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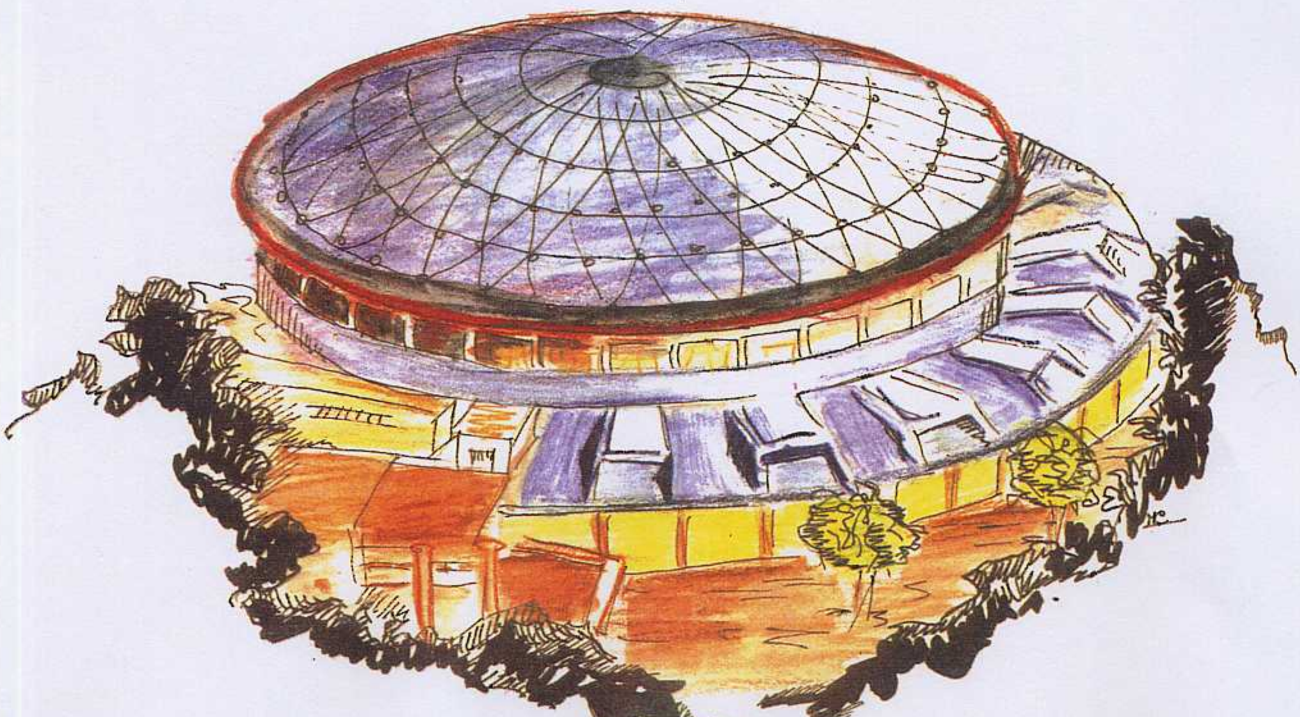

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ABSTRACT

The "no-go" theorem of Nielsen and Ninomiya, forbidding a sensible formulation of the usual electroweak action on a lattice, has been revisited proceeding from the hypothesis that space-time, as a result of the dynamics of quantum gravity, is a four-dimensional random lattice with constant a_p, the Planck length. We find that a sensible electroweak theory at long wave-lengths is recovered by adding a chiral-gauge-invariant quadrilinear fermionic term. Dynamical symmetry breaking of Nambu-Jona-Lasinio(NJL) type is shown to be crucial for making contact with known phenomenology. The relation of this approach to similar recent proposals is briefly discussed.

The very-small-scale structure of space-time, the arena of physical reality, has recently attracted a great deal of attention. The realization of the important role that quantum gravity plays in determining such a structure, coupled with the grave difficulties that this theory faces in the usual, perturbative formulation at distances smaller than the Planck length $a_p = \frac{1}{m_p} \simeq 10^{-33}$cm ($m_p \simeq 10^{19}$GeV is the Planck mass), is the basic motivation of a umber of theoretical proposals to overcome such difficulties and provide a solution of this fascinating problem.

We may conceive that precisely due to the violent quantum fluctuations that the gravitational field *must* exhibit at a_p, space-time somehow "ends" there. Either by the creation of a "foam"[1], or by some other mechanism which we need not discuss here, one may conceive that as a result the physical space-time gets endowed with a fundamental length, a_p, and thus the basic arena of physical reality becomes a (random) lattice with lattice constant a_p.

