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CONDENSATE OF ABEALIAN MONOPOLES AND CONFINEMENT IN LATTICE GAUGE THEORIES

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ABSTRACT

We show that in the confining phase of the $SU(2)$ lattice gauge theory the condensate of abelian magnetic monopoles exists. This condensate vanishes at the critical temperature or (on a symmetric lattice) at the cooling step where the string tension vanishes. These results seem to confirm the confinement mechanism based on the model of the dual superconductor. As a justification of our definition of the monopole condensate we study the $U(1)$ lattice gauge theory.

The confinement mechanism is one of the long standing problems in the quantum field theory. In the present paper, by analyzing the numerical data in lattice gauge theories, we attempt to justify the confinement mechanism suggested by t'Hooft [1] and Mandelstam [2]. In its extremely simplified formulation, this confinement mechanism in nonabelian theory is explained as follows. The nonabelian fields are projected on the compact abelian fields by a partial gauge fixing [3]. The lagrangian of the abelian theory is supposed to have the form:

$$\mathcal{L} = \beta G_{\mu\nu}^2 + |D_\mu \phi|^2 + \lambda(|\phi|^2 - 1)^2; \qquad (1)$$

here ϕ is the monopole field, $D_\mu = \partial_\mu + \imath B_\mu$, $G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$, and B_μ – is the dual of the electromagnetic field: $G_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}(\partial_\alpha A_\beta - \partial_\beta A_\alpha)$. Due to the monopole condensate the flux tube arises between the charges "+" and "–" (between the quark and the antiquark). This flux tube leads to the a linear potential at large distances; at short distances the quark and the antiquark interact via the Coulomb potential. These effects take place at the classical level, in the framework of the classical equations of motion. Therefore the expression (1) plays the role of the effective infrared Lagrangian provided that it can be derived directly from the QCD Lagrangian. At present the said derivation is absent, and therefore we perform the numerical test. The form of effective lagrangians was studied in detail for the $SU(2)$ and $SU(3)$ theories in refs.[4, 5]. In ref.[6] an analytical derivation of the effective Lagrangian was performed by some approximations in the lattice gauge theory.

In this article we give numerical evidence in favor of the existence of a Lagrangian of type (1). Due to the compactness of the abelian gauge group, the monopoles are present in the vacuum of the theory, and it is natural that they interact with the gauge field via the long derivative. Thus in the effective Lagrangian (1), only the interaction $\lambda(|\phi|^2 - 1)^2$ is really nontrivial. This interaction is responsible for the monopole condensate. For the compact $U(1)$ theory the confinement mechanism, discussed here, was suggested in refs.[7, 8, 9, 10]. The existence of the monopole condensate in the confinement phase was numerically proved for the compact $U(1)$ lattice gauge theory by studying the monopole Green function [11] and the effective monopole potential [12].

In refs.[13, 14] the abelian monopoles were studied in the effective $U(1)$ lattice gauge theory obtained from the $SU(2)$ lattice gauge theory by the projection $SU(2) \rightarrow U(1)$. It was found that the monopole density is high in the confinement phase; moreover, there are some indications [15] that the monopole density obeys the scaling law, and therefore the monopoles seem to exist in the continuum limit. However, these facts are still not enough for the proof of the t'Hooft-Mandelstam confinement mechanism. There are a lot of free charges in an ordinary conductor, but the Abrikosov string is formed only in the presence of the condensate of the Cooper pairs. Thus the existence of the monopole condensate is crucial for the discussed model.

The proof of the existence of the monopole condensate in the $U(1)$ lattice gauge theory in refs. [11, 12] was based on an explicit expression for the monopole creation operator [16, 17]. We do not know how to use this approach in the $U(1)$ theory obtained from the $SU(2)$ theory by partial gauge fixing. It seems that the simplest quantity related to the monopole condensate and admitting numerical calculations is the propagator of the monopole-antimonopole pair:

$$\mathcal{G}(x) =< \overline{\phi}(x)\phi(x)\overline{\phi}(0)\phi(0) > . \tag{2}$$

If the monopole condensate exists, $< \phi(x) >= C$, then at $|x| \to \infty$ we have $\mathcal{G}(x) \to A\frac{\exp(-m|x|)}{|x|^\alpha} + C^4$. We used this formula to fit the numerical data for the propagator (2). The numerical procedure for the calculation of the propagator (2) is explained at the end of the paper, but first we discuss the numerical results.

