CHARACTERISTICS OF SYNCHROTRON RADIATION

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1. INTRODUCTION

When relativistic electrons experience centripetal acceleration due to the presence of a magnetic field, they emit electromagnetic waves that are commonly called "synchrotron radiation" because first observed in the visible region, at the 70 MeV small electrosynchrotron, by a research group working in 1947 at the General Electric Research Laboratory (Schenectady - N.Y.).

In 1944, Ivanenko and Pomeranchuk[1,2] drew attention to the fact that the electron beam circulating in the cyclic inductive accelerator, called the "betatron", was interrupted because of the high quantity of energy irradiated by the electrons.

Originally, synchrotron radiation struck the attention of the physicists because of its very interesting and unusual properties, such as high power, high degree of collimation in the vertical plane, very broad spectral distribution and linear polarization on the orbital plane. The emission of radiation from relativistic particles traveling in a magnetic field has, more recently, attracted the attention of astrophysicists as well, since the synchrotron radiation mechanism has been found present, from the radiofrequency range to the X-ray range, also in the radio emission of Crab Nebula and several supernovae.

In 1950, synchrotron radiation became more important in connection with the development and construction of cyclic accelerators like electrosynchrotrons. This new type of accelerator was able to produce increasingly intense photon beams used not only to carry out pioneering works in several different fields,[3,4,5,6] but also as a tool to understand the dynamics of the particles in the accelerator.

In 1955 the spectral and angular distributions of the ultra violet radiation emitted by the 300-MeV Cornell synchrotron were examined by Tomboulian and Hartman for various electron energies over a wide photon energy range.

In 1960 Codling and Madden began to use the 180-MeV electrosynchrotron at NBS in Washington as a light source to perform experiments in the far ultra violet energy range. In Western Europe, first experiments with synchrotron radiation were carried out in 1965 by a French-Italian cooperation utilizing the 1.1 GeV electrosynchrotron in operation at Frascati to measure the mass absorption coefficient of gold thin films in the wavelength region 26 to 120 Å.

At the beginning of 1970, a major advance occurred due to the fact that a new kind of synchrotron radiation source, the so-called "storage ring" for electrons and positrons, became available, with a great improvement of both the electron current circulating in the accelerator and the stability of the photon beams.[7,8,9,10]

Today, several storage rings around the world have been specifically designed, built and put in operation only to serve as synchrotron radiation sources, such as - to mention only a few examples - the 2 GeV storage ring in operation at Daresbury, the Photon Factory machine (E=2.5 GeV) in operation at Tzukuba (Japan), or the two storage rings working at the NSLS - National Synchrotron Light Source at Brookhaven (N.Y.)

The unique properties of synchrotron radiation has opened entirely new horizons in the study of solid state physics, atomic and molecular physics, chemistry, material science, molecular biology,[11,12] and so on.

In 1980, work in such exciting new fields as X-ray lithography,[13] tomography, X-ray microscopy,[14] trace element analysis,[15,16] and digital subtraction angiography started in several laboratories in the world on experiments that are impossible to attempt with conventional X-ray sources.[17,18]

2. RADIATION EMITTED FROM BENDING MAGNETS

In practice, synchrotron radiation is emitted by electrons or positrons forced to move into a vacuum chamber along a closed trajectory under the action of the Lorentz force due to the presence of a magnetic field generated by dipoles or quadrupoles. As we have already mentioned, the main synchrotron radiation sources are "storage rings", whose typical assembly is reported in Fig.1 which shows the vacuum chamber, the radiofrequency apparatus to replace the energy lost from the particles, the bending magnets, and two beam lines to collect the radiation emitted tangentially to the trajectory.

2.a Nonrelativistic case: $\beta=(v/c)\ll 1$

As a starting point, to evaluate the power P radiated by a single nonrelativistic accelerating particle with charge e, we can use the Larmor[19,20,21,22] formula:

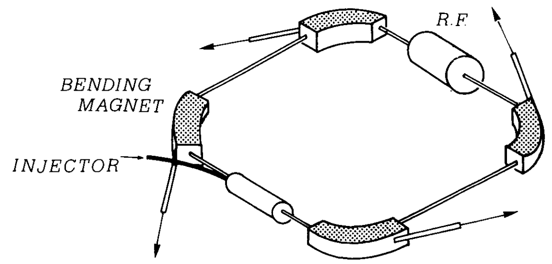

Fig.1 Sketch of an electron storage ring specifically designed as a synchrotron radiation source.

