL} U_{kR} \chi^G + \chi^{G\dagger} \bar{U}_{kR} Q_{kL}) \\ & \left. - g_{kd} (\bar{Q}_{kL} D_{kR} \chi + \chi^\dagger \bar{D}_{kR} Q_{kL}) \right] \\ & - \lambda_0 (\chi^\dagger \chi)^2 \end{aligned} \quad (2.5)$$

Here  $\chi^G = i\tau_2 \chi^*$  is the  $G$ -conjugate field of  $\chi$ . We note that in Eqs.(2.4f) and (2.5) no coupling constants have been introduced along with the gauge fields because they can be absorbed into these fields. This is possible since there is no corresponding kinetic energy term to set the scale for the gauge fields.

The equations of motion obtained by varying  $\chi$  and  $\chi^\dagger$  yield, using Eqs.(2.4) – (2.5),

$$f_i = \left[ (27/16) g_i^4 / \lambda_0^3 \right]^{1/2}, \quad (2.6)$$

where the index  $i$  takes on the values  $k_l, k_u$ , or  $k_d$ . The coupling constant  $\lambda_0$  has been introduced to facilitate connection with the standard theory later.

At this point, we integrate over the fermi fields in the generating functional to obtain the following effective Lagrangian  $\mathbf{L}_{\text{eff}}$  ( for details see reference 4):

$$\int (d^4x) \mathbf{L}_{\text{eff}} = \sum_{k=1}^3 \left[ -i \text{tr} \ln(1 + \mathbf{M}_{k1}) - i \text{tr} \ln(1 + \mathbf{M}_{k2}) \right]. \quad (2.7)$$

Here  $M_{k1}$  arises from integrations over the lepton fields, while  $M_{k2}$  arises from integrations over the quark fields. For each family there is an identical result except for the coupling constants  $g_{kl}$ , etc. Hence suppressing the family labeling momentarily we have

$$M_1 = \begin{pmatrix} (1/(i\bar{\sigma}^\nu \partial_\nu)\bar{\sigma}^\mu[(\tau/2) \cdot \mathbf{V}_\mu + (Y_{lL}/2)S_\mu]) & (1/(i\bar{\sigma}^\nu \partial_\nu)(-ig_l\chi)) \\ (1/(i\sigma^\nu \partial_\nu)(ig_l\chi^\dagger)) & (1/(i\sigma^\nu \partial_\nu)\sigma^\mu(Y_{lR}/2)S_\mu) \end{pmatrix} \quad (2.8)$$

$$M_2 = \begin{pmatrix} (i\bar{\sigma}^\nu \partial_\nu)^{-1}\bar{\sigma}^\mu a_\mu & (i\bar{\sigma}^\nu \partial_\nu)^{-1}(-ig_u\chi^G) & (i\bar{\sigma}^\nu \partial_\nu)^{-1}(-ig_d\chi) \\ (i\sigma^\nu \partial_\nu)^{-1}(ig_u\chi^{G\dagger}) & (i\sigma^\nu \partial_\nu)^{-1}\sigma^\mu[\frac{Y_{uR}}{2}S_\mu + \frac{\lambda_a}{2}T_\mu^a] & 0 \\ (i\sigma^\nu \partial_\nu)^{-1}(ig_d\chi^G) & 0 & (i\sigma^\nu \partial_\nu)^{-1}\sigma^\mu b_\mu \end{pmatrix} \quad (2.9)$$

with

$$a_\mu = [\frac{\tau}{2} \cdot \mathbf{V}_\mu + \frac{\mathbf{Y}_{qL}}{2}\mathbf{S}_\mu + \frac{\lambda_a}{2}\mathbf{T}_\mu^a]$$

and

$$b_\mu = [\frac{Y_{dR}}{2}S_\mu + \frac{\lambda_a}{2}T_\mu^a]$$

Also,  $\chi$  is a 2-column matrix and  $\chi^\dagger$  is a 2-row matrix. In Eq.(2.9),  $\sigma^\mu$  and  $\bar{\sigma}^\mu$  are  $2 \times 2$  matrices defined as

$$\sigma^\mu = (1, \sigma) \quad (2.10a)$$

and

$$\bar{\sigma}^\mu = (1, -\sigma). \quad (2.10b)$$

In terms of Feynman diagrams, the logarithms in Eq.(2.7) are the sum of all one-fermion-loop contributions with any number of external boson legs. Since fermions appear only quadratically and none of the other fields are dynamical, one fermion loop is all there is.