It is this possibility that we wish to explore in this note in connection with a fundamental problem of the standard model, the problems of fermion and intermediate gauge boson masses. As is well known the gauge-symmetry principle as realized in the electroweak sector $SU_L(2) \otimes U_Y(1)$ demands that, at the lagrangian level, *all* fermion and gauge fields must be massless. In order to avoid an obvious theoretical disaster and to save the gauge-principle the Higgs model[2] had to be grafted upon the beautiful gauge lagrangian in a completely "*ad hoc*" manner in order to secure a spontaneous symmetry breaking mechanism for the generation of fermion as well as the gauge boson masses.

Suppose now that, following the considerations reported in the introductory paragraphs, we wish write the standard model lagrangian (without Higgs fields) on a lattice, a profound result obtained more than ten years ago[3], in the form of a no-go theorem, tells us that there is no consistent way to transpose straightforwardly on a lattice the lagrangian of the continuum theory. While this is not the place for the detailed discussion of this important result, we recall that Wilson[4] has shown how can one modify the lagrangian by adding a simple bilinear term so as to remove the unwanted "replicas" from the long wave-length regime. However, this can only be done by sacrificing chiral invariance; and the no-go theorem shows that no bilinear modification can be made to obey the chiral gauge-principle. Thus if we are to go ahead in our program of putting the standard model on a lattice without sacrificing the chiral gauge-principle, the no-go theorem tells us that simple transposition of the continuum lagrangian must be supplemented by extra-terms that are at least quadrilinear in the fermion fields. Familiarity with the original Nambu-Jona-Lasinio model[5] immediately reminds us that such quadrilinear terms can be made to obey the chiral gauge-principle. Thus an $SU(3)_c \otimes SU_L(2) \otimes U_Y(1)$ chiral-gauge invariant lagrangian which evades *in principle* the no-go theorem can be written as (F denotes the generic fermion sectors, F=0 for leptons and F=c for quarks)

$$S_L = S_G + \sum_F (S_D^F + S_{NJL}^{F1} + S_{NJL}^{F2}), \tag{1}$$

where S_G is the usual Wilson gauge action and S_D^F the Dirac action where the expression for gauge link fields will be given elsewhere. The NJL action $S_{NJL}^{F1,2}$

are introduced as (i,j=1,2,3 for fermion families, k,l=1,2 for weak isospin and $a = a_p$)

$$\begin{aligned} S^{F1}_{NJL} &= G_1 \sum_x \{\bar\psi^{Fi}_{Ll}(x)\psi^{Fj}_{Rk}(x)\bar\psi^{Fj}_{Rk}(x)\psi^{Fi}_{Ll}(x)\} \\ S^{F2}_{NJL} &= \frac{G_2}{2}\sum_{\pm\mu,x}\{\bar\psi^{Fi}_{Ll}(x+a_\mu)U^F_\mu(x)\psi^{Fj}_{Rk}(x) \\ &\quad \bar\psi^{Fj}_{Rk}(x)U^F_\mu(x)^\dagger\psi^{Fi}_{Ll}(x+a_\mu)\}, \end{aligned} \quad (2)$$

where $U^c_\mu(x) \in SU_c(3)$ and $\psi_{LR} = P_{LR}\psi$.

In order for the action (1) to avoid *in practice* the no-go theorem it is necessary that the quadrilinear terms that we introduced develop a dynamical chiral symmetry breaking through the following non-zero vacuum expectation values (we omit notation of summation over color indices)

$$\begin{aligned} m^{ij}_{kl} &= \frac{-G_1}{2V_4}\sum_x\langle\bar\psi^{Fi}_l(x)\psi^{Fj}_k(x)\rangle; \\ \bar r^{ij}_{kl} &= \frac{G_2}{4V_4}\sum_{\mu,x}\{\langle\bar\psi^{Fi}_{Ll}(x)U^F_\mu(x)\psi^{Fj}_{Rk}(x+a_\mu)\rangle + h.c.\}, \end{aligned} \quad (3)$$

and

$$\begin{aligned} m^{ij}_{5kl} &= \frac{-iG_1}{2V_4}\sum_x\langle\bar\psi^{Fi}_l(x)\gamma_5\psi^{Fj}_k(x)\rangle; \\ \bar r^{ij}_{5kl} &= \frac{-iG_2}{4V_4}\sum_{\mu,x}\{\langle\bar\psi^{Fi}_{Ll}(x)U^F_\mu(x)\psi^{Fj}_{Rk}(x+a_\mu)\rangle - h.c.\}. \end{aligned} \quad (4)$$