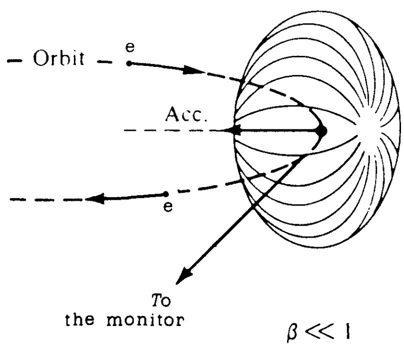

Fig.2 Angular distribution of synchrotron radiation in the nonrelativistic case.

$$P = \frac{2}{3} \frac{e^2}{c^3} \left| \frac{d\vec{v}}{dt} \right|^2 . \qquad (1)$$

In the special case of an electron moving along a circular orbit of radius R with constant velocity v, the power radiated is given by

$$P = \frac{2}{3} \frac{e^2}{c^3} \frac{v^4}{R^2}, \qquad (2)$$

and the radiation is emitted in a dipole pattern like the one shown in Fig.2 The maximum intensity is emitted in a direction perpendicular to the orbital plane.

2.b <u>Relativistic case $\beta=(v/c)\simeq 1$</u>

In the relativistic case,[23,24,25,26] the radiation is emitted in a narrow cone very near to the orbital plane, as shown in Fig.3.

For a particle of mass m and kinetic energy E so that $\beta=(v/c)\simeq 1$, traveling in a circular orbit of radius R, Eq.(1) changes to

$$P=\frac{2}{3}\,\frac{e^2 c}{R^2}\,\beta^4\left(\frac{E}{m_0 c^2}\right)^4, \tag{3}$$

where E/m_0c^2 is the ratio of the particle energy and its rest energy. This quantity, indicated generally by γ, can be written for an electron as

$$\gamma=\frac{E}{m_0 c^2}=1957E\ (\mathrm{GeV}). \tag{4}$$

Remembering that the rest mass of a proton is roughly 2000 times greater than that of an electron, and noting that at very high energy particle momenta become practically independent of the rest mass so that electrons and protons can have comparable R value, it is clear that at the same kinetic energy an electron radiates $(2000)^4\simeq 10^{13}$ times more than a proton and, consequently, electrons, not protons, are used to obtain intense synchrotron radiation sources.

The total instantaneous power increases with the 4^{th} power of the kinetic energy of the particles, which is completely different from the nonrelativistic case.

Finally, the instantaneous power emitted by the particles cannot be neglected in circular accelerators: consequently, suitable apparatus must be inserted into an accelerator to replace the energy lost by the particles to avoid losing the particle beam.

The energy U_0 radiated, per turn, by an electron can be evaluated multiplying Eq.(2) by the quantity $2\pi R/c\beta$ obtaining

$$U_0\ \frac{4\pi}{3}\,\frac{e^2}{R}\,\gamma^4. \tag{5}$$

In practical units (E in GeV, and R in meters) the energy U_0 can be written as

$$U_0=88.5\,\frac{E^4(\mathrm{GeV})}{R(\mathrm{meter})}, \tag{6}$$

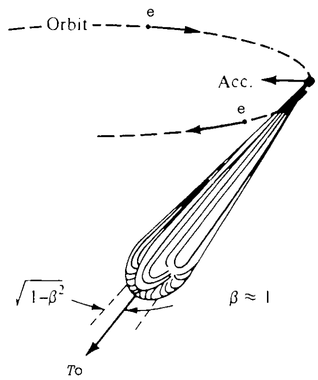

Fig.3 Angular distribution of synchrotron radiation in the relativistic case.

where U_0 is in keV per electron, per turn. For a storage ring like Adone in operation at Frascati (Italy), whose energy E is 1.5 GeV and R is 5 meters, the energy lost per electron, per turn is equal to

$$U_0 = \frac{88.5 \cdot (1.5)^4}{5} \text{ (keV)} = 89.6 \text{keV/electron turn}$$