Our approach was initially used in the 3-dimensional XY model: it was shown [18] that in the high temperature phase, at $\beta < \beta_c$, there exist a condensate of the vortices; at $\beta > \beta_c$ the condensate is absent. In order to justify our approach we also study the value of the condensate in the $U(1)$ lattice gauge theory. The results are shown in Fig.1. It can be seen that the condensate exists at $\beta < \beta_c$ and vanishes at $\beta > \beta_c$. Our statistics being not very high (the statistical errors have the order of the size of the points in Fig.1), and moderate size of the lattice (10^4) do not allow us to distinguish between the week first order phase transition and the second order phase transition.

Next we discuss the results of the numerical experiments in the $SU(2)$ lattice gauge theory. We use cooling of the gauge configurations and variation of the temperature to study the relation between the string tension and the value of the monopole condensate. The results of the finite temperature experiment are shown in Fig.2. We generate the $SU(2)$ gauge fields on the $10^3 \times 4$ lattice using the standard Monte-Carlo algorithm. After that we extract the $U(1)$ gauge fields using the so-called maximal abelian gauge [13, 14]. For this gauge the string tension in the resulting $U(1)$ theory is approximately the same as the string tension in the original $SU(2)$ theory [19, 20]. Thus the abelian fluctuations are mainly responsible for the confinement in the maximal abelian gauge. From Fig.2 it is clearly seen that the condensate of the standard (1^3) monopoles vanishes at the critical temperature. We also calculate the value of the condensate for the extended monopoles (2^3). These monopoles correspond to the measurement of the monopole charge in the cubes with edge $2a$ using the loops $2a \times 2a$ (they are called monopoles of type I in ref. [21]). The monopole condensate of the 2^3 monopoles does not vanish at $T = T_c$ (preliminary results of the experiments on larger lattices show that this is purely a finite size effect, which is obviously stronger for the 2^3 monopoles than for the 1^3 monopoles). We expect that for sufficiently large lattices the condensate of the extended monopoles vanishes above the phase transition.

We also study the value of the monopole condensate under the cooling [22, 23] of the gauge fields. In refs. [24, 25] a flux tube between the static quark and antiquark is observed in the cooled vacuum configuration, but it was found that the density of the monopoles is not correlated with the string tension. The possible explanation of this fact is that the cooling of the gauge fields destroys small monopoles [21], and confinement is due to the extended monopoles. This statement is confirmed by the results shown in Fig.3, where we plot the dependence of the monopole condensate on the value of the string tension. We vary the string tension by cooling the gauge fields. It is well known that at first steps of the cooling the string tension is almost constant [26], during the next cooling steps the string tension decreases [27]. It occurs that the condensate of the 1^3 monopoles vanishes at the initial cooling steps, and the string tension is proportional to the value of the condensate of the extended monopoles (see Fig.3). This fact is in agreement with the results of ref.[28], where it was found that in the cooled configuration the correct value of the string tension can be observed only at sufficiently large distances.

Now we discuss the numerical procedure for the calculation of the Green function (2). From the given configuration of the $U(1)$ gauge fields we extract the monopole world lines by the standard methods [29, 13, 14]. We consider the connected clusters of these world lines; and the Green function is defined as:

$$\mathcal{G}(|x|) = \frac{\sum\limits_{i,j} \left\{ \begin{array}{ll} 1 & \text{if } x, y \in C \\ 0 & \text{otherwise} \end{array} \right.}{\sum\limits_{i,j} 1}, \tag{3}$$

where summation is assumed over all pairs of links $\{i, j\}$ separated by the distance $|x|$ on the lattice. The unity is added to the sum in the numerator if the links i and j belong to the same cluster. The sum in the denominator is the total number of links separated by the distance $|x|$. This definition corresponds to the Green function considered as the probability of the propagation of the monopole-antimonopole pair from the point 0 to the point x. In the numerator of (3) we take into account only the connected clusters, since the contribution of the condensate of type $< \overline{\phi}(0)\phi(0) >$ should be neglected.

Finally we note that the numerical procedures for the calculation of the string tension and the monopole condensate are quite different. So the fact that the monopole condensate vanishes at the same temperature (or at the same cooling step) as the string tension shows that these two quantities, σ and C, are related by some physical mechanism. We consider our results as a fairly strong argument in favor of the confinement mechanism of t'Hooft and Mandelstam.

