INFN - Laboratori Nazionali di Frascati
Servizio Documentazione

LNF-91/017 (R)
3 Maggio 1991

LIGHT FOUR-QUARK STATES WITH $J^{PC}=1^{--}$

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ABSTRACT

The masses of P-wave $q^2\bar{q}^2$ states are reviewed. Starting from QCD it is shown that the interquark confining forces are not of pairwise type as it is usually suggested in potential models, but can be described by means of string junction configuration. Masses of P-wave $q^2\bar{q}^2$ systems obtained lie lower that it was predicted in the bag and potential models.

1. Recently in SLAC in the reaction $K^-p \to \pi^+ \pi^- \Lambda$ the new P-wave state of $\pi^+ \pi^-$ system was discovered[1]. This resonance has a rather low mass (M=1270 MeV) and a typical hadronic width (Γ=166 ± 35 MeV). The observation of this resonance is of a great importance because due to $q\bar{q}$ model prediction only one excited 1^{--} state should exist in the region below 1.5 GeV, but now we have two candidates for the $\rho(2S)$ radially excited state: $\rho_1(1270)$ and $\rho(1450)$. One of them is an extra state in the $q\bar{q}$ classification of levels.

Also remember that in the same region ~ 1.4 ÷ 1.5 GeV there are two other candidates for the non-$q\bar{q}$ states: C(1480) ($J^{PC}=1^{--}$) which was observed in $\phi\pi^0$ system[2], and $\hat{\rho}(1405)$ found in the amplitude analysis of $\eta\pi^0$ system[3], $\hat{\rho}$ has exotic quantum numbers $J^{PC}=1^{-+}$. The quantum numbers of these states and the close values of their masses are rather typical for P-wave four-quark states, so it is interesting to check possibility to interprete them as light $q^2\bar{q}^2$ states.

At first we remind some results about S-wave four-quark states which were studied in details in the bag model in the pioneering paper by R.Jaffe[4]. For S-wave $q^2\bar{q}^2$ states all pairwise angular momenta are equal to zero and so their quantum numbers are 0^{++}, 1^{+-}, 2^{++}. As was already underlined in[4] S-wave resonances should be very broad due to the superallowed decays into two mesons and therefore unobservable. The same result for light quark systems was obtained in[5] in the framework of the nonrelativistic potential model (NPM). Of course, one cannot exclude the situation which probably realizes in a case of a_0

(980) and f_0 (975) when due to the large $\bar{S}\,\bar{S}$ splitting the mass of 0^{++} state (constisting of two light quarks and two strange quarks) lies just below $K\bar{K}$ threshold and this resonance has rather small width.

2. The situation with P-wave four-quark states is quite different. Here we define P-wave states as those ones which have angular momenta inside diquark and antidiquark equal zero but the relative angular momentam between diquark and antidiquark $l_{rel}=1$ (and denote them as λ_{4q}, see details in(6,7)). Since λ_{4q} states need some reconstruction to decay into two mesons they should have a typical hadronic width, $\Gamma \sim 100$ MeV(6). Also note that λ_{4q} represents rather compact systems with the interquark size close to the radius of P-wave meson(6).

The masses of λ_{4q} states were mostly studied in heavy quark systems like $c^2\bar{c}^2$, $\bar{c}\,\bar{s}$ cs and also in heavy - light $qQ\bar{q}\bar{Q}$ systems where NPM can be used(6,7). The detailed analysis of light four-quark states was done in the paper(8) in the framework of bag model approach. In the Table we give masses of different members of different flavor representations (f) taken from(8).

TABLE - Masses (in GeV) and Quantum Numbers of P-wave color irreps $(\bar{3} \times 3)$ four-quark states.

$l^P=1^-$, JC	f	(*) $n_S=0$	$n_s=0$	$n_s=2$
1^-	9	1.50	1.66	1.83
$0^\pm\ 1^\pm\ 2^\pm$	$18 \oplus 18^*$	1.72	1.85 + 1.87	2.02
1^-	36	1.86	2.00	2.14
$0^+\ 1^+\ 2^+$	36	1.90	2.04	2.17
$1^-\ 2^-\ 3^-$	36	1.94	2.07	2.21
$M_{cog}^{(multiplet)}$		1.80	1.95	2.10

(*) n_S is the number of the strange quarks.