From Eq.(6) it follows that the total power in kW radiated in a storage ring for electrons at energy E(GeV), with a circulating current I in ampere, is given by

$$W = 88.5 \quad I \quad \frac{E^4}{R}. \tag{7}$$

For the European Synchrotron Radiation Facility in construction at Grenoble (France), a storage ring with E=6 GeV, R=20 m and I=100 mA, Eq.(7) gives the following value for the total power emitted along all the orbit:

$$W = \frac{88.5 \times 0.1 \times (5)^4}{20} \text{ kW} = 276 \text{kW},$$

an incredible amount of energy.[27,28]

The power given by Eq.(7) is emitted over all the orbit and sometimes it could be more useful to know the power W_1 available for milliradians of horizontal angle, which is a quantity obtainable dividing Eq.(6) by $2\pi \cdot 10^3$, thus obtaining

$$W_1 = \frac{W}{2\pi \cdot 10^3} = 14.1 \ \frac{IE^4}{R}, \tag{8}$$

where W_1 is given in watts for milliradians of orbit.

3. GENERAL PROPERTIES OF THE SYNCHROTRON RADIATION FROM A BENDING MAGNET

For a quantitive description of the properties of synchrotron radiation, a suitable starting point is Schwinger's formula[29,30,31] for the total power P emitted by a monoenergetic relativistic electron traveling on a circular orbit of radius R:

$$P(\lambda \cdot \psi) = \int I(\lambda \cdot \psi) d\lambda \ \ d\psi, \tag{9}$$

where $I(\lambda,\psi)$, given by

$$I(\lambda \cdot \psi) = \frac{27}{32\pi^3} \frac{e^2 c}{R^3} \left(\frac{\lambda_c}{\lambda}\right)^4 \gamma^8 [1 + (\gamma\psi)^2]^2 \left[K^2_{2/3}(\xi) + \frac{(\gamma\psi)^2}{1 + (\gamma\psi)^2} K^2_{1/3}(\xi) \right] \tag{10}$$

is the instantaneous power radiated by one single electron along the whole orbit per revolution in the direction ψ, which is measured from the orbital plane, with wavelength λ. In Eq.(10), λ_c is the so-called "critical wavelength", $K_{1/2}$ and $K_{3/2}$ are modified Bessel functions of the second kind, and ξ is given by

$$\xi = \left(\frac{\lambda_c}{2\lambda}\right) [1 + (\gamma\psi)^2]^{3/2}.$$

Equation (10) can be integrated over the elevation angle ψ obtaining the instantaneous power P_1 radiated by one electron per turn in the whole solid angle as a function of wavelength:

$$P_1(\lambda) = N\frac{c^2 c}{R^3}\gamma^7\left(\frac{\lambda_c}{\lambda}\right)^3\int_{\lambda/c}^{\infty} K_{5/3}(\eta)\,d\eta,$$

where N is a numerical factor. The quantity $P_1(\lambda)d\lambda$ gives the spectral distribution of the synchrotron radiation. Due to the relativistic effect, the frequency distribution of synchrotron radiation extends from the fundamental frequency $\omega_0 = v/R$ of the circulating electrons to very high harmonics. It could be demonstrated that the frequency cut-off, which corresponds to the critical wavelength λ_c, is given approximately by

$$\omega_c \cong \omega_0 \gamma^3. \tag{11}$$

For a storage ring like Adone ($\omega_0 \simeq 2\pi \times 10^7$ Hz, $\gamma = 3\times10^3$), the high frequency cut-off is roughly equal to 10^{18}, which is a value corresponding to photons whose wavelength is equal to few angstrom. Consequently, the radiation is distributed over a large energy range generally extending from the infrared to the X-ray region; the shape of the general spectrum is shown in Fig.4. The horizontal wavelength scale is defined in terms of the ratio λ/λ_c and the vertical scale in terms of the spectral flux defined as the number of the photons per second, emitted in a relative bandwidth $\Delta\lambda/\lambda = 0.1\%$, in a milliradian of horizontal angle and integrated in the vertical plane. As shown in the figure, the spectral distribution is continuous and shows its maximum for $\lambda = 0.42\ \lambda_c$. It falls off exponentially for $\lambda \ll \lambda_c$ and decreases slowly for $\lambda \gg \lambda_c$. In the low energy region, the spectral distribution is practically independent of the energy of the particles. The critical wavelength which divides the spectrum into two parts of equal radiated power is given by