There are ultraviolet divergences in the above expressions only for those diagrams having four or fewer external legs. These are the ones that generate the dynamics for the boson fields and hence are the ones of interest. We employ dimensional regularization since it preserves scale and gauge invariance. For the particular diagrams discussed above, the infra-red divergences disappear as we let the complex dimension  $d$  approach the neighborhood of 4.

The UV-divergences in Eqs.(2.8-2.9) appear as poles at  $d = 4$ . Letting  $d = 4$  in the residue at these poles, we obtain

$$\begin{aligned} \mathbf{L}_{UV} &= \\ &= [Y^2/16\pi^2(4-d)][-1/4S^{\mu\nu}(x)S_{\mu\nu}(x)] \\ &+ [I^2/16\pi^2(4-d)][-1/4\mathbf{V}^{\mu\nu}(x)\mathbf{V}_{\mu\nu}(x)] \\ &+ [L^2/16\pi^2(4-d)][-1/4T^{a\mu\nu}(x)T_{\mu\nu}^a(x)] \\ &+ [G^2/8\pi^2(4-d)][(D^\mu\chi^\dagger)(D_\mu\chi)] \\ &- \left[ \lambda_0 + [1/8\pi^2(4-d)] \sum_{k=1}^3 [g_{kl}^4 + 3(g_{ku}^4 + g_{kd}^4)] \right] (\chi^\dagger\chi)^2 \end{aligned} \quad (2.11)$$

For the group under consideration, although  $I^2$  and  $L^2$  have different origins, they both have the same value  $I^2 = L^2 = 8$ . Also, we have

$$Y^2 = 2Y_{lL}^2 + Y_{lR}^2 + 3(2Y_{qL}^2 + Y_{uR}^2 + Y_{dR}^2) \quad (2.12)$$

and

$$G^2 = \sum_{k=1}^3 [g_{kl}^2 + 3(g_{ku}^2 + g_{kd}^2)]. \quad (2.13)$$

Assigning the standard model hypercharge values

$$Y_{lL} = -1; Y_{lR} = -2; Y_{qL} = -\frac{1}{3}, Y_{uR} = \frac{4}{3} \text{ and } Y_{dR} = -\frac{2}{3},$$

we find that

$$Y^2 = \frac{40}{3}. \quad (2.14)$$

The field-strengths and covariant derivatives of  $\chi$  are given by

$$S_{\mu\nu} = \partial_\mu S_\nu - \partial_\nu S_\mu \quad (2.15)$$

$$\mathbf{V}_{\mu\nu} = \partial_\mu \mathbf{V}_\nu - \partial_\nu \mathbf{V}_\mu + \mathbf{V}_\mu \times \mathbf{V}_\nu \quad (2.16)$$

$$T_{\mu\nu}^a = \partial_\mu T_\nu^a - \partial_\nu T_\mu^a + f^{abc} T_\mu^b \times T_\nu^c \quad (2.17)$$

$$D_\mu \chi = [\partial_\mu - i/2 S_\mu - i/2 \boldsymbol{\tau} \cdot \mathbf{V}_\mu] \chi. \quad (2.18)$$

Now let us tackle the renormalization procedure in order to render Eq.(2.11) finite. We add to  $\mathbf{L}_1$  a counter-term of the form  $\mathbf{L}_{UV}$  so that

$$\mathbf{L}_2 = \mathbf{L}_1 + \mathbf{L}_{UV} \quad (2.19)$$

The generating functional constructed using  $\mathbf{L}_2$  upon integration over the fermi fields gives the same results as is obtained using  $\mathbf{L}_1$  once the fields have been rescaled and the coupling constants properly defined (see below). Needless to say that the limit  $d = 4$  is to be taken only after all the renormalizations have been performed.