Indeed, if this happens, the following effective lattice action of Wilson type is obtained:

$$\begin{aligned} S_L &= S_G + \sum_F(S^F_D + \sum_x\{\bar\psi^F(x)M\psi^F(x) \\ &\quad -\frac{1}{2}\bar\psi^F(x)\bar r U^F_\mu(x)\psi(x+a_\mu) + h.c.\}), \end{aligned} \quad (5)$$

where $M = m + 4\bar r$. It is clear that through dynamical symmetry breaking the obligatory Wilson term ($r = a_p\bar r$) has been produced together with a mass term (m) which must necessarily come with it, if we start from the quadrilinear NJL term introduced in eq.(1). Thus the no-go theorem is evaded and as a necessary consequence a fermion mass term is generated.

As is well known, given the quadrilinear terms (2), a non-trivial dynamical symmetry breaking (3) and (4) may emerge only if the matrices $m, \bar r, m_5$ and $\bar r_5$ obey a coupled set of "gap equations", which has the following approximate matrix-form

$$\begin{aligned} ma + 4r &= 2N^F g_1^2\int_l \frac{ma + 2r\sin^2\frac{l_\mu}{2}}{Den(l)} \\ m_5 a + 4r_5 &= 2N^F g_1^2\int_l \frac{m_5 a + 2r_5\sin^2\frac{l_\mu}{2}}{Den(l)} \\ r &= -N^F g_2^2\int_l(\cos)\frac{ma + 2r\sin^2\frac{l_\mu}{2}}{Den(l)} \\ r_5 &= -N^F g_2^2\int_l(\cos)\frac{m_5 a + 2r_5\sin^2\frac{l_\mu}{2}}{Den(l)} \end{aligned} \quad (6)$$

where we have neglected the gauge degrees of freedom, $g_{1,2}a_p = G_{1,2}$, $r_5 = a\bar{r}_5$, $\int_l = \int_{-\pi}^{\pi} \frac{d^4l}{(2\pi)^4}$, $(\cos) = \sum_\mu \cos l_\mu$ and

$$\begin{aligned} Den(l) &= \sin^2 l_\mu + (am + 2r\sin^2 \frac{l_\mu}{2})^2 \\ &\quad -(am_5 + 2r_5 \sin^2 \frac{l_\mu}{2})^2. \end{aligned} \tag{7}$$

We shall not carry out any detailed analysis of the gap equations and phase diagram in terms of the four fermion couplings g_1 and g_2. We only notice some important points in the physical region where $ma_p \ll 1$ and in the approximation that off-diagonal elements of m, r, m_5 and r_5 are vanishing:

(i) The trivial solution ($m = m_5 = r = r_5 = 0$) can not be realized since it does not correspond to the lowest energy state. No consistent solution of the gap equations exists with $r = 0; m \neq 0$ or $r = 0; m = 0$. Thus whether or not dynamical symmetry breaking is realized, we are guaranteed that the long wave-length doubling of fermion species does not occur.

(ii) There are two types of physically sensible solutions of the gap-equations. In order to see this, we apply the condition $ma_p \ll 1$ to the gap-equations and reduce them to

$$\begin{aligned} m &= 0 \\ 1 &= 2N^F g_1^2 \int_l \frac{1}{Den(l)}, \end{aligned} \tag{8}$$

(similar equations hold for m_5 and r_5), if $R^2 = r^2 - r_5^2$ is equal to a critical value $0 < R_c < 0.4$, which depends on the ratio $\frac{g_2}{g_1}$. The first type of solution, where a fermion mass is generated, is represented by the second line of equation (8), in this case we get a massless Goldstone boson accompanied with $m_5 = 0$. In order for $ma_p \ll 1$ we need to fine-tune $g_1^c(R_c)$. The normal gap-equation is reproduced from (7) by restricting ourself to the "continuum region" where integration variables are confined to $la_p \ll 1$. The second type of solution is represented by the first line of eq.(8) with $r \neq 0; m = 0$.

(iii) Since the fermion spectrum implies that the symmetry of fermion families is broken, we assume that the physical vacuum is realized in such a way that the first solution holds for the top quark due to its particularly heavy mass and the other fermions are associated to the second type of solution ($m = 0; r \neq 0$) and remain massless at this step. Thus all elements of the mass matrix are zero except for the top quark and the Wilson parameter matrix is a non-zero diagonal matrix.

In a future paper we will show how it is possible that the other fermions acquire their masses without extra Goldstone bosons through a massive solution to the Dyson equations with an inhomogeneous term, which stems from the explicit breaking term $r \neq 0$.

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