INFN - Laboratori Nazionali di Frascati
Servizio Documentazione

LNF-91/063 (IR)
3 Ottobre 1991

DAΦNE CAVITY R & D: A SHUNT IMPEDANCE MEASUREMENT SYSTEM BASED ON THE PERTURBATIVE METHOD

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ABSTRACT

RF cavities are likely to drive multibunch instabilities of the *DAΦNE* beam. Therefore resonator characterization is among the crucial R&D activities of the project.
The aim of this paper is to present a perturbative system - based on the Slater theorem - for both longitudinal and transverse impedance measurements of any selected oscillating mode. Theoretical fundamentals, hardware assembling and data acquisition are described, together with the experimental results of the calibration measurement made on a 360 MHz single-cell cavity.

1) Introduction

Collective phenomena affect the operation of every accelerating machine, especially of high-current storage rings - as synchrotron light sources and factories - where multi-bunch instability is a primary source of stored current limitation.
The instability arises from the electromagnetic interaction between the beam and the environment (the vacuum chamber); in fact the e.m. wake fields excited by the beam in the surrounding surfaces react back on the beam itself keeping it unstable as the overall current exceedes a certain threshold.
The RF cavities strongly contribute to the phenomenon by means of their HOMs, i. e. the resonant modes not used for the acceleration that can be excited if they overlap to the beam frequency spectrum.
This kind of interaction can be evaluated once the cavity spectrum has been fully characterized, i.e. once the resonant frequency f, the quality factor Q, the longitudinal and the tranverse shunt impedances R and $R_\perp$ of each mode are known.
Computer codes are very useful in the first evaluation of these parameters. Neverthless experimental check of the code outputs is very important, especially when dealing with 3-D structures.
In this paper we present a perturbative set-up based on the Slater theorem to measure the (R/Q) of $DA\Phi NE$ cavity prototypes; other meaningful parameters - f and Q - are directly measurable with a standard Network Analyzer.

2) Theory Review

The influence of the environment on the beam longitudinal and transverse dynamics is described in the time domain by means of the two functions $w(t)$ and $w_\perp(t)$ called respectively longitudinal and tranverse wake potential ([1]).
The longitudinal wake potential is defined as:

$$w(t) = \frac{U_{21}}{q_1 q_2} \tag{1}$$

where q_1 and q_2 are two charges traveling on axis with a time separation t, and U_{21} is the energy lost by the trailing charge q_2 due to the interaction with the wake field excited by the leading charge q_1.
The Fourier transform of the longitudinal wake potential has the dimension of an impedence and is called longitudinal coupling impedance $Z(\omega)$:

$$Z(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} w(t) e^{-j\omega t} dt \tag{2}$$

The concept of coupling impedance is widely used in frequency domain analysis, well suitable when dealing with circular machines.
The wake potential and the coupling impedance in a resonant structure are uniquely determinated once the resonant frequency f, the quality factor Q and the ratio (R/Q) of each mode are known.
The shunt impedance R is defined as:

$$R = \frac{V^2}{2P} \tag{3}$$

where V is the maximum voltage gain for a fully relativistic particle across the cavity gap and P is the power loss on the cavity walls.
It turns out that V is given by:

$$V = \left| \int_0^L E_z(z) e^{j\omega_n z/c} dz \right| \tag{4}$$

and being Q defined as:

$$Q = \omega_0 \frac{U}{P} \tag{5}$$

where U is the stored e.m. energy in the cavity, we finally obtain:

$$(R/Q)_n = \frac{1}{2\omega_n U_n} \left| \int_0^L E_z(z) e^{j\omega_n z/c} dz \right|^2 \tag{6}$$

where n is the index of the selected mode.
If we imagine now the leading charge q_1 traveling on a path at a distance r apart from the cavity axis, we can define the tranverse wake potential $w_\perp(t)$ as:

$$w_\perp(t) = \frac{M(t)}{q_1 q_2 r} \tag{7}$$

where $M(t)$ is the integral of the transverse force on the trailing charge q_2 over its whole trajectory inside the cavity.
The Fourier transform of the tranverse wake potential is called transverse coupling impedance $Z_\perp(\omega)$ and is given by:

$$Z_\perp(\omega) = jF[w_\perp(t)] = \frac{j}{2\pi} \int_{-\infty}^{+\infty} w_\perp(t) e^{-j\omega t} dt \tag{8}$$