Among many results obtained in(8) we would like to underline the following points.

A. The center of gravity of S-wave multiplet from light quarks (with $J^{PC}=0^{++}, 1^{+-}, 2^{++}$) has mass

$$M_{cog}^{(o)}\,(\text{S-wave}) = 1.458\ \text{GeV} \tag{1}$$

in agreement with bag model predictions in(4).

B. The center of gravity of P-wave multiplet from light quarks lies at the higher mass

$$M^{(1)}_{cog}\,(\text{P-wave}) = 1.80\ \text{GeV} \tag{2}$$

C. The four-quark states λ_{4q} with one or two strange quarks lie in the region ~ 2 GeV.

D. There are many members λ_{4q} of the multiplet with very close values of mass and different quantum numbers ($J^{PC}=1^{--}, 2^{--}, 3^{--}, 0^{-+}, 1^{-+}, 2^{-+}$). So in the experiment we should expect "a whole family" of such states.

E. Only one state with isospin I=0 and $J^{PC}=1^{--}$ has a mass much smaller than $M^{(1)}_{cog}\,(\lambda_{4q})$ due to the large hyperfine splittings. All other states lie in the region $M^{(1)}_{cog} \pm 100$ MeV.

Estimating the accuracy of calculated model mass values in[8] as ± 50 MeV we can conclude that bag model does not predict the low-lying P-wave four-quark states with I=1 as $\rho(M \cong 1.3 \div 1.4\ \text{GeV})$, $\hat{\rho}(1405)$, or C(1480).

3. It is clear that the classification of P-wave $q^2\bar{q}^2$ states and the order of levels are defined by algebra and do not depend on the model with exception of M_{cog}. Therefore we decided to check the value of $M^{(1)}_{cog}$ using the new nonperturbative approach developed in the series of papers[9]. In this approach under some assumptions the proper time Hamiltonian was obtained both for mesons and baryons[10]. For $q_1 q_2 \bar{q}_3 \bar{q}_4$ states $\hat{H}$ is given by the following expression (without spin-dependent forces).

$$\hat{H}(\mu_1 \mu_2 \mu_3 \mu_4) = \frac{m_1^2}{2\mu_1} + \frac{m_2^2}{2\mu_2} + \frac{m_3^2}{2\mu_3} + \frac{m_4^2}{2\mu_4} + \frac{1}{2}(\mu_1 + \mu_2 + \mu_3 + \mu_4) + \sum_{i=1}^{4} \frac{\hat{p}_i^2}{2\tilde{\mu}} + V_{int}(\bar{r}_1, \bar{r}_2, \bar{r}_3, \bar{r}_4) \tag{3}$$

Here m_i is a current quark mass and μ_i a dynamical mass of i-th quark which is defined by means of the steepest descent conditions $\partial M/\partial \mu_i = 0$ where M is the mass term. In the equal mass case M is

$$M = \frac{2m^2}{\mu} + 2\mu + \varepsilon_{nl}(\mu) \tag{4}$$

where $\varepsilon_{nl}(\mu)$ is an eigenvalue of the corresponding Schrödinger equation.

Note that although the kinetic term in $\hat{H}$ has a nonrelativistic form, in (3) the relativistic kinematics is taken into account and this form by itself is a consequence of the proper time representation of the quark Green function[10]. Note also that the structure of mass formula (4) differs from a nonrelativistic one because mass term (4) contains only two dynamical masses instead of four constituent masses in NPM.

Also in QCD approach[10] the dynamical quark mass is strongly dependent on hadron quantum numbers and this effect is basically relativistic.