$$\lambda_c = \frac{4\pi}{3}\ \frac{R}{\gamma^3}. \tag{12}$$

In practical units λ_c can be written as

$$\lambda_c = 5.59\ \frac{R}{E^3}. \tag{13}$$

Taking account that between energy E, radius R, and magnetic field B, there exists the relationship

$$R(m) = 3.33\ \frac{E(GeV)}{B(T)},$$

the critical wavelength can be written as

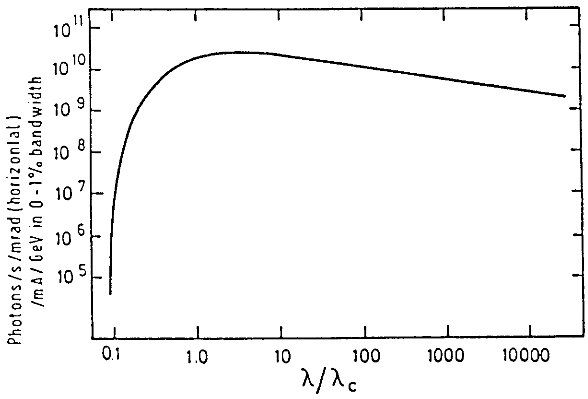

Fig.4 General shape of the spectral distribution of the synchrotron radiation per GeV.

$$\lambda_c = 18.64 \frac{1}{E^2(GeV)B(T)}, \qquad (14)$$

where λ_c is in Å, R in meters, E in GeV, and B in tesla. The shape of the spectrum reported in Fig.4 indicates that in order to obtain a good spectral flux extending to the X-ray region, λ_c must be equal to 1 Å or shorter and this can be achieved with a value of E of some GeV and R no more than a few meters. The radiation emitted in a very small angular cone around the instantaneous direction of flight of the particle is shown in Fig.3. The angular distribution $\Delta\psi$ is given roughly by

$$\Delta\psi = \frac{1}{\gamma}\left(\frac{\lambda}{\lambda_c}\right)^{1/3} \qquad \text{for} \, \lambda \gg \lambda_c,$$

and

$$\Delta\psi = \frac{1}{\gamma}\left(\frac{\lambda}{\lambda_c}\right)^{1/2} \qquad \lambda \ll \lambda_c$$

For $\lambda = \lambda_c$, $\Delta\psi$ is equal to γ^{-1}.

For a storage ring working in the GeV region, the emission angle is of the order of 0.1 milliradians; thus, the emission in the off plane direction is typically confined in a region of only about one milliradian of angular spread. This property together with the wide spectral range makes synchrotron radiation a source with high spectral brilliance over a large energy range.

For a synchrotron radiation source, the spectral brilliance is generally defined as the number of photons per second, per milliradian square, per millimeter square in a relative bandwidth $(\Delta\lambda/\lambda)$ equal to 0.1%. It follows from the definition that to achieve high brilliance, both small angular dispersion and small source size (the electron beam dimensions) are required and only synchrotron radiation shows both these properties.

The synchrotron radiation is 100% polarized in the orbital plane with the electric vector parallel to the orbital plane. Above and below this plane, the radiation is elliptically polarized due to the presence of a polarization component perpendicular to the orbital plane. The two terms present in Eq.(10) in the square brackets are associated with the two components of polarization: the first describes the component with the electrical vector E parallel to the orbital plane; the second one describes the component with E perpendicular to the orbital plane.