It turns out that $Z_\perp(\omega)$ has the dimension of an impedance per unit length.
The contribution of the dangerous dipole modes to transverse wake and impedance can be derived once f, Q, $(R/Q)_\perp$ are known, with $(R/Q)_\perp$ defined as:

$$(R/Q)_\perp = \left(\frac{c}{\omega_n b}\right)^2 \frac{1}{2\omega_n U_n} \left| \int_0^L E_z(z) e^{j\omega_n z/c} dz \right|^2 \tag{9}$$

where c is the light velocity and the integral is now calculated on a longitudinal path at a distance b off axis.

3) The Slater theorem

From the above formulas it is clear that to get the (R/Q) values one has to measure the field distribution in the cavity. This is done by moving some perturbing objects on the selected path and measuring the resulting resonant frequency shift at any position of the object along the path.
In fact the Slater theorem ([2]) states that the frequency shift Δf and the unperturbed field distribution in the removed volume ΔV are related by:

$$\frac{\Delta f}{f} = \frac{1}{4U} \int_{\Delta V} (\mu_0 H_0^2 - \epsilon_0 E_0^2) dV \tag{10}$$

Anyway, the presence of the object also produces a local distortion of the field distribution in the region around it. Hence, to be correctly evaluated, the previous formula has to take into account the actual local fields, including the contribution given by the object itself.
The correction factor depends on the size and the geometry of the perturbing object and is different for different orientations of the fields.
If the object has an axial simmetry we have to introduce 4 correction - or form - factors, for the longitudinal and the transverse components of E and H.
So eq. 10 becomes:

$$\frac{\Delta f}{f} = \frac{1}{4U} \int_{\Delta V} (-k_1 \epsilon_0 E_\parallel^2 - k_2 \epsilon_0 E_\perp^2 + k_3 \mu_0 H_\parallel^2 + k_4 \mu_0 H_\perp^2) dV \tag{11}$$

Form factors can be either measured or calculated. For a perfectly conducting sphere ([3]) it turns out that:

$$k_1 = k_2 = 3 \qquad k_3 = k_4 = 3/2$$

So by means of a sphere one can only measure the quantity $1/2\mu H^2 - \epsilon E^2$ along the selected path.
If the selected mode has longitudinal and transverse components of both E and H it is worth to use a proper perturbing object in order to extract the required information. When dealing with longitudinal components, needle-like objects are preferred ($k_1 \gg k_2$, $k_3 \gg k_4$), while disk-like objects are more suitable for transverse field measurements ($k_1 \ll k_2$, $k_3 \ll k_4$).

If the frequency shift is mainly due to the longitudinal electric field, as in the case of the fundamental $TM010$ mode perturbed along the accelerating gap, and if the object is small enough compared to the wavelength, eq. 11 may be written as:

$$\frac{\Delta f(z)}{f} = -\frac{k_1 \epsilon_0}{4U} E_z^2(z) \Delta V = -\frac{k_1}{3} \left(\frac{\Delta V_{object}}{\Delta V_{sphere}} \right) \frac{\pi \epsilon_0 l^3}{U} E_z^2(z) \tag{12}$$

where l is the length of the object longitudinal semiaxis and ΔV_{sphere} is the volume of a sphere having the same longitudinal size of the perturbing object.
So, by combining eq. 12 and eq. 6, we obtain ([7]):

$$(R/Q)_n = \frac{1}{4\pi^2 G \epsilon_0 l^3 f^2} \left| \int_0^L s(z) |\Delta f(z)|^{1/2} e^{j\omega_n z/c} dz \right|^2 \tag{13}$$

where G is the overall geometric factor ([5]) defined as:

$$G = \frac{k_1}{3} \left(\frac{\Delta V_{object}}{\Delta V_{sphere}} \right)$$

and $s(z)$ is a sign function that has to be introduced in order to recover the sign of the field lost in the square root function.
It turns out that for a perfectly conducting sphere we have $G = 1$.
So by means of eq. 13 one can calculate the (R/Q) parameter once the resonant frequency shift $\Delta f(z)$ has been measured along the selected path.