The required rescaling of the Bose fields are given by

$$\mathbf{A}_\mu = [\mathbf{I}^2/16\pi^2(4-d)]^{1/2} \mathbf{V}_\mu, \quad (2.20)$$

$$B_\mu = [Y^2/16\pi^2(4-d)]^{1/2} S_\mu, \quad (2.21)$$

$$F_\mu^a = [L^2/16\pi^2(4-d)]^{1/2} T_\mu^a, \quad (2.22)$$

$$\Phi = [G^2/8\pi^2(4-d)]^{1/2} \chi. \quad (2.23)$$

To obtain the proper kinetic energy terms for the rescaled bose fields, the coupling constants are defined to be

$$g = [16\pi^2(4-d)/I^2]^{1/2}, \quad (2.24)$$

$$g' = [16\pi^2(4-d)/Y^2]^{1/2}, \quad (2.25)$$

$$g_s = [16\pi^2(4-d)/L^2]^{1/2}, \quad (2.26)$$

$$\lambda = [8\pi^2(4-d)/G^2]^{1/2} [\lambda_0 + 1/8\pi^2(4-d) \sum_{k=1}^3 [g_{kl}^4 + 3(g_{ku}^4 + g_{kd}^4)]], \quad (2.27)$$

$$G_{kl} = [8\pi^2(4-d)/G^2]^{1/2} g_{kl}, \text{ etc.} \quad (2.28)$$

With all of the above, we may finally display the resultant Lagrange density to be

$$\begin{aligned} \mathbf{L}_2 = & \sum_{k=1}^3 \left[ i\bar{\Psi}^{kL} [\not{\partial} - i(g/2)\boldsymbol{\tau} \cdot \mathbf{A} - i(g'/2)Y_{lL}\boldsymbol{B}] \Psi_{kL} + i\bar{E}_{kR} [\not{\partial} - i(g'/2)Y_{lR}\boldsymbol{B}] E_{kR} \right. \\ & + i\bar{Q}_{kL} [\not{\partial} - i(g/2)\boldsymbol{\tau} \cdot \mathbf{A} - i(g'/2)Y_{qL}\boldsymbol{B} - i(g_s/2)\lambda_a F^a] Q_{kL} \\ & + i\bar{U}_{kR} [\not{\partial} - i(g'/2)Y_{uR}\boldsymbol{B} - i(g_s/2)\lambda_a F^a] U_{kR} \\ & + i\bar{D}_{kR} [\not{\partial} - i(g'/2)Y_{dR}\boldsymbol{B} - i(g_s/2)\lambda_a F^a] D_{kR} \\ & - G_{kl} (\bar{\Psi}_{kL} E_{kR} \Phi + \Phi^\dagger \bar{E}_{kR} \Psi_{kL}) - G_{ku} (\bar{Q}_{kL} U_{kR} \Phi^G + \Phi^{G\dagger} \bar{U}_{kR} Q_{kL}) \\ & \left. - G_{kd} (\bar{Q}_{kL} D_{kR} \Phi + \Phi^\dagger \bar{D}_{kR} Q_{kL}) \right] \\ & - \lambda (\Phi^\dagger \Phi)^2 - 1/4 (B^{\mu\nu} B_{\mu\nu}) - 1/4 (\mathbf{A}^{\mu\nu} \cdot \mathbf{A}_{\mu\nu}) - 1/4 (F_{\mu\nu}^a F_{\mu\nu}^a) + (D^\mu \Phi)^\dagger (D_\mu \Phi) \end{aligned} \quad (2.29)$$

where

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \quad (2.30)$$

$$\mathbf{A}_{\mu\nu} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu + \mathbf{g} \mathbf{A}_\mu \mathbf{A}_\nu, \quad (2.31)$$

$$F_{\mu\nu}^a = \partial_\mu F_\nu^a - \partial_\nu F_\mu^a + g_s f^{abc} F_\mu^b F_\nu^c, \quad (2.32)$$

$$D_\mu \Phi = [\partial_\mu - ig'/2 B_\mu - ig/2 \boldsymbol{\tau} \cdot \mathbf{A}_\mu] \Phi. \quad (2.33)$$

$\mathbf{L}_2$  is the standard form that one would be using if starting with a scale- and gauge-invariant theory of  $SU(3) \times SU(2) \times U(1)$ . The difference, of course is that there exist relationships among the coupling constants to which we now turn.