4. The next important result obtained in[9,10] is a new representation of the multiquark forces which differs from usually adopted in NPM. As was suggested by Lipkin[11] long ago the interaction inside a multiquark system may be chosen in the form

$$V_{central}\left(\bar{r}_1, \cdots \bar{r}_n\right) = \sum_{i>j} \left\{ \frac{\alpha_s\left(r_{ij}\right)}{r_{ij}} - \frac{3}{4}\sigma r_{ij} \right\} \frac{\bar{\lambda}_i\,\bar{\lambda}_j}{4} \qquad (5)$$

where both perturbative (P) and nonperturbative (NP) parts of interaction enter with the same factor $(\bar{\lambda}_i\,\bar{\lambda}_j)$. Just this type of interaction was used in many papers[5-7].

Starting from the gauge-invariant representation of meson Green function and separating NP effects in the form of Wilson loop it was shown[9-10] that the confining forces have no any factor like $\bar{\lambda}_i\,\bar{\lambda}_j$ but instead at the large distances they should be written as

$$V_{conf}(\bar{r}_1, \ldots, \bar{r}_n) = \sigma\, L_{min}(\bar{r}_1, \ldots, \bar{r}_n), \qquad (6)$$

where σ is the string tension and L_{min} is a minimal lenght of multiquark configurations to be defined below.

In particular for meson $L_{min} = r$ (interquark distance), and for baryon[12]

$$L_{min} = \frac{\sqrt{3}}{2}\sqrt{r_{ij}^2 + r_k^2 + 2r_{ij}\, r_k \sin\theta} \qquad (7)$$

where i, j, k fixed and θ is defined in[12].

For $q^2\bar{q}^2$ system the situation is more complicated. The minimal lenght shoud be chosen among two types of configurations. The first one corresponds to two separate mesons - the two-string configurations (see Fig. 1)

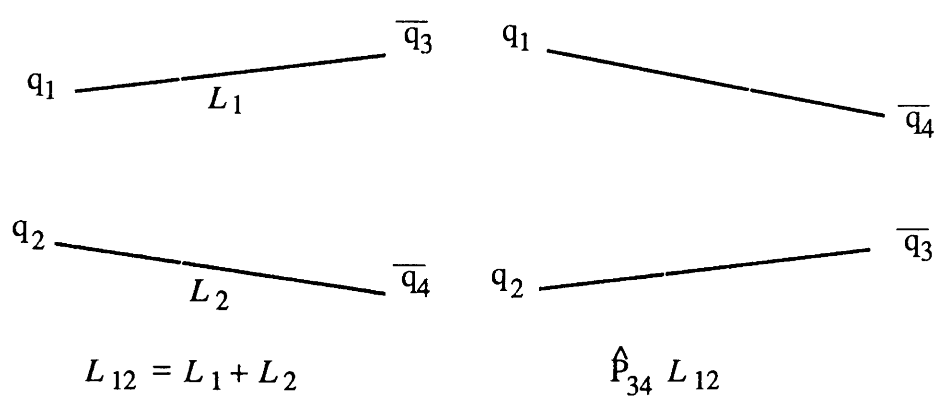

FIG. 1

The second configuration is a string junction configuration shown in Fig. 2, its lenght is denoted as L_3

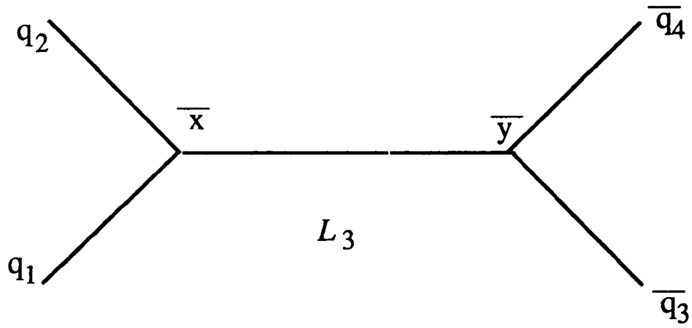

FIG. 2

For $q^2\bar{q}^2$ state the minimal length is defined as

$$L_{\min} = \min \begin{cases} L_{12} = L_1 + L_2 = |\bar{r}_{13}| + |\bar{r}_{24}| \\ L_{3\min} = \min\limits_{x,y} \left\{ |\bar{r}_{1x}| + |\bar{r}_{2x}| + |\bar{x} - \bar{y}| + |\bar{r}_{3y}| + |\bar{r}_{4y}| \right\} \end{cases} \tag{8}$$