4) The experimental set-up

The experimental set-up for frequency shift measurement is based on a Phase Lock system and the sketch is presented in Fig. 1.
In this scheme we profit of the EXT DC FM option of our generator that allows to control the output frequency by means of the voltage connected to an auxiliary port. So any mismatching between exciting and resonant frequencies is converted in a phase error signal that drives the generator to restore the matching condition ([8]).
The task of the acquisition system is to record the error voltage at the EXT DC FM port for a certain number of different position of the perturbing object inside the cavity. The error voltage is software converted in a frequency shift by the scale factor set on the generator front panel.
As the scale factor can be choosen in a large range (from 100 Hz/Volt to 200 KHz/Volt) the system shows a good flexibility and detectes perturbations in a wide dynamic range.
The whole control electronic system has 2 GHz bandwidth and the modes above that frequency are no longer measurable. However most of them will be coupled off by the vacuum chamber. On the other hand many RF devices are avaible in this frequency range and the

resulting system is reliable and simple to use.
The lock procedure is carried out in different steps. First, with the feedback off, the mode resonant frequency has to be found acting on the generator; then the phase error has to be kept to zero by means of the coaxial delay line and the feedback has to be turned on.
A final optimization can be done adjusting the delay line to improve the tuning and adjusting the error amplifier parameters to match the selected mode bandwidth.
The error amplifier is based on standard OP AMP and is mainly an integrator featuring a variable transfer function zero to compensate the mode bandwidth.
The error amplifier transfer function is given by:

$$A(s) = A_\infty \frac{1 + s/\omega_z}{s/\omega_z} \tag{14}$$

where ω_z is the frequency of the variable zero, and A_∞ is the high frequency gain of the amplifier that can be varied indipendently from the ω_z value.
The open loop transfer function $H(s)$ of the whole phase lock system is:

$$H(s) = \frac{A_\infty}{\omega_{bw}} \left(\frac{\Delta V}{\Delta \phi}\right)_{det} \left(\frac{\Delta \omega}{\Delta V}\right)_{gen} \frac{(1 + s/\omega_z)}{(1 + s/\omega_{bw})s/\omega_z} \tag{15}$$

where ω_{bw} is the bandwidth of the mode, $\left(\frac{\Delta V}{\Delta \phi}\right)_{det}$ is the phase detector sensitivity, and $\left(\frac{\Delta \omega}{\Delta V}\right)_{gen}$ is the voltage to frequency scale factor set on the generator.
It turns out that the best loop performances are reached when the error amplifier zero ω_z just matches the mode bandwidth ω_{bw}. In this case the open loop becomes a pure integrator and the gain is the highest possible.

5) The data acquisition system

The data acquisition system is based on the high-performance multifunction analog, digital and timing input-output board NB-M10-16X (National Instruments Corporation) for the Macintosh II computers.
Among the several NB-M10-16X features, one of three indipendent 16 bit counters/timers has been used to generate a pulse output for the steppin' motor driver that leads the perturbing object at the desired position inside the cavity.
The square wave output parameters for the motor driver are set by an interactive graphic interface using the Labview language; this program is also used to process the voltage signal provided by the Error Amplifier and read by one of the I/O analog channel of the NB board. The object speed and the spatial resolution of the data acquisition can be selected for each run.

6) System calibration

Picture 1 shows the whole system under operation. In order to calibrate our equipment we started measuring a 360 MHz single-cell cavity.
We measured the (R/Q) value of some monopoles and compared the results to the URMEL-T code outputs. As the cavity geometry is straightforward 2D we assumed the code results as references and could check the reliability of our system.
Tab. 1 shows the results we obtained for 3 monopoles, namely TM010, TM011 and TM020. The mode field profile is reported in Fig. 2.
We found a very good agreement for the most meaningful modes TM010 and TM011 (within few percent).
The relative error is much larger when the (R/Q) approaches zero, as for TM020 mode; this is because the noise induces a certain amount of absolute error in the measurement, whose relative weight becomes more relevant as the measured value becomes smaller. Anyway, even in this case the results are satysfaying because they show what modes we don't have to worry about.
We carried out the calibration of the system using a conducting sphere (r = 4 mm); once the system has been calibrated we could also measure the overall form factor of two other perturbing objects, a longitudinal asymmetric ellipsoid and a cylindrical cage.
The results we obtained are:

$$G_{ell} = 0.28 \qquad G_{cage} = 0.9$$

As these two objects are mainly extended in the longitudinal direction, they could perform precise measurement of the field axial components.

