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Hard Photon Corrections to Bhabha Scattering Near the Z_0

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ABSTRACT

A previous analysis of QED radiative corrections to Bhabha scattering in the vicinity of the Z_0 is improved by a better analytical treatment of hard photon terms with sizeable effects on the tail of the resonance. A quantitative discussion of two-loop corrections and comparison with some existing Montecarlo codes show that quite generally, the theoretical uncertainty at large angles cannot be reduced much below 1%.

Higher order corrections to Bhabha scattering have been recently investigated [1-8] with increasing accuracy, as demanded by the precision achieved with LEP experiments, now or order of few ‰. The recent calculation [4] of two loop QED corrections has extended previous analytical analysis [1,2] of Bhabha scattering in the vicinity of the Z_0 based on $o(\alpha)$ results implemented by the resummation of soft and collinear effects to all orders. The resulting formulae, valid in the limited kinematical configuration of quasi collinear back-to-back $e^+ e^-$ events, can be used to simply extract from the cross section measured at the peak the Z_0 couplings to $e^+ e^-$ pairs - similarly to what has been achieved for $\mu^+ \mu^-$ pairs and with comparable accuracy - and to test Monte-Carlo calculations in the kinematical domain defined above for the estimation of the QED systematic uncertainties. Indeed various Monte-Carlo codes have been recently proposed [5, 9-12] , mainly based on the formalism of the QED structure functions, with systematic errors of order 1%, which must be tested against detailed analytical results, even in a restricted kinematical region.

The aim of the present note is to improve the accuracy of the theoretical calculation [4] in the energy region above the Z_0 peak with a better description of hard photon effects. The agreement of our formulae with various Monte-Carlo calculations in a extended energy domain is now of order 1%. Furthermore we explicitly show that the effect of $o(\alpha^2)$ corrections, beyond those already included in the exponentiated terms and often not considered in Monte-Carlo codes, is not smaller than ~1%. Therefore a better accuracy of Monte-Carlo results - as sometimes claimed - seems to us quite unrealistic.

The starting formula is given in ref. [4] and, with the same notation, is written as:

$$d\sigma\,(e^+ e^- \to e^+ e^-) \equiv \sum_{i=1}^{10} d\sigma(i) = \sum_{i=1}^{10} d\sigma_0\,(i)\,\left\{ C^{(i)}_{infra}\,(1+\bar{C}^{(i)}_F) + C'^{(i)}_F \right\}, \qquad (1)$$

A few comments are in order.

The coefficients $C^{(i)}_{infra}$ are the usual exponentiated infrared factors and have a different form for the pure QED, interference and resonant terms. The finite factors $\bar{C}^{(i)}_F$ include, up to $o(\alpha^2)$, two groups of terms of different origin. The first one, called $C^{(i)}_1$ in ref. [4], corresponds to form factors and bremsstrahlung contributions in the s and t channels. The second group includes essentially the contribution of the hard terms of the initial and final electron radiators in the formalism of structure functions. The present analysis precisely improves the calculation of that class of terms. Finally the factors $C'^{(i)}_F$ correspond to the various box contributions, do not contain important logarithms and therefore are taken to order α only [1]. Strictly speaking, eq.(1) holds only approximately at the level of the differential cross section, because of the slight different angular dependence of the soft and hard components in the electron radiators, as shown for example in ref. [13], in the reaction $e^+ e^- \rightarrow \mu^+ \mu^-$. However the use of eq. (1), in the vicinity of the Z_0 and in the kinematical domain of quasi collinear back-to-back pairs covered by our formulae, is legitimate within our level of accuracy.

We report now the main lines of the derivation of the hard contributions which improve the analysis of refs. [4, 14]. For the sake of clarity we denote by β_e and β_f the appropriate initial and final states factors, putting of course $\beta_e = \beta_f$ in the final expression for the Bhabha radiative formulae. In the structure function approach the cross section is written as

$$\sigma(s) = \int_0^{\varepsilon} dx \;\; \sigma_0 \,(s(1-x))\, H_e\,(x,s)\, F_f \left[(\varepsilon - x)\,,\,(1\text{-}x)\, s\right], \tag{2}$$

where the initial and final radiators are given, for example, in ref. [15]. Then, for small fractional energy resolution ε and to leading order in β_e and β_f one has

$$H_e\, F_f = \beta_e \Delta_e\, (\Delta_f - \varepsilon\, \beta_f)\, x^{\beta_e - 1}\, (\varepsilon - x)^{\beta_f} - \beta_e \Delta_f (1 + \tfrac{1}{4}\beta_e)\, (\varepsilon - x)^{\beta_f}$$

$$+ \, o(\beta_e^2\, \varepsilon\, ,\, \beta_e\, \varepsilon^2)\, , \tag{3}$$

where [14]