If we adopt as the definition for the degree of polarization the quantity given by

$$P = \frac{I \ \ - I}{I \ \ + I},$$

where $I_{||}$ and $I|$ are the parallel and perpendicular components respectively, for P we obtain

$$P = \frac{K_{2/3}^2(\xi) - [(\gamma\psi)^2/(1+(\gamma\psi)^2)]K_{1/3}^2(\xi)}{K_{2/3}^2(\xi) + [(\gamma\psi)^2/(1+(\gamma\psi)^2)]K_{1/3}^2(\xi)}.$$

This equation shows that for $\psi=0$, the radiation is 100% linearly polarized with the electric vector parallel to the orbital plane. Figure 5 reports the intensities of the parallel and perpendicular polarization components as a function of emission angle for different wavelengths. As can be seen from the figure, the so-called opening angle ψ is approximately γ^{-1} only for $\lambda \simeq \lambda_c$. At shorter wavelengths it becomes smaller and at longer wavelengths it becomes increasingly larger.

Finally, the electrons travel in a storage ring in a compact group - so-called bunches - in synchronism with an applied radiofrequency field which replaces the energy lost by the synchrotron radiation mechanism. Thus, the radiation is emitted in short flashes of bunch length time and shows a very stable intensity from bunch to bunch. To perform particular spectroscopic experiments, the repetition frequency is an important parameter which is determined by the repetition frequency of the filled

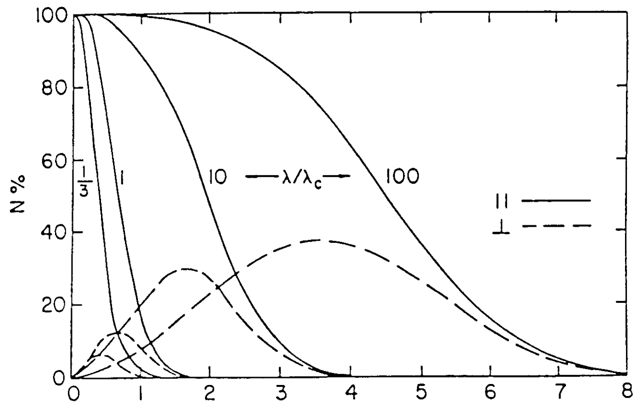

Fig.5 Intensities of parallel and perpendicular polarization components as a function of emission angle for different wavelengths.

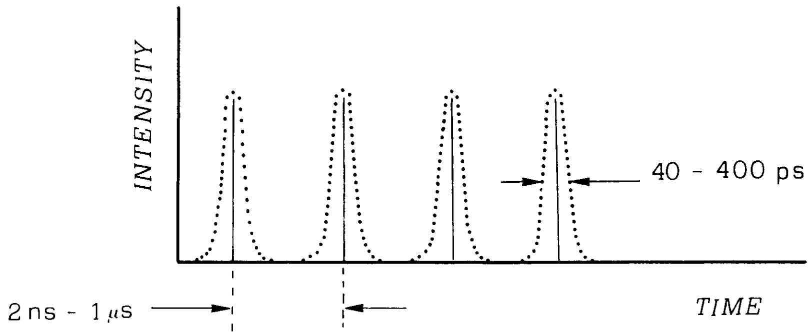

Fig.6 Intensity of the radiation reaching the detector as a function of time for a typical storage ring.

bunches; maximum separation between two contiguous pulses can be obtained working in a single bunch mode. Figure 6 reports the intensity of radiation reaching the detector as a function of time for a typical storage ring.

4. SYNCHROTRON RADIATION FROM WIGGLERS AND UNDULATORS

Up to now we have considered the radiation emitted from the circular trajectories described by electrons moving in the bending magnets of a storage ring. It is, however, possible to design different magnetic structures that are more effective sources of radiation than that coming from a bending magnet. The structure of such devices, known as wigglers[32,33] and used as a synchrotron radiation source, is reported in Fig.7. It is clear, following Eq.(14), that the critical wavelength λ_c can be decreased by increasing the magnetic field B. For a variety of technical and economic reasons, the field in bending magnets is limited to 10-14 kG, whereas wiggler fields equal to 20 kG are possible using conventional magnets and fields of 50 kG or more can be produced using superconducting magnets. Thus, standard wigglers can shift the spectrum of a relatively low-energy machine into the hard X-ray region; in addition, increasing the number N of the particle beam oscillations, the photon flux can be increased by a factor roughly equal to the number of the oscillations.