On the completion of the manuscript the authors received the paper [30] which suggest another approach to the study of the monopole condensate. The conclusion of ref.[30] that the monopole condensate exists in the confining phase of lattice gauge theories agrees with our results.

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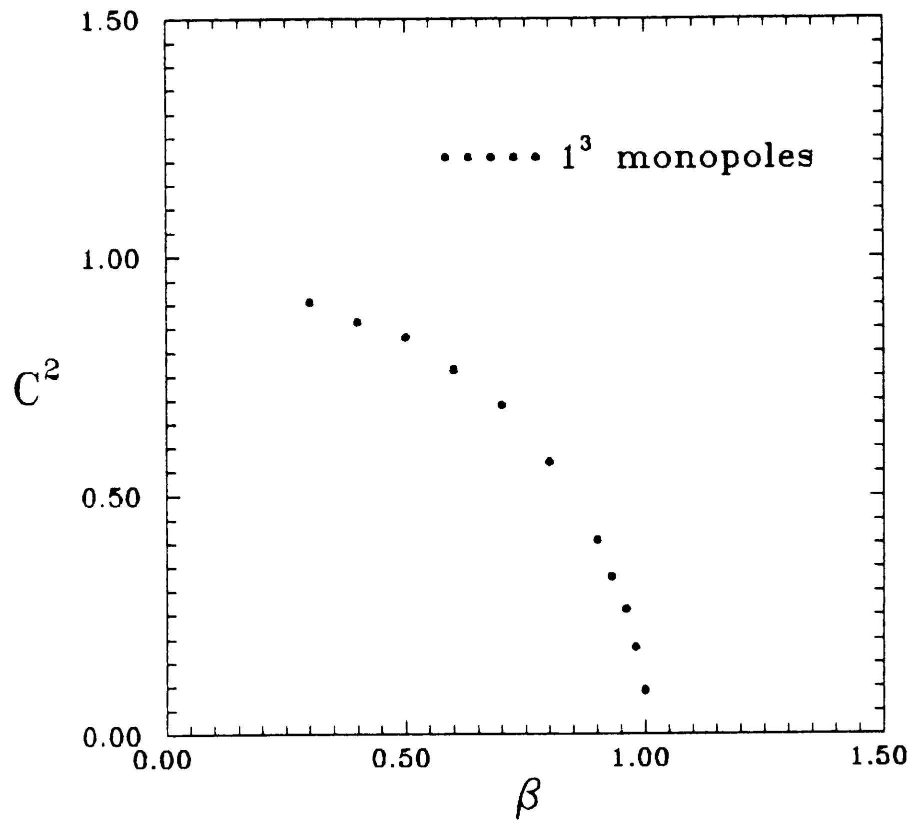

Fig.1 : Square of the monopole condensate versus β for the four dimensional $U(1)$ lattice gauge theory on the 10^4 lattice.