### 3 . Coupling Constant and Mass Relations:

The weak angle  $\theta_W$  is related to the constants  $g$  and  $g'$  by

$$\sin^2 \theta_W = g'^2 / (g^2 + g'^2) = 3/8 \quad (3.1)$$

where we have used Eqs.(2.24-25) with the value  $I^2 = 8$  and  $Y^2 = 40/3$  as calculated in Eq.(2.14). This result for the unrenormalized weak angle is the same as that for  $SU(5)^{(9)}$  even though we do not of course have the complete structure of  $SU(5)$  ( no lepto-quarks, no proton decay etc.). This result was also obtained by Terazawa et al<sup>(2)</sup>. We also find that the bare strong coupling constant  $g_s = g$ , since  $L^2 = 8$ . Hence, there is only one independent gauge-coupling constant.

If we wish we can interpret these "bare" results in terms of a large momentum cut-off  $\Lambda$ , and we find that at the cut-off mass the three coupling constants

$$\alpha_1(\Lambda) = (5/3)(g'^2/4\pi) \quad (3.2)$$

$$\alpha_2(\Lambda) = (g^2/4\pi). \quad (3.3)$$

$$\alpha_3(\Lambda) = (g_s^2/4\pi) \quad (3.3)$$

are indeed equal.

Now let us turn to a discussion of the symmetry breakdown due to radiative corrections and the resultant mass relationships between the various fields. The massless quartic scalar couplings have been analyzed some time ago by Coleman-Weinberg<sup>(10)</sup> and by Weinberg<sup>(11)</sup> and more recently by Branchina and Consoli<sup>(12)</sup>. We shall discuss these recent new results.

For  $\lambda\Phi^4$  interaction, the one-loop potential reads<sup>(10)</sup>

$$V^{1-loop}(\Phi) = (\lambda/24)\Phi^4 + \lambda^2\Phi^4/256\pi^2 (\ln(\lambda\Phi^2/2\Lambda^2) - 1/2) \quad (3.4)$$

The RG equation for the effective potential reads

$$[\Lambda\partial/\partial\Lambda + \beta\partial/\partial\lambda - \gamma\Phi\partial/\partial\Phi]V(\Phi, \lambda, \Lambda) = 0. \quad (3.5)$$

Eq.(3.5) gives rise to a perturbative solution<sup>(10)</sup>

$$\beta = 3\lambda^2/16\pi^2 + O(\lambda^3), \quad (3.6a)$$

and

$$\gamma = O(\lambda^2), \quad (3.6b)$$

as well as a non-perturbative, asymptotically free "exact" solution<sup>(12)</sup>

$$\beta = -3\lambda^2/(16\pi^2), \quad (3.7a)$$

and

$$\gamma = -3\lambda/(32\pi^2). \quad (3.7b)$$

According to the analysis given in ref.(12), one can have a heavy Higgs with predictable mass value near 2.2  $TeV$ . For details, see ref.(12).

We return now to our model to see what we find for the relationship between the vector-boson and quark masses. If  $m_i$  denotes the fermion mass, then

$$m_i = G_i \Phi_v, \quad (3.8)$$

where  $\Phi_v$  is vev of the classical scalar field at the one loop level. Using Eqs.(2.13) and (2.24), we find

$$g^2 = 1/4 \sum_{k=1}^3 [G_{kl}^2 + 3(G_{ku}^2 + G_{kd}^2)]. \quad (3.9)$$

Also, the mass of the  $W$ -meson is given by

$$M_W^2 = g^2 \Phi_v^2, \quad (3.10)$$

so that using Eqs.(3.8-9), we finally obtain  $M_W$  in terms of the fermion masses as

$$M_W^2 = 1/8 \sum_{k=1}^3 [m_{kl}^2 + 3(m_{ku}^2 + m_{kd}^2)]. \quad (3.11)$$

Since all masses except that of the top may be neglected, we find approximately for the top mass

$$m_{top} \approx \sqrt{((8/3)M_W)} \approx 132 GeV, \quad (3.12)$$

a result in accord with the EW radiative corrections.

#### 4. Concluding Remarks

A brief survey of the ideas behind purely fermionic models which generate some (or all) of the bosons has been presented. They go beyond SM in that while all of the verified structure of SM is reproduced, some of the parameters get constrained and which may be experimentally testable. It is also pleasing that some of the "good" results of GUT models such as asymptotic relationships between the gauge coupling constants are obtained as well. Of course, if proton decay is indeed found then the much richer panorama of particles contained in the GUT models would have to be preferred. Until then, it appears that the models discussed here are indeed a viable and attractive alternative.

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### Footnotes and References:

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