$$\Delta_e \equiv \Delta_e(s) = 1 + \frac{\alpha}{\pi}\left\{\frac{3}{2} L_e + 2(\zeta(2) - 1)\right\} + \left[\frac{\alpha}{\pi}\right]^2 \left\{\left[\frac{9}{8} - 2\,\zeta(2)\right] L_e^2 + \right.$$

$$\left. \left[3\,\zeta(3) + \frac{11}{2}\,\zeta(2) - \frac{45}{16}\right] L_e + \left[-\frac{6}{5}\,\zeta^2(2) - \frac{9}{2}\,\zeta(3) - 6\,\zeta(2)\ln 2 + \frac{3}{8}\,\zeta(2) + \frac{57}{12}\right]\right\} \quad (4)$$

with $\beta_e = \frac{2\alpha}{\pi}(L_e - 1)$, $L_e = \ln \frac{s}{m_e^2}$,and Δ_f , β_f similarly defined. The first and the second term in eq. (3) clearly correspond to soft and hard initial state radiation respectively. The insertion of the first contribution in eq. (2) has been carefully evaluated in refs [4, 14]. A better treatment of the second term, on the other hand, is needed when considering the energy region above the Z_0 peak. Then by splitting the Born cross-section $\sigma_0(s)$ in eq. (2) as $\sigma_0^{QED} + \sigma_0^{INT} + \sigma_0^{RES}$ where

$$\sigma_0^{QED} = A\,\frac{1}{s}, \qquad \sigma_0^{INT} = B\,\mathrm{Re}\,\frac{1}{s - M^2 + iM\Gamma}, \qquad \sigma_0^{RES} = C\,\frac{s}{(s-M^2)^2 + M^2\Gamma^2},$$

with $\Gamma \equiv \Gamma(s) = \gamma\, s/M$, the contributions of the second term in the r.h.s. of (3) to the cross sections are

$$\sigma^{QED} = -\,A\,\Delta_f\,\frac{1}{s}\,(\beta_e\,\varepsilon)\,(1 + \tfrac{1}{4}\beta_e)\,\varepsilon^{\beta_f}\,B(1, 1+\beta_f) \quad (5)$$

$$\sigma^{INT} = -\,B\,\Delta_f\,\beta_e\,(1 + \tfrac{1}{4}\beta_e)\,\frac{1}{s}\,\varepsilon^{\beta_f}\;B(1, 1+\beta_f)$$

$$\mathrm{Re}\left\{\frac{1}{1+i\gamma}\,\frac{z}{z-1}\;{}_1F_2\left(1+\beta_f, 1; 2+\beta_f; \frac{z}{z-1}\right)\right\}$$

$$= B\,\Delta_f\,\frac{\beta_e}{s}\,(1 + \tfrac{1}{4}\beta_e)\,\varepsilon^{\beta_f}\,\frac{1}{1+\gamma^2}\left\{\ln|1-z| + \Phi\,\gamma\right\} + o(\beta_e^2\,\varepsilon\,;\beta_e\,\varepsilon^2) \quad (6)$$

$$\sigma^{RES} = - C\,\Delta_f\,\beta_e\,(1+\tfrac{1}{4}\beta_e)\,\varepsilon^{\beta_f}\,\frac{1}{\gamma s}\,B(1,1+\beta_f)\cdot$$

$$\mathrm{Im}\left\{\frac{1}{1+i\gamma}\,\frac{z}{z-1}\;{}_1F_2\left(1+\beta_f,1;2+\beta_f;\frac{z}{z-1}\right)\right\}$$

$$= C\,\Delta_f\,\beta_e\,(1+\tfrac{1}{4}\beta_e)\,\varepsilon^{\beta_f}\,\frac{1}{\gamma s}\,\frac{1}{1+\gamma^2}\left\{\Phi - \gamma\ln|1-z|\right\}$$

$$+\, o(\beta_e^2\,\varepsilon\,;\beta_e\,\varepsilon^2)\,, \qquad (7)$$

where * $z = \varepsilon\, s\,\frac{(1+i\gamma)}{s-M_R^2}$, $M_R^2 = M^2 - iM\Gamma$ and $\Phi = \mathrm{arctg}\,\frac{\varepsilon s + M^2 - s}{M\Gamma(1-\varepsilon)} - \mathrm{arctg}\,\frac{M^2 - s}{M\Gamma}$.

The above eqs. (5-7) give a better description of hard photon effects above the resonance peak than those given in refs. [4, 14].