More generally speaking, similar periodic alternating magnetic field devices, used to force the electrons to make several "wiggles", as shown in Fig.8, are called "wigglers" or undulators depending on their characteristic features. In the approximation that the magnetic field is exactly sinusoidal, they are best characterized by the so-called "deflection parameter K" given by

$$K = eB_0\lambda_0/2\pi m_0 c = \alpha\gamma = 0.0934 \; B_0\lambda_0,$$

where B_0 is the peak magnetic field in tesla, λ_0 is the magnet spatial period, and α the maximum angle deflection. Remembering that between γ and the opening angle ψ there exists the relaxation $\psi \simeq \gamma^{-1}$, it is clear that K is the ratio between the maximum deflection angle and the natural opening angle of the radiation.

The distinction between multipole wigglers and undulators is not so sharp. For $K \gg 1$, the device is called "multipole wiggler" which can, as just mentioned, decrease the critical wavelength and increase the flux by a factor approximately equal to the number of poles. The simplest standard wiggler produces just one full oscillation or "wiggle" of the beam. Generally, a device like this, shown schematically in Fig.9, is called a 1λ standard wiggler or "wavelength shifter" because it can shift the critical wavelength without flux shifting. The comparison between the flux of the radiation emitted by a bending magnet of the Adone (Frascati) storage ring with the flux emitted from the wiggler (single-pole) is shown in Fig.10.

For K less than 1 or of the order of 1, strong interference effects occur which result in a spectrum of the emitted radiation consisting of - for $K \ll 1$ - or several - for $K \simeq 1$ - quasi-monochromatic lines called harmonics. In these cases, the device is called "undulator" and the wavelength of the n^{th} harmonic at an observing angle θ relative to the device axis is given by

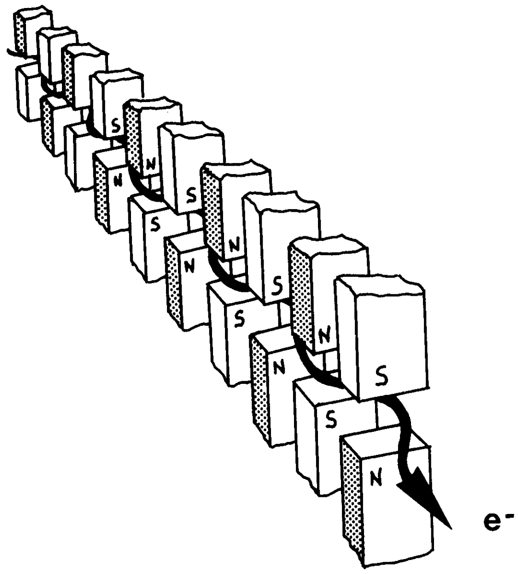

Fig.7 Typical magnetic structure (wiggler) used to obtain a more intense synchrotron radiation source.

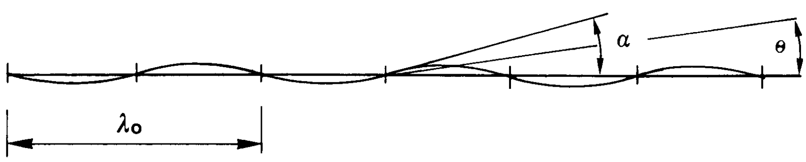

Fig.8 Trajectory of an electron beam traveling in a multipole wiggler.

$$\lambda_n = \frac{\lambda_0}{2n\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right). \qquad n = 1, 2, 3\ldots$$

For θ=0, i.e., on the axis, one has

$$\lambda_n = \frac{\lambda_0}{2n\gamma^2}\left(1 + \frac{K^2}{2}\right).$$

For n=1, the wavelength,

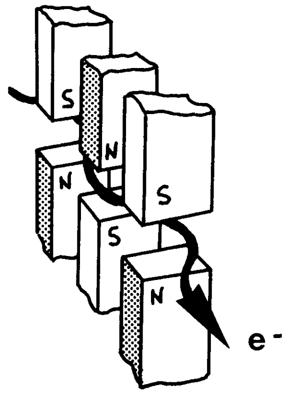

Fig.9 Schematic representation of a wiggler wavelength shifter.