Acknowledgements

The authors would like to thank A. Sprecacenere and T. Tranquilli for the construction of the electronic and mechanical parts of whole equipment.

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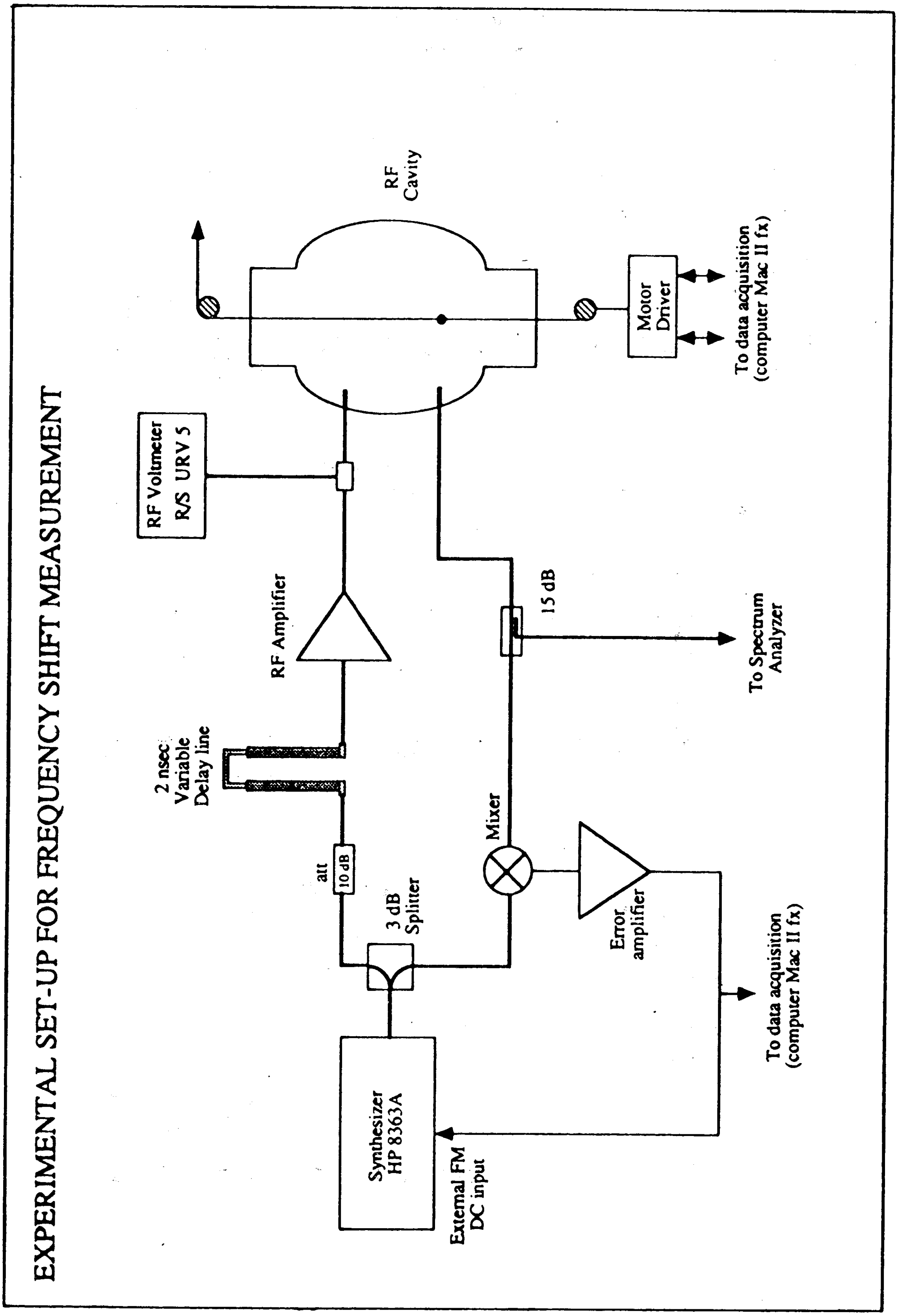