For the specific application to Bhabha scattering eq. (1) is modified correspondingly as follows:

$$d\sigma\,(e^+e^- \to e^+e^-) = \sum_{i=1}^{10} d\sigma_0\,(i)\,\left\{C^{(i)}_{infra}\,(1+\bar{C}^{(i)}_F) + C^{(i)}_H + C'^{(i)}_F\right\} \quad , \qquad (8)$$

where the factors $\bar{C}^{(i)}_F$ contain now the pure soft contributions up to two loops only and differ, therefore, from those reported in ref.[4], the terms $C^{(i)}_H$ include the hard initial state bremsstrahlung discussed above and, finally, the terms $C'^{(i)}_F$ correspond, as in refs [1, 4], to the various box contributions.

Then, including the initial-final state interference β_{int} dependence, as in refs. [4,16] the coefficients $\bar{C}^{(i)}_F$ and $C^{(i)}_H$ in eq. (8) take the form

$$\bar{C}^{(i)}_F = C^{(i)}_1 - \frac{\pi^2}{6}\bar{\beta}_e^{\,2}\,, \qquad (i=1,..,6)$$

$$\bar{C}^{(i)}_F = C^{(i)}_1 - \beta_e\varepsilon - \frac{\pi^2}{6}\bar{\beta}_e^{\,2} \qquad (i=7,8,9) \qquad (9)$$

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* Notice the difference with the analogous definitions in [14] due to the s-dependence in Γ.

$$\tilde{C}_F^{(10)} = C_1^{(10)} - \beta_e \varepsilon - \frac{\pi^2}{6} \beta_e \bar{\bar{\beta}}_e$$

and

$$C_H^{(i)} = - \varepsilon^{\bar{\beta}_e} \beta_e \varepsilon \left(1 + \frac{1}{4} \bar{\beta}_e\right) \qquad (i = 1,..,6)$$

$$C_H^{(i)} = \frac{(s-M^2)^2 + M^2\Gamma^2}{s\,(s-M^2)} \varepsilon^{\beta_e} \bar{\beta}_e \left(1 + \frac{1}{4} \beta_e\right) \cdot$$

$$\frac{1}{1+\gamma^2} \left\{ \ln|1-z| + \Phi \gamma \right\} \qquad (i = 7,8,9) \qquad (10)$$

$$C_H^{(10)} = - \frac{(s-M^2)^2 + M^2\Gamma^2}{s^2} \varepsilon^{\beta_e} \bar{\bar{\beta}}_e \left(1 + \frac{1}{4} \beta_e\right) \frac{1}{\gamma(1+\gamma^2)} \left\{ \Phi - \gamma \ln|1-z| \right\}$$

where $\bar{\beta}_e = \beta_e + \beta_{int}$, $\bar{\bar{\beta}}_e = \beta_e + 2\,\beta_{int}$, Φ and z are defined above and the coefficients $C_1^{(i)}$, as well as $C_{infra}^{(i)}$ and $C'^{(i)}_F$ in eq.(8) are defined in ref.[4] and for sake of brevity will not be reported here again.

When the experimental observation of the final electrons includes the detection of hard photons collinear to the final particles within a small cone of half opening angle δ ($\delta << 1$) then eq. (8) is modified as follows [1, 4, 17]

$$d\tilde{\sigma}\,(e^+ e^- \to e^+ e^-) = \sum_{i=1}^{10} d\sigma_0\,(i) \left\{ \tilde{C}_{infra}^{(i)} (1 + \tilde{C}_F^{(i)}) + \tilde{C}_H^{(i)} + C'^{(i)}_F \right\} , \qquad (11)$$

where

$$\tilde{C}_{infra}^{(i)} = \varepsilon^{\beta_\delta - \beta_e} C_{infra}^{(i)} \qquad (12)$$

$$\tilde{C}_F^{(i)} = \tilde{C}_1^{(i)} + \varepsilon\,(\beta_e - \beta_\delta) - \frac{\pi^2}{6} \bar{\beta}_e\,(\beta_{int} + \beta_\delta), \qquad (i = 1,..,6)$$

$$\tilde{C}_F^{(i)} = \tilde{C}_1^{(i)} - \varepsilon\,\beta_\delta - \frac{\pi^2}{6} \bar{\beta}_e\,(\beta_{int} + \beta_\delta) \qquad (i = 7,8,9) \qquad (13)$$

$$\tilde{C}_F^{(10)} = \tilde{C}_1^{(10)} - \varepsilon\, \beta_\delta - \frac{\pi^2}{6} \beta_\delta \bar{\bar{\beta}}_e$$

and

$$\tilde{C}_H^{(i)} = - \varepsilon^{(\beta_{int} + \beta_\delta)} \beta_e \varepsilon \left(1 + \frac{1}{4} \bar{\beta}_e\right) \qquad (i= 1,..,6)$$