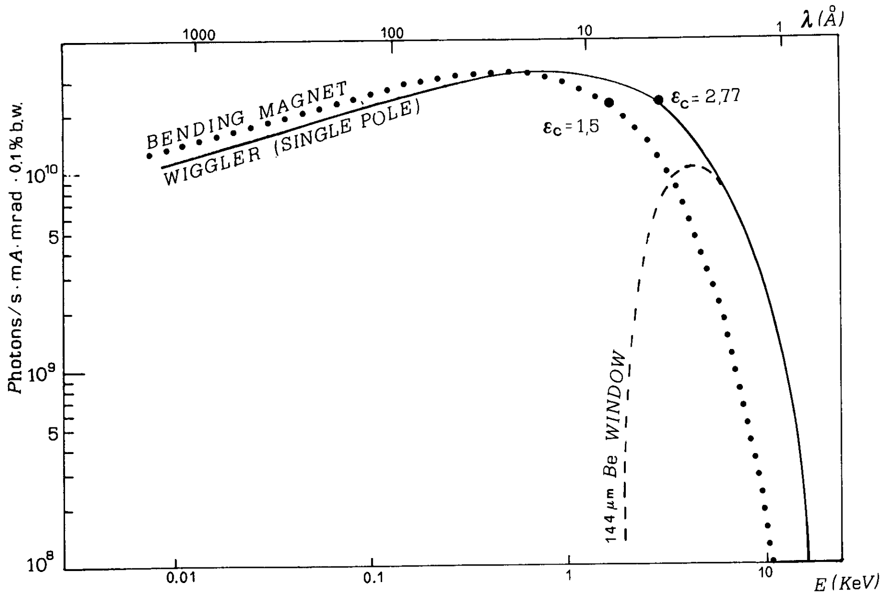

Fig.10 Comparison between the flux of the radiation emitted from a bending magnet of the Adone storage ring with the flux emitted from the wiggler (B=1.85 T - single pole).

$$\lambda = \frac{\lambda_0}{2\gamma^2}\left(1 + \frac{K^2}{2}\right),$$

is called "fundamental".

The natural relative bandwidth of the emitted radiation for an ideally parallel electron beam is given by

$$\frac{\Delta\lambda_n}{\lambda_n} = \frac{1}{nN},$$

where N is the number of poles.

To conclude, some basic synchrotron radiation relationships, which are very useful for any kind of user, are reported in Table 1.

TABLE 1

Basic synchrotron radiation relationships

Total power radiated by a relativistic electron:

$$P = \int I(\lambda, \psi) d\lambda d\psi = \frac{2e^2c}{3R^3} \frac{E^4}{(m_0c^2)^4}.$$

Energy loss per turn, per electron:

$$\delta E(\text{keV}) = 88.5 \frac{E^4(\text{GeV})}{R(\text{m})}.$$

Total power radiated:

$$P_{tot}(\text{kW}) = 26.6 E^3(\text{GeV}) \cdot B(\text{T}) \cdot I(\text{A}).$$

Critical wavelength:

$$\lambda_c(\text{Å}) = 5.59 \frac{R(\text{m})}{E^3(\text{GeV})} = \frac{18.64}{B(\text{T}) \cdot E^2(\text{GeV})}.$$

Critical energy:

$$\epsilon_c(\text{keV}) = 2.218 \frac{E^3(\text{GeV})}{R(\text{m})} = 2.96 \times 10^{-7} \frac{\gamma^3}{R(\text{m})}.$$

Opening angle:

$$\psi \approx \frac{1}{\gamma} = \frac{mc^2}{E},$$

with

$$\gamma = \frac{E}{m_0c^2} = 1957 E \ (\text{GeV}).$$

Energy of a photon:

$$\epsilon(eV) = \frac{12.40}{\lambda(\mathring{A})}.$$

For an undulator
Deflection parameter:

$$K = eB_0\lambda_0/2\pi m_0 c = \alpha\gamma.$$

N^{th} harmonic

$$\lambda_n = \frac{\lambda_0}{2n\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right).$$

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