EXPERIMENTAL SET-UP FOR FREQUENCY SHIFT MEASUREMENT
RF Cavity
Motor Driver
To data acquisition (computer Mac II fx)
RF Voltmeter
R/S URV 5
RF Amplifier
15 dB
To Spectrum Analyzer
2 nsec Variable Delay line
Mixer
att
10 dB
3 dB Splitter
Error amplifier
To data acquisition (computer Mac II fx)
Synthesizer HP 8363A
External FM DC input

Fig. 1

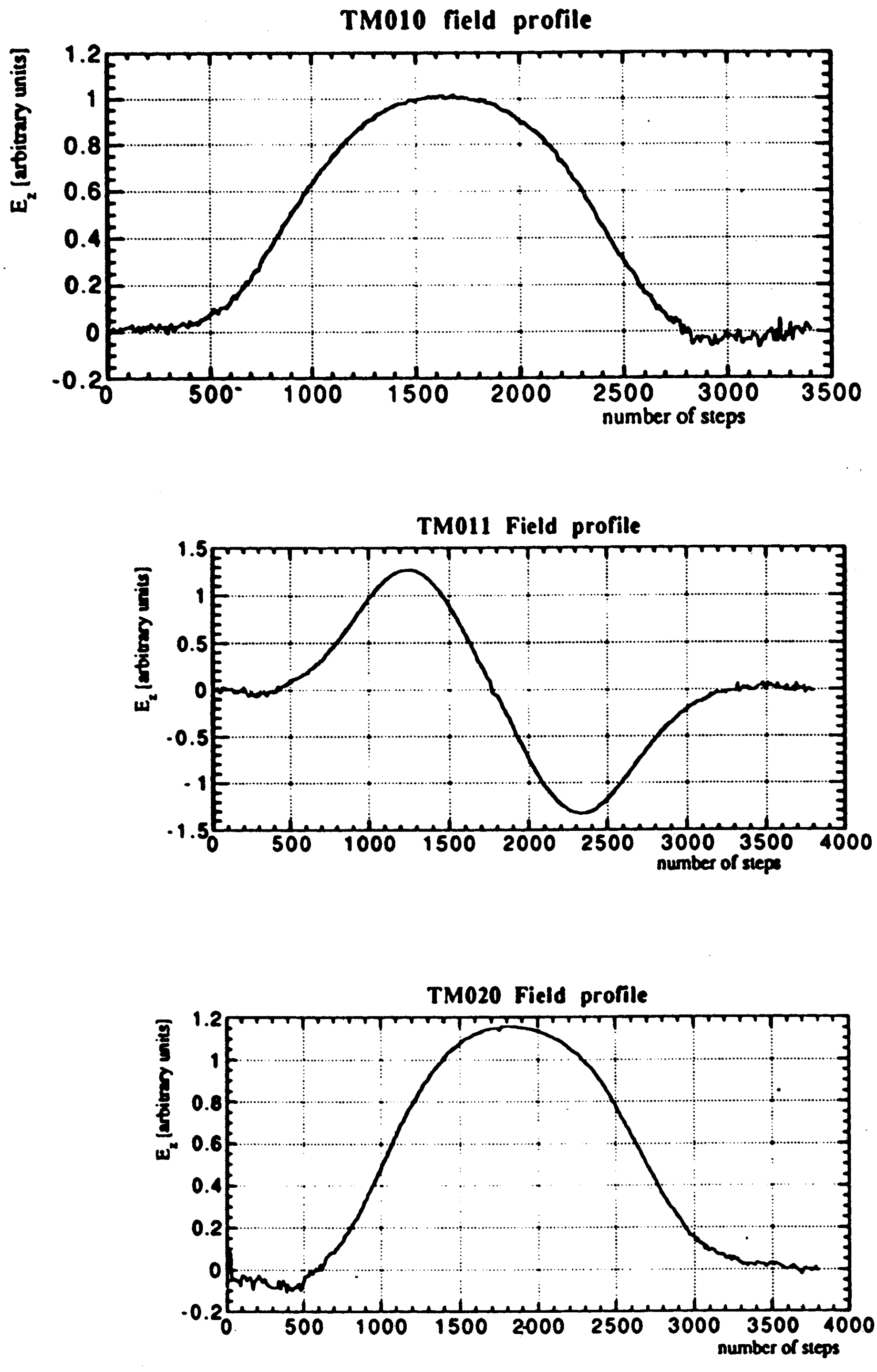
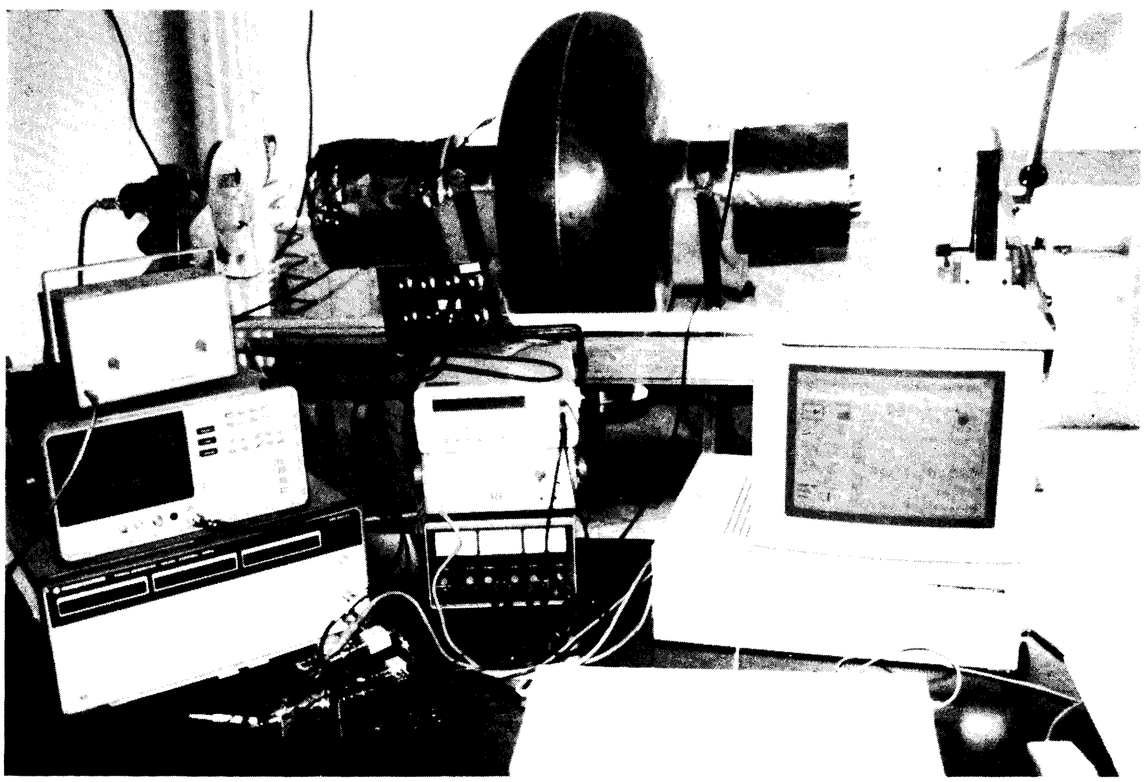

TM010 field profile
E_z [arbitrary units]
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
500
1000
1500
2000
2500
3000
3500
number of steps
TM011 Field profile
E_z [arbitrary units]
1.5
1
0.5
0
-0.5
-1
-1.5
0
500
1000
1500
2000
2500
3000
3500
4000
number of steps
TM020 Field profile
E_z [arbitrary units]
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
500
1000
1500
2000
2500
3000
3500
4000
number of steps

Fig. 2

TABLE I

	URMEL-T CALCULATIONS		MEASUREMENTS	
Mode	Resonant Frequency [MHz]	R/Q [Ohm]	Resonant Frequency [MHz]	R/Q [Ohm]
TM_{010}	360.2	72.2	358.9	72.0
TM_{011}	685.9	19.7	686.4	19.65
TM_{020}	786.3	0.092	782.8	0.58



Picture I: The system under operation