$$\tilde{C}_H^{(i)} = \frac{(s-M^2)^2 + M^2\Gamma^2}{s\,(s-M^2)} \varepsilon^{(\beta_{int} + \beta_\delta)} \bar{\beta}_e \left(1 + \frac{1}{4} \beta_e\right) \cdot$$

$$\cdot \frac{1}{1+\gamma^2} \left\{ \ln|1-z| + \Phi \gamma \right\} \qquad (i= 7,8,9) \qquad (14)$$

$$\tilde{C}_H^{(10)} = - \frac{(s-M^2)^2 + M^2\Gamma^2}{s^2} \varepsilon^{\beta_\delta} \bar{\bar{\beta}}_e \left(1 + \frac{1}{4} \beta_e\right) \frac{1}{\gamma\,(1+\gamma^2)} \left\{ \Phi - \gamma \ln|1-z| \right\},$$

with $\beta_\delta = \frac{2\alpha}{\pi}(L_\delta-1)$, $L_\delta = (4/\delta^2)$ and the factors $\tilde{C}_1^{(i)}$ are also given in ref.[4].

This concludes the discussion of hard photon formulas. We would like to add a few comments on the phenomenological implications for LEP experiments, also in connection with other treatments based on Montecarlo studies.

First we compare the new results with the previous ones of ref. [4], for two different kinematical configurations corresponding to an acollinearity angle of the final electrons of 5 and 10 degrees respectively, with a value of δ - the half opening angle of the collinear hard photons - also given by 5 and 10 degrees. This is shown in table 1, where we give the integrated cross section for $42.3 \leq \theta \leq 137.7$, with M_Z = 91.17 GeV, Γ = 2.484 GeV and $\Gamma_{ee} = 8.38 \cdot 10^{-2}$ GeV.

$\delta=\theta_{ac}$	$\sqrt{s}$	87.17	88.17	89.17	90.17	91.17	92.17	93.17	94.17	95.17
5°	new	0.286	0.354	0.492	0.787	1.045	0.665	0.373	0.241	0.164
	old	0.287	0.355	0.493	0.789	1.049	0.671	0.378	0.241	0.158
10°	new	0.305	0.375	0.516	0.820	1.084	0.696	0.400	0.272	0.211
	old	0.306	0.376	0.518	0.823	1.090	0.704	0.406	0.280	0.218

Table 1

As clearly seen the results practically coincide before and on the peak, whereas they differ up to a few % on the Z tail, where the new corrections tend to generally decrease the cross section. The same behaviour is observed in Table 2, where we compare our analytic results (1) with the previous ones [4] and with those from the event generator BHAGEN [11], based on a non-perturbative formula which reproduces the perturbative approach in the proper limiting configurations. Here the energy resolution ε is varied for fixed $\delta = 10^{-3}$.

ε	$\sqrt{s}$	87.1	89.1	91.1	93.1	95.1
0.2	BHAGEN	0.327	0.539	1.100	0.405	0.217
	BHABHA NEW	0.327	0.538	1.094	0.407	0.222
	BHABHA OLD	0.327	0.537	1.097	0.418	0.237
0.3	BHAGEN	0.339	0.556	1.137	0.418	0.227
	BHABHA NEW	0.342	0.557	1.125	0.423	0.233
	BHABHA OLD	0.342	0.556	1.129	0.434	0.250

Table 2

The agreement within 1% between our formulae and BHAGEN - which in turn has been shown [11] to agree within the same accuracy with the other generator ALIBABA [5] - also gives the realistic estimate of theoretical uncertainty common to the various approaches. Notice that both event generators mentioned above do not consider the two-loop corrections included in our formulae. This very fact already makes a systematic uncertainty of order 1%, as can be seen in Table 3, where we compare, with the same notations of Table 1, the cross section (11) with the factors $\tilde{C}^{(i)}_F$ and $\tilde{C}^{(i)}_H$ evaluated with one or two loop accuracy. This concludes our discussion on the theoretical systematic uncertainties.

$\delta=\theta_{ac}$	$\sqrt{s}$	87.17	89.17	91.17	93.17	95.17
5°	1 loop	0.286	0.494	1.050	0.375	0.164
	2 loop	0.286	0.492	1.045	0.373	0.164
10°	1 loop	0.305	0.516	1.090	0.401	0.211
	2 loop	0.306	0.518	1.084	0.400	0.211

Table 3

To conclude, we have improved a previous analysis of e.m. radiative corrections in the vicinity of the Z_0, in particular for Bhabha scattering, by a better analytical treatment of hard photon effects. The implications for LEP experiments are of particular importance on the tail of the Z_0 distribution, where they generally reduce the previous cross section by order of a few %. A quantitative discussion on the relevance of the two-loop effects, as well as a comparison with some of the existing generators for Bhabha events, clearly show that the theoretical uncertainties cannot be reduced to a level significantly smaller than 1%.

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