Asymptotically the four-quark wave function is a superposition of the wave functions corresponding to two different configurations

$$\Psi(\bar{r}_1, \bar{r}_2, \bar{r}_3, \bar{r}_4,) \underset{\text{asymp.}}{\longrightarrow} A\widehat{\Psi}_{12} + B\widehat{\Psi}_{\text{st. junct.}} \tag{9}$$

and also the wave-functions are (2x2) matrices in the color space. The calculation of exact eigenvalues of multidimensional Schrödinger equation is rather complicated problem. So we simplified our task taking into account the variational principle which states that the exact eigenvalue of equation is smaller than the eigenvalue obtained for the given configuration.

To solve Schrödinger equation we have used the hyperspherical harmonic expansion(6,13) and the approximation of minimal k-harmonic which works with accuracy better than 1%. Our calculations show that for S-wave states the two-string configuration gives a smaller mass of $q^2\bar{q}^2$ system than the configuration L_3. This result supports the old one that S-wave four-quark system (without $\bar{S}\,\bar{S}$ forces) is unstable with respect to decay into two mesons(4,5). Nevertheless it is interesting to note that our calculated value of the center of gravity is smaller than $M^{(0)}_{cog}$ in the bag model. We obtained the following upper bound

$$M^{(0)}_{cog}\ (\text{S-wave}) \leq 1.35\ \text{GeV} \tag{10}$$

which should be compared with 1.46 GeV in (1) taken from(8).

Also we have obtained the upper bound for the center of gravity of P-wave multiplet. In this case the mass for the chosen string junction configuration L_3 has a smaller value than for the two meson configuration L_{12} supporting the idea that λ_{4q} is a metastable state with respect to decay into two mesons. The calculated bound

$$M^{(1)}_{cog}\text{ (P-wave) } < 1.55\text{ GeV} \qquad (11)$$

is much lower than the value 1.80 GeV obtained in the bag model approach[8] (see Eq. 2).

It means that with many-body confining forces of the string-junction type we obtain the four-quark masses smaller than in NPM and bag model.

Our calculations of intervals between $q^2\bar{q}^2$ states with different quantum numbers give values rathar close to those obtained in[8] (see Table). Thus we can conclude that in the region 1.3 ÷ 1.6 GeV there should exist three $q^2\bar{q}^2$ resonances with the quantum numbers of ρ-meson: one of them is a member of $\underline{18} \oplus \underline{18}^*$ with I=1, S=1 and also two members of $\underline{36}$ representation with S=0 and S=2.

We expect that a four-quark P=wave state with S=2 has the largest leptonic width. As was shown for heavy quark system ($m_q \geq 0.6$ GeV)[6] the leptonic width is given by formula

$$\Gamma_{e^+e^-}(S=2) = \frac{128}{27}\pi^3\alpha^2\alpha_s^2 e_q^2 \frac{(C_{\bar{3}3})^2}{m_q^{10}} \qquad (12)$$

and

$$\Gamma_{e^+e^-}(S=0) = \frac{1}{16}\Gamma_{e^+e^-}(S=2)\,. \qquad (13)$$

In (12) the constant C is defined by relation[8]

$$\Psi(\bar{r}_1, \bar{r}_2, \bar{r}_3, \bar{r}_4)\,\big|_{r_i \to 0} = C\,\bar{n}\left(\bar{r}_3 + \bar{r}_4 - \bar{r}_1 - \bar{r}_2\right) \qquad (14)$$

From (12) we can get the upper bound for the width of light quark system: Γ_{e+e-} ($u\bar{d}d\bar{u}$) ≤ 400 eV. Also using the calculated value of Γ_{e+e-} for $s^2\bar{s}^2$ system and the proper value of e_q^2 we can find the lower bound of leptonic wigth for P-wave $q^2\bar{q}^2$ state consisting of light quarks: $\Gamma_{e+e-} > 15$ eV.

We would like to conclude stressing that more detailed calculations of P-wave four-quark systems are needed (including light-strange four-quark systems) and also it is of a great importance to do coupled channel calculations with $\bar{q}q$ channels, which are in progress.

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