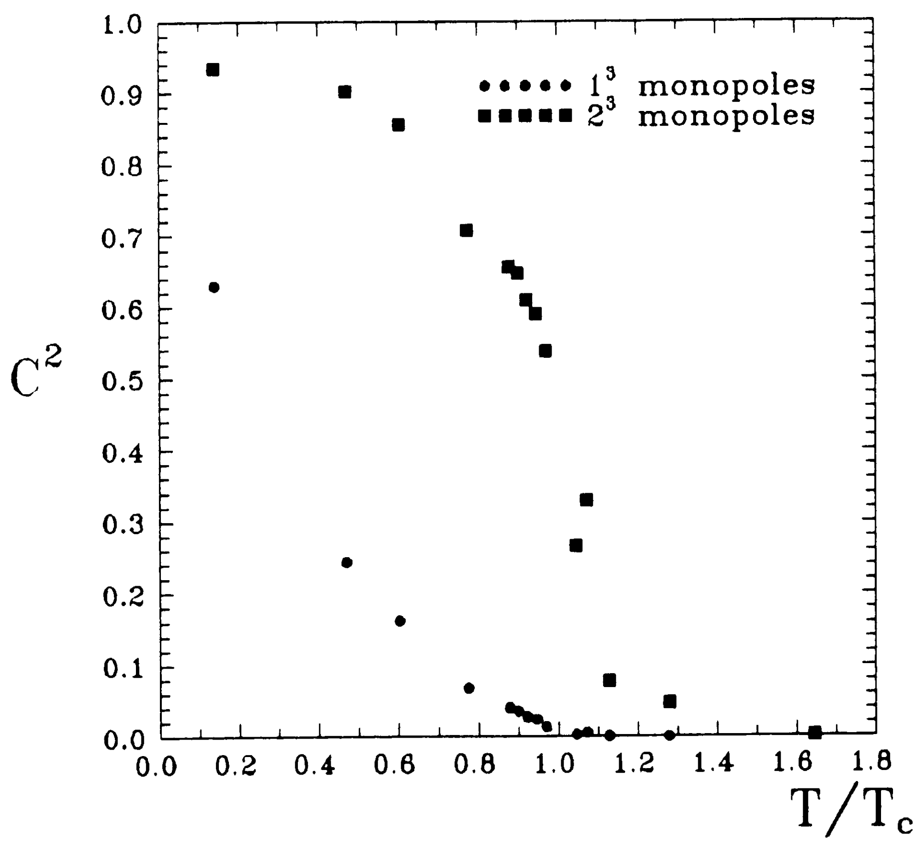

Fig.2 : Square of the monopole condensate versus temperature for the four dimensional $SU(2)$ lattice gauge theory on the $10^3 \times 4$ lattice. Estimated statistical errors are of the order of the size of the graphical symbols.

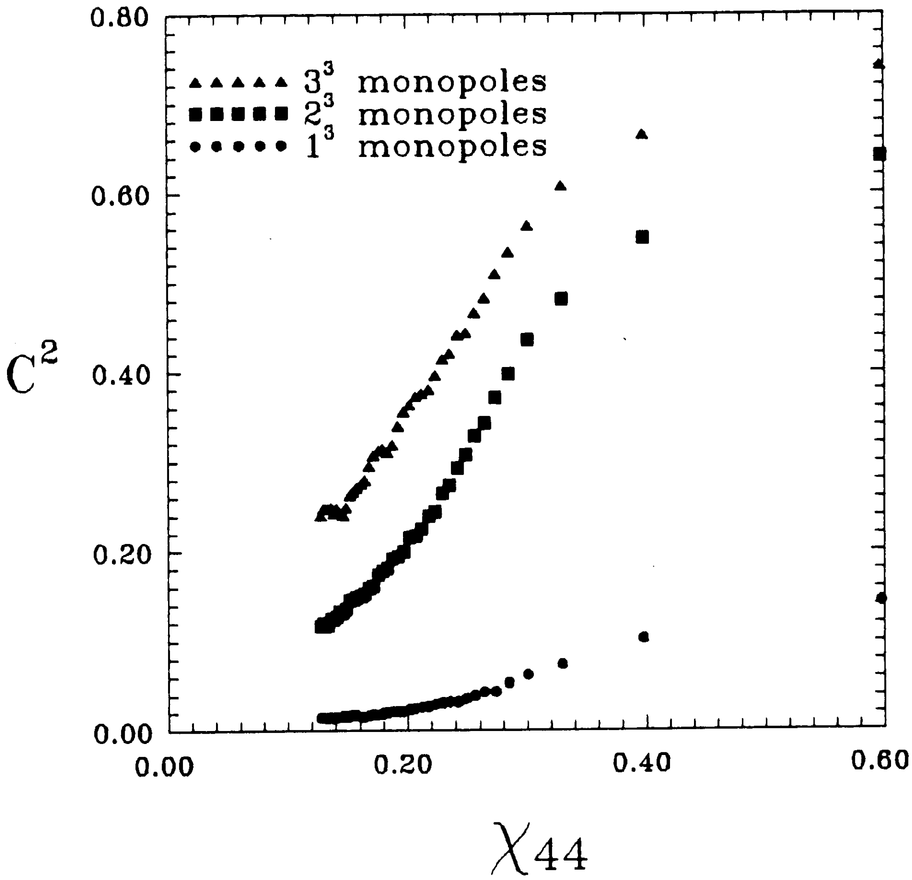

Fig.3 : Square of the monopole condensate versus the Creutz ratio, χ, for the four dimensional $SU(2)$ lattice gauge theory on the 10^4 lattice. We vary the value of χ and C^2 by cooling the gauge fields. Estimated statistical errors are of the order of the size of the graphical symbols.